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Research Article

On the Behavior of Solutions of the System of Rational Difference Equations: $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1)$, $y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$, and $z_{n+1} = z_{n-1}/(y_n z_{n-1} - 1)$

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We investigate the solutions of the system of difference equations $x_{n+1} = x_{n-1}/(y_n x_{n-1} - 1)$, $y_{n+1} = y_{n-1}/(x_n y_{n-1} - 1)$, $z_{n+1} = z_{n-1}/(y_n z_{n-1} - 1)$, where $y_0, y_{-1}, x_0, x_{-1}, z_{-1}, z_0 \in \mathbb{R}$.

1. Introduction

Recently, there has been great interest in studying difference equation systems. One of the reasons for this is a necessity for some techniques which can be used in investigating equations arising in mathematical models describing real life situations in population biology, economic, probability theory, genetics, psychology, and so forth. There are many papers related to the difference equations system, for example, the following papers.

In [1], Çinar studied the solutions of the systems of the difference equations

$$x_{n+1} = \frac{1}{y_n}, \qquad y_{n+1} = \frac{y_n}{x_{n-1} y_{n-1}}. \tag{1.1}$$

In [2] Papaschinopoulos and Schinas studied the oscillatory behavior, the boundedness of the solutions, and the global asymptotic stability of the positive equilibrium of the system of nonlinear difference equations

$$x_{n+1} = A + \frac{y_n}{x_{n-p}}, \quad y_{n+1} = A + \frac{x_n}{y_{n-q}}, \quad n = 0, 1, \ldots, p, q. \tag{1.2}$$

In [3] Papaschinopoulos and Schinas proved the boundedness, persistence, the oscillatory behavior, and the asymptotic behavior of the positive solutions of the system of difference equations

$$x_{n+1} = \sum_{i=0}^{k} \frac{A_i}{y_{n-i}^{p_i}}, \qquad y_{n+1} = \sum_{i=0}^{k} \frac{B_i}{x_{n-i}^{q_i}}. \tag{1.3}$$

In [4, 5] Özban studied the positive solutions of the system of rational difference equations

$$\begin{gathered} x_n = \frac{a}{y_{n-3}}, \qquad y_n = \frac{b y_{n-3}}{x_{n-q} y_{n-q}}, \\ x_{n+1} = \frac{1}{y_{n-k}}, \qquad y_{n+1} = \frac{y_n}{x_{n-m} y_{n-m-k}}. \end{gathered} \tag{1.4}$$

In [6, 7] Clark and Kulenović investigate the global asymptotic stability

$$x_{n+1} = \frac{x_n}{a + c y_n}, \qquad y_{n+1} = \frac{y_n}{b + d x_n}. \tag{1.5}$$

In [8] Camouzis and Papaschinopoulos studied the global asymptotic behavior of positive solutions of the system of rational difference equations

$$x_{n+1} = 1 + \frac{x_n}{y_{n-m}}, \qquad y_{n+1} = 1 + \frac{y_n}{x_{n-m}}. \tag{1.6}$$

In [9] Yang et al. considered the behavior of the positive solutions of the system of the difference equations

$$x_n = \frac{a}{y_{n-p}}, \qquad y_n = \frac{b y_{n-p}}{x_{n-q} y_{n-q}}. \tag{1.7}$$

In [10] Kulenović and Nurkanović studied the global asymptotic behavior of solutions of the system of difference equations

$$x_{n+1} = \frac{a + x_n}{b + y_n}, \qquad y_{n+1} = \frac{c + y_n}{d + z_n}, \qquad z_{n+1} = \frac{e + z_n}{f + x_n}. \tag{1.8}$$

In [11] Zhang et al. investigated the behavior of the positive solutions of the system of difference equations

$$x_n = A + \frac{1}{y_{n-p}}, \qquad y_n = A + \frac{y_{n-1}}{x_{n-r} y_{n-s}}. \tag{1.9}$$

In [12] Zhang et al. studied the boundedness, the persistence, and global asymptotic stability of the positive solutions of the system of difference equations

$$x_{n+1} = A + \frac{y_{n-m}}{x_n}, \qquad y_{n+1} = A + \frac{x_{n-m}}{y_n}. \tag{1.10}$$

In [13] Yalcinkaya studied the global asymptotic behavior of a system of two nonlinear difference equations.

