Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2012, Article ID 429076, 12 pages
doi:10.1155/2012/429076

*Research Article*

# Dynamic Behaviors of a Discrete Two Species Predator-Prey System Incorporating Harvesting

**Ting Wu**

*Department of Mathematics, Minjiang University, Fujian, Fuzhou 350108, China*

Correspondence should be addressed to Ting Wu, wuting723@tom.com

Received 2 May 2012; Revised 19 August 2012; Accepted 29 August 2012

Academic Editor: Jinde Cao



A discrete two species predator-prey captured system is studied. Firstly, a sufficient condition of a positive equilibrium point for this system is obtained. Secondly, we observe that the two nonnegative equilibriums of the system are unstable through the eigenvalue discriminant method, and the positive equilibrium point is asymptotically stable by Jury criterion. Lastly, we obtain the optimal capture strategy of the system from the maximum principle by constructing a discrete Hamiltonian function. To show the feasibility of the main results, a suitable example together with its numerical simulations is illustrated in the last part of the paper. The example with certain practical significance might give an optimal scheme of the greatest economic benefits for the captors.

## 1. Introduction

With economic development, the rational development and management of the biological resources are directly related to its sustainable development. In this situation, more and more scholars considered the problems on ecological balance of biological systems. Also, the stability and permanence of biological systems are well studied recently [1–4]. According to the aim of people's capture, we consider both the economic interest and the permanence of predator-prey system in this paper. The aim of this paper is to make a research on some suitable control of the ecological system in order to obtain the existence and development of the system.

In earlier stage, papers mainly considered the maximum sustainable yield in the field of optimal capture in order to guarantee the maximum of the capture yield, and the biological resources will not lose their reproduction capacity eventually [5]. Recently, much more papers turn their attention to the optimal capture strategy [6–16]. Similarly, the optimal control theory is also a good way.

The predator-prey system is one of the typical ecological systems. For example, Leslie [6, 7] introduced the Leslie-Gower predator-prey systems as follows:

$$\frac{dH}{dt} = (r_1 - a_1 P - b_1 H)H, \qquad \frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P, \tag{1.1}$$

where $H$ and $P$ are the density of prey species and the predator species at time $t$, respectively; $r_1$ and $r_2$ are the intrinsic growth rate of prey and predator; $b_1$ is the density-dependent entry; $a_1 P$ is the number of $H$ eaten by $P$ per unit of time; $a_2 P/H$ represents the "carrying capacity" of the predator's environment which is proportional to the number of prey. The Leslie-Gower system admits a unique equilibrium, and Korobeinikov [8] showed that the positive equilibrium is globally asymptotic stable. Recently, Zhang et al. [9] assumed that the predator and prey in the model have commercial importance, and they are subjected to constant effort harvesting. Let $c_1$, $c_2$ denote the harvest of prey and predator, respectively. Zhang et al. formulated the system as follows:

$$\frac{dH}{dt} = (r_1 - a_1 P - b_1 H)H - c_1 H, \qquad \frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P - c_2 P. \tag{1.2}$$

In addition, they discuss the stability and the optimal harvesting strategy.

Although many scholars considered the economic interest of the captured amount of the continuous system, the distribution with the fish is inhomogeneous and it is not possible to capture successively. Therefore, it is more reasonable to consider the discrete system. The research on the discrete captured model makes a great significance to improve the quality of people's life. The continuous one model is of a continuous time of captured revenue [9]; we consider that the discrete model is divided into time segments of captured revenue. Therefore, it is more reasonable. On the one hand, during the exhausting of the fishing resources, people have to restrict the capture models into discrete ones to avoid the disappearance of those resources. On the other hand, we use the eigenvalue symbol of the coefficient matrix of the linear differential system $dx/dt = Ax$ to determine the stability of continuous models, while for the discrete models, the stability is determined by the eigenvalues of the coefficient matrix of the linear difference system $x(n+1) = Ax(n)$ by Lemma 2.2, Lemma 2.3 and Corollary 2.5. We raise the model as follows:

$$x_{n+1} - x_n = x_n(a - bx_n - cy_n) - h_1 x_n, \qquad y_{n+1} - y_n = y_n(-d + ex_n - fy_n) - h_2 y_n, \tag{1.3}$$

where $x_n$ and $y_n$ are the population densities of prey and predator at time $n$, respectively; $a$ denotes the intrinsic growth rate of prey and predator (or life factor); $d$ is the death rate of the predator $y_n$; $b$ and $f$ are the density-dependent entry; $cx_n y_n$ is the number of $x_n$ eaten by $y_n$ per unit of time; $e$ is the conversion rate ($0 < e < c$); $h_1$ and $h_2$ are the two parameters that measures the effort being spent by a harvesting agency (where $h_1 = q_1E_1$, $h_2 = q_2E_2$, $q_1$, and $q_2$ are the catch-ability coefficients of the prey and predator species, and $E_1$ and $E_2$ denote the effort devoted to the harvesting), also $a > h_1$, $e > d + f$. All the parameters are assumed to be positive.

