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Research Article

Interval Oscillation Criteria of Second-Order Nonlinear Dynamic Equations on Time Scales

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Using functions in some function classes and a generalized Riccati technique, we establish interval oscillation criteria for second-order nonlinear dynamic equations on time scales of the form $(p(t)\varphi(x(t))x^{\Delta}(t))^{\Delta} + f(t, x(\sigma(t))) = 0$. The obtained interval oscillation criteria can be applied to equations with a forcing term. An example is included to show the significance of the results.

1. Introduction

In this paper, we study the second-order nonlinear dynamic equation

$$\left(p(t)\varphi(x(t))x^{\Delta}(t)\right)^{\Delta} + f(t, x(\sigma(t))) = 0, \tag{1.1}$$

on a time scale $\mathbb{T}$.

Throughout this paper we will assume that

(C1) $p \in C_{\mathrm{rd}}(\mathbb{T}, (0, \infty))$;

(C2) $\varphi \in C(\mathbb{R}, (0, \eta])$, where η is an arbitrary positive constant;

(C3) $f \in C(\mathbb{T} \times \mathbb{R}, \mathbb{R})$.

Preliminaries about time scale calculus can be found in [1–3] and hence we omit them here. Without loss of generality, we assume throughout that $\sup \mathbb{T} = \infty$.

Definition 1.1. A solution $x(t)$ of (1.1) is said to have a generalized zero at $t^* \in \mathbb{T}$ if $x(t^*)x(\sigma(t^*)) \leq 0$, and it is said to be nonoscillatory on $\mathbb{T}$ if there exists $t_0 \in \mathbb{T}$ such that

$x(t)x(\sigma(t)) > 0$ for all $t > t_0$. Otherwise, it is oscillatory. Equation (1.1) is said to be oscillatory if all solutions of (1.1) are oscillatory. It is well-known that either all solutions of (1.1) are oscillatory or none are, so (1.1) may be classified as oscillatory or nonoscillatory.

The theory of time scales, which has recently received a lot of attention, was introduced by Hilger in his Ph.D. thesis [4] in 1988 in order to unify continuous and discrete analysis, see also [5]. In recent years, there has been much research activity concerning the oscillation and nonoscillation of solutions of dynamic equations on time scales, for example, see [1–27] and the references therein. In Došlý and Hilger's study [10], the authors considered the second-order dynamic equation

$$\left(p(t)x^{\Delta}(t)\right)^{\Delta} + q(t)x(\sigma(t)) = 0, \tag{1.2}$$

and gave necessary and sufficient conditions for the oscillation of all solutions on unbounded time scales. In Del Medico and Kong's study [8, 9], the authors employed the following Riccati transformation:

$$u(t) = \frac{p(t)x^{\Delta}(t)}{x(t)}, \tag{1.3}$$

and gave sufficient conditions for Kamenev-type oscillation criteria of (1.2) on a measure chain. And in Yang's study [27], the author considered the interval oscillation criteria of solutions of the differential equation

$$(p(t)x'(t))' + q(t)f(x(t)) = g(t). \tag{1.4}$$

In Wang's study [24], the author considered second-order nonlinear differential equation

$$\left(a(t)\psi(x(t))k\left(x'(t)\right)\right)' + p(t)k\left(x'(t)\right) + q(t)f(x(t)) = 0, \quad t \geq t_0, \tag{1.5}$$

used the following generalized Riccati transformations:

$$\begin{aligned} v(t) &= \phi(t)a(t)\left[\frac{\psi(x(t))k(x'(t))}{f(x(t))} + R(t)\right], \quad t \geq t_0, \\ v(t) &= \phi(t)a(t)\left[\frac{\psi(x(t))k(x'(t))}{x(t)} + R(t)\right], \quad t \geq t_0, \end{aligned} \tag{1.6}$$

where $\phi \in C^1([t_0,\infty),\mathbb{R}_+)$, $R \in C([t_0,\infty),\mathbb{R})$, and gave new oscillation criteria of (1.5).

In Huang and Wang's study [16], the authors considered second-order nonlinear dynamic equation on time scales

$$\left(p(t)x^{\Delta}(t)\right)^{\Delta} + f(t,x(\sigma(t))) = 0. \tag{1.7}$$

By using a similar generalized Riccati transformation which is more general than (1.3)

$$u(t) = \frac{A(t)p(t)x^{\Delta}(t)}{x(t)} + B(t), \tag{1.8}$$

where $A \in C^1_{\mathrm{rd}}(\mathbb{T}, \mathbb{R}_+ \setminus \{0\})$, $B \in C^1_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$, the authors extended the results in Del Medico and Kong [8, 9] and Yang [27], and established some new Kamenev-type oscillation criteria and interval oscillation criteria for equations with a forcing term.

In this paper, we will use functions in some function classes and a similar generalized Riccati transformation as (1.8) and was used in [24, 25] for nonlinear differential equations, and establish interval oscillation criteria for (1.1) in Section 2. Finally in Section 3, an example is included to show the significance of the results.

For simplicity, throughout this paper, we denote $(a,b)\bigcap\mathbb{T} = (a,b)$, where $a, b \in \mathbb{R}$, and $[a,b], [a,b), (a,b]$ are denoted similarly.

2. Main Results

In this section, we establish interval criteria for oscillation of (1.1). Our approach to oscillation problems of (1.1) is based largely on the application of the Riccati transformation.