In [14] Yalcinkaya et al. investigated the solutions of the system of difference equations

$$x_{n+1}^{(1)} = \frac{x_n^{(2)}}{x_n^{(2)} - 1}, x_{n+1}^{(2)} = \frac{x_n^{(3)}}{x_n^{(3)} - 1}, \dots, x_{n+1}^{(k)} = \frac{x_n^{(1)}}{x_n^{(1)} - 1}. \tag{1.11}$$

In [15] Yalcinkaya studied the global asymptotic stability of the system of difference equations

$$z_{n+1} = \frac{t_n z_{n-1} + a}{t_n + z_{n-1}}, \qquad t_{n+1} = \frac{z_n t_{n-1} + a}{z_n + t_{n-1}}. \tag{1.12}$$

In [16] Irićanin and Stević studied the positive solutions of the system of difference equations

$$\begin{gathered} x_{n+1}^{(1)} = \frac{1 + x_n^{(2)}}{x_{n-1}^{(3)}}, x_{n+1}^{(2)} = \frac{1 + x_n^{(3)}}{x_{n-1}^{(4)}}, \dots, x_{n+1}^{(k)} = \frac{1 + x_n^{(1)}}{x_{n-1}^{(2)}}, \\ x_{n+1}^{(1)} = \frac{1 + x_n^{(2)} + x_{n-1}^{(3)}}{x_{n-2}^{(4)}}, x_{n+1}^{(2)} = \frac{1 + x_n^{(3)} + x_{n-1}^{(4)}}{x_{n-2}^{(5)}}, \dots, x_{n+1}^{(k)} = \frac{1 + x_n^{(1)} + x_{n-1}^{(2)}}{x_{n-2}^{(3)}}. \end{gathered} \tag{1.13}$$

In [17] Kurbanli et al. studied the behavaior of positive solutions of the system of rational difference equations

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} + 1}, \qquad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} + 1}. \tag{1.14}$$

Also see references.

In this paper, we investigate the behavior of the solutions of the difference equations system

$$x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \qquad y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \qquad z_{n+1} = \frac{z_{n-1}}{y_n z_{n-1} - 1}, \tag{1.15}$$

where the initial conditions are arbitrary real numbers.

Theorem 1.1. *Let $y_0, y_{-1}, x_0, x_{-1}, z_{-1}, z_0 \in \mathbb{R}$ be arbitrary real numbers and $y_0 = a,\ y_{-1} = b,\ x_0 = c,\ x_{-1} = d,\ z_0 = e,\ z_{-1} = f$ and let $\{x_n, y_n, z_n\}$ be a solution of the system* (1.15)*. Also, assume that $af \neq 1$ and $bc \neq 1$ then all solutions of* (1.15) *are*

$$x_n = \begin{cases} \dfrac{d}{(ad-1)^n}, & n\text{-odd}, \\ c(bc-1)^n, & n\text{-even}, \end{cases}$$

$$y_n = \begin{cases} \dfrac{b}{(bc-1)^n}, & n\text{-odd}, \\ a(ad-1)^n, & n\text{-even}, \end{cases}$$

$$z_n = \begin{cases} \dfrac{f}{(-1)^0\binom{n}{0}a^n f d^{n-1} + (-1)^1\binom{n}{1}a^{n-1} f d^{n-2} + \cdots + (-1)^{n-1}\binom{n}{n-1}a^1 f d^0 + (-1)^n\binom{n}{n}}, & n\text{-odd} \\ (-1)^n \dfrac{(bc-1)^n e}{(-1)^n\binom{n}{1}b^1c^0e + \cdots + (-1)^1\binom{n}{n}b^n c^{n-1}e + (-1)^0\binom{n}{0}b^n c^n + \cdots + (-1)^n\binom{n}{n}b^0c^0}, & n\text{-even}. \end{cases} \tag{1.16}$$