In the following context, we will consider the existence of the positive equilibrium, and, by applying the stability theory of linear difference equation, we obtain the stability of the positive equilibrium. We also get a theorem of the trivial solution asymptotic stability for the second-order constant coefficient linear homogeneous difference equations according to Jury criterion. We also use the relation between inhomogeneous difference equations and the corresponding homogeneous difference equations to prove the sufficient conditions of the positive equilibrium stability. Finally, by constructing a discrete Lyapunov function, we obtain the global asymptotic stability of the positive equilibrium. In order to consider economic benefit, we construct a discrete Hamiltonian function and use the maximum principle to get the optimal capture strategy of the system.

## 2. Stability of the Equilibrium Point of System (1.3)

### 2.1. The Equilibrium Point of System (1.3)

By simple calculations, we get that the system (1.3) has three possible nonnegative equilibriums: $O(0,0)$, $P_0((a-h_1)/b, 0)$, $P_1(0, (-d-h_2)/f)$, and $P(x^*, y^*)$, where

$$x^* = fu + cv, \qquad y^* = eu - bv, \qquad u = \frac{(a-h_1)}{bf+ec}, \qquad v = \frac{(d+h_2)}{bf+ec}. \tag{2.1}$$

Since $d + h_2 > 0$, we have $P_1(0, (-d-h_2)/f)$. So, we will only need to study the stability of the equilibria $O$, $P_0$, and $P$ of system (1.3). On these conditions, we have the following theorem.

**Theorem 2.1.** *$P_0$ is a non-negative equilibrium point if and only if $(a-h_1)e > (d+h_2)b$, and $P(x^*, y^*)$ is a positive equilibrium point.*

### 2.2. Stability of the Positive Equilibrium and the Relevant Conclusions

Note that the nonlinear difference system,

$$x(n+1) = Ax(n) + f(x(n)), \tag{2.2}$$

can be determined by the stability of the linear difference system

$$x(n+1) = Ax(n). \tag{2.3}$$

The characteristic equation of the coefficient matrix of the linear difference equation system (2.3) is

$$P(\lambda) = |A - \lambda I| = \lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \cdots + a_{n-1}\lambda + a_n. \tag{2.4}$$

In view of (2.4), we construct the following Jury conditions table:

(1) $a_0 = 1, a_1, a_2, \ldots, a_{n-2}, a_{n-1}, a_n,$

(2) $a_n, a_{n-1}, a_{n-2}, \ldots, a_2, a_1, a_0 = 1,$

(3) $b_0, b_1, b_2, \ldots, b_{n-2}, b_{n-1},$

(4) $b_{n-1}, b_{n-2}, b_{n-3}, \ldots, b_1, b_0,$

(5) $c_0, c_1, c_2, \ldots, c_{n-2},$

(6) $c_{n-2}, c_{n-3}, c_{n-4}, \ldots, c_0,$

$\vdots \ldots\ldots\ldots\ldots\ldots\ldots$

$\vdots$

$(2n-3)$ $s_0, s_1, s_2,$

where

$$\begin{gathered} b_0 = \begin{vmatrix} a_0 & a_n \\ a_n & a_0 \end{vmatrix}, b_1 = \begin{vmatrix} a_0 & a_{n-1} \\ a_n & a_1 \end{vmatrix}, \ldots, b_{n-1} = \begin{vmatrix} a_0 & a_1 \\ a_n & a_{n-1} \end{vmatrix}, \\ c_0 = \begin{vmatrix} b_0 & b_{n-1} \\ b_{n-1} & b_0 \end{vmatrix}, c_1 = \begin{vmatrix} b_0 & b_{n-2} \\ b_{n-1} & b_1 \end{vmatrix}, \ldots, c_{n-2} = \begin{vmatrix} b_0 & b_1 \\ b_{n-1} & b_{n-2} \end{vmatrix}, \ldots, \end{gathered} \tag{2.5}$$

until there are only three elements in the same row.

