

Research Article

Basel III and Asset Securitization

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Asset securitization via special purpose entities involves the process of transforming assets into securities that are issued to investors. These investors hold the rights to payments supported by the cash flows from an asset pool held by the said entity. In this paper, we discuss the mechanism by which low- and high-quality entities securitize low- and high-quality assets, respectively, into collateralized debt obligations. During the 2007–2009 financial crisis, asset securitization was seriously inhibited. In response to this, for instance, new Basel III capital and liquidity regulations were introduced. Here, we find that we can explicitly determine the transaction costs related to low-quality asset securitization. Also, in the case of dynamic and static multipliers, the effects of unexpected negative shocks such as rating downgrades on asset price and input, debt obligation price and output, and profit will be quantified. In this case, we note that Basel III has been designed to provide countercyclical capital buffers to negate procyclicality. Moreover, we will develop an illustrative example of low-quality asset securitization for subprime mortgages. Furthermore, numerical examples to illustrate the key results will be provided. In addition, connections between Basel III and asset securitization will be highlighted.

1. Introduction

Asset securitization involves the process by which securities are created by a special purpose entity (SPE)—hereafter, simply known as an entity—and then issued to investors with a right to payments supported by the cash flows from a pool of financial assets held by the entity. There is broad-based usage of entities by financial institutions of many types, in various jurisdictions, and for many purposes (see, e.g., [1]). Securitization has been popular as an alternative funding source for consumer and asset lending in market economies. Its main objective is to improve credit availability by converting hard-to-trade and nontradable assets into securities that can be traded on capital markets. The categorization of the payment rights into “tranches” paid in a specific order and supported by credit enhancement mechanisms provides investors with diversified credit risk exposure to particular investor risk appetites (see, e.g., [2, 3]). Immediately prior to the securitization market collapse in 2007–2008, structured asset products (SAPs) such as asset-backed securities (ABSs) and collateralized debt obligations (CDOs) as well as covered bonds provided between 25 and 65% of the funding for new residential assets originated in the US and Western Europe

(see, e.g., [4]). In most developed economies, SAP growth peaked by 2007 before declining rapidly due to a lack of liquidity in secondary markets and decreases in primary issuance (see [5] for more details). For example, SAP issuance in the US decreased from about US \$2 trillion in 2007 to around US \$400 billion in 2008. The impact of the financial crisis on securitization in emerging markets was more modest as initial growth had been more subdued.

The contribution of securitization to the financial crisis necessitated changes to banking regulation. In this regard, the introduction of Basel III capital and liquidity regulation includes elements that will potentially affect the incentives for banks to securitize assets (see, e.g., the Basel documents [2, 5–7]). There are several Basel III provisions that address areas of concern that were highlighted during the financial crisis and which supervisors determined as not adequately addressed under the previous framework (see, e.g., [6]). More specifically, in July 2009, the Basel Committee for Banking Supervision (BCBS) published enhancements to the Basel II framework that were intended to strengthen the framework and respond to lessons learned from the financial crisis (see [8] for more details). For instance, because of the higher degree of inherent risk in resecuritization exposures,

the BCBS significantly increased the risk weights applicable to such exposures under both the standardized (SA) and internal ratings based (IRB) approaches relative to the risk weights for other securitization exposures (see, e.g., [3, 9, 10]). As a result, the capital requirements for resecuritization have risen dramatically. In addition, to address the lack of appropriate due diligence on the part of investing institutions and deter them from relying solely on external credit ratings, the Basel framework now requires banks to meet specific operational criteria in order to use the risk weights specified in the Basel II securitization framework. During the financial crisis, credit rating agencies (CRAs) downgraded the ratings of many securitization tranches, including senior tranches, highlighting deficiencies in credit rating agency models originally used to determine the ratings. Capital requirements assigned to highly rated (e.g., AAA) senior and mezzanine securitization exposures were too low and this was illustrated by the poor performance of these securities (see, e.g., [9, 10]). As CRAs downgraded highly rated securitization exposures below investment grade, regulatory capital requirements increased rapidly and significantly due to the presence of cliff effects within the securitization framework (in this context, cliff effects refer to significant increases in capital requirements resulting from a change to a factor used to assign regulatory capital) (see, e.g., [9, 10]). The BCBS also revised the market risk rules to increase the level of capital that must be maintained against securitization exposures held in the trading book. In addition, the BCBS is reviewing whether the risk weights for all securitization exposures should be recalibrated, which could lead to higher capital requirements (see, for instance, the Basel papers [9, 10]).

The main motivation for this paper is that securitization has to be reestablished on a sound basis in order to support credit provision to the real economy and enhance banks' access to funding globally (see, e.g., [4]). Basel III would like to ensure an appropriate risk-sensitive and prudent capitalization of risks arising from securitization exposures while reducing cliff effects and mitigating mechanistic reliance on external credit ratings (see [3, 9]). The interplay between Basel III and asset securitization will be the central theme of this paper. As far as the contribution of this paper is concerned, we investigate the securitization of low- and high-quality assets (denoted by LQAs and HQAs, resp.) into CDOs via low-quality entities (LQEs) and high-quality entities (HQEs), respectively, (see, e.g., the BCBS publications [9, 10]).

1.1. Literature Review about Asset Securitization and Basel III. In this subsection, we provide literature reviews about asset securitization and Basel III capital and liquidity regulation (see [5] for more details).

1.1.1. Literature Review about Asset Securitization. Motivated by the 2007–2009 financial crisis, there is an ever-growing body of literature on LQA-related issues such as shocks to LQEs, investors, and CDOs via, for instance, rating changes. The contribution [11] studies the pricing of LQAs and related SAPs on the basis of data for the ABX.HE family of indices (see [12] for further details). This, of course, is a recurring theme in our contribution where we consider asset and CDO

pricing during the financial crisis. Moreover, [4] addresses the impact of speculative asset funding on the pricing of LQAs (measured by risk premia) and securities backed by these assets (measured by ABX.HE indices). In addition, the paper [13] extends a Kiyotaki-Moore-type model that shows how relatively small shocks might suffice to explain business cycle fluctuations, if credit markets are imperfect (see [14] for more details). Our work has a connection with this paper via the consideration of the effect of shocks on asset parameters although we do not emphasize the imperfection of credit markets (see [15] for additional analysis). The paper [16] studies the impact of penalties on LQ-loans (see, also, [17]). Here, asset prices and penalties are chosen simultaneously with the latter being associated with lower asset prices (see [15] for more details). The paper also contains discussions on prices and penalties and their relationship with loan-to-value ratios (see, also, [18] for more on house equity). In our contribution, we will use the framework introduced in [16] to show how a change in profit subsequent to a negative shock is influenced by subprime mortgage features (see, also, [19]).

1.1.2. Literature Review about Basel III. In July 2009, the BCBS introduced enhancements to the Basel II framework via [8]. This was done in order to address deficiencies identified during the 2007–2009 financial crisis. These measures primarily addressed immediate concerns over resecuritization and was part of reforms known as “Basel 2.5” (see the Basel documents [8, 9] for more details about securitization). The BCBS subsequently agreed to conduct a more fundamental review of the securitization framework, including its reliance on external ratings (see, e.g., [20]). The performance of and role played by securitization exposures during the financial crisis were a key motivation for studying this category of the capital framework. Subsequently, in [2], the BCBS noted that it was “conducting a more fundamental review of the securitization framework, including its reliance on external ratings.” The BCBS has now performed a broader review of the securitization framework for regulatory capital requirements with objectives motivated by events during the financial crisis (see, also, [7]). Furthermore, the consultative document [6] reflects the BCBS's proposal to revise the Basel framework's treatment of securitization exposures. In developing this proposal, the BCBS seeks to make capital requirements more prudent and risk sensitive, mitigate reliance on external credit ratings, and reduce cliff effects (see, e.g., [3, 6]). In particular, our paper adds to the debate about rating downgrades and shocks associated with them. The policy directions set out in [6] form part of the BCBS's broader agenda of reforming bank regulatory capital standards to address the lessons of the crisis. These proposals build on a series of reforms that the BCBS has delivered through Basel III and explain the approaches under consideration by the BCBS to revise the securitization framework (see [6, 21] for further discussion). As in our paper, the features of SPEs are discussed in some detail in [1].

1.2. Preliminaries about Asset Securitization. In this subsection, we provide preliminaries about low- and high-quality

TABLE 1: Defining features of LQAs and HQAs; source: [5].

Feature	LQAs	HQAs
External information		
External ratings	Low	High
Market data	Poor	Rich
Analysts' reports	Negative	Positive
Internal information		
Credit risk	High	Low
Liquidity risk	High	Low
Securitization exposure	Not understood	Understood
Financial obligations	Not easily met	Easily met

classification as well as LQA and HQA securitization (see Table 1 for more information).

1.2.1. Preliminaries about Low- and High-Quality Classification. As indicated in the BCBS documents [2, 7], the characterization of “high quality” is based both, on available external information, such as external ratings, market data, and analyst’s reports, and the entity’s own assessment of credit and liquidity risk, whereby the entity should demonstrate its understanding of the terms of the securitization exposure and the risks of the underlying collateral (see, e.g., [1, 5]). The entity would be required to demonstrate that the credit quality of the position is strong, with very low default risk, and is invulnerable to foreseeable events, implying that financial commitments would be met in a timely manner with a very high probability (see [6, 21] for further discussion). Where this determination could not be made, the position would be assumed to be “low quality.” In particular, in the Basel III document [1], the Joint Forum Working Group on Risk Assessment and Capital (JFWGRAC) consisting of the BCBS, International Organization of Securities Commissions (IOSC), and the International Association of Insurance Supervisors (IAIS) under the support of the Bank for International Settlements (BIS), make recommendations about such entities.

1.2.2. Preliminaries about LQA and HQA Securitization. In this paper, we have that the reference asset portfolios are both a means of generating CDOs and collateral for interentity sponsoring (see, e.g., [1]). Our paper quantifies the effects of unexpected negative shocks such as rating downgrades on asset price and input, CDO price and output, and profit in a Basel III context (see, also, [19]). For instance, the aforementioned result demonstrates how the proportional change in profit subsequent to a rating downgrade is influenced by LQA features such as asset rates. Finally, we present examples that illustrate that asset price is most significantly affected by unexpected negative shocks from asset rates, while, for CDO price, shocks to speculative asset funding, investor risk characteristics, and prepayment rate elicit statistically significant responses (compare with [3]).

LQAs were financed by securitizing these assets into SAPs such as ABSs and CDOs. The lower-rated tranches of low-quality ABSs formed 50 to 60% of the collateral for CDOs

during 2007. These were extremely sensitive to a deterioration in asset credit quality. Housing went through a classic inventory cycle with a worsening of the inventory-to-sales cycle being evident in the midst of the 2007–2009 financial crisis. When this inventory situation worsened, the risk that price would fall more rapidly deepened. The more substantial fall in prices accelerated the delinquency and foreclosure rate and spelt doom for the CDO market which was further revised in Basel III (see, e.g., the BCBS paper [3, 9]). We briefly describe the aforementioned SAPs in turn.

LQ-ABSs are quite different from other securitizations because of the unique features that differentiate low-quality assets from other assets. Like other securitizations, LQ-ABSs of a given transaction differ by seniority. But unlike other securitizations, the amount of *credit enhancement* for and the size of each tranche depend on the *cash flow* coming into the deal in a very significant way. The cash flow comes largely from prepayment of the reference asset portfolios through refinancing. What happens to the cash coming into the deal depends on *triggers* which measure (prepayment and default) performance of the reference asset portfolios. The triggers can potentially divert cash flows within the structure. In some case, this can lead to a leakage of protection for higher-rated tranches. Time tranching in low-quality transactions is contingent on these triggers. The structure makes the degree of credit enhancement dynamic and dependent on the cash flows coming into the deal.