Proof. For $n = 0, 1, 2, 3$, we have

$$\begin{aligned}
x_1 &= \frac{x_{-1}}{y_0 x_{-1} - 1} = \frac{d}{ad-1}, \\
y_1 &= \frac{y_{-1}}{x_0 y_{-1} - 1} = \frac{b}{bc-1}, \\
z_1 &= \frac{z_{-1}}{y_0 z_{-1} - 1} = \frac{f}{af-1}, \\
x_2 &= \frac{x_0}{y_1 x_0 - 1} = \frac{c}{(b/(bc-1))c - 1} = \frac{c(bc-1)}{bc - bc + 1} = c(bc-1), \\
y_2 &= \frac{y_0}{x_1 y_0 - 1} = \frac{a}{(d/(ad-1))a - 1} = \frac{a(ad-1)}{ad - ad + 1} = a(ad-1), \\
z_2 &= \frac{z_0}{y_1 z_0 - 1} = \frac{e}{(b/(bc-1))e - 1} = \frac{e(bc-1)}{be - bc + 1} = -\frac{e(bc-1)}{-1\cdot be + bc - 1}, \\
x_3 &= \frac{x_1}{y_2 x_1 - 1} = \frac{d/(ad-1)}{a(ad-1)\cdot d/(ad-1) - 1} = \frac{d}{(ad-1)^2}, \\
y_3 &= \frac{y_1}{x_2 y_1 - 1} = \frac{b/(bc-1)}{c(bc-1)\cdot b/(bc-1) - 1} = \frac{b/(bc-1)}{bc-1} = \frac{b}{(bc-1)^2}, \\
z_3 &= \frac{z_1}{y_2 z_1 - 1} = \frac{f/(af-1)}{a(ad-1)\cdot f/(af-1) - 1} = \frac{f/(af-1)}{(a^2df - af - af + 1)/(af-1)} = \frac{f}{a^2df - 2af + 1}
\end{aligned} \tag{1.17}$$

for $n = k$ assume that

$$x_k = \begin{cases} \dfrac{d}{(ad-1)^k}, & k\text{-odd}, \\ c(bc-1)^k, & k\text{-even}, \end{cases}$$

$$y_k = \begin{cases} \dfrac{b}{(bc-1)^k}, & k\text{-odd}, \\ a(ad-1)^k, & k\text{-even}, \end{cases}$$

$$z_{2k-1} = \frac{f}{(-1)^0\binom{k}{0}a^k f d^{k-1} + (-1)^1\binom{k}{1}a^{k-1} f d^{k-2} + \cdots + (-1)^{k-1}\binom{k}{k-1}a^1 f d^0 + (-1)^k\binom{k}{k}}, \quad k = 1,2,\ldots,$$

$$z_{2k} = (-1)^k \frac{(bc-1)^k e}{(-1)^k\binom{k}{1}b^1c^0e + \cdots + (-1)^1\binom{k}{k}b^k c^{k-1}e + (-1)^0\binom{k}{0}b^k c^k + \cdots + (-1)^k\binom{k}{k}b^1c^1}, \quad k = 1,2,\ldots \tag{1.18}$$

are true. Then for $n = k + 1$ we will show that (1.16) is true. From (1.15), we have

$$x_{2k+1} = \frac{x_{2k-1}}{y_{2k}x_{2k-1} - 1} = \frac{d/(ad-1)^k}{a(ad-1)^k \cdot d/(ad-1)^k - 1} = \frac{d}{(ad-1)^{k+1}}. \tag{1.19}$$

Also, similarly from (1.15), we have

$$y_{2k+1} = \frac{y_{2k-1}}{x_{2k}y_{2k-1} - 1}\frac{b/(bc-1)^k}{c(bc-1)^k \cdot b/(bc-1)^k - 1} = \frac{b}{(bc-1)^{k+1}},$$

$$\binom{n}{k} = \frac{n!}{(n-k)! \cdot k!},$$

$$\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}, \tag{1.20}$$

$$\binom{n}{0} = \binom{n}{n} = \binom{n+1}{n+1}$$

from properties of Binomial coefficients,

$$
\begin{aligned}
z_{2k+1} &= \frac{z_{2k-1}}{y_{2k}z_{2k-1}-1}\\
&= \frac{f/\left((-1)^0\binom{k}{0}a^k f d^{k-1}+\cdots+(-1)^{k-1}\binom{k}{k-1}a^1 f d^0+(-1)^k\binom{k}{k}\right)}{a(ad-1)^k f/\left((-1)^0\binom{k}{0}a^k f d^{k-1}+\cdots+(-1)^{k-1}\binom{k}{k-1}a^1 f d^0+(-1)^k\binom{k}{k}\right)-1}\\
&= \frac{f}{(-1)^0\binom{k+1}{0}a^{k+1} f d^{k}+(-1)^1\binom{k+1}{1}a^k f d^{k-1}+\cdots+(-1)^{k}\binom{k+1}{k}a^1 f d^0+(-1)^{k+!}\binom{k+1}{k+1}}
\end{aligned}
\tag{1.21}
$$

written and accurate.