**Lemma 2.2** (see [17], page 187, Theorem 5.1). *The zero solution of* (2.3) *is asymptotically stable if and only if eigenvalues modulus of the coefficient matrix $A$ is less than 1.*

**Lemma 2.3** (see [17], page 200, Theorem 6.2). *Setting* $\lim_{x\to 0}(f(x)/x) = 0(f(x(0)) \neq 0)$, *the zero solution of system* (2.2) *is asymptotically stable if $A = (a_{ij})_{n\times n}$ is stable; the zero solution of system* (2.2) *is unstable if* $r(A) > 1$.

**Theorem 2.4** (see [17], page 204, Jury conditions). *All the zero solutions of the polynomial $P(\lambda)$ are in the complex plane of the unit circle if and only if*

$$P(1) > 0, \quad (-1)^n P(-1) > 0, \quad |a_n| < 1, \quad |b_0| > |b_{n-1}|, \quad |c_0| > |c_{n-2}|, \ldots, |s_0| > |s_2|, \tag{2.6}$$

*where $b_i$, $c_j$, and $s_k$ are given by the Jury conditions table.*

To get the second-order constant coefficient linear homogeneous difference equation satisfying Jury criterion in partial if $n = 2$, we get the following criterion form.

**Corollary 2.5.** *Suppose $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$. Then the zero solution of* (2.3) *is asymptotically stable if and only if $(1 + ad - bc)^2 > (a + d)^2$, with $-1 < ad - bc < 1$.*

### 2.3. Local Stability Analysis of the Equilibria

**Theorem 2.6.** *Both of the equilibriums $O(0,0)$ and $P_0((a-h_1)/b,0)$ are unstable equilibrium points. If*

$$4+(fb+ce)(fu+cv)(eu-bv)>2b(fu+cv)+2f(eu-bv), \tag{2.7}$$

*with $0<b(fu+cv)+f(eu+bv)-(fb-ce)(fu+cv)(eu-bv)<2$, then $P(x^*,y^*)$ is a locally stability equilibrium point.*

*Proof.* (1) For $O(0,0)$, the corresponding linear difference equation is

$$\begin{aligned} x_{n+1} &= x_n(1+a-h_1), \\ y_{n+1} &= y_n(1-d-h_2), \end{aligned} \tag{2.8}$$

whose characteristic equation is $(1+a-h_1-\lambda)(1-d-h_2-\lambda)=0$. Thus the eigenvalues are $\lambda_1=1+a-h_1>1$, $\lambda_2=1-d-h_2$. By Lemma 2.2, it follows that $O(0,0)$ is an unstable equilibrium point.

(2) For the non-negative equilibrium point $P_0((a-h_1)/b,0)$, we make translation transformations,

$$\begin{aligned} u_n &= x_n-\frac{a-h_1}{b}, \\ v_n &= y_n. \end{aligned} \tag{2.9}$$

Substituting it into (1.3), getting that $(u_n,v_n)$ charged by $(x_n,y_n)$, and linearizing, we get

$$\begin{aligned} x_{n+1} &= [1-(a-h_1)]x_n-\frac{(a-h_1)c}{b}y_n, \\ y_{n+1} &= \left[1-d-h_2+(a-h_1)\frac{e}{b}\right]y_n. \end{aligned} \tag{2.10}$$

If the variational matrix of the system (1.3) is

$$\begin{pmatrix} 1-(a-h_1)-\lambda & -\dfrac{(a-h_1)c}{b} \\ 0 & (1-d-h_2)+(a-h_1)\dfrac{e}{b}-\lambda \end{pmatrix}, \tag{2.11}$$

then $R(A)=2>1$. By Lemma 2.3, it implies that $P_0((a-h_1)/b,0)$ is an unstable equilibrium point.

(3) For the positive equilibrium point $P(x^*,y^*)$, we make translation transformations,

$$\begin{aligned} u_n &= x_n-x^*, \\ v_n &= y_n-y^*. \end{aligned} \tag{2.12}$$

Substituting it into (1.3), getting that $(u_n, v_n)$ charged by $(x_n, y_n)$, and linearizing, we get

$$\begin{aligned} x_{n+1} - x_n &= \left(1 + a - 2bx^* - cy^* - h_1\right)x_n - cx^* y_n, \\ y_{n+1} - y_n &= \left(1 - d + ex^* - 2fy^* - h_2\right)y_n + ey^* x_n. \end{aligned} \tag{2.13}$$