Let $D_0 = \{s \in \mathbb{T} : s \ge 0\}$ and $D = \{(t,s) \in \mathbb{T}^2 : t \ge s \ge 0\}$. For any function $f(t,s)$: $\mathbb{T}^2 \to \mathbb{R}$, denote by f_1^{Δ} and f_2^{Δ} the partial derivatives of f with respect to t and s, respectively. For $E \subset \mathbb{R}$, denote by $L_{\mathrm{loc}}(E)$ the space of functions which are integrable on any compact subset of E. Define

$$\begin{gathered}
(\mathcal{A}, \mathcal{B}) = \Big\{(A,B) : A(s) \in C^1_{\mathrm{rd}}(D_0, \mathbb{R}_+ \setminus \{0\}),\ B(s) \in C^1_{\mathrm{rd}}(D_0, \mathbb{R}), \\
\eta A(s)p(s) \pm \mu(s)B(s) > 0,\ s \in D_0\Big\}; \\
\mathcal{H}^* = \Big\{H(t,s) \in C^1(D, \mathbb{R}_+) : H(t,t) = 0,\ H(t,s) > 0,\ H_2^{\Delta}(t,s) \le 0,\ t > s \ge 0\Big\}; \\
\mathcal{H}_* = \Big\{H(t,s) \in C^1(D, \mathbb{R}_+) : H(t,t) = 0,\ H(t,s) > 0,\ H_1^{\Delta}(t,s) \ge 0,\ t > s \ge 0\Big\}; \\
\mathcal{H} = \mathcal{H}^* \bigcap \mathcal{H}_*.
\end{gathered} \tag{2.1}$$

These function classes will be used throughout this paper. Now, we are in a position to give our first lemma.

Lemma 2.1. *Assume that (C1)–(C3) hold and that there exist* $c_1 < b_1 < c_2 < b_2$, $\alpha \ge 1$, *functions* $q, g \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$ *such that* $q(t) \ge 0 \not\equiv 0$ *for* $t \in [c_1, b_1] \bigcup [c_2, b_2]$,

$$g(t) \begin{cases} \le 0, & t \in [c_1, b_1], \\ \ge 0, & t \in [c_2, b_2], \end{cases} \tag{2.2}$$

$$\frac{f(t,y)}{y} \ge q(t)|y|^{\alpha-1} - \frac{g(t)}{y}, \tag{2.3}$$

for all $t \in [c_1, b_1] \bigcup [c_2, b_2]$ *and* $y \neq 0$. *If* $x(t)$ *is a solution of* (1.1) *such that* $x(t) > 0$ *on* $[c_1, \sigma(b_1)]$ *(or* $x(t) < 0$ *on* $[c_2, \sigma(b_2)]$*), for any* $(A, B) \in (\mathscr{A}, \mathcal{B})$ *one defines*

$$u(t) = A(t)\frac{p(t)\psi(x(t))x^{\Delta}(t)}{x(t)} + B(t), \tag{2.4}$$

on $[c_i, b_i]$, $i = 1, 2$, *and* $\Phi_1(t) = A^{\sigma}(t)(q(t) - (B(t)/A(t))^{\Delta})$, $A^{\sigma}(t) = A(\sigma(t))$. *Then for any* $(A, B) \in (\mathscr{A}, \mathcal{B})$, $H \in \mathscr{H}^*$, *and* $M_1(t, \cdot) \in L([0, \rho(t)])$, *one has*

$$\Psi_1(c_i, b_i) \leq H(b_i, c_i)u(c_i), \quad i = 1, 2, \tag{2.5}$$

where $\Phi_2(s) = A^{\sigma}(s)(\alpha(\alpha-1)^{(1-\alpha)/\alpha}[q(s)]^{1/\alpha}|g(s)|^{1-1/\alpha} - (B(s)/A(s))^{\Delta})$ *for* $\alpha > 1$, $\Phi_2(s) = \Phi_1(s)$ *for* $\alpha = 1$, *and*

$$\begin{aligned}
\Psi_1(c_i, b_i) = &\int_{c_i}^{b_i} H(b_i, \sigma(s))\Phi_2(s)\Delta s - \int_{c_i}^{\rho(b_i)} M_1(b_i, s)\Delta s \\
&+ H_2^{\Delta}\big(b_i, \rho(b_i)\big)\big(\eta A\big(\rho(b_i)\big)p\big(\rho(b_i)\big) - \mu\big(\rho(b_i)\big)B\big(\rho(b_i)\big)\big), \quad i = 1, 2,
\end{aligned}$$

$$\begin{aligned}
&M_1(t, s) \\
&\quad \triangleq \frac{\Big(H(t,s)A(s)B(s) + H(t,\sigma(s))A^{\sigma}(s)B(s) + \eta A(s)p(s)(H(t,s)A(s))^{\Delta_s}\Big)^2}{4H(t,\sigma(s))A(s)\min\big\{A(s)\big[\eta A(s)p(s) - \mu(s)B(s)\big], A^{\sigma}(s)\big[\eta A(s)p(s) + \mu(s)B(s)\big]\big\}}.
\end{aligned} \tag{2.6}$$

Proof. Suppose that $x(t)$ is a solution of (1.1) such that $x(t) > 0$ on $[c_1, \sigma(b_1)]$. First,

$$\mu u - \mu B + Ap\psi(x) = \mu\frac{Ap\psi(x)x^{\Delta}}{x} + Ap\psi(x) = Ap\psi(x)\frac{x^{\sigma}}{x} > 0. \tag{2.7}$$

Hence, we always have

$$\mu u - \mu B + \eta Ap \geq \mu u - \mu B + Ap\psi(x) > 0, \tag{2.8}$$

$$\frac{x}{x^{\sigma}} = \frac{Ap\psi(x)}{\mu u - \mu B + Ap\psi(x)} \geq \frac{Ap\psi(x)}{\mu u - \mu B + \eta Ap}. \tag{2.9}$$