Also, we have

$$
\begin{aligned}
x_{2k+2} &= \frac{x_{2k}}{y_{2k+1}x_{2k}-1} = \frac{c(bc-1)^k}{\left(b/(bc-1)^{k+1}\right)c(bc-1)^k-1} = \frac{c(bc-1)^k}{bc/(bc-1)-1} = c(bc-1)^{k+1},\\
y_{2k+2} &= \frac{y_{2k}}{x_{2k+1}y_{2k}-1} = \frac{a(ad-1)^k}{\left(d/(ad-1)^{k+1}\right)\cdot a(ad-1)^k-1} = \frac{a(ad-1)^k}{ad/(ad-1)-1} = a(ad-1)^{k+!},\\
z_{2k+2} &= \frac{z_{2k}}{y_{2k+1}z_{2k}-1}\\
&= \frac{(-1)^k(bc-1)^k e/\left(\mathcal{A}+(-1)^0\binom{k}{0}b^kc^k+\cdots+(-1)^k\binom{k}{k}b^0c^0\right)}{\left(b/(bc-1)^{k+1}\right)\cdot(-1)^k(bc-1)^k e/\left(\mathcal{A}+(-1)^0\binom{k}{0}b^kc^k+\cdots+(-1)^k\binom{k}{k}b^0c^0\right)-1} = (-1)^{k+1}\\
&\quad\times\frac{(bc-1)^{k+1}e}{(-1)^{k+1}\binom{k+1}{1}b^1c^0e+\cdots+(-1)^1\binom{k+1}{k+1}b^{k+1}c^ke+(-1)^0\binom{k+1}{0}b^{k+1}c^{k+1}+\cdots+(-1)^{k+1}\binom{k+1}{k+1}b^1c^1},
\end{aligned}
\tag{1.22}
$$

where $\mathcal{A}$ denotes $(-1)^k\binom{k}{1}b^1c^0e+\cdots+(-1)^1\binom{k}{k}b^kc^{k-1}e$. □

Corollary 1.2. *Let a,b,c,d be only positive real numbers or negative real numbers and let e,f be arbitrary nonnegative real numbers, and let $\{x_n,y_n,z_n\}$ be a solution of the system* (1.15). *If $ad\neq 1,\ bc\neq 1,\ 0<ad<1$ and $0<bc<1$ then one has*

$$
\begin{aligned}
\lim_{n\to\infty}x_{2n-1} &= \lim_{n\to\infty}y_{2n-1}=\infty,\\
\lim_{n\to\infty}x_{2n} &= \lim_{n\to\infty}y_{2n}=0.
\end{aligned}
\tag{1.23}
$$

Proof. From $ad \neq 1,\ bc \neq 1,\ 0 < ad < 1$ and $0 < bc < 1$ we have $-1 < ad - 1 < 0$ and $-1 < bc - 1 < 0$. Hence, we have

$$x_{2n-1} = \frac{d}{(ad-1)^n}, \qquad y_{2n-1} = \frac{b}{(bc-1)^n},$$
$$x_{2n} = c(bc-1)^n, \qquad y_{2n} = a(ad-1)^n. \tag{1.24}$$

Then,

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = \begin{cases} +\infty, & d > 0,\ n\text{-even} \\ +\infty, & d < 0,\ n\text{-odd} \\ -\infty, & d > 0,\ n\text{-odd} \\ -\infty, & d < 0,\ n\text{-even,} \end{cases}$$
$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(bc-1)^n} = \begin{cases} +\infty, & b > 0,\ n\text{-even} \\ +\infty, & b < 0,\ n\text{-odd} \\ -\infty, & b > 0,\ n\text{-odd} \\ -\infty, & b < 0,\ n\text{-even.} \end{cases} \tag{1.25}$$

Also,

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(bc-1)^n = 0,$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = 0. \tag{1.26}$$

□

Example 1.3. If $y(0) = -1/2,\ y(-1) = -1/2,\ x(0) = -1/2,\ x(-1) = -1/2$, then the solutions of (1.15) can be represented by Table 1.

