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Research Article

Two Positive Periodic Solutions for a Neutral Delay Model of Single-Species Population Growth with Harvesting

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By coincidence degree theory for k-set-contractive mapping, this paper establishes a new criterion for the existence of at least two positive periodic solutions for a neutral delay model of single-species population growth with harvesting. An example is given to illustrate the effectiveness of the result.

1. Introduction

In 1993, Kuang [1] proposed the following open problem (Open Problem 9.2): obtain sufficient conditions for the existence of positive periodic solutions for

$$N'(t) = N(t)\left[a(t) - \beta(t)N(t) - b(t)N(t-\tau(t)) - c(t)N'(t-\tau(t))\right], \tag{1.1}$$

where all parameters are nonnegative continuous T-periodic functions. Fang and Li [2] gave an answer to the above open problem. In recent years, many papers have been published on the existence of multiple positive periodic solutions for some population systems with periodic harvesting terms by using Mawhin's coincidence degree theory (see, e.g., [3–7]). However, to our knowledge, few papers deal with the existence of multiple positive periodic solutions for neutral delay population models with harvesting. The main difficulty is that Mawhin's coincidence degree theory is generally not available to neutral delay models. Moreover, it is also hard to obtain a *priori* bounds on solutions for neutral delay models.

In this paper, we consider the following neutral delay model of single-species population growth with harvesting

$$N'(t) = N(t)\left[a(t) - \beta(t)N(t) - b(t)N(t-\tau(t)) - c(t)N'(t-\tau(t))\right] - h(t), \tag{1.2}$$

where $a(t), \beta(t), b(t), \tau(t), c(t)$, and $h(t)$ are nonnegative continuous T-periodic functions, and $h(t)$ denotes the harvesting rate.

The purpose of this paper is to establish the existence of at least two positive periodic solutions for neutral delay model (1.2). To show the existence of solutions to the considered problems, we will use the coincidence degree theory for k-set contractions [8–10] and a *priori* bounds on solutions.

2. Preliminaries

We now briefly state the part of coincidence degree theory for k-set-contractive mapping (see [8–10]).

Let Z be a Banach space. For a bounded subset $A \subset Z$, let $\Gamma_Z(A)$ denote the (Kuratowski) measure of noncompactness defined by

$$\Gamma_Z(A) = \inf\left\{\delta > 0 : \exists \text{ a finite number of subsets } A_i \subset A, A = \bigcup_i A_i, \operatorname{diam}(A_i) \leq \delta\right\}. \tag{2.1}$$

Here, diam (A_i) denotes the maximum distance between the points in the set A_i.

Let X and Z be Banach spaces with norms $\|\cdot\|_x$ and $\|\cdot\|_z$ respectively, and Ω a bounded open subset of $X\cdot$ A continuous and bounded mapping $N : \overline{\Omega} \to Z$ is called k-set-contractive if, for any bounded $A \subset \overline{\Omega}$, we have

$$\Gamma_Z(N(A)) \leq k\Gamma_X(A). \tag{2.2}$$

Also, for a continuous and bounded map $T : X \to Y$, we define

$$l(T) = \sup\{r \geq 0 : \forall \text{ bounded subset } A \subset \mathrm{X}, r\Gamma_X(A) \leq \Gamma_Y(T(A))\}. \tag{2.3}$$

Let $L : \operatorname{dom} L \subset X \to Z$ be a linear mapping and $N : X \to Z$ a continuous mapping. The mapping L will be called a Fredholm mapping of index zero if dim Ker$L$ = codim$ImL < +\infty$ and ImL is closed in Z. If L is a Fredholm mapping of index zero, then there exist continuous projectors $P : X \to X$ and $Q : Z \to Z$ such that $ImP = \operatorname{Ker} L, ImL = \operatorname{Ker} Q = Im(I - Q)$. If we define $L_P : \operatorname{dom} L \cap \operatorname{Ker} P \to ImL$ as the restriction $L|_{\operatorname{dom} L \cap \operatorname{Ker} P}$ of L to $\operatorname{dom} L \cap \operatorname{Ker} P$, then L_P is invertible. We denote the inverse of that map by K_P. If Ω is an open bounded subset of X, the mapping N will be called L-k-set-contractive on $\overline{\Omega}$ if $QN(\overline{\Omega})$ is bounded and $K_P(I - Q)N : \overline{\Omega} \to X$ is k-set contractive. Since ImQ is isomorphic to $\operatorname{Ker} L$, there exists isomorphism $J : ImQ \to \operatorname{Ker} L$.

