

Research Article Global Asymptotic Stability of a Family of Nonlinear Difference Equations

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In this note, we consider global asymptotic stability of the following nonlinear difference equation $x_n = (\prod_{i=1}^{v} (x_{n-k_i}^{\beta_i} + 1) + \prod_{i=1}^{v} (x_{n-k_i}^{\beta_i} - 1))/(\prod_{i=1}^{v} (x_{n-k_i}^{\beta_i} + 1) - \prod_{i=1}^{v} (x_{n-k_i}^{\beta_i} - 1)), n = 0, 1, ..., where <math>k_i \in \mathbb{N}$ $(i = 1, 2, ..., v), v \ge 2, \beta_1 \in [-1, 1], \beta_2, \beta_3, ..., \beta_v \in (-\infty, +\infty), x_{-m}, x_{-m+1}, ..., x_{-1} \in (0, \infty)$, and $m = \max_{1 \le i \le v} \{k_i\}$. Our result generalizes the corresponding results in the recent literature and simultaneously conforms to a conjecture in the work by Berenhaut et al. (2007).

1. Introduction

The study of dynamical properties of nonlinear difference equations has been an area of intense interest in recent years (e.g., see [1–13]).

In [4], by analysis of semicycle structure, the authors discussed the global asymptotic stability of rational difference equation

$$x_{n+1} = \frac{x_n x_{n-1} + 1}{x_n + x_{n-1}}, \quad n = 0, 1, \dots,$$
(1)

where the initial values $x_{-1}, x_0 \in (0, +\infty)$.

Li [5, 6] investigated the qualitative behavior of the rational difference equations

$$x_{n} = \frac{x_{n-1} + x_{n-2} + x_{n-4} + x_{n-1}x_{n-2}x_{n-4} + a}{1 + x_{n-1}x_{n-2} + x_{n-2}x_{n-4} + x_{n-1}x_{n-4} + a},$$

$$x_{n} = \frac{x_{n-2} + x_{n-3} + x_{n-4} + x_{n-2}x_{n-3}x_{n-4} + a}{1 + x_{n-2}x_{n-3} + x_{n-3}x_{n-4} + x_{n-2}x_{n-4} + a},$$
(2)

$$n=0,1,2,\ldots,$$

 $n = 0, 1, 2, \ldots,$

with $x_{-4}, x_{-3}, \ldots, x_{-1} \in (0, \infty)$ and $a \in [0, \infty)$ via analysis of semicycle structure and verified that every solution of (2) converges to equilibrium 1.

By using the transformation method, Berenhaut et al. [1] studied the behavior of positive solutions to the rational difference equation

$$x_n = \frac{x_{n-k} + x_{n-m}}{1 + x_{n-k}x_{n-m}}, \quad n = 0, 1, 2, \dots,$$
(3)

with $x_{-m}, x_{-m+1}, \ldots, x_{-1} \in (0, \infty)$ and $1 \le k < m$ and proved that every solution of (3) converges to the unique equilibrium 1. Based on the above facts, Berenhaut et al. [1] put forward the following two conjectures.

Conjecture 1. Suppose that $1 \le k < l < m$ and that $\{x_n\}$ satisfies

$$x_{n} = \frac{x_{n-k} + x_{n-l} + x_{n-m} + x_{n-k}x_{n-l}x_{n-m}}{1 + x_{n-k}x_{n-l} + x_{n-l}x_{n-m} + x_{n-m}x_{n-k}},$$

$$n = 0, 1, 2, \dots$$
(4)

with $x_{-m}, x_{-m+1}, \ldots, x_{-1} \in (0, \infty)$. Then, the sequence $\{x_n\}$ converges to the unique equilibrium 1.