Then differentiating (2.4) and using (1.1), it follows that

$$
\begin{aligned}
u^{\Delta} &= A^{\Delta}\left(\frac{p\psi(x)x^{\Delta}}{x}\right) + A^{\sigma}\left(\frac{p\psi(x)x^{\Delta}}{x}\right)^{\Delta} + B^{\Delta} \\
&= \frac{A^{\Delta}}{A}(u-B) + A^{\sigma}\frac{\left(p\psi(x)x^{\Delta}\right)^{\Delta}x - p\psi(x)\left(x^{\Delta}\right)^{2}}{xx^{\sigma}} + B^{\Delta} \\
&= \frac{A^{\Delta}}{A}u + B^{\Delta} - \frac{A^{\Delta}}{A}B - A^{\sigma}\frac{f(t,x^{\sigma})}{x^{\sigma}} - A^{\sigma}p\psi(x)\frac{\left(x^{\Delta}\right)^{2}}{x^{2}}\frac{x}{x^{\sigma}}.
\end{aligned}
\tag{2.10}
$$

(i) $\alpha > 1$. Noting that $g(t) \leq 0$ on $[c_1, b_1]$, from (2.10), we have

$$
\begin{aligned}
u^{\Delta} &\leq \frac{A^{\Delta}}{A}u + A^{\sigma}\left(\frac{B}{A}\right)^{\Delta} - A^{\sigma}\left[\frac{|g|}{x^{\sigma}} + q(x^{\sigma})^{\alpha-1}\right] - A^{\sigma}p\psi(x)\frac{\left(x^{\Delta}\right)^{2}}{x^{2}}\frac{x}{x^{\sigma}} \\
&\leq \frac{A^{\Delta}}{A}u + A^{\sigma}\left(\frac{B}{A}\right)^{\Delta} - \alpha(\alpha-1)^{(1-\alpha)/\alpha}A^{\sigma}[q]^{1/\alpha}|g|^{1-1/\alpha} - A^{\sigma}p\psi(x)\frac{\left(x^{\Delta}\right)^{2}}{x^{2}}\frac{x}{x^{\sigma}} \\
&\leq \frac{A^{\Delta}}{A}u - \frac{A^{\sigma}}{A}\frac{(u-B)^{2}}{\mu u - \mu B + \eta A p} - \Phi_2.
\end{aligned}
\tag{2.11}
$$

That is, for $\alpha > 1$,

$$
u^{\Delta}(t) + \Phi_2(t) + \frac{A(t)u^{2}(t) - \left[(A^{\sigma}(t) + A(t))B(t) + \eta A^{\Delta}(t)A(t)p(t)\right]u(t) + A^{\sigma}(t)B^{2}(t)}{A(t)\left(\mu(t)u(t) - \mu(t)B(t) + \eta A(t)p(t)\right)} \leq 0.
\tag{2.12}
$$

(ii) For $\alpha = 1$, from (2.10), we have

$$
\begin{aligned}
u^{\Delta} &\leq \frac{A^{\Delta}}{A}u + A^{\sigma}\left(\frac{B}{A}\right)^{\Delta} - A^{\sigma}\left[\frac{|g|}{x^{\sigma}} + q\right] - A^{\sigma}p\psi(x)\frac{\left(x^{\Delta}\right)^{2}}{x^{2}}\frac{x}{x^{\sigma}} \\
&\leq \frac{A^{\Delta}}{A}u - A^{\sigma}p\psi(x)\frac{\left(x^{\Delta}\right)^{2}}{x^{2}}\frac{x}{x^{\sigma}} + A^{\sigma}\left[\left(\frac{B}{A}\right)^{\Delta} - q\right].
\end{aligned}
\tag{2.13}
$$

Then (2.12) also holds.

From (i) and (ii) above, we see that (2.12) holds for $\alpha \geq 1$. For simplicity in the following, we let $H_{\sigma} = H(b_1, \sigma(s))$, $H = H(b_1, s)$, $H_2^{\Delta} = H_2^{\Delta}(b_1, s)$, and omit the arguments in the integrals. For $s \in \mathbb{T}$,

$$
H_{\sigma} - H = H_2^{\Delta}\mu.
\tag{2.14}
$$

Since $H_2^\Delta \leq 0$ on D, we see that $H_\sigma \leq H$. Multiplying (2.12), where t is replaced by s, by H_σ, and integrating it with respect to s from c_1 to b_1, we obtain

$$\int_{c_1}^{b_1} H_\sigma \Phi_2 \Delta s \leq -\int_{c_1}^{b_1} \left(H_\sigma u^\Delta + H_\sigma \frac{Au^2 - \left[(A^\sigma + A)B + \eta A^\Delta Ap\right]u + A^\sigma B^2}{A(\mu u - \mu B + \eta Ap)} \right) \Delta s. \tag{2.15}$$

Noting that $H(t,t) = 0$, by the integration by parts formula, we have

$$\begin{aligned}
\int_{c_1}^{b_1} H_\sigma \Phi_2 \Delta s &\leq H(b_1, c_1)u(c_1) + \int_{c_1}^{b_1} \left(H_2^\Delta u - H_\sigma \frac{Au^2 - \left[(A^\sigma + A)B + \eta A^\Delta Ap\right]u + A^\sigma B^2}{A(\mu u - \mu B + \eta Ap)} \right) \Delta s \\
&\leq H(b_1, c_1)u(c_1) + \int_{\rho(b_1)}^{b_1} H_2^\Delta u \Delta s \\
&\quad + \int_{c_1}^{\rho(b_1)} \left(H_2^\Delta u - H_\sigma \frac{Au^2 - \left[(A^\sigma + A)B + \eta A^\Delta Ap\right]u}{A(\mu u - \mu B + \eta Ap)} \right) \Delta s.
\end{aligned} \tag{2.16}$$