Lemma 2.1 (see[8], Proposition XI.2.). *Let L be a closed Fredholm mapping of index zero, and let $N : \overline{\Omega} \to Z$ be k'-set contractive with*

$$0 \leq k' < l(L). \tag{2.4}$$

Then $N : \overline{\Omega} \to Z$ is a L-k-set contraction with constant $k = k'/l(L) < 1$.

The following lemma (see [8], page 213) will play a key role in this paper.

Lemma 2.2. *Let L be a Fredholm mapping of index zero, and let $N : \overline{\Omega} \to Z$ be L-k-set contractive on $\overline{\Omega}, k < 1$. Suppose that*

(i) *$Lx \neq \lambda Nx$ for every $x \in \operatorname{dom} L \cap \partial\Omega$ and every $\lambda \in (0,1)$;*

(ii) *$QNx \neq 0$ for every $x \in \partial\Omega \cap \operatorname{Ker} L$;*

(iii) *Brouwer degree $\deg_B(JQN, \Omega \cap \operatorname{Ker} L, 0) \neq 0$.*

Then $Lx = Nx$ has at least one solution in $\operatorname{dom} L \cap \overline{\Omega}$.

3. Main Result

Let C_T^0 denote the linear space of real-valued continuous T-periodic functions on R. The linear space C_T^0 is a Banach space with the usual norm for $x \in C_T^0$ given by $|x|_0 = \max_{t\in R}|x(t)|$. Let C_T^1 denote the linear space of T-periodic functions with the first-order continuous derivative. C_T^1 is a Banach space with norm $|x|_1 = \max\{|x|_0, |x'|_0\}$.

Let $X = C_T^1$ and $Y = C_T^0$, and let $L : X \to Y$ be given by $Lx = dx/dt$. Since $|Lx|_0 = |x'|_0 \leq |x|_1$, we see that L is a bounded (with bound = 1) linear map.

Since we are concerning with positive solutions of (1.2), we make the change of variables as follows:

$$N(t) = e^{x(t)}. \tag{3.1}$$

Then (1.2) is rewritten as

$$x'(t) = a(t) - \beta(t)e^{x(t)} - b(t)e^{x(t-\tau(t))} - c(t)x'(t-\tau(t))e^{x(t-\tau(t))} - \frac{h(t)}{e^{x(t)}}. \tag{3.2}$$

Next define a nonlinear map $N : X \to Y$ by

$$N(x)(t) = a(t) - \beta(t)e^{x(t)} - b(t)e^{x(t-\tau(t))} - c(t)x'(t-\tau(t))e^{x(t-\tau(t))} - \frac{h(t)}{e^{x(t)}}. \tag{3.3}$$

Now, if $Lx = Nx$ for some $x \in$ X, then the problem (3.2) has a periodic solution $x(t)$.

In the following, we denote that

$$\overline{g} = \frac{1}{T}\int_0^T g(t)dt, \qquad g^l = \min_{t\in[0,T]} g(t), \qquad g^u = \max_{t\in[0,T]} g(t), \tag{3.4}$$

where $g(t)$ is a continuous nonnegative T-periodic solution.