Conjecture 2. Suppose that *m* is odd and $1 \le k_1 < k_2 < \cdots < k_m$, and define $S = \{1, 2, \dots, m\}$. If $\{x_n\}$ satisfies

$$x_{n} = \frac{f_{1}\left(x_{n-k_{1}}, x_{n-k_{2}}, \dots, x_{n-k_{m}}\right)}{f_{2}\left(x_{n-k_{1}}, x_{n-k_{2}}, \dots, x_{n-k_{m}}\right)},$$
(5)

 $n = 0, 1, 2, \ldots,$

with $x_{-k_{m}}, x_{-k_{m}+1}, \dots, x_{-1} \in (0, \infty)$, where

$$f_{1}(x_{1}, x_{2}, ..., x_{m}) = \sum_{j \in \{1,3,...,m\}} \sum_{\substack{\{t_{1},t_{2},...,t_{j}\} \in S; \\ t_{1} < t_{2} < \cdots < t_{j}}} x_{t_{1}}, x_{t_{2}}, ..., x_{t_{j}},$$

$$f_{2}(x_{1}, x_{2}, ..., x_{m}) = 1 + \sum_{\substack{j \in \{2,4,...,m-1\}}} \sum_{\substack{\{t_{1},t_{2},...,t_{j}\} \in S; \\ t_{1} < t_{2} < \cdots < t_{j}}} x_{t_{1}}, x_{t_{2}}, ..., x_{t_{j}},$$
(6)

then the sequence $\{x_n\}$ converges to the unique equilibrium 1.

Recently, by method used in [4–6], the authors of [12] studied the global asymptotic stability of the following nonlinear difference equation.

$$x_{n+1} = \frac{F(x_n, x_{n-1}, x_{n-2}, x_{n-3})}{G(x_n, x_{n-1}, x_{n-2}, x_{n-3})},$$

$$n = 0, 1, \dots,$$
(7)

where

$$F(x, y, z, w) = x^{\alpha_1} y^{\alpha_2} + x^{\alpha_1} z^{\alpha_3} + x^{\alpha_1} w^{\alpha_4} + y^{\alpha_2} z^{\alpha_3} + y^{\alpha_2} w^{\alpha_4} + z^{\alpha_3} w^{\alpha_4} + x^{\alpha_1} y^{\alpha_2} z^{\alpha_3} w^{\alpha_4} + 1,$$

$$G(x, y, z, w) = x^{\alpha_1} + y^{\alpha_2} + z^{\alpha_3} + w^{\alpha_4} + x^{\alpha_1} y^{\alpha_2} z^{\alpha_3} + x^{\alpha_1} y^{\alpha_2} w^{\alpha_4} + x^{\alpha_1} z^{\alpha_3} w^{\alpha_4} + y^{\alpha_2} z^{\alpha_3} w^{\alpha_4},$$

(8)

the parameter $\alpha_1 \in (0, 1]$, $\alpha_2, \alpha_3, \alpha_4 \in (0, +\infty)$, and the initial values $x_{-3}, x_{-2}, x_{-1}, x_0 \in (0, +\infty)$.

Motivated by the above studies, in this note, we propose and consider the following nonlinear difference equation.

$$x_{n} = \frac{\prod_{i=1}^{\nu} \left(x_{n-k_{i}}^{\beta_{i}} + 1 \right) + \prod_{i=1}^{\nu} \left(x_{n-k_{i}}^{\beta_{i}} - 1 \right)}{\prod_{i=1}^{\nu} \left(x_{n-k_{i}}^{\beta_{i}} + 1 \right) - \prod_{i=1}^{\nu} \left(x_{n-k_{i}}^{\beta_{i}} - 1 \right)}, \qquad (9)$$

$$n = 0, 1, \dots,$$

where $k_i \in \mathbb{N}$ $(i = 1, 2, ..., v), v \ge 2, \beta_1 \in [-1, 1], \beta_2, \beta_3, ..., \beta_v \in (-\infty, +\infty), x_{-m}, x_{-m+1}, ..., x_{-1} \in (0, \infty),$ and $m = \max_{1 \le i \le v} \{k_i\}.$