Since $H_2^\Delta \leq 0$ on D, from (2.8), we see that

$$\begin{aligned}
\int_{\rho(b_1)}^{b_1} H_2^\Delta u \Delta s &= H_2^\Delta (b_1, \rho(b_1)) u(\rho(b_1)) \mu(\rho(b_1)) \\
&\leq -H_2^\Delta (b_1, \rho(b_1)) \left(\eta A(\rho(b_1)) p(\rho(b_1)) - \mu(\rho(b_1)) B(\rho(b_1))\right).
\end{aligned} \tag{2.17}$$

For $s \in [c_1, \rho(b_1))$, and $u(s) \leq 0$, we have

$$\begin{aligned}
&H_2^\Delta u - H_\sigma \frac{Au^2 - \left[(A^\sigma + A)B + \eta A^\Delta Ap\right]u}{A(\mu u - \mu B + \eta Ap)} \\
&\quad = -\frac{H}{\mu u - \mu B + \eta Ap} u^2 + \frac{HAB + H_\sigma A^\sigma B + \eta Ap(HA)^\Delta}{A(\eta Ap - \mu B)} u \\
&\qquad - \frac{HAB + H_\sigma A^\sigma B + \eta Ap(HA)^\Delta}{A(\eta Ap - \mu B)} \frac{\mu u^2}{\mu u - \mu B + \eta Ap}
\end{aligned}$$

$$\leq -\frac{H_\sigma A^\sigma(\eta Ap+\mu B)}{A(\eta Ap-\mu B)^2}u^2+\frac{HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta}{A(\eta Ap-\mu B)}u$$
$$=-\frac{H_\sigma A^\sigma(\eta Ap+\mu B)}{A(\eta Ap-\mu B)^2}\left[u-\frac{(\eta Ap-\mu B)\left(HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta\right)}{2H_\sigma A^\sigma(\eta Ap+\mu B)}\right]^2$$
$$+\frac{\left(HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta\right)^2}{4H_\sigma A^\sigma A(\eta Ap+\mu B)}$$
$$\leq \frac{\left(HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta\right)^2}{4H_\sigma A\min\{A(\eta Ap-\mu B),A^\sigma(\eta Ap+\mu B)\}}=M_1. \tag{2.18}$$

For $s\in[c_1,\rho(b_1))$, and $u(s)>0$, we have

$$H_2^\Delta u-H_\sigma\frac{Au^2-[(A^\sigma+A)B+\eta A^\Delta Ap]u}{A(\mu u-\mu B+\eta Ap)}$$
$$=-\frac{H}{\mu u-\mu B+\eta Ap}\left[u-\frac{HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta}{2HA}\right]^2$$
$$+\frac{\left(HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta\right)^2}{4HA^2(\mu u-\mu B+\eta Ap)} \tag{2.19}$$
$$\leq \frac{\left(HAB+H_\sigma A^\sigma B+\eta Ap(HA)^\Delta\right)^2}{4H_\sigma A\min\{A(\eta Ap-\mu B),\ A^\sigma(\eta Ap+\mu B)\}}=M_1.$$

Therefore, for $s\ \in\ [c_1,\rho(b_1))$, we have

$$H_2^\Delta u-H_\sigma\frac{Au^2-[(A^\sigma+A)B+\eta A^\Delta Ap]u}{A(\mu u-\mu B+\eta Ap)}\leq M_1. \tag{2.20}$$

Then from (2.16), (2.17), and (2.20), we obtain that (2.5) holds for $i=1$.

If $x(t)<0$ on $[c_2,\sigma(b_2)]$, then we see that $g(t)\geq 0$ on $[c_2,b_2]$ and

$$u^\Delta\leq\frac{A^\Delta}{A}u+A^\sigma\left(\frac{B}{A}\right)^\Delta-A^\sigma\left[\frac{g}{|x^\sigma|}+q|x^\sigma|^{\alpha-1}\right]-A^\sigma p\psi(x)\frac{(x^\Delta)^2}{x^2}\frac{x}{x^\sigma}. \tag{2.21}$$

Following the steps above, we have that (2.5) holds for $i=2$. The proof is complete. □

Next, we have the second lemma.

Lemma 2.2. *Assume that (C1)–(C3) hold, and that there exist* $a_1 < c_1 < a_2 < c_2$, $\alpha \geq 1$, *functions* $q, g \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$ *such that* $q(t) \geq 0 \not\equiv 0$ *for* $t \in [a_1, c_1] \bigcup [a_2, c_2]$ *and*

$$g(t) \begin{cases} \leq 0, & t \in [a_1, c_1], \\ \geq 0, & t \in [a_2, c_2], \end{cases} \tag{2.22}$$

and (2.3) *holds for all* $t \in [a_1, c_1] \bigcup [a_2, c_2]$ *and* $y \neq 0$. *If* $x(t)$ *is a solution of* (1.1) *such that* $x(t) > 0$ *on* $[a_1, \sigma(c_1)]$*(or* $x(t) < 0$ *on* $[a_2, \sigma(c_2)]$*), define* $u(t)$ *as in* (2.4) *on* $[a_i, c_i]$, $i = 1, 2$. *Then for any* $(A, B) \in (\mathcal{A}, \mathcal{B})$, $H \in \mathcal{H}_*$, $M_2(\cdot, t) \in L_{\mathrm{loc}}([\sigma(t), \infty))$, *one has*

$$\Psi_2(a_i, c_i) \leq -H(c_i, a_i)u(c_i), \quad i = 1, 2, \tag{2.23}$$

where Φ_2 *is defined as before, and*

$$\begin{aligned} \Psi_2(a_i, c_i) = & \int_{a_i}^{c_i} H(\sigma(s), a_i)\Phi_2(s)\Delta s - \int_{\sigma(a_i)}^{c_i} M_2(s, a_i)\Delta s \\ & - \left[\eta p(a_i) H_1^{\Delta}(a_i, a_i) A^{\sigma}(a_i) + \frac{H(\sigma(a_i), a_i)A^{\sigma}(a_i)B(a_i)}{A(a_i)}\right], \quad i = 1, 2, \end{aligned}$$

$$\begin{aligned} & M_2(s, t) \\ & \triangleq \frac{\left(H(s,t)A(s)B(s) + H(\sigma(s),t)A^{\sigma}(s)B(s) + \eta A(s)p(s)(H(s,t)A(s))^{\Delta_s}\right)^2}{4H(s,t)A(s)\min\{A(s)[\eta A(s)p(s) - \mu(s)B(s)], A^{\sigma}(s)[\eta A(s)p(s) + \mu(s)B(s)]\}} \end{aligned} \tag{2.24}$$

