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Research Article

Projective Synchronization of *N*-Dimensional Chaotic Fractional-Order Systems via Linear State Error Feedback Control

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Based on linear feedback control technique, a projective synchronization scheme of N-dimensional chaotic fractional-order systems is proposed, which consists of master and slave fractional-order financial systems coupled by linear state error variables. It is shown that the slave system can be projectively synchronized with the master system constructed by state transformation. Based on the stability theory of linear fractional order systems, a suitable controller for achieving synchronization is designed. The given scheme is applied to achieve projective synchronization of chaotic fractional-order financial systems. Numerical simulations are given to verify the effectiveness of the proposed projective synchronization scheme.

1. Introduction

The fractional calculus, as a very old mathematical topic, has been in existence for more than 300 years [1], but it has not been widely used in the science and engineering for many years, because its geometrical or physical interpretation has been not widely accepted [2, 3]. However, due to the long memory advantage, in the recent past, the fractional calculus has been widely applied to diffusion processes [4], Sprott chaotic systems [5], happiness and love [6], economics and finances [7, 8], and so on.

Chaos synchronization has been widely investigated in science and engineering such as humanistic community [9], physical science [10], and secure communications [11]. The chaos projective synchronization was first reported by Mainieri and Rehacek [12]. This type of projective synchronization is interesting due to its property of proportionally diminished or enlarged synchronizing responses, but the early work was limited to a certain kind of

nonlinear systems with partly linear properties. Chaos projective synchronization has been an active research topic in nonlinear science until Wen and Xu [13, 14] proposed an observer-based control method and showed "no special limitation" to nonlinear systems themselves to achieve this type of chaos synchronization. Wen and coauthors tried to explore the potential applications of projective synchronization to noise reduction in mechanical engineering [15, 16] or design bifurcation solutions based on the property of projective synchronization [17]. Synchronization of fractional-order chaotic systems was first presented by Deng and Li [18]. There has been an increasing interest in fractional-order chaos synchronization during the last few years because of its potentials in both theory and applications [19]. Peng et al. [20] proposed the generalized projective synchronization scheme of fractional order chaotic systems via a transmitted signal. Shao [21] proposed a method to achieve general projective synchronization of two fractional order Rossler systems. Odibat et al. [22] studied synchronization of 3-dimensional chaotic fractional-order systems via linear control. The advantage of the linear feedback controller is that it is robust and linear, and moreover, it is easier to be designed and implemented for chaos synchronization than standard PID feedback controller, sliding mode controller, nonlinear feedback controller, and so on [23–25].

Huang and Li [26] reported an integer order financial model as follows:

$$\begin{aligned} \dot{x} &= z + (y - a)x, \\ \dot{y} &= 1 - by - x^2, \\ \dot{z} &= -x - cz, \end{aligned} \tag{1.1}$$

where x is the interest rate, y is the investment demand, z is the price index, a is the saving amount, b is the cost per investment, c is the demand elasticity of commercial markets, and all three constants $a, b, c \geq 0$.

Chen [7] considered the generalization of system (1.1) for the fractional-order model which takes the following form:

$$\begin{aligned} \frac{d^{q_1}x}{dt^{q_1}} &= z + (y - a)x, \\ \frac{d^{q_2}y}{dt^{q_2}} &= 1 - by - x^2, \\ \frac{d^{q_3}z}{dt^{q_3}} &= -x - cz, \end{aligned} \tag{1.2}$$

where the q_ith-order fractional derivative is given by the following Caputo definition, $i = 1, 2, 3$.

Definition 1.1 (see [27]). The qth-order fractional derivative of function $f(t)$ with respect to t and the terminal value 0 is written as

$$\frac{d^q f(t)}{dt^q} = \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{q-m+1}} d\tau, \tag{1.3}$$

where m is an integer and satisfies $m - 1 \leq q < m$.

Remark 1.2. If $q_1 = q_2 = q_3 = 1$, the system (1.2) degenerates into the system (1.1).

The remainder of this paper is organized as follows. In Section 2, a projective synchronization scheme of n-dimensional chaotic fractional-order systems is proposed. In Section 3, a projective synchronization scheme of chaotic fractional-order financial systems is studied. In Section 4, the Adams-Bashforth-Moulton predictor-corrector scheme of a fractional-order system is described. Numerical simulations are given in Section 5 to show the effectiveness of the proposed synchronization scheme. Finally, the paper is concluded in Section 6.

