Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 373904, 9 pages
doi:10.1155/2012/373904

Research Article

Approximation of Homomorphisms and Derivations on non-Archimedean Lie C^*-Algebras via Fixed Point Method

Yeol Je Cho,[1] Reza Saadati,[2] and Javad Vahidi[2]

[1] *Department of Mathematics Education and RINS, Gyeongsang National University, Chinju 660-701, Republic of Korea*
[2] *Department of Mathematics, Iran University of Science and Technology, Tehran, Iran*

Correspondence should be addressed to Reza Saadati, rsaadati@eml.cc

Received 19 October 2011; Accepted 12 January 2012

Academic Editor: Yong Zhou

Using fixed point methods, we prove the generalized Hyers-Ulam stability of homomorphisms in C^*-algebras and Lie C^*-algebras and of derivations on non-Archimedean C^*-algebras and Non-Archimedean Lie C^*-algebras for an m-variable additive functional equation.

1. Introduction and Preliminaries

By a *non-Archimedean field* we mean a field K equipped with a function (valuation) $|\cdot|$ from K into $[0,\infty)$ such that $|r|=0$ if and only if $r=0$, $|rs|=|r||s|$, and $|r+s|\le\max\{|r|,|s|\}$ for all $r,s\in K$. Clearly $|1|=|-1|=1$ and $|n|\le 1$ for all $n\in\mathbb{N}$. By the trivial valuation we mean the mapping $|\cdot|$ taking everything but 0 into 1 and $|0|=0$. Let X be a vector space over a field K with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\|: X\to[0,\infty)$ is called a *non-Archimedean norm* if it satisfies the following conditions:

(i) $\|x\|=0$ if and only if $x=0$;

(ii) for any $r\in K$, and $x\in X$, $\|rx\|=|r|\|x\|$;

(iii) the strong triangle inequality (ultrametric) holds; namely,

$$\|x+y\|\le\max\{\|x\|,\|y\|\}. \tag{1.1}$$

for all $x,y\in X$.

Then $(X, \|\cdot\|)$ is called a *non-Archimedean normed space*. From the fact that

$$\|x_n - x_m\| \le \max\{\|x_{j+1} - x_j\| : m \le j \le n-1\} \quad (n > m) \tag{1.2}$$

holds, a sequence $\{x_n\}$ is a Cauchy sequence if and only if $\{x_{n+1} - x_n\}$ converges to zero in a non-Archimedean normed space. By a complete non-Archimedean normed space we mean one in which every Cauchy sequence is convergent.

For any nonzero rational number x, there exists a unique integer $n_x \in \mathbb{Z}$ such that $x = (a/b)p^{n_x}$, where a and b are integers not divisible by p. Then $|x|_p := p^{-n_x}$ defines a non-Archimedean norm on $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to the metric $d(x, y) = |x - y|_p$ is denoted by $\mathbb{Q}_p$, which is called the *p-adic number field*.

A non-Archimedean Banach algebra is a *complete non-Archimedean algebra* $\mathcal{A}$ which satisfies $\|ab\| \le \|a\| \cdot \|b\|$ for all $a, b \in \mathcal{A}$. For more detailed definitions of non-Archimedean Banach algebras, we refer the reader to [1, 2].

If $\mathcal{U}$ is a non-Archimedean Banach algebra, then an *involution* on $\mathcal{U}$ is a mapping $t \to t^*$ from $\mathcal{U}$ into $\mathcal{U}$ which satisfies

(i) $t^{**} = t$ for $t \in \mathcal{U}$;

(ii) $(\alpha s + \beta t)^* = \overline{\alpha} s^* + \overline{\beta} t^*$;

(iii) $(st)^* = t^* s^*$ for $s, t \in \mathcal{U}$.

If, in addition, $\|t^* t\| = \|t\|^2$ for $t \in \mathcal{U}$, then $\mathcal{U}$ is a *non-Archimedean C^*-algebra*.

The stability problem of functional equations was originated from a question of Ulam [3] concerning the stability of group homomorphisms: let $(G_1, *)$ be a group and let $(G_2, \diamond, d)$ be a metric group (a metric which is defined on a set with group property) with the metric $d(\cdot, \cdot)$. Given $\epsilon > 0$, does there exist a $\delta(\epsilon) > 0$ such that, if a mapping $h : G_1 \to G_2$ satisfies the inequality $d(h(x * y), h(x) \diamond h(y)) < \delta$ for all $x, y \in G_1$, then there is a homomorphism $H : G_1 \to G_2$ with $d(h(x), H(x)) < \epsilon$ for all $x \in G_1$? If the answer is affirmative, we would say that the equation of homomorphism $H(x * y) = H(x) \diamond H(y)$ is stable (see also [4–6]).