Proof. Suppose that $x(t)$ is a solution of (1.1) such that $x(t) > 0$ on $[a_1, \sigma(c_1)]$. For simplicity in the following, we let $H'_{\sigma} = H(\sigma(s), a_1)$, $H' = H(s, a_1)$, $H_1^{\Delta} = H_1^{\Delta}(s, a_1)$, and omit the arguments in the integrals. Multiplying (2.12), where t is replaced by s, by H'_{σ}, and integrating it with respect to s from a_1 to c_1 and then using the integration by parts formula we have that

$$\begin{aligned} \int_{a_1}^{c_1} H'_{\sigma}\Phi_2\Delta s \leq & -\int_{a_1}^{c_1}\left(H'_{\sigma}u^{\Delta} + H'_{\sigma}\frac{Au^2 - [(A^{\sigma} + A)B + \eta A^{\Delta}Ap]u + A^{\sigma}B^2}{A(\mu u - \mu B + \eta Ap)}\right)\Delta s \\ \leq & -H(c_1, a_1)u(c_1) \\ & + \left(\int_{a_1}^{\sigma(a_1)} + \int_{\sigma(a_1)}^{c_1}\right)\left(H_1^{\Delta}u - H'_{\sigma}\frac{Au^2 - [(A^{\sigma} + A)B + \eta A^{\Delta}Ap]u}{A(\mu u - \mu B + \eta Ap)}\right)\Delta s. \end{aligned} \tag{2.25}$$

For $s \in [a_1, c_1)$,

$$H'_\sigma - H_1^\Delta \mu = H'. \tag{2.26}$$

Hence,

$$\begin{aligned}
&\int_{a_1}^{\sigma(a_1)} \left(H_1^\Delta u - H'_\sigma \frac{Au^2 - [(A^\sigma + A)B + \eta A^\Delta Ap]u}{A(\mu u - \mu B + \eta Ap)} \right) \Delta s \\
&\quad = \left. \frac{\left(H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta\right)u\mu}{A(\mu u - \mu B + \eta Ap)} \right|_{s=a_1} \\
&\quad \le \eta p(a_1) H_1^\Delta(a_1, a_1) A^\sigma(a_1) + \frac{H(\sigma(a_1), a_1) A^\sigma(a_1) B(a_1)}{A(a_1)}.
\end{aligned} \tag{2.27}$$

Furthermore, for $s \in [\sigma(a_1), c_1)$, and $u(s) \le 0$,

$$\begin{aligned}
&H_1^\Delta u - H'_\sigma \frac{Au^2 - [(A^\sigma + A)B + \eta A^\Delta Ap]u}{A(\mu u - \mu B + \eta Ap)} \\
&\quad = -\frac{H'}{\mu u - \mu B + \eta Ap} u^2 + \frac{H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta}{A(\eta Ap - \mu B)} u \\
&\qquad - \frac{H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta}{A(\eta Ap - \mu B)} \frac{\mu u^2}{\mu u - \mu B + \eta Ap} \\
&\quad \le -\frac{H'_\sigma A^\sigma(\eta Ap + \mu B)}{A(\eta Ap - \mu B)^2} u^2 + \frac{H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta}{A(\eta Ap - \mu B)} u \\
&\quad = -\frac{H'_\sigma A^\sigma(\eta Ap + \mu B)}{A(\eta Ap - \mu B)^2} \left[u - \frac{(\eta Ap - \mu B)\left(H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta\right)}{2H'_\sigma A^\sigma(\eta Ap + \mu B)} \right]^2 \\
&\qquad + \frac{\left(H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta\right)^2}{4H'_\sigma A^\sigma A(\eta Ap + \mu B)} \\
&\quad \le \frac{\left(H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta\right)^2}{4H'A \min\{A(\eta Ap - \mu B), A^\sigma(\eta Ap + \mu B)\}} = M_2.
\end{aligned} \tag{2.28}$$

For $s \in [\sigma(a_1), c_1)$, and $u(s) > 0$,

$$\begin{aligned} H_1^\Delta u - H'_\sigma \frac{Au^2 - \left[(A^\sigma + A)B + \eta A^\Delta Ap\right]u}{A(\mu u - \mu B + \eta Ap)} &= -\frac{H'}{\mu u - \mu B + \eta Ap}\left[u - \frac{H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta}{2H'A}\right]^2 \\ &\quad + \frac{\left(H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta\right)^2}{4H'A^2(\mu u - \mu B + \eta Ap)} \\ &\le \frac{\left(H'AB + H'_\sigma A^\sigma B + \eta Ap(H'A)^\Delta\right)^2}{4H'A\min\{A(\eta Ap - \mu B), A^\sigma(\eta Ap + \mu B)\}} = M_2. \end{aligned} \tag{2.29}$$

Hence, for $s \in [\sigma(a_1), c_1)$, we have

$$H_1^\Delta u - H'_\sigma \frac{Au^2 - \left[(A^\sigma + A)B + \eta A^\Delta Ap\right]u}{A(\mu u - \mu B + \eta Ap)} \le M_2. \tag{2.30}$$

From (2.25), (2.27), and (2.30), we have that (2.23) holds for $i = 1$.