2. A Projective Synchronization Scheme of n-Dimensional Chaotic Fractional-Order Systems

Definition 2.1. The projective synchronization discussed in this paper is defined as two relative chaotic dynamical systems can be synchronous with a desired scaling factor.

Consider a fractional-order chaotic system as follows:

$$\frac{d^q x(t)}{dt^q} = Ax(t) + C + f(x(t)), \quad 0 < q < 1, \tag{2.1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in \mathbb{R}^n$ is an n-dimensional state vector of the system, A is an $n \times n$ linear constant matrix, C is an $n \times 1$ linear constant matrix, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear vector function.

For the given system (2.1), one can construct the following new system

$$\frac{d^q y(t)}{dt^q} = Ay(t) + \alpha\big(C + f(x(t))\big) + u(t), \quad 0 < q < 1, \tag{2.2}$$

where $y = (y_1, y_2, \ldots, y_n)^T \in \mathbb{R}^n$ is an n-dimensional state vector of the system, $f : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous nonlinear vector function, A, C are linear constant matrix, α is a desired scaling factor, $u(t)$ is a linear state error feedback controller.

The synchronization error between the master system (2.1) and the slave system (2.2) is defined as

$$e(t) = y(t) - \alpha x(t), \quad i = 1, 2, \ldots, n. \tag{2.3}$$

The linear state error feedback controller is defined as

$$u(t) = \widehat{A} e(t), \tag{2.4}$$

where $\widehat{A}$ is an $n \times n$ linear constant matrix.

Then the error system can be written as

$$\frac{d^q e(t)}{dt^q} = \frac{d^q y(t)}{dt^q} - \alpha \frac{d^q x(t)}{dt^q} = Ae(t) + u(t) = Be(t), \tag{2.5}$$

where $B = A + \widehat{A}$ is an $n \times n$ linear constant matrix. Obviously the orginal point is the equilibrium point of system (2.5).

According to the stability criterion of linear fractional-order dynamical system, one can directly obtain the following theorem.

Theorem 2.2. *If B is an upper or lower triangular matrix and all eigenvalues $\lambda_I, \lambda_2, \ldots, \lambda_n$ satisfy $\lambda_i, \lambda_2, \ldots, \lambda_n < 0$, then the equilibrium point of synchronization error $e(t)$ is asymptotically stable and $\lim_{t\to\infty} e(t) = 0$, that is, the master system* (2.1) *and the slave system* (2.2) *achieve projective synchronization.*

Remark 2.3. If $\alpha = 1$ and $n = 3$, the above synchronization scheme is similar to the synchronization scheme in [28].

Remark 2.4. If $\alpha = 1$ and $n = 3$, the above synchronization scheme degenerates into the synchronization scheme proposed by Odibat et al. [22].

Remark 2.5. If $n = 3$, the above synchronization scheme degenerates into the synchronization scheme proposed by Xin et al. [29].

3. A Projective Synchronization Scheme of Chaotic Fractional-Order Financial Systems

In order to investigate the synchronization behaviors of two chaotic fractional-order financial systems, one can set a master-slave configuration with a master system given by the fractional-order financial systems (1.2) and with a slave system (denoted by the subscript s) as follows:

$$\begin{aligned} \frac{d^{q_1} x_s}{dt^{q_1}} &= -a x_s + z_s + \alpha x y + u_1, \\ \frac{d^{q_2} y_s}{dt^{q_2}} &= -b y_s + \alpha\left(1 - x^2\right) + u_2, \\ \frac{d^{q_3} z_s}{dt^{q_3}} &= -x_s - c z_s + u_3, \end{aligned} \tag{3.1}$$