We recall a fundamental result in fixed point theory. Let Ω be a set. A function $d : \Omega \times \Omega \to [0, \infty]$ is called a *generalized metric* on Ω if d satisfies

(1) $d(x, y) = 0$ if and only if $x = y$;

(2) $d(x, y) = d(y, x)$ for all $x, y \in \Omega$;

(3) $d(x, z) \le d(x, y) + d(y, z)$ for all $x, y, z \in \Omega$.

Theorem 1.1 (see [7]). *Let (Ω, d) be a complete generalized metric space and let $J : \Omega \to \Omega$ be a contractive mapping with Lipschitz constant $L < 1$. Then for each given element $x \in \Omega$, either $d(J^n x, J^{n+1} x) = \infty$ for all nonnegative integers n or there exists a positive integer n_0 such that*

(1) *$d(J^n x, J^{n+1} x) < \infty$ for all $n \ge n_0$;*

(2) *the sequence $\{J^n x\}$ converges to a fixed point y^* of J;*

(3) *y^* is the unique fixed point of J in the set $\Gamma = \{y \in \Omega \mid d(J^{n_0} x, y) < \infty\}$;*

(4) *$d(y, y^*) \le (1/(1 - L)) d(y, Jy)$ for all $y \in \Gamma$.*

In this paper, using the fixed point method, we prove the generalized Hyers-Ulam stability of homomorphisms and derivations in non-Archimedean C^*-algebras and non-Archimedean Lie C^*-algebras for the following additive functional equation (see [8]):

$$\sum_{i=1}^{m} f\left(mx_i + \sum_{j=1, j\neq i}^{m} x_j\right) + f\left(\sum_{i=1}^{m} x_i\right) = 2f\left(\sum_{i=1}^{m} mx_i\right) \quad (m \in \mathbb{N},\ m \geq 2). \tag{1.3}$$

2. Stability of Homomorphisms and Derivations in C^*-Algebras

Throughout this section, assume that $\mathcal{A}$ is a non-Archimedean C^*-algebra with norm $\|\cdot\|_{\mathcal{A}}$ and that $\mathcal{B}$ is a non-Archimedean C^*-algebra with norm $\|\cdot\|_{\mathcal{B}}$.

For a given mapping $f : \mathcal{A} \to \mathcal{B}$, we define

$$D_\mu f(x_1, \ldots, x_m) := \sum_{i=1}^{m} \mu f\left(mx_i + \sum_{j=1, j\neq i}^{m} x_j\right) + f\left(\mu\sum_{i=1}^{m} x_i\right) - 2f\left(\mu\sum_{i=1}^{m} mx_i\right) \tag{2.1}$$

for all $\mu \in \mathbb{T}^1 := \{\nu \in \mathbb{C} : |\nu| = 1\}$ and all $x_1, \ldots, x_m \in \mathcal{A}$.

Note that a $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a *homomorphism* in non-Archimedean C^*-algebras if H satisfies $H(xy) = H(x)H(y)$ and $H(x^*) = H(x)^*$ for all $x, y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of homomorphisms in non-Archimedean C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 2.1. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^m \to [0,\infty)$, $\psi : \mathcal{A}^2 \to [0,\infty)$ and $\eta : \mathcal{A} \to [0,\infty)$ such that $|m| < 1$ is far from zero and*

$$\left\|D_\mu f(x_1, \ldots, x_m)\right\|_{\mathcal{B}} \leq \varphi(x_1, \ldots, x_m), \tag{2.2}$$

$$\left\|f(xy) - f(x)f(y)\right\|_{\mathcal{B}} \leq \psi(x, y), \tag{2.3}$$

$$\left\|f(x^*) - f(x)^*\right\|_{\mathcal{B}} \leq \eta(x) \tag{2.4}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \ldots, x_m, x, y \in A$. If there exists an $L < 1$ such that

$$\varphi(mx_1, \ldots, mx_m) \leq |m|\ L\varphi(x_1, \ldots, x_m), \tag{2.5}$$

$$\psi(mx, my) \leq |m|^2 L\psi(x, y), \tag{2.6}$$

$$\eta(mx) \leq |m|L\eta(x) \tag{2.7}$$

for all $x, y, x_1, \ldots, x_m \in \mathcal{A}$, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\left\|f(x) - H(x)\right\|_{\mathcal{B}} \leq \frac{1}{|m| - |m|L}\varphi(x, 0, \ldots, 0) \tag{2.8}$$

for all $x \in \mathcal{A}$.