If $x(t) < 0$ on $[a_2, \sigma(c_2)]$, then we see that $g(t) \ge 0$ on $[a_2, c_2]$. Following the steps above, we have that (2.23) holds for $i = 2$. The proof is complete. □

Theorem 2.3. *Assume that (C1)–(C3) and the following two conditions hold:*

(C4) *For any $T \ge t_0$, there exist $T \le a_1 < b_1 \le a_2 < b_2$, $\alpha \ge 1$, functions $q, g \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$ such that $q(t) \ge 0 \not\equiv 0$ for $t \in [a_1, b_1] \bigcup [a_2, b_2]$,*

$$g(t)\begin{cases} \le 0, & t \in [a_1, b_1], \\ \ge 0, & t \in [a_2, b_2], \end{cases} \tag{2.31}$$

and (2.3) holds for all $t \in [a_1, b_1] \bigcup [a_2, b_2]$ and $y \ne 0$.

(C5) *There exist $c_i \in (a_i, b_i)$, $i = 1, 2$, $(A, B) \in (\mathcal{A}, \mathcal{B})$, $H \in \mathcal{H}$, $M_1(t, \cdot) \in L([0, \rho(t)])$, $M_2(\cdot, t) \in L_{\mathrm{loc}}([\sigma(t), \infty))$ such that for $i = 1, 2$,*

$$\frac{1}{H(b_i, c_i)}\Psi_1(c_i, b_i) + \frac{1}{H(c_i, a_i)}\Psi_2(a_i, c_i) > 0, \tag{2.32}$$

where M_1, M_2, $\Psi_1(c_i, b_i)$ and $\Psi_2(a_i, c_i)$ are defined as before.

Then (1.1) is oscillatory.

Proof. Suppose that $x(t)$ is a nonoscillatory solution of (1.1) which is eventually positive, say $x(t) > 0$ when $t \ge T \ge t_0$ for some T depending on the solution $x(t)$. From the assumption (C4), we can choose a_1, $b_1 \ge T$ so that $g(t) \le 0$ on the interval $I = [a_1, b_1]$ with $a_1 < b_1$.

From Lemmas 2.1 and 2.2, we see that (2.5) and (2.23) hold for $i = 1$. By dividing (2.5) and (2.23) by $H(b_1, c_1)$ and $H(c_1, a_1)$, respectively, and then adding them, we obtain a contradiction to assumption (2.32) with $i = 1$.

When $x(t)$ is eventually negative, we choose $a_2, b_2 \geq T$ so that $g(t) \geq 0$ on $[a_2, b_2]$ to reach a similar contradiction. Hence, every solution of (1.1) has at least one generalized zero in (a_1, b_1) or (a_2, b_2).

Pick a sequence $\{T_j\} \subset \mathbb{T}$ such that $T_j \geq T$ and $T_j \to \infty$ as $j \to \infty$. By assumption, for each $j \in \mathbb{N}$ there exists $a_j, b_j, c_j \in \mathbb{R}$ such that $T_j \leq a_j < c_j < b_j$ and (2.32) holds, where $a, b,$ and c are replaced by $a_j, b_j,$ and c_j, respectively. Hence, every solution $x(t)$ has at least one generalized zero $t_j \in (a_j, b_j)$. Noting that $t_j > a_j \geq T_j, j \in \mathbb{N}$, we see that every solution has arbitrarily large generalized zeros. Thus, (1.1) is oscillatory. The proof is complete. □

Corollary 2.4. *Assume that (C1)–(C4) hold and that*

(C6) there exist $c_i \in (a_i, b_i)$, $i = 1, 2$, $(A, B) \in (\mathcal{A}, \mathcal{B})$, $H \in \mathcal{H}$, $M_1(t, \cdot) \in L([0, \rho(t)])$, $M_2(\cdot, t) \in L_{\text{loc}}([\sigma(t), \infty))$ such that for $i = 1, 2$,

$$\Psi_1(c_i, b_i) > 0, \tag{2.33}$$

$$\Psi_2(a_i, c_i) > 0, \tag{2.34}$$

where M_1, M_2, $\Psi_1(c_i, b_i)$ and $\Psi_2(a_i, c_i)$ are defined as before. Then (1.1) *is oscillatory.*

Proof. By (2.33) and (2.34), we get (2.32). Therefore, (1.1) is oscillatory by Theorem 2.3. The proof is complete. □

When $q \in C_{\text{rd}}(\mathbb{T}, \mathbb{R}_+)$, $g(t) \equiv 0$, $\alpha = 1$, we have the following corollary.

Corollary 2.5. *Assume that (C1)–(C3) hold and that there exists a function $q \in C_{\text{rd}}(\mathbb{T}, \mathbb{R}_+)$ such that $uf(t, u) \geq q(t)u^2$. Also, suppose that there exist $(A, B) \in (\mathcal{A}, \mathcal{B})$, $H \in \mathcal{H}$, $M_1(t, \cdot) \in L([0, \rho(t)])$, $M_2(\cdot, t) \in L_{\text{loc}}([\sigma(t), \infty))$ such that for any $l \in \mathbb{T}$*

$$\begin{aligned}\limsup_{t \to \infty} \Bigg\{ &\int_l^t H(\sigma(s), l)\Phi_1(s)\Delta s - \int_{\sigma(l)}^t M_2(s, l)\Delta s \\ &- \left[\eta p(l) H_1^{\Delta}(l, l) A^{\sigma}(l) + \frac{H(\sigma(l), l) A^{\sigma}(l) B(l)}{A(l)}\right] \Bigg\} > 0,\end{aligned} \tag{2.35}$$

$$\begin{aligned}\limsup_{t \to \infty} \Bigg[&\int_l^t H(t, \sigma(s))\Phi_1(s)\Delta s - \int_l^{\rho(t)} M_1(t, s)\Delta s \\ &+ H_2^{\Delta}(t, \rho(t))\big(\eta A(\rho(t)) p(\rho(t)) - \mu(\rho(t)) B(\rho(t))\big) \Bigg] > 0.\end{aligned} \tag{2.36}$$