Corollary 1.4. *Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system* (1.15)*. If $ad \neq 1,\ bc \neq 1,\ 1 < ad < 2$ and $1 < bc < 2$ then one has*

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = \infty,$$
$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = 0. \tag{1.27}$$

Table 1

i	1	2	3	4	5	6	7	8
x_{2i-1}	0.6667	−0.8889	1.1852	−1.5803	2.1070	−2.8093	3.7458	−4.9944
x_{2i}	0.3750	−0.2813	0.2109	−0.1582	0.1187	−0.0890	0.0667	−0.0501
y_{2i-1}	0.6667	−0.8889	1.1852	−1.5803	2.1070	−2.8093	3.7458	−4.9944
y_{2i}	0.3750	−0.2813	0.2109	−0.1582	0.1187	−0.0890	0.0667	−0.0501
z_{2i-1}	0.6667	−0.8889	1.1852	−1.5803	2.1070	−2.8093	3.7458	−4.9944
z_{2i}	0.3750	−0.2813	0.2109	−0.1582	0.1187	−0.0890	0.0667	−0.0501

Proof. From $ad \neq 1$, $bc \neq 1$, $1 < ad < 2$ and $1 < bc < 2$ we have $0 < ad-1 < 1$ and $0 < bc-1 < 1$. Hence, we have

$$\begin{gathered} x_{2n-1} = \frac{d}{(ad-1)^n}, \qquad y_{2n-1} = \frac{b}{(bc-1)^n}, \\ x_{2n} = c(bc-1)^n, \qquad y_{2n} = a(ad-1)^n. \end{gathered} \tag{1.28}$$

Then,

$$\begin{gathered} \lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = \begin{cases} +\infty, & d > 0 \\ -\infty, & d < 0, \end{cases} \\ \lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(bc-1)^n} = \begin{cases} +\infty, & b > 0 \\ -\infty, & b < 0. \end{cases} \end{gathered} \tag{1.29}$$

Also,

$$\begin{gathered} \lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(bc-1)^n = 0, \\ \lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = 0. \end{gathered} \tag{1.30}$$

□

Example 1.5. If $y(0) = 3/2$, $y(-1) = 3/4$, $x(0) = 3/2$, $x(-1) = 3/4$, $z(0) = 3/2$, $z(-1) = 3/4$ then the solutions of (1.15) can be represented by Table 2.

Corollary 1.6. *Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system* (1.15).

If $-\infty < ad < 0$ *and* $-\infty < bc < 0$ *then one has*

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = 0,$$
$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = \infty. \tag{1.31}$$

Proof. From $-\infty < ad < 0$ and $-\infty < bc < 0$ we have $-\infty < ad - 1 < -1$ and $-\infty < bc - 1 < -1$. Hence, we have

$$x_{2n-1} = \frac{d}{(ad-1)^n}, \qquad y_{2n-1} = \frac{b}{(bc-1)^n},$$
$$x_{2n} = c(bc-1)^n, \qquad y_{2n} = a(ad-1)^n. \tag{1.32}$$

Then,

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = 0,$$
$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(bc-1)^n} = 0. \tag{1.33}$$

Also,

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(bc-1)^n = \begin{cases} +\infty, & c > 0,\ n\text{-even} \\ +\infty, & c < 0,\ n\text{-odd} \\ -\infty, & c > 0,\ n\text{-odd} \\ -\infty, & c < 0,\ n\text{-even}, \end{cases}$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = \begin{cases} +\infty, & a > 0,\ n\text{-even} \\ +\infty, & a < 0,\ n\text{-odd} \\ -\infty, & a > 0,\ n\text{-odd} \\ -\infty, & a < 0,\ n\text{-even}. \end{cases} \tag{1.34}$$

□

Example 1.7. If $y(0) = -3/2,\ y(-1) = -3/4,\ x(0) = 3/2,\ x(-1) = 3/4,\ z(0) = 3/2,\ z(-1) = 3/4$ then the solutions of (1.15) can be represented by Table 3.