Proof. It follows from (2.5), (2.6), (2.7) and $L < 1$ that

$$\lim_{n\to\infty} \frac{1}{|m|^n}\varphi(m^n x_1, \dots, m^n x_m) = 0, \tag{2.9}$$

$$\lim_{n\to\infty} \frac{1}{|m|^{2n}}\psi(m^n x, m^n y) = 0, \tag{2.10}$$

$$\lim_{n\to\infty} \frac{1}{|m|^n}\eta(m^n x) = 0 \tag{2.11}$$

for all $x, y, x_1, \dots, x_m \in \mathcal{A}$.

Let us define Ω to be the set of all mappings $g : \mathcal{A} \to \mathcal{B}$ and introduce a generalized metric on Ω as follows

$$d(g,h) = \inf\{k \in (0,\infty) : \|g(x) - h(x)\|_{\mathcal{B}} < k\phi(x,0,\dots,0),\ \forall x \in \mathcal{A}\}. \tag{2.12}$$

It is easy to show that (Ω, d) is a generalized complete metric space (see [9]).

Now we consider the function $J : \Omega \to \Omega$ defined by $Jg(x) = (1/m)g(mx)$ for all $x \in \mathcal{A}$ and $g \in \Omega$. Note that for all $g, h \in \Omega$ we have

$$\begin{aligned} d(g,h) < k &\Longrightarrow \|g(x) - h(x)\|_{\mathcal{B}} < k\phi(x,0,\dots,0) \\ &\Longrightarrow \left\|\frac{1}{m}g(mx) - \frac{1}{m}h(mx)\right\|_{\mathcal{B}} < \frac{k}{|m|}\phi(mx,0,\dots,0) \\ &\Longrightarrow \left\|\frac{1}{m}g(mx) - \frac{1}{m}h(mx)\right\|_{\mathcal{B}} < kL\phi(mx,0,\dots,0) \\ &\Longrightarrow d(Jg, Jh) < kL. \end{aligned} \tag{2.13}$$

From this it is easy to see that $d(Jg, Jk) \le Ld(g,h)$ for all $g, h \in \Omega$, that is, J is a self-function of Ω with the Lipschitz constant L.

Putting $\mu = 1$, $x = x_1$ and $x_2 = x_3 = \cdots = x_m = 0$ in (2.2), we have

$$\|f(mx) - mf(x)\|_{\mathcal{B}} \le \phi(x,0,\dots,0) \tag{2.14}$$

for all $x \in \mathcal{A}$. Then

$$\left\|f(x) - \frac{1}{m}f(mx)\right\|_{\mathcal{B}} \le \frac{1}{|m|}\phi(x,0,\dots,0) \tag{2.15}$$

for all $x \in \mathcal{A}$, that is, $d(Jf, f) \le 1/|m| < \infty$. Now, from the fixed point alternative, it follows that there exists a fixed point H of J in Ω such that

$$H(x) = \lim_{n\to\infty} \frac{1}{|m|^n} f(m^n x) \tag{2.16}$$

for all $x \in \mathcal{A}$ since $\lim_{n\to\infty} d(J^n f, H) = 0$.

On the other hand, it follows from (2.2), (2.9), and (2.16) that

$$\begin{aligned} \left\| D_\mu H(x_1, \ldots, x_m) \right\|_{\mathcal{B}} &= \lim_{n\to\infty} \left\| \frac{1}{m^n} Df(m^n x_1, \ldots, m^n x_m) \right\|_{\mathcal{B}} \\ &\le \lim_{n\to\infty} \frac{1}{|m|^n} \phi(m^n x_1, \ldots, m^n x_m) = 0. \end{aligned} \tag{2.17}$$

By a similar method to the above, we get $\mu H(mx) = H(m\mu x)$ for all $\mu \in \mathbb{T}^1$ and $x \in \mathcal{A}$. Thus one can show that the mapping $H : \mathcal{A} \to \mathcal{B}$ is $\mathbb{C}$-linear.