From now on, we always assume that

(H_1) $a(t) \in C(R,(0,+\infty)), \beta(t), b(t) \in C(R,R^+), c(t), \tau(t) \in C^1(R,R^+), \tau' < 1$;

(H_2) $c_0'(t) < b(t)$, where $c_0(t) = c(t)/(1-\tau'(t))$;

(H_3) $a^l > c^u M_0 + 2\sqrt{[\beta^u + b^u]h^u}$, $c^u e^{R_1} < 1$, where

$$M_0 = \frac{a^u e^{R_1} + (\beta^u + b^u)e^{2R_1} + h^u}{1 - c^u e^{R_1}},$$
$$R_1 = \ln\frac{\overline{a}}{\beta^l} + \frac{c_0^u \overline{a}}{(b - c_0')^l} + 2\overline{a}T. \tag{3.5}$$

For further convenience, we introduce 6 positive numbers as below

$$l^{\pm} = \frac{a^u + c^u M_0 \pm \sqrt{[a^u + c^u M_0]^2 - 4\beta^l h^l}}{2\beta^l},$$
$$x^{\pm} = \frac{\overline{a} \pm \sqrt{(\overline{a})^2 - 4\left[\overline{\beta} + \overline{b}\right]\overline{h}}}{2\left[\overline{\beta} + \overline{b}\right]}, \tag{3.6}$$
$$u^{\pm} = \frac{a^l - c^u M_0 \pm \sqrt{\left[a^l - c^u M_0\right]^2 - 4\left[\beta^u + b^u\right]h^u}}{2\left[\beta^u + b^u\right]}.$$

Set the following:

$$g^-(x,y,z) = \frac{y - \sqrt{y^2 - 4xz}}{2x} = \frac{2z}{y + \sqrt{y^2 - 4xz}} \quad (x > 0,\ y > 0,\ z > 0),$$
$$g^+(x,y,z) = \frac{y + \sqrt{y^2 - 4xz}}{2x} \quad (x > 0,\ y > 0,\ z > 0), \tag{3.7}$$

where $y^2 > 4xz$.

By the monotonicity of the functions $g^-(x,y,z), g^+(x,y,z)$ on x, y, z, it is not difficult to see that

$$l^- < x^- < u^- < u^+ < x^+ < l^+. \tag{3.8}$$

Theorem 3.1. *In addition to* (H_1), (H_2), (H_3), *assume further that the following condition holds:*

(H_4) $k^* = c^u \max\{e^{R_1}, l^+\} < 1$.

Then (1.2) *has at least two positive T-periodic solutions.*

Before proving Theorem 3.1, we need the following lemmas.

Lemma 3.2 (see [11]). *L is a Fredholm map of index* 0 *and satisfies*

$$l(L) \geq 1. \tag{3.9}$$

Lemma 3.3. *Under the assumptions of Theorem 3.1, let*

$$\Omega = \left\{ x \in X \;\middle|\; \begin{array}{l} \max\limits_{t\in[0,T]} x(t) \in \left(\ln(l^- - \delta),\ \ln\left(\max\left\{e^{R_1}, l^+\right\} + \delta\right)\right), \\ \min\limits_{t\in[0,T]} x(t) \in \left(\ln(l^- - \delta),\ \ln\left(\max\left\{e^{R_1}, l^+\right\} + \delta\right)\right), \\ \max\limits_{t\in[0,T]} |x'(t)| < M_1. \end{array} \right\}, \tag{3.10}$$

where

$$M_1 = \frac{a^u + \left(\beta^u + b^u\right)e^{R_1} + \left(h^u/l^-\right)}{1 - c^u e^{R_1}}, \tag{3.11}$$

and $0 < \delta < l^-$ *such that*

$$k_0 = c^u\left[\max\left\{e^{R_1}, l^+\right\} + \delta\right] < 1. \tag{3.12}$$

Then $N : \overline{\Omega} \to Y$ *is a* k_0*-set-contractive map.*

Proof. The proof is similar to that of lemma 3.3 in [9], but for the sake of completeness we give the proof here. Let $A \subset \overline{\Omega}$ be a bounded subset and let $\eta = \Gamma_X(A)$. Then for any $\varepsilon > 0$, there is a finite family of subsets $\{A_i\}$ with $A = \bigcup_i A_i$ and $\operatorname{diam}_1(A_i) \leq \eta + \varepsilon$.