Table 2

i	1	2	3	4	5	6	7	8
x_{2i-1}	6	48	384	3072	24576	$1.967{\cdot}10^5$	$1.573 \cdot 10^6$	$1.258 \cdot 10^7$
x_{2i}	0.188	0.023	0.003	0.0004	0.00004	0.00001	$7.153 \cdot 10^{-7}$	$8.941 \cdot 10^{-8}$
y_{2i-1}	6	48	384	3072	24576	$1.967{\cdot}10^5$	$1.573 \cdot 10^6$	$1.258 \cdot 10^7$
y_{2i}	0.188	0.023	0.003	0.0004	0.00004	0.00001	$7.153 \cdot 10^{-7}$	$8.941 \cdot 10^{-8}$
z_{2i-1}	6	48	384	3072	24576	$1.967{\cdot}10^5$	$1.573 \cdot 10^6$	$1.258 \cdot 10^7$
z_{2i}	0.188	0.023	0.003	0.0004	0.00004	0.00001	$7.153 \cdot 10^{-7}$	$8.941 \cdot 10^{-8}$

Table 3

i	1	2	3	4	5	6	7	8
x_{2i-1}	−0.3529	0.1661	−0.0782	0.0368	−0.0173	0.0081	−0.0038	0.0018
x_{2i}	−3.1875	6.7734	−14.3936	30.5863	−64.9959	138.1163	−293.4971	623.6813
y_{2i-1}	0.3529	−0.1661	0.0782	−0.0368	0.0173	−0.0081	0.0038	−0.0018
y_{2i}	3.1875	−6.7734	14.3936	−30.5863	64.9959	−138.1163	293.4971	−623.6813
z_{2i-1}	−0.3529	0.1661	−0.0782	0.0368	−0.0173	0.0081	−0.0038	0.0018
z_{2i}	−3.1875	6.7734	−14.3936	30.5863	−64.9959	138.1163	−293.4971	623.6813

Corollary 1.8. *Let a, b, c, d be only positive real numbers or negative real numbers and let e, f be arbitrary nonnegative real numbers, and let $\{x_n, y_n, z_n\}$ be a solution of the system* (1.15). *If* $2 < ad < +\infty$ *and* $2 < bc < +\infty$ *then one has*

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} y_{2n-1} = 0,$$
$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} y_{2n} = \infty. \tag{1.35}$$

Proof. From $2 < ad < +\infty$ and $2 < bc < +\infty$ we have $1 < ad - 1 < +\infty$ and $1 < bc - 1 < +\infty$. Hence, we have

$$x_{2n-1} = \frac{d}{(ad-1)^n}, \qquad y_{2n-1} = \frac{b}{(bc-1)^n},$$
$$x_{2n} = c(bc-1)^n, \qquad y_{2n} = a(ad-1)^n. \tag{1.36}$$

Then,

$$\lim_{n\to\infty} x_{2n-1} = \lim_{n\to\infty} \frac{d}{(ad-1)^n} = 0,$$
$$\lim_{n\to\infty} y_{2n-1} = \lim_{n\to\infty} \frac{b}{(bc-1)^n} = 0. \tag{1.37}$$

Table 4

i	1	2	3	4	5	6	7	8
x_{2i-1}	0.546	0.198	0.072	0.026	0.010	0.004	0.001	0.0005
x_{2i}	4.125	11.344	31.195	85.787	235.915	648.765	1784.104	4906.285
y_{2i-1}	0.909	0.331	0.120	0.044	0.016	0.006	0.002	0.001
y_{2i}	6.875	18.906	51.992	142.979	393.191	1081.275	2973.506	8177.142
z_{2i-1}	0.546	0.198	0.072	0.026	0.010	0.004	0.001	0.0005
z_{2i}	4.125	11.344	31.195	85.787	235.915	648.765	1784.104	4906.285

Also,

$$\lim_{n\to\infty} x_{2n} = \lim_{n\to\infty} c(bc-1)^n = \begin{cases} +\infty, & c > 0 \\ -\infty, & c < 0, \end{cases}$$
$$\lim_{n\to\infty} y_{2n} = \lim_{n\to\infty} a(ad-1)^n = \begin{cases} +\infty, & a > 0 \\ -\infty, & a < 0. \end{cases} \tag{1.38}$$

□

Example 1.9. If $y(0) = 5/2,\ y(-1) = 5/2,\ x(0) = 3/2,\ x(-1) = 3/2,\ z(0) = 3/2,\ z(-1) = 3/2$ then the solutions of (1.15) can be represented by Table 4.

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