Suppose that $a_1 = 1 + a - 2bx^* - cy^* - h_1$, $b_1 = -cx^*$, $c_1 = ey^*$, and $d_1 = 1 - d + ex^* - 2fy^* - h_2$. By Corollary of Theorem 2.4, it implies that if $(1 + a_1 d_1 - b_1 c_1)^2 > (a_1 + d_1)^2$, $-1 < a_1 d_1 - b_1 c_1 < 1$, then

$$4 + \left(fb + ce\right)\left(fu + cv\right)(eu - bv) > 2b\left(fu + cv\right) + 2f(eu - bv), \tag{2.14}$$

also $0 < b(fu + cv) + f(eu + bv) - (fb - ce)(fu + cv)(eu - bv) < 2$. By Corollary 2.5, it implies that if $(1 + a_1 d_1 - b_1 c_1)^2 > (a_1 + d_1)^2$, $-1 < a_1 d_1 - b_1 c_1 < 1$, $P(x^*, y^*)$ is a locally stability equilibrium point. □

## *2.4. Global Stability*

**Theorem 2.7.** *Under the conditions of Theorem 2.6, if there are positive numbers $\delta$ and $n_i (i = 1, 2)$ satisfying the following two inequalities,*

(i) $(2bn_1 - en_2)(fu + cv) + cn_1(eu - bv) - n_1(a - h_1) > \delta$,

(ii) $(2fn_2 - cn_1)(eu - bv) + n_2(d + h_1) - en_2(fu + cv) > \delta$,

*then the positive equilibrium point $P(x^*, y^*)$ of system* (1.3) *is globally stable.*

*Proof.* We make translation transformations,

$$\begin{aligned} u_n &= x_n - x^*, \\ v_n &= y_n - y^*. \end{aligned} \tag{2.15}$$

Substituting it into (1.3), getting that $(u_n, v_n)$ charged by $(x_n, y_n)$, we have

$$\begin{aligned} x_{n+1} - x_n &= (x_n + x^*)\left[a - b(x_n + x^*) - c\left(y_n + y^*\right) - h_1\right], \\ y_{n+1} - y_n &= \left(y_n + y^*\right)\left[-d + e(x_n + x^*) - f\left(y_n + y^*\right) - h_2\right], \end{aligned} \tag{2.16}$$

where $O(0,0)$ is an equilibrium point of (2.16). Make Taylor expanding the right side of (2.16) on the equilibrium point $O(0,0)$, we have

$$\begin{aligned} x_{n+1} &= \left(1 + a - 2bx^* - cy^* - h_1\right)x_n - cx^* y_n + g_1\left(n, x_n, y_n\right), \\ y_{n+1} &= \left(1 - d + ex^* - 2fy^* - h_2\right)y_n + ey^* x_n + g_2\left(n, x_n, y_n\right), \end{aligned} \tag{2.17}$$

where $X_n = (x_n, y_n)$ and $\|X_n\| = |x_n| + |y_n|$. If $\|X_n\| \longrightarrow 0$, then

$$\frac{|g_i(n, x_n, y_n)|}{\|X_n\|} \longrightarrow 0. \tag{2.18}$$

For every $n \in N$, both of them are consistent ($i = 1, 2$.) Then (2.17) can be rewritten as follows:

$$\begin{aligned} x_{n+1} - x_n &= x^* \left[ (a - 2bk_1 - ck_2 - h_1)\frac{x_n}{x^*} - ck_2\frac{y_n}{y^*} + \frac{g_1(n, x_n, y_n)}{x^*} \right], \\ y_{n+1} - y_n &= y^* \left[ (-d + ek_1 - 2fk_2 - h_2)\frac{y_n}{y^*} + ek_1\frac{x_n}{x^*} + \frac{g_2(n, x_n, y_n)}{y^*} \right], \end{aligned} \tag{2.19}$$

where $k_1 = fu + cv$, $k_2 = eu - bv$.

We have the following Lyapunov function:

$$V(x_n, y_n) = n_1\left|\frac{x_n}{x^*}\right| + n_2\left|\frac{y_n}{y^*}\right|. \tag{2.20}$$

By condition (i), (ii), and (2.19), we get the difference of Lyapunov function as follows:

$$\begin{aligned} \Delta V(x_n, y_n) \leq{} & n_1[a - 2bk_1 - ck_2 - h_1]\left|\frac{x_n}{x^*}\right| \\ & + n_1ck_2\left|\frac{y_n}{y^*}\right| + n_1\left|\frac{g_1(n, x_n, y_n)}{x^*}\right| \\ & + n_2[-d + ek_1 - 2fk_2 - h_2]\left|\frac{y_n}{y^*}\right| \\ & + n_2ek_1\left|\frac{x_n}{x^*}\right| + n_2\left|\frac{g_2(n, x_n, y_n)}{y^*}\right| \\ ={} & -[(2bn_1 - en_2)k_1 + cn_1k_2 - n_1(a - h_1)]\left|\frac{x_n}{x^*}\right| \\ & - [(2fn_2 - cn_1)k_2 + n_2(d + h_2) - en_2k_1]\left|\frac{y_n}{y^*}\right| \\ & + n_1\left|\frac{g_1(n, x_n, y_n)}{x^*}\right| + n_2\left|\frac{g_2(n, x_n, y_n)}{y^*}\right|. \end{aligned} \tag{2.21}$$