Then (1.1) *is oscillatory.*

Proof. When (C3) holds and there exists a function $q \in C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R}_+)$ such that $uf(t,u) \geq q(t)u^2$, it follows that (C4) holds for $g(t) \equiv 0$ and $\alpha = 1$. Now $\Phi_1(s) = \Phi_2(s)$. For any $T \geq t_0$, let $a_1 = T$. In (2.35), we choose $l = a_1$. Then there exists $c_1 > a_1$ such that

$$\Psi_2(a_1, c_1) > 0. \tag{2.37}$$

In (2.36), we choose $l = c_1$. Then there exists $b_1 > c_1$ such that

$$\Psi_1(c_1, b_1) > 0. \tag{2.38}$$

Combining (2.37) and (2.38) we obtain (2.32) with $i = 1$.

Next, in (2.35) we choose $l = a_2 = b_1$. Then there exists $c_2 > a_2$ such that

$$\Psi_2(a_2, c_2) > 0. \tag{2.39}$$

In (2.36), we choose $l = c_2$. Then there exists $b_2 > c_2$ such that

$$\Psi_1(c_2, b_2) > 0. \tag{2.40}$$

Combining (2.39) and (2.40) we obtain (2.32) with $i = 2$. The conclusion thus follows from Theorem 2.3. The proof is complete. □

3. Example

In this section, we will show the application of our oscillation criteria in an example. The example is to demonstrate Theorem 2.3.

Example 3.1. Consider the equation

$$\left(p(t)\left(2 + \cos 2x(t) + \frac{\sin x(t)}{1 + x^2(t)}\right)x^{\Delta}(t)\right)^{\Delta} + q(t)x^3(\sigma(t))\left[\frac{2 + x^2(\sigma(t))}{1 + x^2(\sigma(t))}\right] + \cos\frac{\pi}{16}t = 0, \tag{3.1}$$

where $p \in C_{\mathrm{rd}}(\mathbb{T}, (0, \eta_0])$, $t \in \mathbb{T}$, $\psi(x(t)) = 2 + \cos 2x(t) + \sin x(t)/(1 + x^2(t))$,

$$q(t) = \begin{cases} \cos\dfrac{\pi}{16}t, & t \in [32n, 32n + 12], \\ \dfrac{2 + \sqrt{2}}{8}(t - 32n - 12), & t \in [32n + 12, 32n + 16], \\ -\cos\dfrac{\pi}{16}t, & t \in [32n + 16, 32n + 28], \\ \dfrac{2 + \sqrt{2}}{8}(t - 32n - 28), & t \in [32n + 28, 32n + 32], \ n \in \mathbb{N}_0, \end{cases} \tag{3.2}$$

and $g(t) = -\cos(\pi/16)t$. So we have $\eta = 4$.

For any $T > 0$, there exists $n \in \mathbb{N}_0$ such that $32n > T$. Let $\alpha = 3$, $a_1 = 32n$, $b_1 = 32n+8$, $c_1 = 32n+4$, $a_2 = 32n+16$, $b_2 = 32n+24$, $c_2 = 32n+20$, $(A, B) = (1, 0)$, $H(t, s) = (t-s)^2$, we have

$$g(t) \begin{cases} \leq 0, & t \in [32n, 32n+8], \\ \geq 0, & t \in [32n+16, 32n+24]. \end{cases} \tag{3.3}$$

(i) Consider $\mathbb{T} = \mathbb{R}_+$,

$$\begin{aligned}
\Psi_2(a_1, c_1) &\geq \frac{3}{\sqrt[3]{4}} \int_{32n}^{32n+4} (s-32n)^2 \cos \frac{\pi}{16} s \, ds - \int_{32n}^{32n+4} \frac{4\eta_0 (s-32n)^2}{(s-32n)^2} \, ds \\
&= \frac{192\sqrt[6]{32}}{\pi^3} \left(\pi^2 + 8\pi - 32 \right) - 16\eta_0, \\
\Psi_2(a_2, c_2) &\geq -\frac{3}{\sqrt[3]{4}} \int_{32n+16}^{32n+20} (s-32n-16)^2 \cos \frac{\pi}{16} s \, ds \\
&\quad - \int_{32n+16}^{32n+20} \frac{4\eta_0 (s-32n-16)^2}{(s-32n-16)^2} \, ds \\
&= \frac{192\sqrt[6]{32}}{\pi^3} \left(\pi^2 + 8\pi - 32 \right) - 16\eta_0, \\
\Psi_1(c_1, b_1) &\geq \frac{3}{\sqrt[3]{4}} \int_{32n+4}^{32n+8} (32n+8-s)^2 \cos \frac{\pi}{16} s \, ds - \int_{32n+4}^{32n+8} \frac{4\eta_0 (32n+8-s)^2}{(32n+8-s)^2} \, ds \\
&= \frac{192\sqrt[6]{32}}{\pi^3} \left(-\pi^2 + 8\pi - 32\left(\sqrt{2}-1\right) \right) - 16\eta_0, \\
\Psi_1(c_2, b_2) &\geq -\frac{3}{\sqrt[3]{4}} \int_{32n+20}^{32n+24} (32n+24-s)^2 \cos \frac{\pi}{16} s \, ds \\
&\quad - \int_{32n+20}^{32n+24} \frac{4\eta_0 (32n+24-s)^2}{(32n+24-s)^2} \, ds \\
&= \frac{192\sqrt[6]{32}}{\pi^3} \left(-\pi^2 + 8\pi - 32\left(\sqrt{2}-1\right) \right) - 16\eta_0.
\end{aligned} \tag{3.4}$$