In our case, a CDO issues debt and equity and uses the income to invest in financial assets such as assets and ABSs. It distributes the cash flow from its asset portfolio to holders of various liabilities—usually a capital structure consisting of equity or preferred shares, subordinated debt, mezzanine debt and AAA-rated senior debt, and borrowings—in set ways taking into account the relative seniority of the aforementioned liabilities. A key feature of such CDOs is that they are mainly constituted by ABS portfolios that are rated according to their prevailing credit risk and put into tranches (compare with [3]). CDO tranches include LQ- and Alt-A deals and consist of three categories, namely, senior, mezzanine, and equity or low tranches according to increasing credit risk. This risk is spread to investors who invest in these risky CDO tranches. It is difficult for investors to locate the risk exposure of these CDOs because of their complex design structure. Despite this, CDOs have additional structural credit protection which can be characterized as either cash flow or market value protection. Finally, all CDOs are created to fulfill a given purpose that can be classified as arbitrage, balance sheet, or origination.

1.3. Main Contributions and Outline. The main contributions about Basel III regulation and asset securitization in this paper are constituted by the answers to the questions listed below.

Question 1 (LQA securitization). How can we characterize the securitization of LQAs into CDOs by the LQE? For instance, can we determine the LQE cash flow constraint? (see Lemma 3 in Section 2.1).

Question 2 (HQA securitization). How can we characterize the securitization of HQAs into CDOs by HQEs? For instance, can we determine the HQEs’ cash flow constraint? (see Lemma 5 in Section 2.2).

Question 3 (LQE at steady state). How can we characterize the behavior of an LQE at steady state? (see Theorem 7 in Section 2.3).

Question 4 (dynamic multiplier: negative shocks to asset securitization). For a dynamic multiplier, how do negative shocks affect asset price and input, CDO price and output, and profit? (see Theorem 8 in Section 3.1).

Question 5 (static multiplier: temporary shocks to asset securitization). For a static multiplier, how do negative shocks affect asset price and input? (see Corollary 9 in Section 3.2).

Question 6 (example of subprime mortgage securitization). How can we provide an example to illustrate the main features of LQA securitization specifically for subprime mortgages? (see Corollary 10 in Section 4.1).

Question 7 (numerical examples of asset securitization). How can we provide numerical examples to illustrate the main results obtained in this paper? (see Section 4.2).

Question 8 (Basel III and asset securitization in general). In general, how does Basel III capital and liquidity regulation assist in eliminating flaws in existing asset securitization practice in banking? (see Sections 2, 3, and 4 for more details).

This paper is arranged as follows. Section 2 describes HQA and LQA securitization. Also, Section 3 sheds light on the effect of negative shocks like rating downgrades on the asset securitization process in a Basel III context while Section 4 provides numerical examples of the aforementioned. Finally, Section 5 provides some concluding remarks and possible topics for future research.

2. Low- and High-Quality Entities

In this section, we consider LQEs and HQEs and their equilibrium features. We study an economy consisting of LQAs with a fixed total supply of \bar{A} and CDOs that cannot be retained by the entity. In the sequel, for the sake of argument, we assume that the CDOs correspond to senior CDO tranches. In this model, CDOs are taken as the numeraire. There is a continuum of infinitely lived LQEs and HQEs, with population sizes 1 and n , respectively. Both these entities take one period to securitize assets into CDOs—the LQEs and HQEs produce CDOs from HQAs and LQAs, respectively—but they differ in their securitization technologies (refer to Table 1). At each date, t , there is a competitive spot market in which assets for CDOs are purchased by entities at a price of p_t^A . The only other market is a one-period credit market in which one CDO unit at date t is exchanged for a claim to $1 + r_t^B$ units of CDOs at date $t + 1$. These markets are opaque and are dominated by a handful of interests. During

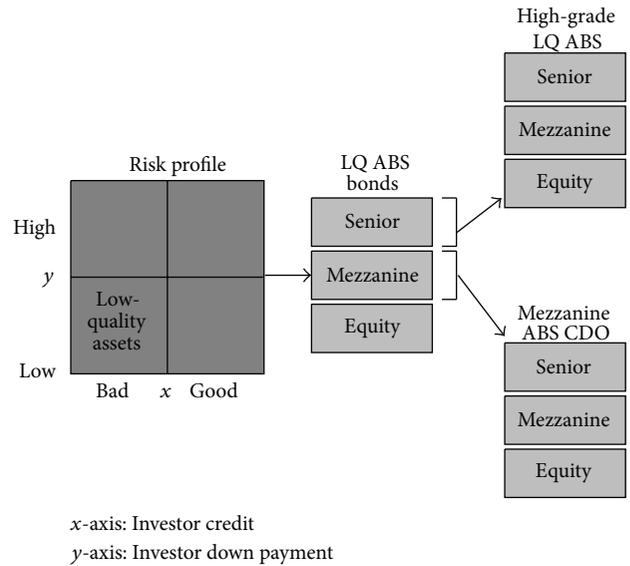


FIGURE 1: Chain of LQAs and their SAPs; source: [4].

the 2007–2009 financial crisis, because CDOs were lightly regulated their details often went undisclosed. This created major problems in the monitoring of these credit derivatives and the new regulatory framework in Basel III focuses on correcting such problems (see [9] for further explanation).

2.1. The LQE. Figure 1 illustrates the securitization of assets into ABSs and ABS CDOs by the LQE.

We notice from Figure 1 that LQAs are securitized into ABSs that, in turn, get securitized into ABS CDOs. As far as the latter is concerned, it is clearly shown that senior ABS bonds rated AAA, AA, and A constitute the *high-grade* ABS CDO portfolio. On the other hand, the mezzanine-rated ABS bonds are securitized into *mezzanine ABS CDOs*, since its portfolio is based on BBB-rated ABSs and their tranches which expose the portfolio to an increase in credit risk (see, e.g., [3]). From Figure 1, it is clear that the LQEs (and any other entities), rather than banks, hold assets and ABSs. As a result, there are reductions in the incentives of banks to play their traditional monitoring function. The fundamental role of banks in financial intermediation according to Basel III in the document [22] (see, also, [21]) makes them inherently vulnerable to liquidity risk, of both an institution-specific and market nature (see [5] for more details). Financial market developments have increased the complexity of liquidity risk and its management. During the early “liquidity phase” of the financial crisis that began in 2007, many banks—despite adequate capital levels—still experienced difficulties because they did not manage their liquidity in a prudent manner (see [5] for further discussion). The difficulties experienced by some banks, which, in some cases, created significant contagion effects on the broader financial system, were due to lapses in basic principles of liquidity risk measurement and management (see, e.g., [5] for more details). During the 2007–2009 financial crisis, systemic risk from CDOs was problematic. In this case, the default of one or more collateral

assets or bond classes generated a ripple effect on CDO defaults. Figure 1 suggest how this may have happened. The LQE is risk neutral, with its expected utilities being

$$\mathbf{E}_t \left(\sum_{s=0}^{\infty} \beta^s x_{t+s} \right), \quad \mathbf{E}_t \left(\sum_{t=0}^{\infty} \beta^t x_t \right), \quad (1)$$

where x_{t+s} and x_t are their respective LQE CDO consumption at dates $t + s$ and t , with \mathbf{E}_t denoting the expectation formed at date t . These entities have constant returns on scale securitization function of

$$C_{t+1} = S(A_t) \equiv (\mu + \nu) A_t, \quad \nu = c + r^f, \quad (2)$$

$$\left(\frac{\mu}{\mu + \nu} \right) < \beta,$$

where A_t are the input assets securitized at date t and C_{t+1} is the CDO output at date $t + 1$. Also, r^f is the fraction of assets that have refinanced and c the fraction of CDOs consumed. However, only μA_t of the CDO output is marketable. Here, νA_t is nonmarketable and can be consumed by the LQE. We introduce νA_t in order to avoid the situation in which the LQE continually postpones consumption. The ratio $\mu(\mu + \nu)^{-1}$ may be thought of as a technological upper bound on the LQE's retention rate. Since β is near 1, the inequality in (2) amounts to a weak assumption. We shall see later that this inequality ensures that in equilibrium the LQE will not want to consume more than illiquid CDOs as observed in Basel III (see, e.g., [21]). The overall return from investment, $\mu + \nu$, is high enough that all its marketable CDO outputs are used for investment. There is a further critical assumption we make about investing.

Assumption 1 (LQE CDO technology and labor). We assume that each LQE's CDO technology is distinct in the sense that, once securitization has started at date t with assets, A_t , only the LQE has the skill necessary for securitizing assets into CDOs at date $t + 1$, subject to the availability of appropriate technology and labor. Secondly, we assume that an LQE always has the option to withdraw its labor.

In other words, if the LQE were to withdraw its labor between dates t and $t + 1$, there would be no CDO output at $t + 1$. Assumption 1 leads to the fact that if an LQE is highly leveraged, it may find it advantageous to threaten the HQEs by withdrawing its labor and repudiating its debt contract. HQEs as intertently lenders protect themselves from the threat of repudiation by collateralizing the LQAs. However, because assets yield no SAPs without the LQE's labor, the asset liquidation value (outside value) are less than what the assets would earn under its control (inside value). Thus, following a repudiation, it is efficient for the LQE to persuade the sponsoring HQE into letting it keep the assets. In effect, the LQE can renegotiate a smaller loan. HQEs know of this possibility in advance, and so take care never to allow the size of the debt (gross of interest) to exceed the value of the collateral as in the following assumption.

Assumption 2 (credit limit). If at date t , the LQE possesses the assets, A_t , then it can borrow B_t in total, as long as the

repayment does not exceed the market value of assets at date $t + 1$ given by

$$(1 + r^B) B_t \leq p_{t+1}^A A_t, \quad (3)$$

where p_{t+1}^A represents the asset price in period $t + 1$ while A_t represents the LQE's asset holdings in period t . In this case, given rational expectations, agents have perfect foresight of future asset prices.

As noted in the Basel III document [21], the objective of the LCR is to promote the short-term resilience of the liquidity risk profile of banks (see [5] for additional). It does this by ensuring that banks have an adequate stock of unencumbered high-quality liquid assets (HQLA) that can be converted easily and immediately in private markets into cash to meet their liquidity needs for a 30-calendar-day liquidity stress scenario. Of course, during the 2007–2009 financial crisis, when monitoring incentives was reduced, it is unlikely that the HQE monitored the LQE closely. The LQE's balance sheet consists of assets and marketable securities (assets) as well as borrowings and capital (liabilities). Therefore, an LQE's balance sheet constraint can be represented at time t as

$$p_t^A A_{t-1} + B_t = B_t + K_t, \quad (4)$$

where p^A , A , B , B , and K represent the LQE's asset price, asset holdings, marketable securities, borrowings, and capital, respectively. As we have mentioned before, the entities' capital structure consisting of equity or preferred shares, subordinated debt, and mezzanine debt LQE enforces a price cap (PC), with the weighted average PC being denoted by \bar{p} (see, e.g., [4] for more details). In this case, we have that the CDO price is given by

$$p_t^C = \min [p_t^A, \bar{p}_t], \quad (5)$$

where C_{t-1} denotes the quantity of CDOs in period $t - 1$. Hedge funds and other sophisticated investors have incentives to manipulate the pricing and structuring of CDOs. Some studies suggest that CDO managers manipulate collateral in order to shift risks among various tranches. The potential for this can be clearly seen in (5) where the PC offers a means of changing collateral features that is important in determining the CDO price, p_t^C . During the 2007–2009 financial crisis, collateral according to Basel III was also manipulated via the violation of restrictions on asset portfolio composition, rating category, weighted average life, weighted average weighting factor, correlation factors, and the number of obligors (see [2]). Nevertheless, in our case, the value of assets in period t can be represented as

$$p_t^A A_{t-1} = B_t + K_t - B_t. \quad (6)$$

In general, the asset rate, r^A , for profit maximizing entities (see, also, [19]), may be represented as

$$r_t^A = r_t^L + \varrho_t, \quad (7)$$

where r^L is, for instance, the 6-month LIBOR rate and ρ is the risk premium, that is indicative of asset price (see, e.g., [3]). Next, the LQE's profit may be expressed as

$$\Pi_t = (r_t^A + c_t^p r_t^f - (1 - r_t^R) r_t^S) p_t^A A_{t-1} + r^B B_t - r^B B_t, \quad (8)$$

where r^A , c^p , r^f , r^R , r^S , r^B , and r^B represent the asset rate, prepayment costs, fraction of assets that refinance, recovery rate, default rate, returns on marketable securities, and borrowing rate in period t , respectively. In this case, asset value can be represented by

$$p_t^A A_{t-1} = \frac{\Pi_t - r^B B_t + r^B B_t}{r_t^A + c_t^p r_t^f - (1 - r_t^R) r_t^S}. \quad (9)$$

From (9), it is clear that, as recognized by Basel III regulation, even a relatively small default rate can trigger a crisis. The unwinding of contracts involving the securitization of such assets—such as CDO contracts—created serious liquidity problems during the 2007–2009 financial crisis as detailed in Basel III (see [5, 21, 22]). Since the CDO market was quite large, the crisis caused convulsions throughout global financial markets. By considering the above, we can deduce an appropriate LQE cash flow constraint in the following result.

Lemma 3 (LQE cash flow constraint). *Suppose that the credit constraint (3) as well as (6) to (9) holds. In this case, the LQE's cash flow is subject to the constraint*

$$\begin{aligned} \Pi_t \geq & (r_t^A + c_t^p r_t^f - (1 - r_t^R) r_t^S) p_t^A A_{t-1} + r^B B_t \\ & - p_{t+1}^A A_t + B_t. \end{aligned} \quad (10)$$

Proof. The proof follows from taking constraint (3) from Assumption 2 and (10) into consideration. \square

The LQE can expand its scale of securitization by investing in more assets. Consider an LQE that holds A_{t-1} assets at the end of date $t - 1$ and incurs a total debt of B_{t-1} . At date t , the LQE harvests μA_{t-1} marketable CDOs, which, together with a new loan B_t , is available to cover the cost of purchasing new assets, to repay the accumulated debt $(1 + r^B)B_{t-1}$ (which includes interest), and to meet any additional consumption $x_t - \nu A_{t-1}$ that exceeds the normal consumption of non-marketable output νA_{t-1} . The LQE's flow-of-funds constraint is thus

$$p_t^A (A_t - A_{t-1}) + (1 + r^B) B_{t-1} + x_t - \nu A_{t-1} = \mu A_{t-1} + B_t. \quad (11)$$

2.2. HQEs. For HQEs, Figure 2 shows the chain formed by HQAs, ABSs, and ABS CDOs.

As we proceed from left to right in Figure 2, HQAs are securitized into ABSs that, in turn, get securitized into ABS CDOs. Only the higher-grade ABS bonds rated AAA, AA, and A are securitized that make out the high-grade ABS CDO portfolio. Figure 2 also suggests that HQE ABSs and CDOs are not as risky as those of the LQE since the reference asset

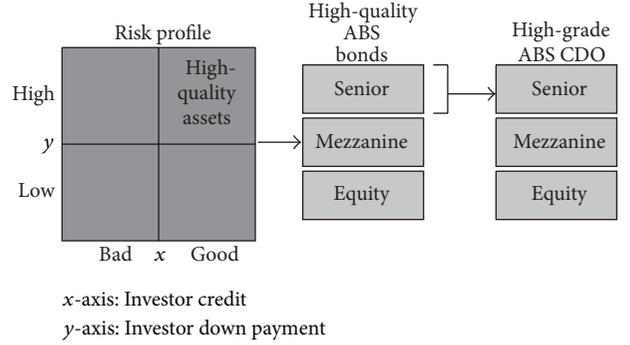


FIGURE 2: Chain of HQAs and their SAPs; source: [4].

portfolios have higher credit quality. HQE capital levels will also be greater than those of the LQE, in the sense that the LQE used its capital to provision for low-quality default. In this regard, we have the secondary effect of securitization where credit risk is transferred to investors. Basel III identifies a number of shortcomings within the current securitization framework some of which were categorised broadly as too low-risk weights for highly rated securitizations and too high-risk weights for low-rated senior securitization exposures (see [9] for detailed explanation). Furthermore, we assume that HQEs are risk neutral, with expected utilities:

$$\mathbf{E}_t \left(\sum_{s=0}^{\infty} \beta'^s x'_{t+s} \right), \quad \mathbf{E}_t \left(\sum_{t=0}^{\infty} \beta'^t x'_t \right), \quad (12)$$

where x'_{t+s} and x'_t are their respective consumptions of CDOs at dates $t + s$ and t . For the discount factors β^s and β'^s , we have that $0 < \beta^s, \beta'^s < 1$ and suppose that $\beta < \beta'$. This inequality ensures that, in equilibrium, the LQEs will not want to postpone securitization because they are relatively impatient (compare with [13] and the references contained therein). The following assumption is made for ease of computation.

Assumption 4 (HQE price, asset, default and borrowing rate).

For HQEs, suppose that p^A , r^A , and r^B are the asset price, asset rate, and borrowing rate, respectively. For all t , we assume that

$$p_t^A = p_t^A, \quad r_t^A = r_t^A, \quad r_t^B = r_t^B, \quad (13)$$

where p^A , r^A , and r^B are as before for HQEs. Also, we assume that the assets held by HQEs do not default or refinance.

In reality, this assumption may be violated since LQAs are more expensive than HQAs. However, this adjustment can be catered for in the sequel. We shall see that in equilibrium the LQE borrows from HQEs and that the rate of interest always equals the HQEs' constant rate of time preference so that

$$r^B = r_t^B \equiv \frac{1}{\beta'} - 1. \quad (14)$$

All HQEs have an identical securitization function that exhibits decreasing returns to scale. In a Basel III context, the

additional quantitative information that HQEs may consider disclosing could include customised measurement tools or metrics that assess the structure of HQEs' balance sheets, as well as metrics that project cash flows and future liquidity positions, taking into account off-balance sheet risks, which are specific to that HQE (see the BCBS publications [5, 21, 22]). In this case, per unit of population, an asset input of A'_t at date t yields an output of \bar{C}_{t+1} marketable CDOs at date $t + 1$, according to

$$\begin{aligned} \bar{C}_{t+1} &= P(A'_t), \quad \text{where } P' > 0, \\ P'' < 0, \quad P'\left(\frac{\bar{A}}{n}\right) &< \mu(1 + r^B) < P'(0). \end{aligned} \quad (15)$$

The last two inequalities in (15) are included to ensure that both the LQE and HQEs are producing in the neighborhood of the steady-state equilibrium. HQEs securitization does not require any specific skill nor do they produce any non-marketable CDOs. As a result, no HQE is credit constrained as noted in Basel III (see [9]). At date t , such entities' budget constraint can be expressed as

$$p_t^A(A'_t - A'_{t-1}) + (1 + r^B)B'_{t-1} + x'_t = P(A'_{t-1})_{t-1} + B'_t, \quad (16)$$

where x'_t is secondary securitization at date t , $(1 + r^B)B'_{t-1}$ is debt repayment, and B'_t is new interbank sponsoring. The HQEs' balance sheet constraint

$$p_t^A A'_{t-1} + B'_t = B'_t + K'_t, \quad (17)$$

is the same as in the case for an LQE, but the ratios of these variables will differ from those of the LQEs with much lower risk (compare with (4)). In this regard, assets held by HQEs are less risky, long-term loans with fixed rates (compare with [3]). Next, the HQEs' profit may be expressed as

$$\Pi'_t = r_t^A p_t^A A'_{t-1} + r^B B'_t - r^B B'_t, \quad (18)$$

where r^A , r^B , and r^B represent the asset rate, returns on marketable securities, and borrowing rate in period t , respectively. Notice that the prepayment cost is zero in the case for HQEs (see (10)). Thus, the value of HQAs is represented by

$$p_t^A A'_{t-1} = \frac{\Pi'_t - r^B B'_t + r^B B'_t}{r^A}. \quad (19)$$

We provide an appropriate HQE cash flow constraint in the following result.

Lemma 5 (HQE cash flow constraint). *Suppose that the credit constraint (3) as well as (16) to (19) holds. In this case, the HQEs' cash flow constraint is given by*

$$\Pi'_t \geq r^A p_t^A A'_{t-1} + r^B B'_t - p_{t+1}^A A'_t + B'_t. \quad (20)$$

2.3. Market Equilibrium. In equilibrium, B'_{t-1} and B'_t are negative, reflecting the fact that HQEs lend the LQE. For our purposes, market equilibrium is defined as follows.

Definition 6 (market equilibrium). Market equilibrium is a sequence of asset prices and allocations, debt, and securitization by the LQE and HQEs, given by

$$\{p_t^A, A_t, A'_t, B_t, B'_t, x_t, x'_t\}, \quad (21)$$

such that each LQE chooses (A_t, B_t, x_t) to maximize the expected discounted utilities of the LQE and HQEs subject to the securitization function, sponsoring constraint and flow-of-funds constraint given by (2), (3), and (11), respectively. On the other hand, each HQE chooses (A'_t, B'_t, x'_t) to maximize the above expected discounted utilities subject to the securitization function (15) and budget constraint (16). Also, in the case of the HQE, we have that the markets for assets, CDOs, and debt clear.

2.3.1. LQE at Equilibrium. In the sequel, we assume that the asset price bubble does not burst during securitization. In this case, it turns out that there is a locally unique perfect-foresight equilibrium path starting from initial values A_{t-1} and B_{t-1} in the neighborhood of the steady state. In this state, the LQE's marketable output, μA^* , is just enough to cover the interest on their debt, $r^B B^*$. Equivalently, the required screening costs per asset unit, u^* , equal the LQE's securitization of marketable output, μ . As a result, entities neither expand nor shrink. To further characterize entity equilibrium, we provide the following Kiyotaki-Moore-type result (see [14] for further discussion).

Theorem 7 (LQE behavior at steady state). *Assume that the asset bubble does not burst during the securitization process. In the neighborhood of the steady state, the LQE prefers to borrow up to the maximum and invest in assets, consuming no more than its current output of non-marketable CDOs. In this case, there is a unique steady-state (p^{A^*}, A^*, B^*) , with the associated transaction cost, u^* , being given by*

$$u^* = \frac{r^B}{1 + r^B} p^{A^*} = \frac{1}{1 + r^B} P' \left[\frac{1}{n} (\bar{A} - A^*) \right] = \mu, \quad (22)$$

$$B^* = \frac{\mu}{r^B} A^*. \quad (23)$$

Proof. The result follows by considering the LQE's marginal unit of marketable CDOs at date t . This entity can invest in $1/u_t$ assets, which yields c/u_t non-marketable CDOs and a/u_t marketable CDOs at date $t + 1$. The former are consumed while the latter are reinvested. This, in turn, yields

$$\frac{a}{u_t} \frac{c}{u_{t+1}} \quad (24)$$

non-marketable CDOs and

$$\frac{a}{u_t} \frac{a}{u_{t+1}} \quad (25)$$

marketable CDOs at date $t + 2$ and so on.

Next, we appeal to the principle of unimprovability, which states that we need to only consider single deviations at date t to show that this investment strategy is optimal. There are two alternatives open to the LQE at date t . Either it can save the marginal unit—equivalently, reduce its current borrowing by one—and use the return R to commence a strategy of maximum levered investment from date $t + 1$ onward or the LQE can simply consume the marginal unit. Its choice is equivalent to choosing one of the following consumption paths:

$$\text{Invest: } 0, \frac{c}{u_t}, \frac{a}{u_t} \frac{c}{u_{t+1}}, \frac{a}{u_t} \frac{a}{u_{t+1}} \frac{c}{u_{t+2}}, \dots \quad (26)$$

$$\text{Save: } 0, 0, r^B \frac{c}{u_t}, r^B \frac{a}{u_t} \frac{c}{u_{t+2}}, \dots \quad (27)$$

$$\text{Consumption: } 1, 0, 0, 0, \dots \quad (28)$$

at dates $t, t + 1, t + 2, t + 3, \dots$, respectively.

To complete the proof of the result, we need to confirm that, given the LQE's discount factor, β , consumption path (26) offers a strictly higher utility than (27) or (28), in the neighborhood of the steady-state. We shall be in a position to show this once we have found the steady-state value of u_t in (34) below.

Since the optimal a_t and b_t are linear in a_{t-1} and b_{t-1} , we can aggregate across entities to find the equations of motion of the aggregate asset holding and borrowing, A_t and B_t , respectively, of entities may be given by

$$A_t = \frac{1}{u_t} \left[(\mu + p_t^A) A_{t-1} - (1 + r^B) B_{t-1} \right], \quad (29)$$

$$B_t = \frac{1}{1 + r^B} p_{t+1}^A A_t. \quad (30)$$

Next, we consider market clearing. Since all HQEs have identical securitization functions, their aggregate asset demand equals A_t' times their population n . The sum of the aggregate demand for assets by the LQE and HQEs is equal to the total supply given by

$$\bar{A} = A_t + nA_t'. \quad (31)$$

In this case, from (35), we obtain the asset market (clearing) equilibrium condition

$$u_t = p_t^A - \frac{p_{t+1}^A}{1 + r^B} = u(A_t), \quad (32)$$

$$\text{where } u(A) \equiv \frac{1}{1 + r^B} P' \left[\frac{1}{n} (\bar{A} - A) \right].$$

Next, it is useful to look at the steady-state equilibrium. From (29), (30), and (32), it is easily shown that there is a unique steady-state (p^*, A^*, B^*) , with associated steady-state transaction cost u^* , where (22) and (23) hold. \square

Theorem 7 postulates that at each date t , the LQE's optimal choice of (A_t, B_t, x_t) satisfies $x_t = \nu A_{t-1}$ in (11), and the borrowing constraint (3) is binding so that

$$B_t = \frac{p_{t+1}^A}{1 + r^B} A_t,$$

$$A_t = \frac{1}{p_t^A - 1/(1 + r^B) p_{t+1}^A} \left[(\mu + p_t^A) A_{t-1} - (1 + r^B) B_{t-1} \right]. \quad (33)$$

Here, the term $(\mu + p_t^A) A_{t-1} - (1 + r^B) B_{t-1}$ is the LQE's net worth at the beginning of date t . This corresponds to the value of its marketable CDOs and assets held from the previous period net of debt repayment. In effect, (33) says that the LQE uses all its net worth to finance the difference between the asset price, p_t^A , and the amount the entity can borrow against each asset unit, $p_{t+1}^A / 1 + r^B$. This difference is given by

$$u_t = p_t^A - \frac{p_{t+1}^A}{1 + r^B} \quad (34)$$

and can be thought of as the screening costs required to purchase an asset unit. The equations of motion of the aggregate asset holding and borrowing, A_t and B_t , respectively, of entities may be given by (29) and (30). Notice from (29) that if, for example, present and future asset prices, p_t^A and p_{t+1}^A , were to rise, then the LQE's asset demand at date t would also rise—provided that leverage is sufficient that debt repayments $(1 + r^B) B_{t-1}$ exceed current output μA_{t-1} , which holds in equilibrium. The usual notion that a higher asset price p_t^A reduces the LQE's demand is more than offset by the facts that they can borrow more when p_{t+1}^A is higher and their net worth increases as p_t^A rises. Even though the required screening costs, u_t , per asset unit rises proportionately with p_t^A and p_{t+1}^A , the LQE's net worth is increasing more than proportionately with p_t^A because of the leverage effect of the outstanding debt.

2.3.2. HQEs at Equilibrium. Next, we examine the HQEs' behavior at equilibrium. Such entities are not credit constrained, and so their asset demand is determined at the point at which the present value of the marginal product of assets is equal to the transaction fee associated with assets (refer to Basel III in [1]). In this case, we have that

$$u_t = p_t^A - \frac{p_{t+1}^A}{1 + r^B} = \frac{1}{1 + r^B} P' (A_t'). \quad (35)$$

In the model, u_t is both the HQEs' opportunity cost of holding an asset unit and the required screening costs per unit of assets held by the LQE.

2.3.3. LQE and HQE Market Clearing. In this subsection, we consider market clearing that refers to either a simplifying assumption made that markets always go to where the assets supplied equal the assets demanded or the process of getting there via price adjustment. A market clearing price is the price

of goods or a service at which assets supplied are equal to assets demanded, also called the equilibrium price. Another market clearing price may be a price below equilibrium price to stimulate demand.

Since all HQEs have identical securitization functions (see Basel III and revision to securitization in [9]), their aggregate asset demand equals A'_t times their population n . The sum of the aggregate demand for assets by the LQE and HQEs is equal to the total supply given by (31). In this case, from (35), we obtain the asset market (clearing) equilibrium condition (32). The function $u(\cdot)$ is increasing. This arises from the fact that if the LQE's asset demand, A_t , goes up, then in order for the asset market to clear, the HQEs' demand has to be stymied by a rise in the transaction fee, u_t . Given that the HQEs have linear preferences and are not credit constrained, in equilibrium they must be indifferent about any path of consumption and debt (or credit). In this case, the interest rate equals their rate of time preference so that

$$r^B = \frac{1}{\beta'} - 1. \quad (36)$$

Moreover, given (32), the CDO markets and credit are in equilibrium.

We restrict attention to perfect-foresight equilibria in which, without unanticipated shocks, the expectations of future variables realize themselves. For a given level of the LQE's asset holding and debt at the previous date, A_{t-1} and B_{t-1} , an equilibrium from date t onward is characterized by the path of asset price, LQE asset holding, and interbank borrowings given by

$$\left\{ (p_{t+s}^A, A_{t+s}, B_{t+s}) \geq 0 \right\}, \quad (37)$$

satisfying (29), (30), and (32) at dates $t, t+1, t+2, \dots$

2.4. LQE and HQE Equilibrium Summary. Figure 3 displays the main features of market equilibrium for the LQE and HQEs.

The horizontal axis represents LQA and HQA demand from the left-hand side and right-hand-side, respectively. We note that the total asset supply is denoted by \bar{A} . The vertical axis represents the marginal products of assets for LQE and HQEs given by $\mu + \nu$ and $P'(A'/n)$, respectively. The HQEs' marginal product decreases with asset use. If there are no credit limits, then E^0 would be the best allocation for where the LQE and HQE marginal products are in equilibrium. The asset price would then be $p^0 = (\mu + \nu)(r^B)^{-1}$. On the other hand, when credit limits exist, then the equilibrium is at point E^* , where the marginal product of the LQE is greater than that of the HQE. In this case, we have

$$\mu + \nu > P' \left[\frac{(\bar{A} - A^*)}{n} \right] = \mu(1 + r^B). \quad (38)$$

This means that the LQE's asset use is not enough. The output of CDOs per period in equilibrium is represented by the light gray area under the thick line, whereas the gray triangle represents the CDO loss per period. In this case, CDO output increases relative to LQA holdings. If A_t increases, then the CDO output will also increase in period $t+1$.

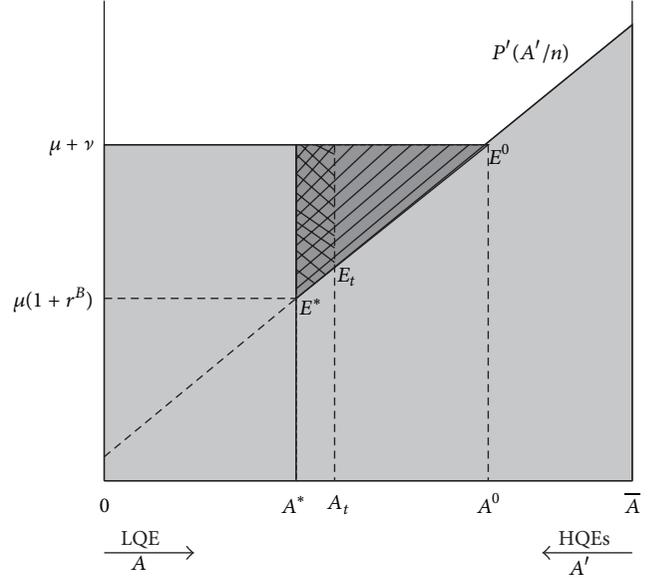


FIGURE 3: LQE and HQE market equilibrium.

3. Asset Securitization Shocks

In this section, we describe the effect of unexpected negative shocks on LQA price and input, CDO price in a Basel III context and output, and profit (see, e.g., [19]). In this regard, two kinds of multiplier processes are considered. The first is the within-period or static multiplier process. Here, the shock such as a ratings downgrade reduces the net worth of the constrained LQE and compels it to reduce its asset demand. In this case, by keeping the future constant, the transaction fees decrease to clear the market and the asset price drops by the same amount. In turn, this lowers the value of the LQE's existing assets and reduces their net worth even more. Since the future is not constant, this multiplier misses the intuition offered by the more realistic intertemporal or dynamic multiplier. In this case, the decrease in asset prices results from the cumulative decrease in present and future opportunity costs, stemming from the persistent reductions in the constrained LQE's net worth and asset demand, which are in turn exacerbated by a decrease in asset price and net worth in period t .

3.1. Dynamic Multiplier: Response to Temporary Shock. In order to understand the effect of unexpected inter-temporal shocks on the economy, suppose at date $t-1$ that it is in steady state with

$$A^* = A_{t-1}, \quad B^* = B_{t-1}. \quad (39)$$

3.1.1. Dynamic Multiplier: Shock Equilibrium Path. We introduce an unexpected inter-temporal shock where the CDO output of the LQE and HQEs at date t are $1 - \Sigma$ times their expected levels. In order for our model to resonate with the 2007–2009 financial crisis, we take Σ to be positive. Eventually, the LQE and HQEs' securitization technologies between dates t and $t+1$ (and thereafter) return to (2) and (15),

respectively. Combining the market-clearing condition (32) with LQA demand under a temporary shock and borrowing constraint given by (29) and (30), respectively, we obtain

$$u(A_t)A_t = (\mu - \mu\Sigma + p_t^A - p^{A*})A^*, \quad (\text{date } t), \quad (40)$$

$$u(A_{t+s})A_{t+s} = \mu A_{t+s-1}, \quad (\text{dates } t+1, t+2, \dots). \quad (41)$$

The formulae (40) and (41) imply that at each date the LQE can hold assets up to the level A at which the required cost of funds, $u(A)A$, is covered by its nett worth. Notice that in (41), at each date $t+s$, $s \geq 1$, the LQE's nett worth is just its ambient output of marketable CDOs, μA_{t+s-1} . In this case, from the borrowing constraint at date $t+s-1$, the value of the LQE's assets at date $t+s$ is exactly offset by the amount of debt outstanding. From (40), subsequent to the shock, we see that the LQE's nett worth at date t is more than only their current output given by

$$(1 - \Sigma)\mu A^* \quad (42)$$

because p_t^A changes in response to the shock and unexpected capital gains of

$$(p_t^A + p^{A*})A^* \quad (43)$$

result in their asset holdings. In this case, the asset value held from date $t-1$ is now $p_t^A A^*$, while the debt repayment is

$$(1 + r^B)B^* = p^{A*}A^*. \quad (44)$$

To find closed-form expressions for the new equilibrium path, we take Σ to be small and linearized around the steady state. In the sequel, we let the proportional changes in A_t , p_t^A , and Π_t relative to their steady-state values A^* , p^{A*} , and Π^* , respectively, be given by

$$\widehat{A}_t = \frac{A_t - A^*}{A^*}, \quad \widehat{p}_t^A = \frac{p_t^A - p^{A*}}{p^{A*}}, \quad \widehat{\Pi}_t = \frac{\Pi_t - \Pi^*}{\Pi^*}, \quad (45)$$

respectively. For our purpose, assume that steady-state profit, Π_t^* , represents profit when the asset value and borrowings are in steady state (compare with [19]). Thus, steady-state profit for the LQE and HQEs are represented by

$$\begin{aligned} \Pi_t^* &= (r_t^A + c_t^p r_t^f - (1 - r_t^R) r_t^S) p_t^{A*} A_{t-1}^* + r^B B_t - r^B B_t^*, \\ \Pi_t'^* &= r^A p_t^{A*} A_{t-1}' + r^B B_t' - r^B B_t'^*, \end{aligned} \quad (46)$$

respectively. Then, by using the steady state, transaction fee, and (22), we have from (40) and (41) that

$$\left(1 + \frac{1}{\eta}\right) \widehat{A}_t = \frac{1 + r^B}{r^B} \widehat{p}_t^A - \Sigma, \quad (\text{date } t), \quad (47)$$

$$\left(1 + \frac{1}{\eta}\right) \widehat{A}_{t+s} = \widehat{A}_{t+s-1}, \quad (48)$$

for $s \geq 1$, (dates $t+1, t+2, \dots$),

where $\eta > 0$ denotes the elasticity of the residual asset supply to the LQE with respect to the transaction fee at the steady state. Here, we have

$$\begin{aligned} \frac{1}{\eta} &= \frac{d \log u(A)}{d \log A} \Big|_{A=A^*} \\ &= - \frac{d \log P'(A')}{d \log A'} \Big|_{A'=1/n(\bar{A}-A^*)} \times \frac{A^*}{\bar{A} - A^*}. \end{aligned} \quad (49)$$

The right-hand side of (47) divides the change in the LQE's nett worth at date t into two components: the direct effect of the securitization shock, Σ , and the indirect effect of the capital gain arising from the unexpected rise in price, \widehat{p}_t^A . In order to compute (47), from (22), (40), and (45), we have that the RHS of (47) is given by

$$\frac{1 + r^B}{r^B} \widehat{p}_t^A - \Sigma = \frac{u(A_t)A_t}{\mu A^*} - 1. \quad (50)$$

Also, from (22), (40), and (45), we have that the LHS of (47) is given by

$$\begin{aligned} &\left(1 + \frac{1}{\eta}\right) \widehat{A}_t \\ &= \left(\frac{A_t - A^*}{A^*}\right) + \left(- \frac{d \log P'(A')}{d \log A'} \Big|_{A'=1/n(\bar{A}-A^*)}\right. \\ &\quad \left. \times \frac{A^*}{\bar{A} - A^*}\right) \left(\frac{A_t - A^*}{A^*}\right). \end{aligned} \quad (51)$$

Crucially, the impact of \widehat{p}_t^A is scaled up by the factor $(1 + r^B)/(r^B)$ because of leverage. Furthermore, the factor $1 + 1/\eta$ on the left-hand sides of (47) and (48) reflects the fact that as LQA demand rises, the transaction fee must rise for the market to clear and, this in turn, partially chokes off the increase in the LQE's demand. The key point to note from (48) is that, except for the limit case of a perfectly inelastic supply $\eta = 0$, the effect of a shock persists into the future. The reason is that the LQE's ability to invest at each date $t+s$ is determined by how much screening costs they can afford from their nett worth at that date, which in turn is historically determined by their level of securitization at the previous date $t+s-1$.

3.1.2. Dynamic Multiplier: Asset Price and Input, CDO Price and Output, and Profit. We will determine the size of the initial change in the LQE's asset holdings, \widehat{A}_t , which, from (47), can be jointly determined with the change in asset price, \widehat{p}_t^A . Also, we would like to compute the proportional change in CDO output and profit denoted by \widehat{C}_{t+1} and $\widehat{\Pi}_t$, respectively (see, e.g., [19]).

Theorem 8 (dynamic multiplier: shocks to asset price and input, CDO price and output, and profit). *Assume that the asset bubble does not burst during the securitization process and that $p_t^A \leq \bar{p}$, for all t in (5). In this case, one has that the*

proportional change in asset price and input, CDO price and output, and profit subject to a negative shock is given by

$$\widehat{p}_t^A = -\frac{1}{\eta}\Sigma, \quad (52)$$

$$\widehat{A}_t = -\frac{1}{1 + 1/\eta} \left(1 + \frac{1 + r^B}{\eta r^B} \right) \Sigma, \quad (53)$$

$$\widehat{p}_t^C = -\frac{1}{\eta}\Sigma \quad (54)$$

$$\widehat{C}_{t+1} = \frac{\mu + \nu - (1 + r^B)\mu}{\mu + \nu} \frac{(\mu + \nu) A^*}{C^*} \widehat{A}_t, \quad (55)$$

$$\widehat{\Pi}_t = \frac{(r_t^A + c_t^p r_t^f - (1 - r_t^R) r_t^S) p_t^A A_{t-1} + r^B B_t - r^B B_t^*}{(r_t^A + c_t^p r_t^f - (1 - r_t^R) r_t^S) p_t^A A_{t-1}^* + r^B B_t - r^B B_t^*} - 1, \quad (56)$$

respectively.

Proof. Since there are no bursting bubbles, (32) intimates that the asset price, p_t^A , is the discounted sum of future opportunity costs given by

$$u_{t+s} = u(A_{t+s}), \quad s \geq 0. \quad (57)$$

Linearizing around the steady state and then substituting from (48) given by

$$\left(1 + \frac{1}{\eta} \right) \widehat{A}_{t+s} = \widehat{A}_{t+s-1}, \quad \text{for } s \geq 1, \quad (58)$$

(dates $t + 1, t + 2, \dots$),

we obtain

$$\begin{aligned} \widehat{p}_t^A &= \frac{1}{\eta} \frac{r^B}{1 + r^B} \sum_{s=0}^{\infty} (1 + r^B)^{-s} \widehat{A}_{t+s} \\ &= \frac{1}{\eta} \frac{r^B}{1 + r^B} \frac{1}{1 - \eta/(1 + r^B)(1 + \eta)} \widehat{A}_t. \end{aligned} \quad (59)$$

We have to verify that

$$\begin{aligned} \sum_{s=0}^{\infty} (1 + r^B)^{-s} \widehat{A}_{t+s} &= \frac{1}{1 - \eta/(1 + r^B)(1 + \eta)} \widehat{A}_t \\ &= \frac{(1 + r^B)(1 + \eta)}{(1 + r^B)(1 + \eta) - \eta} \widehat{A}_t \end{aligned} \quad (60)$$

is standard for infinite series. The dynamic multiplier

$$\left[1 - \frac{\eta}{(1 + r^B)(1 + \eta)} \right]^{-1} = \frac{(1 + r^B)(1 + \eta)}{(1 + r^B)(1 + \eta) - \eta} \quad (61)$$

in (59) captures the effects of persistence in entities' reference asset portfolio holdings and has a dramatic effect on the sizes of \widehat{p}_t^A and \widehat{A}_t . In order to find \widehat{p}_t^A and \widehat{A}_t in terms of the size of

the shock Σ , we utilize (47) and (59). The calculations above verify that (52) and (53) as well as (54) hold.

Next, we prove that (55) holds. As we saw in Figure 3, aggregate CDO output—the combined harvest of the LQE and HQEs—is positively correlated to the LQA holdings, since such entities' marginal product is higher than the HQEs'. Suppose that the proportional change in aggregate output, \widehat{C}_{t+s} , is given (compare with \widehat{p}_t^A and \widehat{A}_t above) by

$$\widehat{C}_t = \frac{C_t - C^*}{C^*}, \quad C_t = (\widehat{C}_t + 1) C^*, \quad C^* = \frac{C_t}{\widehat{C}_t + 1}. \quad (62)$$

In this case, we can verify that at each date $t + s$ the proportional change in aggregate output, \widehat{C}_{t+s} , is given by

$$\widehat{C}_{t+s} = \frac{\mu + \nu - (1 + r^B)\mu}{\mu + \nu} \frac{(\mu + \nu) A^*}{C^*} \widehat{A}_{t+s-1}, \quad \text{for } s \geq 1. \quad (63)$$

The RHS of (63) yields

$$\begin{aligned} &\frac{\mu + \nu - (1 + r^B)\mu}{\mu + \nu} \frac{(\mu + \nu) A^*}{C^*} \widehat{A}_{t+s-1} \\ &= \frac{C_{t+s} - [(1 + r^B)\mu A_{t+s-1} + (\mu + \nu - (1 + r^B)\mu) A^*]}{C^*}. \end{aligned} \quad (64)$$

In order to verify (63), we have to show that

$$\begin{aligned} C^* &= (1 + r^B)\mu A_{t+s-1} + (\mu + \nu - (1 + r^B)\mu) A^* \\ &= [(1 + r^B)\mu \widehat{A}_{t+s-1} + \mu + \nu] A^*. \end{aligned} \quad (65)$$

This, of course, is true since

$$\begin{aligned} C^* &= (\mu + \nu) A^*, \quad (1 + r^B)\mu A_{t+s-1} = (1 + r^B)\mu A^* \\ \text{or } A_{t+s-1} &= A^*. \end{aligned} \quad (66)$$

The proportional change in profit, $\widehat{\Pi}_t$, given by (56), is a direct consequence of its definition. \square

The proportional changes in CDO output, \widehat{C} , and profit, $\widehat{\Pi}$, given by (55) and (56), respectively, have important connections with the 2007–2009 financial crisis and Basel III. For mortgage loans, this relationship stems from the terms involving the asset and prepayment rates, refinancing, and house equity.

At date t , (52) tells us that, in percentage terms, the effect on the asset price is of the same order of magnitude as the temporary securitization shock. As a result, the effect of the shock on the LQA holdings at date t is large. In this case, the multiplier in (53) exceeds unity, and can do so by a sizeable margin, thanks to the factor $(1 + r^B)(r^B)^{-1}$. In terms of (47), the indirect effect of \widehat{p}_t^A , scaled up by the leverage factor $(1 + r^B)(r^B)^{-1}$, is easily enough to ensure that the overall effect on \widehat{A}_t , is more than one-for-one.

3.2. Static Multiplier: Response to Temporary Shocks. At the beginning of this section, we made a distinction between static and dynamic multipliers. Imagine, hypothetically, that there was no dynamic multiplier. In this case, suppose p_{t+1}^A were artificially pegged at the steady-state level p^{A*} . Equation (47) would remain unchanged. However, the right-hand side of (59) would contain only the first term of the summation—the term relating to the change in transaction fee at date t —so that the multiplier (61) would disappear. Combining the modified equation, we have

$$\widehat{p}_t^A = \left[\frac{r^B}{\eta(1+r^B)} \right] \widehat{A}_t. \quad (67)$$

The following result follows from the above.

Corollary 9 (static multiplier: shocks to asset price and input). *For the static multiplier, suppose that the hypothesis of Theorem 8 holds. Then, one has that*

$$\widehat{p}_t^A \Big|_{p_{t+1}^A = p^{A*}} = -\frac{r^B}{\eta(1+r^B)} \Sigma, \quad (68)$$

$$\widehat{A}_t \Big|_{p_{t+1}^A = p^{A*}} = -\Sigma. \quad (69)$$

Proof. We prove the result by considering (47) and (59) where the changes in the asset price and the LQA holdings can be solely traced to the static multiplier. \square

Subtracting (68) from (52), we find that the additional movement in asset price attributable to the dynamic multiplier is $(1+r^B)^{-1}$ times the movement due to the static multiplier. And a comparison of (53) with (69) shows that the dynamic multiplier has a similarly large proportional effect on LQA holdings. The term

$$\frac{\mu + \nu - (1+r^B)\mu}{\mu + \nu} \quad (70)$$

reflects the difference between LQA (equal to $\mu + \nu$) and HQA securitization (equal to $(1+r^B)\mu$ in the steady state). The ratio $(\mu + \nu)A^*C^{*-1}$ is the share of the LQE's output. If aggregate securitization was measured by $C_{t+s} \overline{A}^{-1}$, it would be persistently above its steady-state level, even though there are no positive securitization shocks after date t . The explanation lies in a composition effect. In this regard, there is a persistent change in asset usage between the LQE and HQEs, which is reflected in increased aggregate output.

4. Illustrative Examples of Asset Securitization

In this section, we present examples of asset securitization. Firstly, we consider a LQA securitization example involving subprime mortgages. Next, we illustrate LQE and HQE equilibrium from Section 2 as well as the effects of shocks to asset and CDO prices as discussed in Section 3.

4.1. Example of Subprime Mortgage Securitization. In this subsection, we provide a specific example of LQA securitization involving subprime mortgages (see, e.g., [4, 15]). For such securitization, we bring into play the main results contained in [16]. Subprime mortgages are usually adjustable rate mortgages (ARMs), where high step-up rates are charged in period $t+1$ after low teaser rates apply in period t . Secondly, this higher step-up rate causes an incentive to refinance in period $t+1$. Refinancing is subject to the fluctuation in house prices. When house prices rise, the entity is more likely to refinance. This means that investors could receive further assets with lower interest rates as house prices increase. Thirdly, a high prepayment penalty is charged to dissuade investors from refinancing (see [4] for more details).

In subprime mortgage context, the paper [16] provides a relationship between the mortgage rate, r^M , loan-to-value ratio (LTVR), L , and prepayment cost, c^P , by means of the simultaneous equations model

$$\begin{aligned} r_t^M &= \alpha^0 L_t + \alpha^1 c_t^P + \alpha^2 X_t + \alpha^3 Z_t^M + u_t, \\ L_t &= \psi^1 r_t^M + \psi^2 X_t + \psi^3 Z_t^L + v_t, \\ c_t^P &= \gamma^1 r_t^M + \gamma^2 X_t + \gamma^3 Z_t^P + w_t. \end{aligned} \quad (71)$$

Investors typically have a choice of r^M and L , while the choice of c^P triggers an adjustment to the mortgage rate, r^M . Thus, L and c^P are endogenous variables in the r^M -equation. There is no reason to believe that L and c^P are simultaneously determined. Therefore, c^P does not appear in the L -equation and L does not make an appearance in the c^P -equation. From [16], X comprises explanatory variables such as asset characteristics (owner occupation, asset purpose, and documentation requirements); investor characteristics (income and Fair Isaac Corporation (FICO) score); and distribution channel (broker origination). The last term in each equation Z^M , Z^L , or Z^P comprises the instruments excluded from either of the other equations. Reference [16] points out that the model is a simplification with other terms such as type of interest rate, the term to maturity, and distribution channel possibly also being endogenous.

Corollary 10 (dynamic multiplier: shocks to profit for subprime mortgages). *Suppose that the hypothesis of Theorem 8 holds. Then, the relative change in profit may be expressed in terms of r^M , c^P , and L as*

$$\widehat{\Pi}_t(r^M) = \frac{F_t p_t^A A_{t-1} + r^B B_t - r^B B_t}{F_t p_t^{A*} A_{t-1}^* + r^B B_t - r^B B_t^*} - 1, \quad (72)$$

$$\widehat{\Pi}_t(c^P) = \frac{G_t p_t^A A_{t-1} + r^B B_t - r^B B_t}{G_t p_t^{A*} A_{t-1}^* + r^B B_t - r^B B_t^*} - 1, \quad (73)$$

$$\widehat{\Pi}_t(L) = \frac{H_t p_t^A A_{t-1} + r^B B_t - r^B B_t}{H_t p_t^{A*} A_{t-1}^* + r^B B_t - r^B B_t^*} - 1, \quad (74)$$

respectively. Here, one has in (72), (73), and (74) that

$$\begin{aligned}
F_t &= r_t^M (1 + \gamma^1 r_t^f) + (\gamma^2 X_t + \gamma^3 Z_t^{c^p} + w_t) r_t^f \\
&\quad - (1 - r_t^R) r_t^S, \\
G_t &= c_t^p \left(\frac{1}{\gamma^1} + r_t^f \right) - \frac{1}{\gamma^1} (\gamma^2 X_t + \gamma^3 Z_t^{c^p} + w_t) \\
&\quad - (1 - r_t^R) r_t^S, \\
H_t &= \left[\gamma^1 \left\{ \frac{1}{\psi^1} L_t - \frac{\psi^2}{\psi^1} X_t - \frac{\psi^3}{\psi^1} Z_t^L - \frac{1}{\psi^1} v_t \right\} + \gamma^2 X_t + \gamma^3 Z_t^{c^p} \right. \\
&\quad \left. + w_t \right] [\gamma^1 + r_t^f] + \alpha^0 L_t + \alpha^2 X_t + \alpha^3 Z_t^{r^M} \\
&\quad + u_t - (1 - r_t^R) r_t^S,
\end{aligned} \tag{75}$$

respectively.

The most important contribution of the aforementioned result is that it demonstrates how the proportional change in profit subsequent to a negative shock is influenced by quintessential low-quality asset features such as asset and prepayment rates, refinancing, and house equity given by r^M , c^p , r^f , and L , respectively. The default rate is also implicitly embedded in formulas (72) to (74) in Corollary 10. In this regard, by consideration of simultaneity in the choice of r^M and c^p , it is possible to address the issue of possible bias in estimates of the effect of c^p on r^M .

4.2. Numerical Examples of Asset Securitization. In this subsection, we provide numerical examples to illustrate LQE and HQE equilibrium as described in Section 2 as well as the effects of shocks on asset and CDO prices as in Section 3. Initial asset securitization parameter choices are given in Table 2.

The financial crisis has exposed the limits of liability management and the proposed regulation will make the retreat from liability management permanent. To a much greater extent than at any time since the 1970s, banks will be forced back towards “asset management,” in other words towards a business model in which balance sheet size is determined from the liabilities side of the balance sheet, by the amount of funding which the bank can raise, and in which asset totals have to be adjusted to meet the available liabilities. This amounts to a “macroprudential” policy—that is, a policy designed to prevent credit creation from spiralling out of control as it did in the run-up to the recent crisis as expressed in Basel III on the the macroprudential overlay (see, e.g., [21, 22]).

4.2.1. Numerical Example: LQE and HQE Equilibrium. Suppose that the LQE’s and HQE’s deposits, borrowings, marketable securities and capital are equal. In this case, notice that the LQE’s and HQE’s asset holdings, A and A' , are a

TABLE 2: Asset securitization parameter choices.

Parameter	Value
μ	0.002
$\frac{P_{t+1}}{A_t}$	0.113
r^R	720 000
r^B	0.5
B_{t-1}	\$2 600
r^S	0.205
Σ	0.002
ν	0.2
c^p	0.03
\bar{A}_{t-1}	460 000
r^S	0.15
r^B	0.2
K_t	\$3 000
C^*	240 000
α	0.3
r^f	0.2
r^A	0.061
B_t	\$4 800
B_t	\$5 000
n	1
$P(A'_{t-1})$	240 000

proportion, α and $1-\alpha$, of the aggregate assets, \bar{A} , respectively. Thus, we have that

$$A = \alpha \bar{A} = 0.3 \times 720000 = 216000;$$

$$\begin{aligned}
A' &= (1 - \alpha) \bar{A} = (1 - 0.3) 720000 \\
&= 504000.
\end{aligned} \tag{76}$$

We compute the LQE’s debt obligation output in period $t + 1$ by considering the securitization function (2). Therefore, the CDO output can be computed by

$$\begin{aligned}
C_{t+1} &= (\mu + \nu) A_t = (\mu + \nu) \alpha \bar{A}_t \\
&= (0.002 + 0.2) \times 0.3 \times 720000 = 43632.
\end{aligned} \tag{77}$$

Next, the upper bound of the LQE’s retention rate should be less that the discount factor β ; thus,

$$\beta \left(\frac{0.002}{0.002 + 0.2} \right) = 0.0099099. \tag{78}$$

The value of LQAs in period t is computed by using (6). Thus, we have

$$\begin{aligned}
P_t^A A_{t-1} &= D_t + B_t + K_t - B_t = 1200 + 4800 + 3000 - 5000 \\
&= 4000.
\end{aligned} \tag{79}$$

The asset price in period t is therefore

$$\begin{aligned}
P_t^A &= \frac{4000}{A_{t-1}} = \frac{4000}{\alpha \bar{A}_{t-1}} = \frac{4000}{(0.3 \times 460000)} \\
&= 0.0289855072.
\end{aligned} \tag{80}$$

The LQE's profit is computed by considering the cash flow constraint (10):

$$\begin{aligned}\Pi_t &= (r^A + c_t^p r_t^f) p_t^A A_{t-1} + r^B B_t - r^D D_t - r^B B_t \\ \Pi_t &= (0.061 + 0.03 \times 0.2) \times 4000 + 0.205 \times 5000 - 0.205 \\ &\quad \times 1200 - 0.2 \times 4800 = 87.\end{aligned}\quad (81)$$

Furthermore, the LQE's profit is subject to the constraint (15); thus,

$$\begin{aligned}\Pi_t &= (0.061 + 0.03 \times 0.2) \times 4000 + 0.205 \times 5000 - 0.205 \\ &\quad \times 1200 - 0.113 \times 0.3 \times 720000 + 4800 = -18561.\end{aligned}\quad (82)$$

We compute the LQE's additional consumption, $x_t - \nu A_{t-1}$ by considering the flow of funds constraint given by

$$\begin{aligned}0.002 \times 0.3 \times 460000 + 8000 - (1 + 0.2) \times 2600 \\ - 0.011904761 (0.3 \times 720000 - 0.3 \times 460000) \\ = 4227.4286.\end{aligned}\quad (83)$$

Next, we concentrate on HQE constraints. In this case, HQEs' secondary securitization at date t is computed by using the budget constraint (16); thus,

$$\begin{aligned}x_t' &= 280000 + 4800 - 0.011904761 (1 - 0.3) 720000 \\ &\quad - (1 - 0.3) \times 460000 - (1 + 0.2) \times 2600 \\ &= -46319.99954.\end{aligned}\quad (84)$$

Thus, $x_t = 54103.64210$. Next, we consider the HQE's profit (18) at face value to compute profit at date t ; thus,

$$\begin{aligned}\Pi_t' &= 0.061 \times 4000 + 0.205 \times 5000 - 0.205 \times 1500 \\ &\quad - 0.2 \times 4800 = 123.5.\end{aligned}\quad (85)$$

In this regard, the value of assets can be computed as

$$\begin{aligned}p_t^A A_{t-1}' &= \frac{123.5 - 0.205 \times 5000 + 0.205 \times 1200 + 0.2 \times 4800}{0.061} \\ &= 4991.8.\end{aligned}\quad (86)$$

The HQEs' cash flow constraint (20) is given by

$$\begin{aligned}\Pi_t' &\geq 0.061 \times 4000 + 0.205 \times 5000 - 0.205 \times 1200 - 0.113 \\ &\quad \times (1 - 0.3) \times 720000 + 4800 = -51007.\end{aligned}\quad (87)$$

The screening cost an LQE has to pay to purchase an asset unit is financed by the HQE's nett worth. This screening cost is represented by

$$\begin{aligned}u_t &= p_t^A - \frac{p_{t+1}^A}{1 + r^B} = 0.011904761 - \frac{0.113}{1 + 0.2} \\ &= -0.082261905.\end{aligned}\quad (88)$$

The motion of the aggregate asset holding and borrowing, A_t and B_t , of the entity may be computed as

$$\begin{aligned}A_t &= \frac{1}{0.082261905} [(0.002 + 0.011904761) \times 0.3 \times 460000 \\ &\quad - (1 + 0.2) \times 2600 = 14601.44865], \\ B_t &= \frac{1}{1 + 0.2} 0.113 \times 0.3 \times 720000 = 20340.\end{aligned}\quad (89)$$

The sum of the aggregate asset demand from asset originators by the LQE and HQEs' is computed by

$$\bar{A} = A_t + nA_t' = 0.3 \times 720000 + 1 (1 - 0.3) 720000 = 720000.\quad (90)$$

The steady-state asset price and borrowings for the LQE are

$$p^{A*} = 0.002 \frac{1 + 0.2}{0.2} = 0.012, \quad B^* = \frac{0.002}{0.2} 260000 = 2600.\quad (91)$$

Notice that the required screening costs per asset unit equals the LQE's securitization of marketable output, $u^* = \mu = 0.001$. Also, we have $A_{t-1} = A^*$ and $B_{t-1} = B^*$.

4.2.2. Numerical Example: Negative Shocks to LQAs and Their CDOs. LQA demand and borrowings under a temporary shock at date t are computed by

$$\begin{aligned}A_t &= \frac{1}{-0.082261905} [(0.002 - 0.002 \times 0.002 + 0.011904761) \\ &\quad \times 0.3 \times 460000 - (1 + 0.2) \times 2600] \\ &= 14608.15893, \\ B_t &= \frac{1}{1 + 0.2} 0.113 \times 0.3 \times 720000 = 20340,\end{aligned}\quad (92)$$

respectively. In this regard, we compute the cost of funds in period t as

$$\begin{aligned}u(A_t) A_t &= (0.002 - 0.002 \times 0.002 + 0.011904761 - 0.022) \\ &\quad \times 0.3 \times 460000 = -1117.69.\end{aligned}\quad (93)$$

Also, we see that the LQE's nett worth at date t is more than their current output just after the shock; thus,

$$(1 - \Sigma) u A^* = (1 - 0.002) 0.002 \times 0.3 \times 460000 = 275.448.\quad (94)$$

With unexpected capital gains,

$$\begin{aligned}(p_t^A + p^{A*}) A^* &= (0.011904761 + 0.022) \times 0.3 \times 460000 \\ &= 4679.\end{aligned}\quad (95)$$

While the debt repayment is given by

$$(1 + r^B)B^* = p^{A^*}A^* = (1 + 0.2)2600 = 3120. \quad (96)$$

Proportional change in A_t and p_t^A can be computed as

$$\begin{aligned} \widehat{A}_t &= \frac{0.3(720000 - 460000)}{0.3 \times 460000} = 0.565217391, \\ \widehat{p}_t^A &= \frac{0.011904761 - 0.022}{0.022} = -0.4588745. \end{aligned} \quad (97)$$

The steady-state profit for LQE is

$$\begin{aligned} \Pi_t^* &= (0.061 + 0.03 \times 0.2)0.022 \times 0.3 \times 460000 + 0.205 \\ &\quad \times 5000 - 0.205 \times 1200 - 0.2 \times 2600 = 462.412. \end{aligned} \quad (98)$$

and steady-state profit for HQEs is

$$\begin{aligned} \Pi_t'^* &= 0.061 \times 0.022 \times (1 - 0.3) \times 460000 + 0.205 \times 5000 \\ &\quad - 0.205 \times 1200 - 0.2 \times 2600 = 691.124. \end{aligned} \quad (99)$$

Thus, the proportional changes in Π_t and Π_t' are

$$\begin{aligned} \widehat{\Pi}_t &= \frac{87 - 462.412}{462.412} = -0.81186, \\ \widehat{\Pi}_t' &= \frac{123.5 - 691.124}{691.124} = -0.82131. \end{aligned} \quad (100)$$

Elasticity of the residual asset supply to the LQE with respect to the monitoring cost at the steady-state at date t is

$$\begin{aligned} \eta &= \left[\frac{((2 + 0.2)/0.2)0.4588745 - 0.002}{0.565217391} - 1 \right]^{-1} \\ &= 0.126718931. \end{aligned} \quad (101)$$

The proportional changes for \widehat{p}_t^A and \widehat{A}_t in terms of the size of the shock Σ are computed by

$$\begin{aligned} \widehat{p}_t^A &= -\frac{1}{0.126718931}0.002 = -0.015782961, \\ \widehat{A}_t &= -\frac{1}{1 + 1/0.126718931} \left(1 + \frac{1 + 0.2}{0.126718931 \times 0.2} \right) 0.002 \\ &= -0.010875327, \end{aligned} \quad (102)$$

respectively. By considering (47), we see from (63) and (71) that \widehat{p}_t^A and \widehat{A}_t become

$$\begin{aligned} \widehat{p}_t^A \Big|_{p_{t+1}^A = p^{A^*}} &= -\frac{0.2}{0.126718931(2 + 0.2)}0.002 \\ &= -0.001434814, \\ \widehat{A}_t \Big|_{p_{t+1}^A = p^{A^*}} &= -0.002, \end{aligned} \quad (103)$$

TABLE 3: Computed asset securitization parameters.

Parameter	Value
C_{t+1}	43 632
$p_t^A A_{t-1}$	\$4 000
$\Pi_t \geq$	\$-18 561
x_t'	\$-46 319.9995
$p_t^A A'_{t-1}$	\$4 991.8
u_t	-0.082261905
Aggregate B_t	\$20 340
p^{A^*}	0.012
A_t under shock	\$14 608.15893
$u(A_t)A_t$	-1 117.69
$(p_t^A + p^{A^*})A^*$	\$4 679
\widehat{A}_t	0.565217391
Π_t^*	\$462.412
$\widehat{\Pi}_t$	\$-2.1118
η	0.126718931
\widehat{A}_t in terms of shock	0.01
\widehat{A}_t where $p_{t+1}^A = p^{A^*}$	-0.002
$\beta >$	0.0099099
Π_t	\$87
x_t	\$54 103.6421
Π_t'	\$123.5
$\Pi_t' \geq$	\$-51 007
Aggregate A_t	14 601.44865
\bar{A}	720 000
B^*	\$2 600
B_t under shock	\$20 340
$(1 - \Sigma)\mu A^*$	275.448
$(1 + r^B)B^*$	\$3 120
$\widehat{p}_t^A = \widehat{p}_t^C$	-0.4588745
$\Pi_t'^*$	\$275.31
$\widehat{\Pi}_t'$	\$691.124
\widehat{p}_t^A in terms of shock	-0.015782961
\widehat{p}_t^A where $p_{t+1}^A = p^{A^*}$	-0.001434814
\widehat{C}_{t+1}	0.064869

respectively. The proportional change in aggregate output, C_{t+1} , represented by (54) is given by

$$\begin{aligned} \widehat{C}_{t+1} &= \frac{0.002 + 0.2 - (1 + 0.2)0.002}{0.002 + 0.2} \\ &\quad \times \frac{(0.002 + 0.2)0.3 \times 460000}{240000} 0.565217391 \\ &= 0.064869. \end{aligned} \quad (104)$$

We provide a summary of computed asset securitization parameters in Table 3.

An analysis of the computed shock parameters in Table 3 shows that the aggregate output, \widehat{C}_{t+1} , increases to \$43632. The value of LQAs, $p_t^A A_{t-1}$, increases to \$5 000. LQA demand, A_t , has declined to \$14 608.15893 while the borrowings B_t have increased to \$20 340. This implies that HQAs

were more sensitive to changes in market conditions and that asset transformation may have been a greater priority. The proportional negative change in profit for the LQE subsequent to a temporary shock is higher than that of the HQEs. In addition, the steady-state asset price p^{A^*} and the borrowings B^* for the LQE increased to \$0.012 and \$2 600, respectively. In summary, this example shows that when parameter choices are altered and the size of shock increased, the LQE suffers bigger losses on their asset holdings and the rate of borrowing to refinance increases. This explains why most people could not pay back their loans during the 2007–2009 financial crisis.

4.2.3. Numerical Example: Financial Intuition. A financial motivation for the numerical example discussed in Section 4.2.1 is outlined in the current paragraph. According to Basel III capital and liquidity regulation, LQEs and HQEs have a couple of obligations in terms of transaction costs (refer to (22)). The first is towards the originator for acquiring the assets while the second is for using the assets for securitization into CDOs, a type of user cost. In Section 4.2.1, we compute LQE and HQE equilibrium variables to add credence to the aforementioned statements.

The financial intuition behind the numerical example discussed in Section 4.2.2 is given below. Investors sought SAPs because they met certain prudential standards such as restrictions to purchase only investment grade debt (see the Basel documents [8, 9] for more details about securitization). Investors could meet relative “safety requirements” since securitization is essentially a form of bankruptcy-remote secured lending (as assets are legally isolated in a SPE) with credit enhancement (or government guarantees for RABS in some jurisdictions) that often resulted in the securities being highly rated (e.g., AAA, AA, or A). However, as noted before, the reliance on credit ratings sowed the seeds of later difficulties, as many investors relied too heavily on credit ratings and effectively “outsourced” their due diligence to the credit rating agencies (see the BCBS papers [8, 9] for further discussion). It is also worth noting that investor requirements for HQLAs created liquidity concerns during the crisis when policies or requirements forced investors to sell assets after ratings downgrades (see [5] for more details). This tested the liquidity assumptions about securitized products when many investors were simultaneously forced to sell, driving prices ever lower in a down market (see, e.g., [5, 21, 22]). Investors could avoid exceeding concentration limits, both regulatory restrictions and internal limits on exposures to a single name, by purchasing SAPs. Further, investors could manage risk in the entire portfolio by holding securitized assets that had a low correlation with other components of the investors’ portfolios, such as equities and corporate bonds (see [3] for further discussion). Also, investors could meet their internal portfolio diversification requirements by increasing the types of assets as well as the geographical location of the assets’ origination. Synthetic securitizations also allowed investors to increase the variety and volume of instruments they could acquire without funding the credit exposure (see [8, 9] for further discussion on asset securitization). However,

as the crisis unfolded, investors learned that their expected diversification benefits were partially false and that in some cases the underlying assets were highly correlated. SAPs helped investors achieve higher yield thresholds since the risk-adjusted return on ABS was typically higher relative to a similarly rated nonsecuritization investment. In the period before the crisis, investors were seeking higher yields at the same time spreads in the broader fixed income markets were narrowing (see the Basel documents [8, 9]).

5. Conclusions about Basel III and Asset Securitization

The main conclusions about LQA and HQA securitization, an LQE at equilibrium, negative shocks to asset securitization for dynamic and static multipliers, and illustrative examples involving asset securitization are summarized in this section.

5.1. Conclusions about LQA Securitization. We answered Question 1 in Section 2.1 by characterizing the securitization of LQAs by an LQE. In this case, the cash flow constraint is given by (10). The discussion on LQE cash flow has connections with Basel III capital and liquidity regulation (see [5] for further discussion). In this regard, it is clear that investors may be motivated to purchase securities issued by LQEs to avert Basel III regulatory limits, such as those relating to LQAs. In the case of synthetic transactions, investors may find it beneficial that they would not have to fund credit exposures at the outset (see, e.g., the Joint Forum paper [1]). In cases where parties to entities possessed a comprehensive understanding of the associated risks and possible structural behaviors of these LQEs under various scenarios, they have effectively engaged in and reaped benefits from their LQE activities (see [1, 3, 6] for more details). However, it is unclear that LQAs sold into the LQE can be attributed to the existence of these structures, that were simply the legal form in which such assets were held to issue bonds backed by them. Nonetheless, it is important for Basel III to address why some of the recent failures of LQE usage occurred (see, e.g., the Basel publications [1, 6]).

5.2. Conclusions about HQA Securitization. Next, we answered Question 2 in Section 2.2 by describing the securitization of HQAs by HQEs. In this case, the HQE cash flow constraint is expressed as in (20). In Section 2.2, the discussion on HQEs has relationships with Basel III capital and liquidity regulation (see [5] for more details). For example, a bank might view the Basel III risk-based capital charge applied to HQAs as being excessive and therefore might engage in securitization activities of these receivables to benefit from a slightly lower capital charge resulting from other aspects of the risk-based capital rules (see, e.g., [1]). In the bank’s view, this slightly lower Basel III charge might be more rational in terms of the true risks and capital needed for corporate lending activities (see, e.g., the Basel documents [1, 6]).

5.3. Conclusions about an LQE at Steady State. Theorem 7 in Section 2.3 provides the answer to Question 3. Here, a characterization of the behaviour of an LQE in equilibrium

is given. According to the Basel III securitization framework, LQEs and HQEs have at least two obligations in terms of transaction costs (refer to (22)). The first is towards the originator for acquiring the assets while the second is for using the assets for securitization into CDOs, a type of user cost. The latter fee involves, for instance, an external credit rating agency (see Section 2.4 for more details). Also, because of information asymmetry and regulatory dysfunction, CDOs open up opportunities for arbitrage, a point that has been considered in Basel III for balanced information distribution (see, e.g., [9]). In this regard, sophisticated CDO entities, often circumvent regulatory constraints. This type of arbitrage is accompanied by high costs with originators and other financial intermediaries earning huge transaction fees and eroding value for entities and investors (see, e.g., the Basel papers [9, 10]).

5.4. Conclusions about Negative Shocks to Asset Securitization (Dynamic Multiplier). In the presence of a dynamic multiplier, in Theorem 8 from Section 3.1, we quantify changes to asset price and holdings, CDO output, and profit subsequent to negative shocks. Also, we quantify changes to profit in terms of asset and prepayment rates as well as equity subsequent to negative shocks (refer to Question 4). We conclude that the proportional change in asset price, \hat{p}_t^A , and input, \hat{A}_t , as well as CDO price, \hat{p}_t^C , is negative in response to a negative shock like a ratings downgrade. Basel III endeavours to make provision for these negative changes in a countercyclical way (see, e.g., [1, 6, 23]). The same observation can be made about CDO output, \hat{C}_{t+1} , and profit, $\hat{\Pi}_t$, where a ratings downgrade has a negative impact (see, e.g., Basel III's (see, e.g., the BCBS documents [1, 6]).

5.5. Conclusions about Temporary Shocks to Asset Securitization (Static Multiplier). When a static multiplier is present, in Corollary 9 in Section 3.2, we determine changes to asset price, $\hat{p}_t^A|_{p_{t+1}^A}$, and input, $\hat{A}_t|_{p_{t+1}^A}$, given by (68) and (69), respectively, subsequent to a negative shock such as a ratings downgrade (refer to Question 5). As before, Basel III capital regulation attempts to negate the deleterious effects of ratings downgrades on price and input in a countercyclical manner (see, e.g., [23]).

5.6. Conclusions about Example of Subprime Mortgage Securitization. Question 6 is answered in Corollary 10 from Section 4.1. Here, an example of LQA securitization for subprime mortgages is provided. Of course, the subprime mortgage crisis preceded the 2007–2009 financial crisis and as such its connection with Basel III is very important (see, e.g., [4, 15]). The BCBS reports in Basel III documents that risk weights for low-rated senior subprime securitization exposures were too high. Many senior securitization exposures were downgraded during the crisis (compare with [3]). While some of these exposures resulted in total loss to investors, most of these exposures have resulted in recovery of some principal (see the Basel documents [8, 9]).

5.7. Conclusions about Example of Numerical Examples of Asset Securitization. Section 4.2 produces a numerical example to illustrate the impact of negative shocks on asset securitization by the LQE and HQEs. In particular, the example illustrates the impact of negative shocks on assets and CDOs. It shows that when parameter choices are altered and the size of the shock increased, the LQE suffers significant losses to their asset holdings and the rate of borrowing increases (compare with Question 7). Also, we illustrated that asset price is most significantly affected by unexpected negative shocks from asset rates, while, for CDO price, shocks to speculative asset funding, investor risk characteristics, and prepayment rate elicit statistically significant responses (compare with [3]). Problems from the financial crisis relate to our models for assets and CDOs with respect to the reduction in incentives for banks to monitor entities, transaction costs, manipulation of CDO price and structure, CDO market opacity, self-regulation, systemic risks associated with CDOs, and the mispricing of debt (see the BCBS papers [3, 8, 9]).

According to [8, pages 9–11], the role of Basel III in the numerical example from Section 4.2 can be considered from two perspectives which are the

- (i) quantitative perspective—the amount of HQLAs that the banks will have to amass in the next few years, both to meet the new requirements and to repay special facilities provided by governments and central banks, which is assumed to not be renewed. There is clearly a potential transition problem which needs to be considered in determining the timing of implementation of new regulations (see the BCBS publication [8] for more details);
- (ii) structural perspective—this refers to the ways in which the funding and liquidity management, not only of banks but also of their customers, will be permanently affected by the proposed regulations (see [5] for further details). What may appear to be rather detailed aspects of the proposed regulations may in fact have very serious implications for financial stability in the future (see the Basel document [8] for further discussion).

5.8. General Conclusions about Basel III and Asset Securitization. In answering Question 8, we note that in the Basel III document [1], the JFWGRAC makes the following recommendations about the supervision of the LQE and HQEs discussed in previous subsections.

5.8.1. LQE and HQE Economic Risks and Purposes. Firstly, supervisors should make sure that market participants assess all economic risks and business purposes of LQEs and HQEs during the lifespan of a transaction distinguishing between risk transfer and transformation. We note that, over time, the nature of these risks can change. Supervisors should ensure that such assessment is ongoing and that management has sufficient understanding of the risks (see the Basel document [1, 3] for more details).

5.8.2. *LQE and HQE Transaction Complexity.* Secondly, market participants should be able to assess and risk manage factors that increase transaction complexity, such as structural features of the LQE and HQEs including triggers and the roles of parties involved (see, e.g., the Basel documents [1, 6]).

5.8.3. *LQE and HQE Governance Processes.* In the third place, banks and supervisors should ensure the governance process of LQEs and HQEs commensurate with the degree of active intervention and discretion of the parties participating in these entities (see [1] for more discussions).

5.8.4. *LQE and HQE Portfolio Risk Profiles.* The fourth recommendation is that banks should continually monitor the quality of transferred exposures in relation to the risk profile of LQEs' and HQEs' remaining portfolios and the impact on its balance sheet components, and supervisors should where appropriate assess systemic implications of risk dispersion to transferees (see the Basel documents [1, 3, 6] for more details).

5.8.5. *LQE and HQE Risk Exposures.* Fifthly, banks should have the capability of aggregating, assessing and reporting all their LQE and HQE risk exposures in conjunction with all other bank-wide risks (see, e.g., [1, 3]).

5.8.6. *LQE and HQE Support.* In the sixth place, if at inception or at any point during LQE and HQE existence there is a likelihood or evidence of support by the bank, including noncontractual support, then the activities and risks of those entities should be aggregated with those of the bank for both supervisory assessment and internal risk management purposes (see the Basel documents [1, 6] for more discussions).

5.8.7. *LQE and HQE Transaction Standardization.* The seventh recommendation is that supervisors should support market participants' efforts towards greater standardization of definitions, documentation, and disclosure requirements of LQE and HQE transactions and provide for the communication of any material divergence from these standards to investors in individual transactions (see [1, 3] for more details).

5.8.8. *LQE and HQE Weakness, Contagion, or Procyclicality.* Finally, supervisors should regularly oversee and monitor the use of all LQE and HQE activities and assess the implications for regulated banks of the activities of such entities, in order to identify developments that can lead to systemic weakness and contagion or that can exacerbate procyclicality (see, e.g., the Basel discussion documents [1, 6]).

5.9. *Future Directions Related to Basel III and Asset Securitization.* A topic of future research may be the effects of shocks on high-quality liquid assets (HQLA) as in [21]. In this regard, we recall that an asset is considered to be a HQLA if it can be easily and immediately converted into cash at little or no

loss of value. Asset liquidity depends on the underlying stress scenario, the volume to be monetized, and the time-frame considered (see [5] for more details). Nevertheless, there are certain assets that are more likely to generate funds without incurring large discounts in sale or repurchase agreement (repo) markets due to fire-sales even in times of stress (see, e.g., [6, 21] for more details). While [6] lays out proposed revisions to the securitization framework, it does not include proposed rules text that would effectuate these changes. The BCBS is seeking industry feedback on some key elements of the proposed changes and will, in the coming months, conduct a quantitative impact study (QIS) of the proposals before deciding on a definitive way forward (see, e.g., [1]). The BCBS will consider all comments received along with the results of the QIS before determining the appropriate next steps in the process of moving forward with revisions to the securitization framework. This would mean that the contents of the current paper would have to be refined quite considerably.

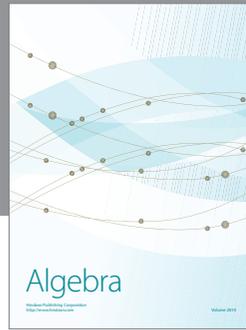
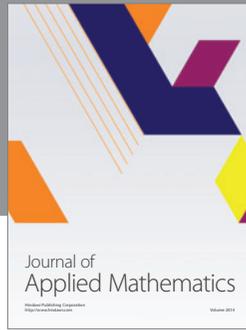
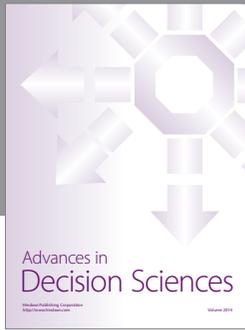
Conflict of Interests

The authors of this paper, do not have a direct financial relationship with the commercial entities mentioned in this paper that might lead to a conflict of interests.

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