As if $\|X_n\| \rightarrow 0$, then $|g_i(n, x_n, y_n)|/\|X_n\| \rightarrow 0$ ($i = 1, 2$). If $n$ is great enough, then there exists a positive $\delta$ such that $\Delta V \leq -\delta\|X_n\|/2$. So, if the interior equilibrium $O(0, 0)$ of system (2.16) is globally stable, then the interior equilibrium $P(x^*, y^*)$ of system (1.3) is also globally stable. □

## 3. Harvesting the Optimal Economic Benefit

The fishermen or the fishing companies must consider the cost effectiveness when harvesting all kinds of fish. It is necessary to consider not only the sale price, but also the injecting funds capture. If the largest capture intensity is $h_m$, then $0 < h_1 + h_2 = h \le h_m$. Suppose the cost is $c_1$, $c_2$, and suppose the prices of the two kinds of group are $p_1$, $p_2$. To obtain the optimal capture, we need to seek for the best efforts of the degrees $h_1^*$, $h_2^*$. Since the optimal balance point is $P(x^*, y^*)$, the goal function is given by

$$L = \sum_{n=1}^{\infty} \alpha^{n-1} \left[ (p_1 x_n - c_1) h_1 + (p_2 y_n - c_2) h_2 \right]. \tag{3.1}$$

According to the discrete maximum principle, to seek optimal control $h_1$, $h_2$, we need the following Hamilton function:

$$\begin{aligned} H_n = {} & \alpha^{n-1} \left[ (p_1 x_n - c_1) h_1 + (p_2 y_n - c_2) h_2 \right] \\ & + \lambda_{1,n} (a - b x_n - c y_n - h_1) x_n + \lambda_{2,n} (-d + e x_n - f y_n - h_2) y_n, \end{aligned} \tag{3.2}$$

where $\alpha = 1/(1+i)$; $i$ is the instantaneous discount rate for periods; $\lambda_{1,n}$, $\lambda_{2,n}$ are with ariables; $h_1$, $h_2$ get maximum value $H_n$, respectively. Consider the following equations:

$$\Delta\lambda_{1,n} = \lambda_{1,n} - \lambda_{1,n-1} = -\frac{\partial H}{\partial x_n} = \alpha^{n-1} p_1 h_1 + b x_n \lambda_{1,n} - e y_n \lambda_{2,n}, \tag{3.3}$$

$$\Delta\lambda_{2,n} = \lambda_{2,n} - \lambda_{2,n-1} = -\frac{\partial H}{\partial y_n} = \alpha^{n-1} p_2 h_2 + c x_n \lambda_{1,n} + f y_n \lambda_{2,n}. \tag{3.4}$$

By (3.3) $\times$ $f$ $-$ (3.4) $\times$ $e$, we get

$$e\Delta\lambda_{2,n} = -\alpha^{n-1} (f p_1 h_1 + e p_2 h_2) + (bf + ce) x_n \lambda_{1,n} - f \Delta\lambda_{1,n}. \tag{3.5}$$

By (3.3) $\times$ $c$ $-$ (3.4) $\times$ $b$, we get

$$c\Delta\lambda_{1,n} = -\alpha^{n-1} (b p_2 h_2 - c p_1 h_1) - (bf + ce) y_n \lambda_{2,n} + b\Delta\lambda_{2,n}. \tag{3.6}$$

So,

$$\begin{aligned} \Delta^2\lambda_{1,n} &= \Delta\lambda_{1,n} - \Delta\lambda_{1,n-1} = \lambda_{1,n} - 2\lambda_{1,n-1} + \lambda_{1,n-2} \\ &= -\Delta\left(\alpha^{n-1} p_1 h_1\right) + b x_n \Delta\lambda_{1,n} - e y_n \Delta\lambda_{2,n}, \\ \Delta^2\lambda_{2,n} &= \Delta\lambda_{2,n} - \Delta\lambda_{2,n-1} = \lambda_{2,n} - 2\lambda_{2,n-1} + \lambda_{2,n-2} \\ &= -\Delta\left(\alpha^{n-1} p_2 h_2\right) + c x_n \Delta\lambda_{1,n} + f y_n \Delta\lambda_{2,n}. \end{aligned} \tag{3.7}$$