Set the following:

$$g(t, x, x_1, x_2) = a(t) - \beta(t)e^x - b(t)e^{x_1} - c(t)x_2 e^{x_1} - \frac{h(t)}{e^x}. \tag{3.13}$$

Now it follows from the fact that $g(t, x, x_1, x_2)$ is uniformly continuous on any compact subset of $R \times R^3$, and from the fact A and A_i are precompact in C_T^0 with norm $|\cdot|_0$, that there is a finite family of subsets $\{A_{ij}\}$ of A_i such that $A_i = \bigcup_j A_{ij}$ with

$$\left|g\left(t, x(t), x(t-\tau(t)), u'(t-\tau(t))\right) - g\left(t, u(t), u(t-\tau(t)), u'(t-\tau(t))\right)\right| < \varepsilon \tag{3.14}$$

for any $x, u \in A_{ij}$. Therefore, for $x, u \in A_{ij}$ we have

$$\begin{aligned}
&|Nx - Nu|_0 \\
&\quad = \sup_{0\le t\le T} \left| g(t, x(t), x(t-\tau(t)), x'(t-\tau(t))) - g(t, u(t), u(t-\tau(t)), u'(t-\tau(t))) \right| \\
&\quad \le \sup_{0\le t\le T} \left| g(t, x(t), x(t-\tau(t)), x'(t-\tau(t))) - g(t, x(t), x(t-\tau(t)), u'(t-\tau(t))) \right| \\
&\qquad + \sup_{0\le t\le T} \left| g(t, x(t), x(t-\tau(t)), u'(t-\tau(t))) - g(t, u(t), u(t-\tau(t)), u'(t-\tau(t))) \right| \\
&\quad \le c^u \left[\max\left\{ e^{R_1}, l^+ \right\} + \delta \right] \sup_{0\le t\le T} \left| x'(t-\tau(t)) - u'(t-\tau(t)) \right| + \varepsilon \\
&\quad \le k_0 \left| x' - u' \right|_0 + \varepsilon \\
&\quad \le k_0 \eta + (k_0 + 1)\varepsilon.
\end{aligned} \tag{3.15}$$

That is $\Gamma_Y(N(A)) \le k_0 \Gamma_X(A)$. The proof is complete. □

Lemma 3.4. *If the assumptions of Theorem 3.1 hold, then every solution $x \in X$ of the problem*

$$Lx = \lambda N x, \quad \lambda \in (0,1) \tag{3.16}$$

satisfies

$$\begin{gathered}
\max_{t\in[0,T]} x(t) \in \left[\ln l^-, \ln u^-\right] \cup \left[\ln u^+, \ln l^+\right], \\
\min_{t\in[0,T]} x(t) \in \left[\ln l^-, \ln l^+\right], \\
\max_{t\in[0,T]} \left| x'(t) \right| < M_1.
\end{gathered} \tag{3.17}$$

Proof. Let $Lx = \lambda N x$ for $x \in X$, that is,

$$x'(t) = \lambda \left[a(t) - \beta(t) e^{x(t)} - b(t) e^{x(t-\tau(t))} - c(t) x'(t-\tau(t)) e^{x(t-\tau(t))} - \frac{h(t)}{e^{x(t)}} \right], \quad \lambda \in (0,1). \tag{3.18}$$

Therefore, we have

$$x'(t) = \lambda \left[a(t) - \beta(t) e^{x(t)} - b(t) e^{x(t-\tau(t))} - c_0(t) \left[e^{x(t-\tau(t))} \right]' - \frac{h(t)}{e^{x(t)}} \right], \quad \lambda \in (0,1), \tag{3.19}$$

where $c_0(t) = c(t)/(1-\tau'(t))$.

By (3.19), we have

$$\left[x(t) + \lambda c_0(t) e^{x(t-\tau(t))} \right]' = \lambda \left[a(t) - \beta(t) e^{x(t)} - \left(b(t) - c_0'(t) \right) e^{x(t-\tau(t))} - \frac{h(t)}{e^{x(t)}} \right]. \tag{3.20}$$

Integrating this identity leads to

$$\int_0^T \left[\beta(t)e^{x(t)} + \left(b(t) - c_0'(t)\right)e^{x(t-\tau(t))} + \frac{h(t)}{e^{x(t)}}\right]dt = \int_0^T a(t)dt. \tag{3.21}$$