It follows from (2.3), (2.10) and (2.16) that

$$\begin{aligned} \left\| H(xy) - H(x)H(y) \right\|_{\mathcal{B}} &= \lim_{n\to\infty} \frac{1}{|m|^{2n}} \left\| f\left(m^{2n} xy\right) - f(m^n x) f(m^n y) \right\|_{\mathcal{B}} \\ &\le \lim_{n\to\infty} \frac{1}{|m|^{2n}} \psi(m^n x, m^n y) = 0 \end{aligned} \tag{2.18}$$

for all x, $y \in \mathcal{A}$. So $H(xy) = H(x)H(y)$ for all x, $y \in \mathcal{A}$. Thus $H : \mathcal{A} \to \mathcal{B}$ is a homomorphism, satisfying (2.8), as desired.

Also, by (2.4), (2.11), (2.16) and by a similar method, we have $H(x^*) = H(x)^*$. □

Corollary 2.2. *Let $r > 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\begin{aligned} \left\| D_\mu f(x_1, \ldots, x_m) \right\|_{\mathcal{B}} &\le \theta \cdot \left(\|x_1\|_{\mathcal{A}}^r + \|x_2\|_{\mathcal{A}}^r + \cdots + \|x_m\|_{\mathcal{A}}^r \right), \\ \left\| f(xy) - f(x)f(y) \right\|_{\mathcal{B}} &\le \theta \cdot \left(\|x\|_{\mathcal{A}}^r \cdot \|y\|_{\mathcal{A}}^r \right), \\ \left\| f(x^*) \; - f(x)^* \right\|_{\mathcal{B}} &\le \theta \cdot \|x\|_{\mathcal{A}}^r, \end{aligned} \tag{2.19}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \ldots, x_m, x, y \in \mathcal{A}$. Then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\left\| f(x) - H(x) \right\|_{\mathcal{B}} \le \frac{\theta}{|m| - |m|^r} \|x\|_{\mathcal{A}}^r \tag{2.20}$$

for all $x \in \mathcal{A}$.

Proof. The proof follows from Theorem 2.1 by taking

$$\begin{aligned} \varphi(x_1, \ldots, x_m) &= \theta \cdot \left(\|x_1\|_{\mathcal{A}}^r + \|x_2\|_{\mathcal{A}}^r + \cdots + \|x_m\|_{\mathcal{A}}^r \right), \\ \psi(x, y) &:= \theta \cdot \left(\|x\|_{\mathcal{A}}^r \cdot \|y\|_{\mathcal{A}}^r \right), \\ \eta(x) &= \theta \cdot \|x\|_{\mathcal{A}}^r \end{aligned} \tag{2.21}$$

for all $x_1, \ldots, x_m$, $x, y \in A$, $L = |m|^{r-1}$ and so we get the desired result. □

Note that a $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a *derivation* on $\mathcal{A}$ if δ satisfies $\delta(xy) = \delta(x)y + x\delta(y)$ for all $x, y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of derivations on non-Archimedean C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 2.3. *Let $f : \mathcal{A} \to \mathcal{A}$ be a mapping for which there are functions $\varphi : \mathcal{A}^m \to [0, \infty)$, $\psi : \mathcal{A}^2 \to [0, \infty)$ and $\eta : \mathcal{A} \to [0, \infty)$ such that $|m| < 1$ is far from zero and*

$$\begin{gathered}\left\|D_\mu f(x_1, \ldots, x_m)\right\|_{\mathcal{A}} \leq \varphi(x_1, \ldots, x_m), \\ \left\|f(xy) - f(x)y - xf(y)\right\|_{\mathcal{A}} \leq \psi(x, y), \left\|f(x^*) - f(x)^*\right\|_{\mathcal{A}} \leq \eta(x)\end{gathered} \tag{2.22}$$

for all $\mu \in \mathbb{T}^1$ and $x_1, \ldots, x_m, x, y \in A$. If there exists an $L < 1$ such that (2.5), (2.6) *and* (2.7) *hold, then there exists a unique derivation $\delta : \mathcal{A} \to \mathcal{A}$ such that*

$$\|f(x) - \delta(x)\|_{\mathcal{A}} \leq \frac{1}{(|m| - |m|L)}\varphi(x, 0, \ldots, 0) \tag{2.23}$$

for all $x \in \mathcal{A}$.