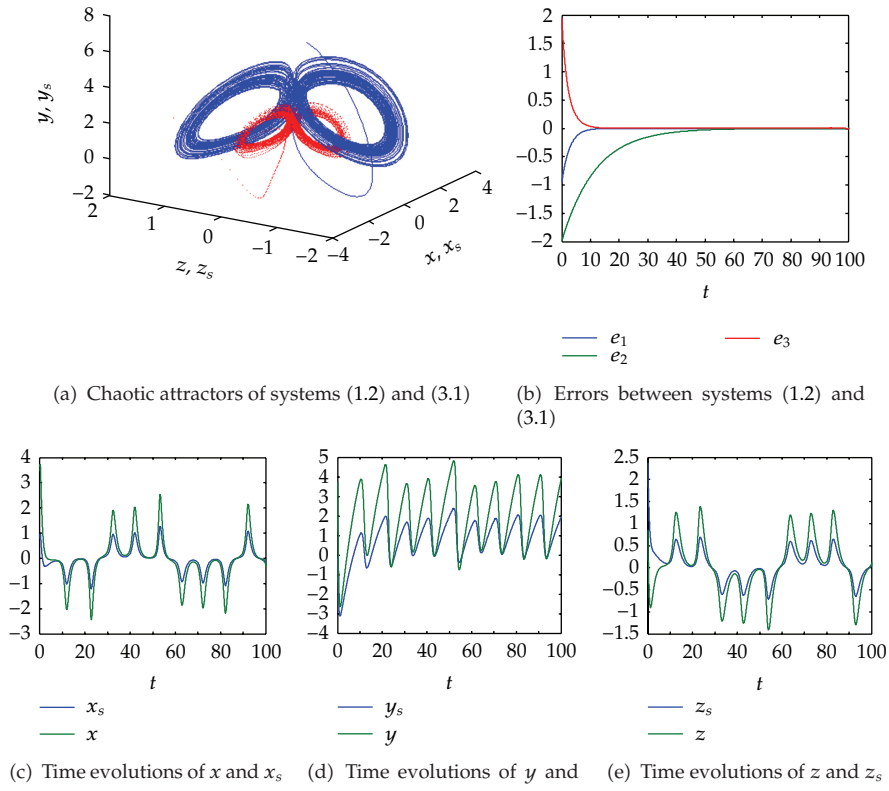
where $x_s, y_s, z_s \in \mathbb{R}^n$ have the same meanings as x, y, z of system (1.2), α is a desired scaling factor, u_1, u_2, u_3 are linear state error feedback controllers.

Proposition 3.1. *The drive system* (1.2) *and the slave system* (3.1) *will approach global synchronization for any initial condition if anyone of the following control laws holds:*

$$(1)\ u_1 = a_u(x_s - \alpha x) - z_s + \alpha z, \qquad u_2 = b_u(y_s - \alpha y), \qquad u_3 = c_u(z_s - \alpha z), \tag{3.2}$$

$$(2)\ u_1 = a_u(z_s - \alpha z), \qquad u_2 = b_u(y_s - \alpha y), \qquad u_3 = x_s - \alpha x + c_u(z_s - \alpha z), \tag{3.3}$$

where $a_u < a$, $b_u < b$ and $c_u < c$.



(a) Chaotic attractors of systems (1.2) and (3.1)

(b) Errors between systems (1.2) and (3.1)

(c) Time evolutions of x and x_s

(d) Time evolutions of y and y_s

(e) Time evolutions of z and z_s

Figure 1: Synchronization errors between the master system (1.2) and the slave system (3.1) with $a = 1$, $b = 0.1$, $c = 1.2$, $q_1 = 0.88$, $q_2 = 0.98$, $q_3 = 0.96$, $a = 0.5$, $x(0) = 3$, $y(0) = 4$, $z(0) = 1$, $x_s(0) = 0.5$, $y_s(0) = 0$, $z_s(0) = 2.5$.

Proof. The synchronization errors between the master system (1.2) and the slave system (3.1) are defined as $e_x = x_s - \alpha x$, $e_y = y_s - \alpha y$, $e_z = z_s - \alpha z$. Subtracting (1.2) from (3.1) yields the error system as follows.

$$\begin{aligned} \frac{d^{q_1} e_x}{dt^{q_1}} &= e_z - a e_x + u_1, \\ \frac{d^{q_2} e_y}{dt^{q_2}} &= -b e_y + u_2, \\ \frac{d^{q_3} e_z}{dt^{q_3}} &= -e_x - c e_z + u_3. \end{aligned} \tag{3.4}$$