So for $i = 1, 2$, we have

$$\frac{1}{H(b_i, c_i)} \Psi_1(c_i, b_i) + \frac{1}{H(c_i, a_i)} \Psi_2(a_i, c_i) \geq \frac{192\sqrt[6]{32}}{\pi^3} \left(\pi - 2\sqrt{2} \right) - 2\eta_0. \tag{3.5}$$

When $0 < \eta_0 < (96\sqrt[6]{32}/\pi^3)(\pi - 2\sqrt{2}) \approx 1.728$, we have $(192\sqrt[6]{32}/\pi^3)(\pi - 2\sqrt{2}) - 2\eta_0 > 0$, so (2.32) holds, which means that (C5) holds. By Theorem 2.3, we have that (3.1) is oscillatory. However, when $\eta_0 \geq (96\sqrt[6]{32}/\pi^3)(\pi - 2\sqrt{2})$, we do not know whether (3.1) is oscillatory.

(2) Consider $\mathbb{T} = \mathbb{N}_0$,

$$\begin{aligned}
\Psi_2(a_1, c_1) &\geq \frac{3}{\sqrt[3]{4}} \sum_{k=32n}^{32n+3} (k+1-32n)^2 \cos\frac{\pi}{16}k - \eta_0 \sum_{k=32n+1}^{32n+3} \frac{(2k-64n+1)^2}{(k-32n)^2} - 4\eta_0 \\
&= \frac{3}{\sqrt[3]{4}}\left(1 + 4\cos\frac{\pi}{16} + 9\cos\frac{\pi}{8} + 16\cos\frac{3\pi}{16}\right) - \frac{889}{36}\eta_0, \\
\Psi_2(a_2, c_2) &\geq -\frac{3}{\sqrt[3]{4}} \sum_{k=32n+16}^{32n+19} (k+1-32n-16)^2 \cos\frac{\pi}{16}k - \eta_0 \sum_{k=32n+17}^{32n+19} \frac{(2k-64n-32+1)^2}{(k-32n-16)^2} - 4\eta_0 \\
&= \frac{3}{\sqrt[3]{4}}\left(1 + 4\cos\frac{\pi}{16} + 9\cos\frac{\pi}{8} + 16\cos\frac{3\pi}{16}\right) - \frac{889}{36}\eta_0, \\
\Psi_1(c_1, b_1) &\geq \frac{3}{\sqrt[3]{4}} \sum_{k=32n+4}^{32n+7} (32n+8-k-1)^2 \cos\frac{\pi}{16}k - \eta_0 \sum_{k=32n+4}^{32n+6} \frac{(64n+16-2k-1)^2}{(32n+8-k-1)^2} - 4\eta_0 \\
&= \frac{3}{\sqrt[3]{4}}\left(9\cos\frac{\pi}{4} + 4\cos\frac{5\pi}{16} + \cos\frac{3\pi}{8}\right) - \frac{889}{36}\eta_0, \\
\Psi_1(c_2, b_2) &\geq -\frac{3}{\sqrt[3]{4}} \sum_{k=32n+20}^{32n+23} (32n+24-k-1)^2 \cos\frac{\pi}{16}k \\
&\quad - \eta_0 \sum_{k=32n+20}^{32n+22} \frac{(64n+48-2k-1)^2}{(32n+24-k-1)^2} - 4\eta_0 \\
&= \frac{3}{\sqrt[3]{4}}\left(9\cos\frac{\pi}{4} + 4\cos\frac{5\pi}{16} + \cos\frac{3\pi}{8}\right) - \frac{889}{36}\eta_0.
\end{aligned} \tag{3.6}$$

So we have

$$\begin{aligned}
&\frac{1}{H(b_i, c_i)}\Psi_1(c_i, b_i) + \frac{1}{H(c_i, a_i)}\Psi_2(a_i, c_i) \\
&\quad \geq \frac{3}{16\sqrt[3]{4}}\left[\left(1 + 4\cos\frac{\pi}{16} + 9\cos\frac{\pi}{8} + 16\cos\frac{3\pi}{16}\right)\right. \\
&\qquad \left. + \left(9\cos\frac{\pi}{4} + 4\cos\frac{5\pi}{16} + \cos\frac{3\pi}{8}\right)\right] - \frac{889}{288}\eta_0, \quad i = 1, 2.
\end{aligned} \tag{3.7}$$

When $0 < \eta_0 < (54/889\sqrt[3]{4})(1+4\cos(\pi/16)+9\cos(\pi/8)+16\cos(3\pi/16)+9\cos(\pi/4)+4\cos(5\pi/16) + \cos(3\pi/8)) \approx 1.359$, we have $(3/16\sqrt[3]{4})[(1 + 4\cos(\pi/16) + 9\cos(\pi/8) + 16\cos(3\pi/16)) + (9\cos(\pi/4) + 4\cos(5\pi/16) + \cos(3\pi/8))] - (889/288)\eta_0 > 0$, so (2.32) holds, which means that (C5) holds. By Theorem 2.3, we have that (3.1) is oscillatory. However, when $\eta_0 \geq (54/889\sqrt[3]{4})(1+4\cos(\pi/16)+9\cos(\pi/8)+16\cos(3\pi/16)+9\cos(\pi/4)+4\cos(5\pi/16) + \cos(3\pi/8))$, we do not know whether (3.1) is oscillatory.

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