From (3.20),(3.21), we have

$$\begin{aligned}
&\int_0^T \left|\left[x(t) + \lambda c_0(t)e^{x(t-\tau(t))}\right]'\right|dt \\
&\quad\le \lambda\left(\int_0^T a(t)dt + \int_0^T \left[\beta(t)e^{x(t)} + \left(b(t) - c_0'(t)\right)e^{x(t-\tau(t))} + \frac{h(t)}{e^{x(t)}}\right]dt\right) \\
&\quad< 2\int_0^T a(t)dt = 2T\overline{a}.
\end{aligned} \tag{3.22}$$

By (3.21), we have

$$\int_0^T a(t)dt \ge \int_0^T \left[\beta(t)e^{x(t)} + \left(b(t) - c_0'(t)\right)e^{x(t-\tau(t))}\right]dt. \tag{3.23}$$

It follows that

$$\int_0^T a(t)dt \ge T\beta^l e^{x(\xi)} + T\left(b - c_0'\right)^l e^{x(\xi-\tau(\xi))}, \tag{3.24}$$

for some $\xi \in [0, T]$.

Therefore, we have

$$x(\xi) \le \ln\frac{\overline{a}}{\beta^l}, \qquad e^{x(\xi-\tau(\xi))} \le \frac{\overline{a}}{\left(b - c_0'\right)^l}. \tag{3.25}$$

From (3.22) and (3.25), we see that

$$\begin{aligned}
x(t) + \lambda c_0(t)e^{x(t-\tau(t))} &\le x(\xi) + \lambda c_0(\xi)e^{x(\xi-\tau(\xi))} + \int_0^T \left|\left[x(t) + \lambda c_0(t)e^{x(t-\tau(t))}\right]'\right|dt \\
&< \ln\frac{\overline{a}}{\beta^l} + \frac{c_0^u\overline{a}}{\left(b - c_0'\right)^l} + 2\overline{a}T = R_1.
\end{aligned} \tag{3.26}$$

Hence, we have $x(t) < R_1$.

Let $s = t - \tau(t)$. It follows from (3.19) that

$$x'(t) = \lambda\left[a(t) - \beta(t)e^{x(t)} - b(t)e^{x(t-\tau(t))} - c(t)\frac{de^{x(s)}}{ds} - \frac{h(t)}{e^{x(t)}}\right], \quad \lambda \in (0, 1). \tag{3.27}$$

Then from (3.27) and the inequality $x(t) < R_1$, we obtain that

$$\begin{aligned}\left|\left[e^{x(t)}\right]'\right| &\le \lambda\left[a(t)e^{x(t)} + \beta(t)e^{2x(t)} + b(t)e^{x(t)+x(t-\tau(t))} + c(t)e^{x(t)}\left|\frac{de^{x(s)}}{ds}\right| + h(t)\right] \\ &< a^u e^{R_1} + \left(\beta^u + b^u\right)e^{2R_1} + c^u e^{R_1}\left|\frac{de^{x(s)}}{ds}\right| + h^u, \quad \forall t \in R.\end{aligned} \tag{3.28}$$

So that

$$\left|\left[e^{x(t)}\right]'\right| < a^u e^{R_1} + \left[\beta^u + b^u\right]e^{2R_1} + c^u e^{R_1}\left|\left[e^x\right]'\right|_0 + h^u, \quad \forall t \in R. \tag{3.29}$$

Since $c^u e^{R_1} < 1$, we have

$$\left|\left[e^x\right]'\right|_0 < \frac{a^u e^{R_1} + \left(\beta^u + b^u\right)e^{2R_1} + h^u}{1 - c^u e^{R_1}} = M_0. \tag{3.30}$$

Choose $t_M,\ t_m \in [0,T]$, such that

$$x(t_M) = \max_{t\in[0,T]} x(t), \qquad x(t_m) = \min_{t\in[0,T]} x(t). \tag{3.31}$$