For the first control law in Proposition 3.1, substituting (3.2) into the error system (3.4), the system (3.4) can be rewritten as follows:

$$\begin{aligned} \frac{d^{q_1} e_x}{dt^{q_1}} &= (a_u - a) e_x, \\ \frac{d^{q_2} e_y}{dt^{q_2}} &= (b_u - b) e_y, \\ \frac{d^{q_3} e_z}{dt^{q_3}} &= -e_x + (c_u - c) e_z, \end{aligned} \tag{3.5}$$

which has one equilibrium point at $E^* = (0,0,0)$. Its Jacobian matrix evaluated at equilibrium point E^* is given by

$$J(E^*) = \begin{pmatrix} a_u - a & 0 & 0 \\ 0 & b_u - b & 0 \\ -1 & 0 & c_u - c \end{pmatrix}, \tag{3.6}$$

which is a lower triangular matrix and its eigenvalues satisfy $\lambda_1, \lambda_2, \lambda_3 < 0$.

For the second control law in Proposition 3.1, substituting (3.3) into the error system (3.4), the system (3.4) can be rewritten as follows:

$$\begin{aligned} \frac{d^{q_1} e_x}{dt^{q_1}} &= (a_u - a)e_x + e_z, \\ \frac{d^{q_2} e_y}{dt^{q_2}} &= (b_u - b)e_y, \\ \frac{d^{q_3} e_z}{dt^{q_3}} &= (c_u - c)e_z, \end{aligned} \tag{3.7}$$

which has one equilibrium point at $E^* = (0,0,0)$. Its Jacobian matrix evaluated at equilibrium point E^* is given by

$$J(E^*) = \begin{pmatrix} a_u - a & 0 & 1 \\ 0 & b_u - b & 0 \\ 0 & 0 & c_u - c \end{pmatrix}, \tag{3.8}$$

which is an upper triangular matrix and its eigenvalues satisfy $\lambda_1, \lambda_2, \lambda_3 < 0$.

It follows from Theorem 2.2 that system (3.4) is asymptotically stable, that is, the master system (1.2) and the slave system (3.1) are synchronized finally.

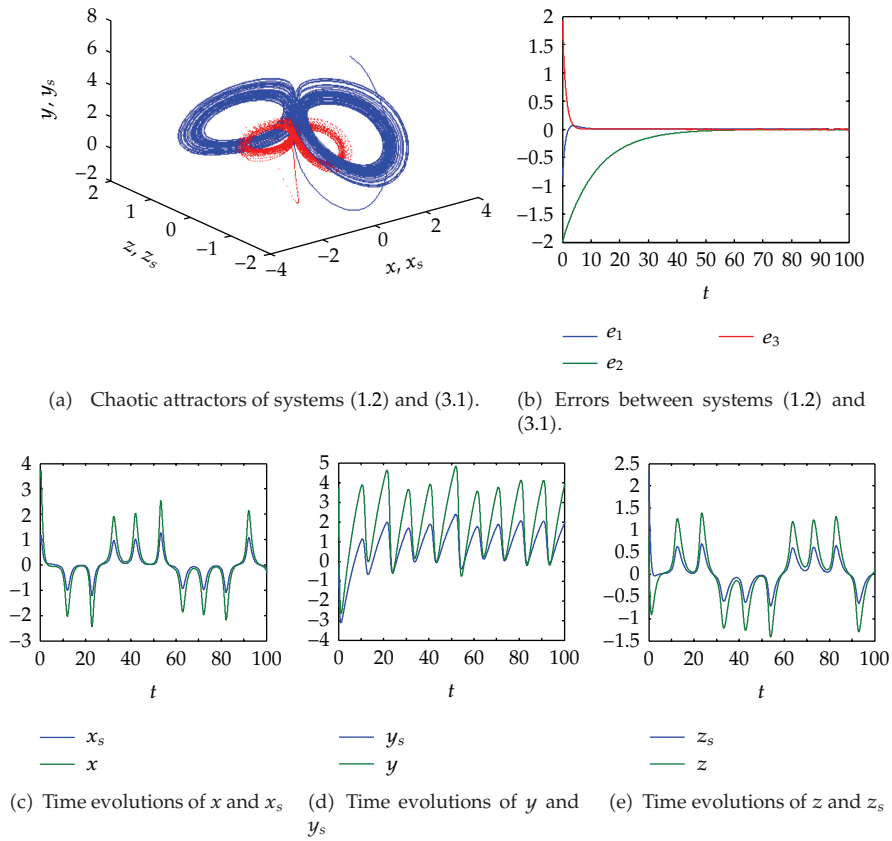
The Proposition 3.1 is proved. □

4. Numerical Method for Solving System (1.2)

An improved Adams-Bashforth-Moulton predictor-corrector scheme [30] can be employed to solve fractional-order ordinary differential equations. The improved predictor-corrector scheme of system (1.2) can be described as follows.

With the initial value $(x_0^{(k)}, y_0^{(k)}, z_0^{(k)})$, $k = 0, 1, \ldots, [m] - 1$, system (1.2) is equivalent to the Volterra integral equations as follows:

$$\begin{aligned} x(t) &= \sum_{k=0}^{[m]-1} x_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q_1)} \int_0^t (t-\tau)^{q_1-1} \big(z(\tau) + (y(\tau) - a)x(\tau)\big) d\tau, \\ y(t) &= \sum_{k=0}^{[m]-1} y_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q_2)} \int_0^t (t-\tau)^{q_2-1} \Big(1 - by(\tau) - x^2(\tau)\Big) d\tau, \\ z(t) &= \sum_{k=0}^{[m]-1} z_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q_3)} \int_0^t (t-\tau)^{q_3-1} (-x(\tau) - cz(\tau)) d\tau. \end{aligned} \tag{4.1}$$



(a) Chaotic attractors of systems (1.2) and (3.1).

(b) Errors between systems (1.2) and (3.1).

(c) Time evolutions of x and x_s

(d) Time evolutions of y and y_s

(e) Time evolutions of z and z_s

Figure 2: Synchronization errors between the master system (1.2) and the slave system (3.1) with $a = 1$, $b = 0.1$, $c = 1.2$, $q_1 = 0.88$, $q_2 = 0.98$, $q_3 = 0.96$, $a = 0.5$, $x(0) = 3$, $y(0) = 4$, $z(0) = 1$, $x_s(0) = 0.5$, $y_s(0) = 0$, $z_s(0) = 2.5$.

Consider the uniform grid $\{t_n = nh, n = 0, 1, \ldots, N\}$ for some integers $N \in Z^+$ and $h = T/N$, system (4.1) can be approximated to the following difference equations:

$$x_{n+1} = x_0 + \frac{h^{q_1}}{\Gamma(q_1+2)}\left(z_{n+1}^p + \left(y_{n+1}^p - a\right)x_{n+1}^p\right) + \frac{h^{q_1}}{\Gamma(q_1+2)}\sum_{j=0}^{n}\alpha_{1,j,n+1}\left(z_j + (y_j - a)x_j\right),$$

$$y_{n+1} = y_0 + \frac{h^{q_2}}{\Gamma(q_2+2)}\left(1 - by_{n+1}^p - x_{n+1}^{2p}\right) + \frac{h^{q_2}}{\Gamma(q_2+2)}\sum_{j=0}^{n}\alpha_{2,j,n+1}\left(1 - by_j - x_j^2\right), \tag{4.2}$$

$$z_{n+1} = z_0 + \frac{h^{q_3}}{\Gamma(q_3+2)}\left(-x_{n+1}^p - cz_{n+1}^p\right) + \frac{h^{q_3}}{\Gamma(q_3+2)}\sum_{j=0}^{n}\alpha_{3,j,n+1}\left(-x_j - cz_j\right),$$

where

$$x_{n+1}^{p} = x_0 + \frac{1}{\Gamma(q_1)}\sum_{j=0}^{n}\beta_{1,j,n+1}\left(z_j + (y_j - a)x_j\right),$$

$$y_{n+1}^{p} = y_0 + \frac{1}{\Gamma(q_2)}\sum_{j=0}^{n}\beta_{2,j,n+1}\left(1 - by_j - x_j^2\right),$$

$$z_{n+1}^{p} = z_0 + \frac{1}{\Gamma(q_3)}\sum_{j=0}^{n}\beta_{3,j,n+1}\left(-x_j - cz_j\right), \tag{4.3}$$

$$\alpha_{i,j,n+1} = \begin{cases} n^{q_i+1} - (n - q_i)(n+1)^{q_i}, & j = 0, \\ (n - j + 2)^{q_i+1} + (n - j)^{q_i+1} - 2(n - j + 1)^{q_i+1}, & 1 \le j \le n, \\ 1, & j = n + 1, \end{cases}$$

$$\beta_{i,j,n+1} = \frac{h^{q_i}}{q_i}(n - j + 1)^{q_i} - \frac{h^{q_i}}{q_i}(n - j)^{q_i}, \quad 0 \le j \le n,\ i = 1, 2, 3.$$