It is noticed that, letting v = 2, $\beta_1 = \beta_2 = 1$, $k_1 = 1$, and $k_2 = 2$, (9) reduces to (1); letting v = 3, $\beta_1 = \beta_2 = \beta_3 = 1$, $k_1 = 1, k_2 = 2$, and $k_3 = 4$ and v = 3, $\beta_1 = \beta_2 = \beta_3 = 1$, $k_1 = 2$, $k_2 = 3$, and $k_3 = 4$, (9) reduces to (2); letting v = 2, $\beta_1 = \beta_2 = 1, k_1 = k$, and $k_2 = m$, (9) reduces to (3); letting v = 3, $\beta_1 = k$, $\beta_2 = l$, and $\beta_3 = m$, (9) reduces to (4); letting v = 4, $\beta_1 \in (0, 1]$, $\beta_i = \alpha_i$ (i = 1, 2, 3, 4), $k_1 = 1$, $k_2 = 2$, $k_3 = 3$, and $k_4 = 4$, (9) reduces to (7); letting v = m be odd, $1 \le k_1 < k_2 < \cdots < k_m$, and $\beta_1 = \beta_2 = \cdots = \beta_m = 1$, (9) reduces to (5). Clearly, (5) is a special example of (9).

In 2007, Berenhaut and Stević [2] had proved Conjecture 1. In this paper, by making full use of analytical

techniques, we mainly prove that the unique positive equilibrium point of (9) is globally asymptotically stable. It is clear that our result generalizes the corresponding works in [1, 2, 4-9, 12] and simultaneously conforms to Conjecture 2.

2. Existence of a Unique Positive Equilibrium

In this section, we mainly show the existence of a unique positive equilibrium of (9).

Theorem 3. In (9) there exists a unique positive equilibrium point $\overline{x} = 1$.

Proof. A positive equilibrium point \overline{x} of (9) satisfies the next equation:

$$\overline{x} = \frac{\prod_{i=1}^{\nu} \left(\overline{x}^{\beta_i} + 1\right) + \prod_{i=1}^{\nu} \left(\overline{x}^{\beta_i} - 1\right)}{\prod_{i=1}^{\nu} \left(\overline{x}^{\beta_i} + 1\right) - \prod_{i=1}^{\nu} \left(\overline{x}^{\beta_i} - 1\right)},$$
(10)

from which we may get

$$(\overline{x}-1)\prod_{i=1}^{\nu}\left(\overline{x}^{\beta_i}+1\right) = (\overline{x}+1)\prod_{i=1}^{\nu}\left(\overline{x}^{\beta_i}-1\right); \quad (11)$$

that is,

$$(\overline{x} - 1) \left(\overline{x}^{\beta_1} + 1 \right) \prod_{i=2}^{\nu} \left(\overline{x}^{\beta_i} + 1 \right)$$

$$= (\overline{x} + 1) \left(\overline{x}^{\beta_1} - 1 \right) \prod_{i=2}^{\nu} \left(\overline{x}^{\beta_i} - 1 \right).$$
(12)

From the above equation, we can get

$$\left(\overline{x}^{\beta_{1}+1}-1\right)\left(\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}+1\right)-\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}-1\right)\right)$$

$$+\left(\overline{x}-\overline{x}^{\beta_{1}}\right)\left(\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}+1\right)+\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}-1\right)\right)=0.$$
(13)

One can see that for any $\overline{x} > 0$ and $v \ge 2$,

$$\prod_{i=2}^{\nu} \left(\overline{x}^{\beta_i} + 1\right) - \prod_{i=2}^{\nu} \left(\overline{x}^{\beta_i} - 1\right) > 0,$$

$$\prod_{i=2}^{\nu} \left(\overline{x}^{\beta_i} + 1\right) + \prod_{i=2}^{\nu} \left(\overline{x}^{\beta_i} - 1\right) > 0.$$
(14)

- (i) If $\beta_1 = -1, 0, 1$, from (13) and (14), we can get that (9) has a unique positive equilibrium $\overline{x} = 1$.
- (ii) If $-1 < \beta_1 < 0$ or $0 < \beta_1 < 1$ and $0 < \overline{x} < 1$, we have

$$\overline{x} < \overline{x}^{\beta_1}, \qquad \overline{x}^{\beta_1+1} < 1.$$
 (15)

Further, we have

$$(\overline{x}^{\beta_1+1}-1)\left(\prod_{i=2}^{\nu} (\overline{x}^{\beta_i}+1)-\prod_{i=2}^{\nu} (\overline{x}^{\beta_i}-1)\right) + (\overline{x}-\overline{x}^{\beta_1})\left(\prod_{i=2}^{\nu} (\overline{x}^{\beta_i}+1)+\prod_{i=2}^{\nu} (\overline{x}^{\beta_i}-1)\right) < 0.$$

$$(16)$$

(iii) If $-1 < \beta_1 < 0$ or $0 < \beta_1 < 1$ and $\overline{x} > 1$, we have

$$\left(\overline{x}^{\beta_{1}+1}-1\right)\left(\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}+1\right)-\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}-1\right)\right) +\left(\overline{x}-\overline{x}^{\beta_{1}}\right)\left(\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}+1\right)+\prod_{i=2}^{\nu}\left(\overline{x}^{\beta_{i}}-1\right)\right)>0.$$
(17)

It is clear that (9) has a unique positive equilibrium $\overline{x} = 1$. The proof is complete.

3. Global Asymptotic Stability for the Unique **Positive Equilibrium Point**

In this section, we give our main result.

Theorem 4. The unique positive equilibrium point $\overline{x} = 1$ of (9) is globally asymptotically stable.

In order to prove Theorem 4, we introduce the following lemma by Kruse and Nesemann [3] and make full use of analytical techniques.

Lemma 5. Consider the difference equation

$$x_{n+k} = f(x_{n+k-1}, \dots, x_n), \quad n = 0, 1, 2, \dots,$$
 (18)

where $k \in \mathbb{N}$ and $f : (0, \infty)^k \rightarrow (0, \infty)$ is a continuous function with some unique equilibrium \overline{x} . Suppose that there is a $p \in \mathbb{N}$ such that for all solutions $\{x_n\}$ of (18)

$$\left(x_n - x_{n+p}\right) \left(\frac{\overline{x}^2}{x_n} - x_{n+p}\right) \le 0, \tag{19}$$

where equality holds if and only if $x_n = \overline{x}$. Then \overline{x} is globally asymptotically stable.

Proof of Theorem 4. Let $\{x_n\}_{n=-m}^{\infty}$ be any solution of (9). We have

$$\begin{split} & x_n - x_{n-k_1}^{\beta_1} \\ &= \frac{\prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} + 1 \right) + \prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} - 1 \right)}{\prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} + 1 \right) - \prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} - 1 \right)} - x_{n-k_1}^{\beta_1} \\ &= \frac{\left(1 - x_{n-k_1}^{\beta_1} \right) \prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} + 1 \right) + \left(1 + x_{n-k_1}^{\beta_1} \right) \prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} - 1 \right)}{\prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} + 1 \right) - \prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} - 1 \right)} \\ &= \frac{\left(1 - x_{n-k_1}^{\beta_1} \right) \left(1 + x_{n-k_1}^{\beta_1} \right) \left(\prod_{i=2}^{\nu} \left(x_{n-k_i}^{\beta_i} + 1 \right) - \prod_{i=2}^{\nu} \left(x_{n-k_i}^{\beta_i} - 1 \right)}{\prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} + 1 \right) - \prod_{i=1}^{\nu} \left(x_{n-k_i}^{\beta_i} - 1 \right)} \\ &= \frac{n = 0, 1, \dots, n = 0, 1, \dots,$$

$$\begin{split} &\frac{1}{x_{n-k_{1}}^{\beta_{1}}} - x_{n} \\ &= \frac{1}{x_{n-k_{1}}^{\beta_{1}}} - \frac{\prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} + 1\right) + \prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} - 1\right)}{\prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} + 1\right) - \prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} - 1\right)} \\ &= \frac{\left(1 - x_{n-k_{1}}^{\beta_{1}}\right) \prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} + 1\right) - \left(1 + x_{n-k_{1}}^{\beta_{1}}\right) \prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} - 1\right)}{\prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} + 1\right) - \prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} - 1\right)}, \\ &= \frac{\left(1 - x_{n-k_{1}}^{\beta_{1}}\right) \left(1 + x_{n-k_{1}}^{\beta_{1}}\right) \left(\prod_{i=2}^{v} \left(x_{n-k_{i}}^{\beta_{i}} + 1\right) + \prod_{i=2}^{v} \left(x_{n-k_{i}}^{\beta_{i}} - 1\right)\right)}{\prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} + 1\right) - \prod_{i=1}^{v} \left(x_{n-k_{i}}^{\beta_{i}} - 1\right)}, \\ &= \frac{n = 0, 1, \dots, n = 0, \dots, n = 0, 1, \dots, n = 0, \dots, n = 0, 1, \dots, n$$

It follows from (20) that

1

$$\begin{split} x_{n+k_1} &- x_n^{\beta_1} \\ &= \frac{\left(1 - x_n^{\beta_1}\right) \left(1 + x_n^{\beta_1}\right) \left(\prod_{i=2}^{\nu} \left(x_{n+k_1-k_i}^{\beta_i} + 1\right) - \prod_{i=2}^{\nu} \left(x_{n+k_1-k_i}^{\beta_i} - 1\right)\right)}{\prod_{i=1}^{\nu} \left(x_{n+k_1-k_i}^{\beta_i} + 1\right) - \prod_{i=1}^{\nu} \left(x_{n+k_1-k_i}^{\beta_i} - 1\right)}, \\ &n = -k_1, -k_1 + 1, \dots, \end{split}$$

$$\frac{1}{x_{n}^{\beta_{1}}} - x_{n+k_{1}} = \frac{\left(1 - x_{n}^{\beta_{1}}\right) \left(1 + x_{n}^{\beta_{1}}\right) \left(\prod_{i=2}^{\nu} \left(x_{n+k_{1}-k_{i}}^{\beta_{i}} + 1\right) + \prod_{i=2}^{\nu} \left(x_{n+k_{1}-k_{i}}^{\beta_{i}} - 1\right)\right)}{\prod_{i=1}^{\nu} \left(x_{n+k_{1}-k_{i}}^{\beta_{i}} + 1\right) - \prod_{i=1}^{\nu} \left(x_{n+k_{1}-k_{i}}^{\beta_{i}} - 1\right)}, \quad n = -k_{1}, -k_{1} + 1, \dots.$$
(21)

Clearly, from (21), we have

$$\left(x_n^{\beta_1} - x_{n+k_1} \right) \left(\frac{1}{x_n^{\beta_1}} - x_{n+k_1} \right) \le 0,$$

$$n = -k_1, -k_1 + 1, \dots$$
(22)

From (22), we have

$$1 - x_{n+k_1} \left(\frac{1}{x_n^{\beta_1}} + x_n^{\beta_1} \right) + x_{n+k_1}^2 \le 0,$$

$$n = -k_1, -k_1 + 1, \dots$$
(23)

If $\beta_1 = \pm 1$, it is clear that

$$\frac{1}{x_n^{\beta_1}} + x_n^{\beta_1} = \frac{1}{x_n} + x_n.$$
(24)

If $0 < x_n < 1$ and $-1 < \beta_1 < 1$, we have $x_n < x_n^{\beta_1}$ and $0 < x_n^{\beta_1 + 1} < 1$, so that

$$(x_n - x_n^{\beta_1}) \left(1 - \frac{1}{x_n x_n^{\beta_1}}\right) > 0.$$
 (25)

(20)

$$\left(x_n - x_n^{\beta_1}\right) \left(1 - \frac{1}{x_n x_n^{\beta_1}}\right) > 0.$$
 (26)

Hence, for $-1 \le \beta_1 \le 1$, we always have

$$\frac{1}{x_n^{\beta_1}} + x_n^{\beta_1} \le \frac{1}{x_n} + x_n.$$
(27)

Further, from (23) and (27), we have

$$1 - x_{n+k_1} \left(\frac{1}{x_n} + x_n\right) + x_{n+k_1}^2$$

$$\leq 1 - x_{n+k_1} \left(\frac{1}{x_n^{\beta_1}} + x_n^{\beta_1}\right) + x_{n+k_1}^2 \leq 0, \qquad (28)$$

$$n = -k_1, -k_1 + 1, \dots.$$

Therefore,

$$(x_n - x_{n+k_1}) \left(\frac{1}{x_n} - x_{n+k_1}\right) \le 0,$$

 $n = -k_1, -k_1 + 1, \dots,$ (29)

where equality holds if and only if $x_n = \overline{x} = 1$. By Lemma 5 and (29), with $p = k_1 \in \mathbb{N}$, it follows that the unique positive equilibrium point $\overline{x} = 1$ of (9) is globally asymptotically stable. The proof is complete.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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