Then, it is clear that

$$x'(t_M) = 0, \qquad x'(t_m) = 0. \tag{3.32}$$

From this and (3.27), we obtain that

$$\begin{aligned}a(t_M)e^{x(t_M)} &= \beta(t_M)e^{2x(t_M)} + b(t_M)e^{x(t_M)+x(t_M-\tau(t_M))} \\ &\quad + c(t_M)e^{x(t_M)}\left[\frac{de^{x(s)}}{ds}\right]_{s=t_M-\tau(t_M)} + h(t_M),\end{aligned} \tag{3.33}$$

$$\begin{aligned}a(t_m)e^{x(t_m)} &= \beta(t_m)e^{2x(t_m)} + b(t_m)e^{x(t_m)+x(t_m-\tau(t_m))} \\ &\quad + c(t_m)e^{x(t_m)}\left[\frac{de^{x(s)}}{ds}\right]_{s=t_m-\tau(t_m)} + h(t_m).\end{aligned} \tag{3.34}$$

It follows from (3.33) that

$$\left[\beta^u + b^u\right]e^{2x(t_M)} - \left[a^l - c^u M_0\right]e^{x(t_M)} + h^u \ge 0. \tag{3.35}$$

By (H_3), we have

$$x(t_M) \le \ln u^- \quad \text{or} \quad x(t_M) \ge \ln u^+. \tag{3.36}$$

It also follows from (3.33) that

$$\beta^l e^{2x(t_M)} - [a^u + c^u M_0] e^{x(t_M)} + h^l \leq 0. \tag{3.37}$$

By (H_3), we have

$$\ln l^- \leq x(t_M) \leq \ln l^+. \tag{3.38}$$

Similarly, it follows from (3.34) that

$$\beta^l e^{2x(t_m)} - [a^u + c^u M_0] e^{x(t_m)} + h^l \leq 0. \tag{3.39}$$

By (H_3), we have

$$\ln l^- \leq x(t_m) \leq \ln l^+. \tag{3.40}$$

Hence, it follows from (3.36), (3.38), and (3.40) that

$$\begin{gathered} x(t_M) \in \left[\ln l^-, \ln u^-\right] \cup [\ln u^+, \ln l^+], \\ x(t_m) \in \left[\ln l^-, \ln l^+\right]. \end{gathered} \tag{3.41}$$

From the above inequality and (3.18), we obtain that

$$\begin{aligned} \left|x'(t)\right| &\leq \lambda \left[a(t) + \beta(t) e^{x(t)} + b(t) e^{x(t-\tau(t))} + c(t) \left|x'(t-\tau(t))\right| e^{x(t-\tau(t))} + \frac{h(t)}{e^{x(t)}}\right] \\ &< a^u + \left(\beta^u + b^u\right) e^{R_1} + c^u e^{R_1} \left|x'\right|_0 + \frac{h^u}{e^{x(t_m)}}. \end{aligned} \tag{3.42}$$

So that

$$\left|x'\right|_0 < a^u + \left[\beta^u + b^u\right] e^{R_1} + c^u e^{R_1} \left|x'\right|_0 + \frac{h^u}{l^-}. \tag{3.43}$$

Since $c^u e^{R_1} < 1$, we have

$$\left|x'\right|_0 < \frac{a^u + \left(\beta^u + b^u\right) e^{R_1} + h^u / l^-}{1 - c^u e^{R_1}} = M_1. \tag{3.44}$$

The proof is complete. □

The Proof of Theorem 3.1

Clearly, $l^{\pm}, u^{\pm}$ are independent of λ. Now, let us consider $QN(x)$ with $x \in R$. Note that

$$QN(x) = \overline{a} - \left[\overline{\beta} + \overline{b}\right] e^{x} - \frac{\overline{h}}{e^{x}}. \tag{3.45}$$

It is easy to see that $QN(x) = 0$ has two distinct solutions:

$$\widetilde{u}_1 = \ln x^{-}, \qquad \widetilde{u}_2 = \ln x^{+}. \tag{3.46}$$

By (3.8), one can take $v^{-}, v^{+} > 0$ such that

$$u^{-} < v^{-} < v^{+} < u^{+}. \tag{3.47}$$

Let

$$\begin{gathered}
\Omega_1 = \left\{ x \in X \;\middle|\; \begin{array}{l} \max\limits_{t\in[0,T]} x(t) \in (\ln(l^{-} - \delta),\ \ln v^{-}), \\ \min\limits_{t\in[0,T]} x(t) \in (\ln(l^{-} - \delta),\ \ln(l^{+} + \delta)), \\ \max\limits_{t\in[0,T]} |x'(t)| < M_1. \end{array} \right\}, \\
\Omega_2 = \left\{ x \in X \;\middle|\; \begin{array}{l} \max\limits_{t\in[0,T]} x(t) \in (\ln v^{+},\ \ln(l^{+} + \delta)), \\ \min\limits_{t\in[0,T]} x(t) \in (\ln(l^{-} - \delta),\ \ln(l^{+} + \delta)), \\ \max\limits_{t\in[0,T]} |x'(t)| < M_1. \end{array} \right\}.
\end{gathered} \tag{3.48}$$

Then Ω_1 and Ω_2 are bounded open subsets of X. Clearly, $\Omega_i \subset \Omega \ (i = 1, 2)$. It follows from Lemma 3.3 that $N : \overline{\Omega}_i \to Y$ is a k_0-set-contractive map $(i = 1, 2)$. Therefore, it follows from Lemmas 2.1 and 3.2 that $N : \overline{\Omega}_i \to Z$ is L-k-set contractive on $\overline{\Omega}_i$ $(i = 1, 2)$ with $k = k_0 / l(L) \le k_0 < 1$.

It follows from (3.8) and (3.46) that $\widetilde{u}_i \in \Omega_i$ $(i = 1, 2)$. From (3.8), (3.47) and Lemma 3.4, it is easy to see that $\overline{\Omega}_1 \cap \overline{\Omega}_2 = \emptyset$ and Ω_i satisfies (i) in Lemma 2.2 for $i = 1, 2$. Moreover, $QN(x) \neq 0$ for $x \in \partial\Omega_i \cap \operatorname{Ker} L \ (i = 1, 2)$.

A direct computation gives the following:

$$\deg\{JQN, \Omega_1 \cap \operatorname{Ker} L, 0\} = 1, \qquad \deg\{JQN, \Omega_2 \cap \operatorname{Ker} L, 0\} = -1. \tag{3.49}$$

Here, J is taken as the identity mapping since $ImQ = \operatorname{Ker} L$. So far we have proved that Ω_i satisfies all the assumptions in Lemma 2.2 $(i = 1, 2)$. Hence, (3.2) has at least two T-periodic solutions: $x_i^{*}(t)$ and $x_i^{*} \in \operatorname{dom} L \cap \overline{\Omega}_i$ $(i = 1, 2)$. Obviously, x_i^{*} $(i = 1, 2)$ are different. Let $N_i^{*}(t) = e^{x_i^{*}(t)}$ $(i = 1, 2)$. Then $N_i^{*}\ (t)(i = 1, 2)$ are two different positive T-periodic solutions of (1.2). The proof is complete.

Example 3.5. Take the following:

$$\tau(t) = 1 + 0.5\sin t, \qquad a(t) = 2 - \sin t, \qquad \beta(t) = b(t) = 1 + 0.5\sin t,$$
$$c(t) = \epsilon(1 - 0.5\cos t), \qquad h(t) = \epsilon(2 + \sin t), \tag{3.50}$$

where the constant $\epsilon > 0$.

Clearly, we have

$$a^l = 1, \qquad \beta^l = 0.5, \qquad \beta^u = b^u = 1.5, \qquad c^u = 1.5\epsilon, \qquad h^u = 3\epsilon, \qquad c_0(t) = \epsilon. \tag{3.51}$$

Therefore, we have $c_0'(t) < b(t)$. Moreover, it is easy to see that $R_1, M_0,$ and l^+ are bounded with respect to ϵ. Hence, for some sufficiently small $\epsilon > 0$, we have

$$a^l > c^u M_0 + 2\sqrt{[\beta^u + b^u]h^u}, \qquad k^* = c^u \max\left\{e^{R_1}, l^+\right\} = 1.5\epsilon \max\left\{e^{R_1}, l^+\right\} < 1. \tag{3.52}$$

In this case, all necessary conditions of Theorem 3.1 hold. By Theorem 3.1, (1.2) has at least two positive 2π-periodic solutions.

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