Errors of the above method are

$$\Delta x = \max_{j=0,1,\ldots,N}\left|x(t_j) - x_h(t_j)\right| = O(h^{p_1}),$$

$$\Delta y = \max_{j=0,1,\ldots,N}\left|y(t_j) - y_h(t_j)\right| = O(h^{p_2}), \tag{4.4}$$

$$\Delta z = \max_{j=0,1,\ldots,N}\left|z(t_j) - z_h(t_j)\right| = O(h^{p_3}),$$

where $p_i = \min(2, 1 + q_i)$.

5. Numerical Simulations

Based on the Adams-Bashforth-Moulton predictor-corrector scheme, one can let the master system (1.2) and the slave system (3.1) with parameters $a = 1$, $b = 0.1$, $c = 1.2$, $q_1 = 0.88$, $q_2 = 0.98$, $q_3 = 0.96$, $a = 0.5$, initial values $x(0) = 3$, $y(0) = 4$, $z(0) = 1$, $x_s(0) = 0.5$, $y_s(0) = 0$, $z_s(0) = 2.5$. The following numerical simulations are carried out to illustrate the main results.

From the first control law of Proposition 3.1, the linear controllers have the following form: $u_1 = z - \alpha z_s$, $u_2 = 0$, $u_3 = 0$. The chaotic attractors of the master system (1.2) and the slave system (3.1) are shown in Figure 1(a). Synchronization errors between systems (1.2) and (3.1) are shown in Figure 1(b). Time evolutions of x, x_s, y, y_s, z and z_s are shown in Figures 1(c)–1(e), respectively. From Figures 1(a)–1(e), it is clear that the projective synchronization is achieved for all these values.

From the second control law of Proposition 3.1, the linear controllers have the following form: $u_1 = 0$, $u_2 = 0$, $u_3 = x_s - \alpha x$. The chaotic attractors of the master system (1.2) and the slave system (3.1) are shown in Figure 2(a). Synchronization errors between systems (1.2) and (3.1) are shown in Figure 2(b). Time evolutions of x, x_s, y, y_s, z, and z_s are shown in Figures 2(c)–2(e), respectively. From Figures 2(a)–2(e), it is clear that the projective synchronization is achieved for all these values.

6. Conclusions

In this paper, we propose a projective synchronization scheme of n-dimensional chaotic fractional-order systems via line error feedback control, and apply the scheme to achieve synchronization of the chaotic fractional-order financial systems. Numerical simulations validate the main results of this work.

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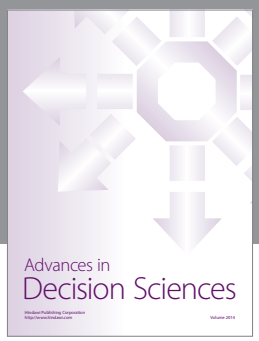

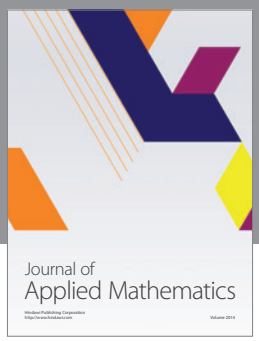

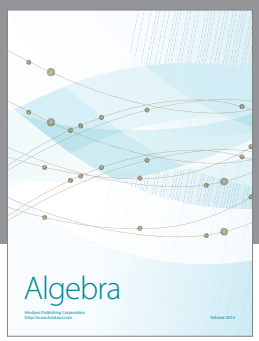

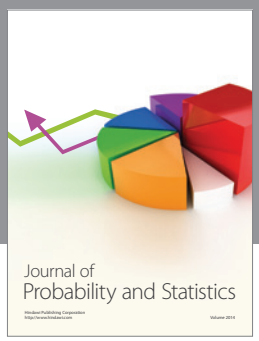

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