3. Stability of Homomorphisms and Derivations in Non-Archimedean Lie C^*-Algebras

A non-Archimedean C^*-algebra $\mathcal{C}$, endowed with the Lie product

$$[x, y] := \frac{xy - yx}{2} \tag{3.1}$$

on $\mathcal{C}$, is called a *Lie non-Archimedean C^*-algebra*.

Definition 3.1. Let $\mathcal{A}$ and $\mathcal{B}$ be Lie C^*-algebras. A $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a *non-Archimedean Lie C^*-algebra homomorphism* if $H([x, y]) = [H(x), H(y)]$ for all $x, y \in \mathcal{A}$.

Throughout this section, assume that $\mathcal{A}$ is a non-Archimedean Lie C^*-algebra with norm $\|\cdot\|_{\mathcal{A}}$ and $\mathcal{B}$ is a non-Archimedean Lie C^*-algebra with norm $\|\cdot\|_{\mathcal{B}}$.

We prove the generalized Hyers-Ulam stability of homomorphisms in non-Archimedean Lie C^*-algebras for the functional equation $D_\mu f(x_1, \ldots, x_m) = 0$.

Theorem 3.2. *Let $f : \mathcal{A} \to \mathcal{B}$ be a mapping for which there are functions $\varphi : \mathcal{A}^m \to [0, \infty)$ and $\psi : \mathcal{A}^2 \to [0, \infty)$ such that* (2.2) *and* (2.4) *hold and*

$$\|f([x, y]) - [f(x), f(y)]\|_{\mathcal{B}} \leq \psi(x, y) \tag{3.2}$$

for all $\mu \in \mathbb{T}^1$ and $x, y \in \mathcal{A}$. If there exists an $L < 1$ and (2.5), (2.6), *and* (2.7) *hold, then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that* (2.8) *holds.*

Proof. By the same reasoning as in the proof of Theorem 2.1, we can find the mapping $H : \mathcal{A} \to \mathcal{B}$ given by

$$H(x) = \lim_{n\to\infty} \frac{f(m^n x)}{|m|^n} \tag{3.3}$$

for all $x \in \mathcal{A}$. It follows from (2.6) and (3.3) that

$$\begin{aligned} \|H([x,y]) - [H(x),H(y)]\|_{\mathcal{B}} &= \lim_{n\to\infty} \frac{1}{|m|^{2n}} \left\| f\left(m^{2n}[x,y]\right) - [f(m^n x), f(m^n y)\right\|_{\mathcal{B}} \\ &\leq \lim_{n\to\infty} \frac{1}{|m|^{2n}} \varphi(m^n x, m^n y) = 0 \end{aligned} \tag{3.4}$$

for all $x, y \in \mathcal{A}$ and so

$$H([x,y]) = [H(x), H(y)], \tag{3.5}$$

for all $x, y \in \mathcal{A}$. Thus $H : \mathcal{A} \to \mathcal{B}$ is a Lie C^*-algebra homomorphism satisfying (2.8), as desired. □

Corollary 3.3. *Let $r > 1$ and θ be nonnegative real numbers, and let $f : \mathcal{A} \to \mathcal{B}$ be a mapping such that*

$$\begin{aligned} \left\|D_\mu f(x_1,\ldots,x_m)\right\|_{\mathcal{B}} &\leq \theta(\|x_1\|_A^r + \|x_2\|_A^r + \cdots + \|x_m\|_A^r), \\ \|f([x,y]) - [f(x),f(y)]\|_{\mathcal{B}} &\leq \theta\cdot\|x\|_A^r\cdot\|y\|_A^r, \\ \|f(x^*) - f(x)^*\|_{\mathcal{B}} &\leq \theta\cdot\|x\|_{\mathcal{A}}^r \end{aligned} \tag{3.6}$$

all $\mu \in \mathbb{T}^1$ and $x_1,\ldots,x_m, x, y \in \mathcal{A}$. Then there exists a unique homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\|f(x) - H(x)\|_{\mathcal{B}} \leq \frac{\theta}{|m| - |m|^r}\|x\|_{\mathcal{A}}^r \tag{3.7}$$

for all $x \in \mathcal{A}$.