That is,

$$\begin{aligned}
&\left[1-(bx_n+fy_n)+(bf+ce)x_ny_n\right]\lambda_{1,n}+(bx_n+fy_n-2)\lambda_{1,n-1}+\lambda_{1,n-2}=\alpha^{n-2}\phi_1,\\
&\left[1-(bx_n+fy_n)+(bf+ce)x_ny_n\right]\lambda_{2,n}+(bx_n+fy_n-2)\lambda_{2,n-1}+\lambda_{2,n-2}=\alpha^{n-2}\phi_2,
\end{aligned}\tag{3.8}$$

where $\phi_1=(1-\alpha+\alpha f)p_1h_1+\alpha ep_2h_2$, $\phi_2=(\alpha b+\alpha-1)p_2h_2-\alpha cp_1h_1$.

Substitute $n-2$ by $n$ in the following two equations, we have

$$\begin{aligned}
&\left[1-(bx_n+fy_n)+(bf+ce)x_ny_n\right]\lambda_{1,n+2}+(bx_n+fy_n-2)\lambda_{1,n+1}+\lambda_{1,n}=\alpha^{n}\phi_1,\\
&\left[1-(bx_n+fy_n)+(bf+ce)x_ny_n\right]\lambda_{2,n+2}+(bx_n+fy_n-2)\lambda_{2,n+1}+\lambda_{2,n}=\alpha^{n}\phi_2.
\end{aligned}\tag{3.9}$$

Suppose $\phi=\alpha^2[1-bx_n-fy_n+(bf+ce)x_ny_n]+\alpha(bx_n+fy_n-2)+1$, that is,

$$\phi=\alpha^2(bf+ce)x_ny_n+\alpha(1-\alpha)(bx_n+fy_n)+(1-\alpha)^2.\tag{3.10}$$

If $b^2x_n^2+f^2y_n^2-4cex_ny_n>0$, then

$$\begin{aligned}
\lambda_{1,n}&=\frac{\alpha^n\phi_1}{\phi},\\
\lambda_{2,n}&=\frac{\alpha^n\phi_2}{\phi}.
\end{aligned}\tag{3.11}$$

By $\partial H/\partial h_1=0$, $\partial H/\partial h_2=0$, it implies

$$\lambda_{1,n}=\frac{\alpha^{n-1}(p_1x_n-c_1)}{x_n},\qquad \lambda_{2,n}=\frac{\alpha^{n-1}(p_2y_n-c_2)}{y_n}.\tag{3.12}$$

Substitute (3.11) into (3.12), we have

$$(p_1x_n-c_1)\phi=\alpha\phi_1x_n,\qquad (p_2y_n-c_2)=\alpha\phi_2y_n,\tag{3.13}$$

that is,

$$\begin{aligned}
&(p_1x_n-c_1)\left[\alpha^2(bf+ce)x_ny_n+\alpha(1-\alpha)(bx_n+fy_n)+(1-\alpha)^2\right]\\
&\quad=\alpha(1-\alpha)p_1h_1x_n+\alpha^2(ep_2h_2+fp_1h_1)x_ny_n,
\end{aligned}\tag{3.14}$$

$$\begin{aligned}
&(p_2y_n-c_2)\left[\alpha^2(bf+ce)x_ny_n+\alpha(1-\alpha)(bx_n+fy_n)+(1-\alpha)^2\right]\\
&\quad=\alpha(1-\alpha)p_2h_2y_n+\alpha^2(bp_2h_2-cp_1h_1)x_ny_n.
\end{aligned}\tag{3.15}$$

From (3.12) and (3.14), it follows that the $P_\alpha^*(x_\alpha, y_\alpha)$ is the optimal equilibrium solution, whose best efforts of degrees are

$$h_{1,\alpha} = a - bx_\alpha - cy_\alpha, \qquad h_{2,\alpha} = -d + ex - \alpha - fy_\alpha, \tag{3.16}$$

which is also the optimal equilibrium program. Thus the economic profits of the captured populations are completely determined by the discount rates $\alpha$, $c_i$, $p_i$ $(i = 1, 2)$.

## 4. Number Simulations

In this section, we consider some numerical simulations examples.

*Example 4.1.* Let $a = 1.6$, $b = 0.5$, $c = 0.2$, $d = 0.01$, $e = 0.2$, $f = 0.2$, $h_1 = 0.6$, and $h_2 = 0.35$ into the system (1.3), then

$$\begin{aligned} x_{n+1} &= x_n\left(1.6 - 0.5x_n - 0.5y_n\right), \\ y_{n+1} &= y_n\left(-0.35 + 0.2x_n - 0.2y_n\right). \end{aligned} \tag{4.1}$$

Through calculating, we have

$$\begin{gathered} 4 + (fb + ce)(fu + cv)(eu - bv) = 4.1056, \qquad 2b(fu + cv) + 2f(eu - bv) = 1.755 < 4.1056, \\ 0 < b(fu + cv) + f(eu + bv) - (fb - ce)(fu + cv)(eu - bv) = 0.7719 < 2. \end{gathered} \tag{4.2}$$

Let $n_1 = 10$, $n_2 = 5$, $\delta = 0.8$, we have

$$\begin{aligned} (2bn_1 - en_2)(fu + cv) + cn_1(eu - bv) - n_1(a - h_1) = 11.31 > 0.8, \\ (2fn_2 - cn_1)(eu - bv) + n_2(d + h_1) - en_2(fu + cv) = 0.875 > 0.8. \end{aligned} \tag{4.3}$$

In this case, the positive equilibrium point is $P(1.625, 0.325)$. And the system (1.3) is globally asymptotically stable (Figure 1).

*Example 4.2.* Let $a = 1.6$, $b = 0.5$, $c = 0.2$, $d = 0.01$, $e = 0.2$, $f = 0.2$, $p_1 = 0.5$, $p_2 = 0.5$, $c_1 = 0.2$, $c_2 = 0.2$, and $\alpha = 0.5$ into (3.14) and (3.15), we have

$$\begin{aligned} 10x_n^2y_n + 25x_n^2 - 3.4x_ny_n + 10x_n - 4y_n - 20 = 0, \\ 10x_ny_n^2 + 10y_n^2 + 25.5x_ny_n - 10x_n + 29y_n - 20 = 0. \end{aligned} \tag{4.4}$$

Solve (3.14) and (3.15) by Maple, we get only one optimal equilibrium point:
$P_\alpha(0.72883022165025259637411853051416, 0.72883022165025259637411853051416)$,
meeting the condition $x_\alpha < c_1/p_1 = 0.4$, $y_\alpha < c_2/p_2 = 0.4$, $b^2x_n^2 + f^2y_n^2 - 4cex_ny_n > 0$.

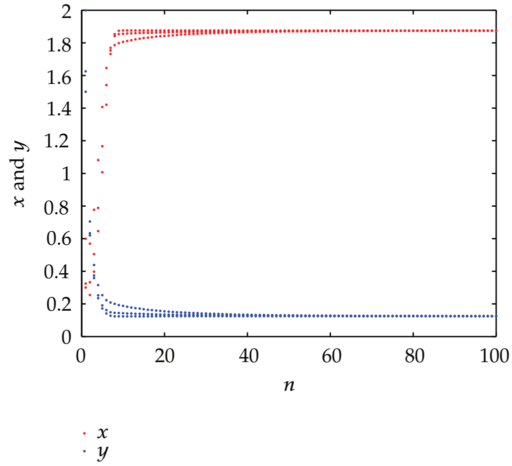


**Figure 1:** Solution curves of system (4.1) with initial conditions $(1.625, 1.325)$, $(1.5, 0.6)$, and $(2.0, 0.3)$.

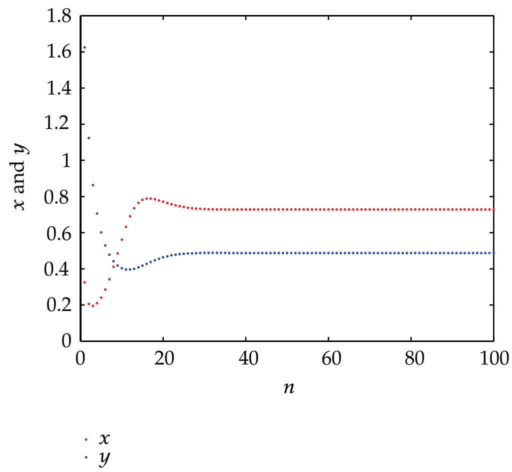


**Figure 2:** System (3.2) in the optimal balance point $P_\alpha(0.7288302216502525963741185305141 6, 0.72883022165025259637411853051416)$. There is an optimal degree of capture effort $(0.99197566015688, 0.03832235272285)$.

In the above values of parameters, we found that the optimal equilibrium point $P_\alpha^*(x_\alpha, y_\alpha)$ exists, and the corresponding optimal harvesting efforts are $h_{1,\alpha} = 0.99197566015688$ and $h_{2,\alpha} = 0.03832235272285$ (Figure 2).

## 5. Epilogue

The paper studies optimal capture problems of the predator-prey system. Firstly, we consider the existence and the stability of the positive equilibrium of system, and, by the maximum principle of a discrete model and Hamilton function, we obtain the optimal capture strategy that is under the condition $b^2x_n^2 + f^2y_n^2 - 4cex_ny_n > 0$. Finally, by applying numerical simulations, we show that system (1.3) is a globally stable positive equilibrium and an optimal harvesting policy. At last, we notice that there are corresponding results if the conditions switch to $b^2x_n^2 + f^2y_n^2 - 4cex_ny_n = 0$ or $b^2x_n^2 + f^2y_n^2 - 4cex_ny_n < 0$, respectivelty.

## Acknowledgment

This work is supported by the Technology Innovation Platform Project of Fujian Province (2009J1007).

## References

[1] F. D. Chen, "Persistence and global stability for nonautonomous cooperative system with diffusion and time delay," *Acta Scientiarum Naturalium Universitatis Pekinensis*, vol. 39, no. 1, pp. 22–28, 2003.

[2] T. Wu, "Permanence for nonautonomous Lotka-Volterra tow species cooperatative systems," *Bulletin of Science and Technology*, vol. 25, no. 6, pp. 743–746, 2009 (Chinese).

[3] T. Wu, "Permanence and global stability of a discrete competition feedback-control system with Beddington-DeAngelis functional response," *Journal of Minjiang University (Natural Science Edition)*, vol. 31, no. 2, pp. 16–20, 2010 (Chinese).

[4] T. Wu, "Permanence of discrete predator-prey system with infinite delay Beddington-DeAngelis functional response," *Journal of Minjiang University (Natural Science Edition)*, vol. 31, no. 5, pp. 1–5, 2010 (Chinese).

[5] C. W. Clark, *Mathematical Bioeconomics: The Optimal Management of Renewable Resources*, John Wiley & Sons, New York, NY, USA, 2nd edition, 1990.

[6] P. H. Leslie, "Some further notes on the use of matrices in population mathematics," *Biometrika*, vol. 35, pp. 213–245, 1948.

[7] P. H. Leslie, "A stochastic model for studying the properties of certain biological systems by numerical methods," *Biometrika*, vol. 45, pp. 16–31, 1958.

[8] A. Korobeinikov, "A Lyapunov function for Leslie-Gower predator-prey models," *Applied Mathematics Letters*, vol. 14, no. 6, pp. 697–699, 2001.

[9] N. Zhang, F. D. Chen, Q. Q. Su, and T. Wu, "Dynamic behaviors of a harvesting Leslie-Gower predator-prey model," *Discrete Dynamics in Nature and Society*, vol. 655, no. 10, pp. 1–10, 2011.

[10] D. Hu and Z. Zhang, "Four positive periodic solutions of a discrete time delayed predator-prey system with nonmonotonic functional response and harvesting," *Computers & Mathematics with Applications*, vol. 56, no. 12, pp. 3015–3022, 2008.

[11] K. S. Chaudhuri and S. S. Ray, "On the combined harvesting of a prey-predator system," *Journal of Biological Systems*, vol. 4, no. 3, pp. 373–389, 1996.

[12] K. S. Chaudhuri, "A bioeconomic model of harvesting amultispecies fishery," *Ecological Modelling*, vol. 32, no. 4, pp. 267–279, 1986.

[13] K. S. Chaudhuri, "Dynamic optimization of combined harvesting of a two-species fishery," *Ecological Modelling*, vol. 41, no. 1-2, pp. 17–25, 1988.

[14] T. K. Kar, "Modelling and analysis of a harvested prey-predator system incorporating a prey refuge," *Journal of Computational and Applied Mathematics*, vol. 185, no. 1, pp. 19–33, 2006.

[15] M. Fan and K. Wang, "Optimal harvesting policy for single population with periodic coefficients," *Mathematical Biosciences*, vol. 152, no. 2, pp. 165–177, 1998.

[16] C. W. Clark, *Bioeconomic Modeling and Fisheries Management*, A Wiley-Interscience Publication, John Wiley & Sons, New York, NY, USA, 1985.

[17] L. Wang and M. Q. Wang, *Ordinary Difference Equation*, Xinjiang University Press, Xinjiang, China, 1991.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
Algebra
Journal of Probability and Statistics
The Scientific World Journal
Hindawi
Submit your manuscripts at http://www.hindawi.com
International Journal of Differential Equations
International Journal of Combinatorics
Advances in Mathematical Physics
Journal of Complex Analysis
Journal of Mathematics
Mathematical Problems in Engineering
Abstract and Applied Analysis
Discrete Dynamics in Nature and Society
International Journal of Mathematics and Mathematical Sciences
Journal of Discrete Mathematics
Journal of Function Spaces
International Journal of Stochastic Analysis
Journal of Optimization