Proof. The proof follows from Theorem 3.2 and a method similar to Corollary 3.3. □

Definition 3.4. Let $\mathcal{A}$ be a non-Archimedean Lie C^*-algebra. A $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ is called a *Lie derivation* if $\delta([x,y]) = [\delta(x),y] + [x,\delta(y)]$ for all $x,y \in \mathcal{A}$.

We prove the generalized Hyers-Ulam stability of derivations on non-Archimedean Lie C^*-algebras for the functional equation $D_\mu f(x_1,\ldots,x_m) = 0$.

Theorem 3.5. *Let $f : \mathcal{A} \to \mathcal{A}$ be a mapping for which there are functions $\varphi : A^m \to [0,\infty)$ and $\psi : A^2 \to [0,\infty)$ such that* (2.2) *and* (2.4) *hold and*

$$\|f([x,y]) - [f(x),y] - [x,f(y)]\|_{\mathcal{A}} \leq \psi(x,y) \tag{3.8}$$

for all $x,y \in \mathcal{A}$. If there exists an $L<1$ and (2.5), (2.6) *and* (2.7) *hold, then there exists a unique Lie derivation $\delta : \mathcal{A} \to \mathcal{A}$ such that such that* (2.8) *holds.*

Proof. By the same reasoning as the proof of Theorem 2.3, there exists a unique $\mathbb{C}$-linear mapping $\delta : \mathcal{A} \to \mathcal{A}$ satisfying (2.8) and the mapping $\delta : \mathcal{A} \to \mathcal{A}$ is given by

$$\delta(x) = \lim_{n\to\infty} \frac{f(m^n x)}{|m|^n} \tag{3.9}$$

for all $x \in \mathcal{A}$.

It follows from (2.6) and (3.9) that

$$\begin{aligned} &\|\delta([x,y]) - [\delta(x),y] - [x,\delta(y)]\|_{\mathcal{A}} \\ &\quad = \lim_{n\to\infty} \frac{1}{|m|^{2n}} \left\| f\left(m^{2n}[x,y]\right) - [f(m^n x), m^n y] - [m^n x, f(m^n y)] \right\|_{\mathcal{A}} \\ &\quad \leq \lim_{n\to\infty} \frac{1}{|m|^{2n}} \psi(m^n x, m^n y) = 0, \end{aligned} \tag{3.10}$$

for all $x,y \in \mathcal{A}$ and so

$$\delta([x,y]) = [\delta(x),y] + [x,\delta(y)] \tag{3.11}$$

for all $x,y \in \mathcal{A}$. Thus $\delta : \mathcal{A} \to \mathcal{A}$ is a Lie derivation satisfying (2.8). □

Acknowledgment

This paper was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 2011-0021821).

References

[1] M. Eshaghi Gordji and Z. Alizadeh, "Stability and superstability of ring homomorphisms on non-Archimedean Banach algebras," *Abstract and Applied Analysis*, Article ID 123656, 10 pages, 2011.
[2] N. Shilkret, *Non-Archimedian Banach algebras*, Ph.D. thesis, Polytechnic University, 1968.
[3] S. M. Ulam, *A Collection of Mathematical Problems*, Interscience Publishers, London, UK, 1960.
[4] D. O'Regan, J. M. Rassias, and R. Saadati, "Approximations of ternary Jordan homomorphisms and derivations in multi-C* ternary algebras," *Acta Mathematica Hungarica*, vol. 134, no. 1-2, pp. 99–114, 2012.

[5] J. M. Rassias, "On approximation of approximately linear mappings by linear mappings," *Journal of Functional Analysis*, vol. 46, no. 1, pp. 126–130, 1982.
[6] T. M. Rassias, "On the stability of the linear mapping in Banach spaces," *Proceedings of the American Mathematical Society*, vol. 72, no. 2, pp. 297–300, 1978.
[7] J. B. Diaz and B. Margolis, "A fixed point theorem of the alternative, for contractions on a generalized complete metric space," *Bulletin of the American Mathematical Society*, vol. 74, pp. 305–309, 1968.
[8] G. Z. Eskandani, "On the Hyers-Ulam-Rassias stability of an additive functional equation in quasi-Banach spaces," *Journal of Mathematical Analysis and Applications*, vol. 345, no. 1, pp. 405–409, 2008.
[9] L. Cădariu and V. Radu, "Fixed points and the stability of Jensen's functional equation," *Journal of Inequalities in Pure and Applied Mathematics*, vol. 4, no. 1, 2003.

Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
International Journal of Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization