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Research Article

Mean-Square Exponential Synchronization of Stochastic Complex Dynamical Networks with Switching Topology by Impulsive Control

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This paper investigates the mean-square exponential synchronization issues of delayed stochastic complex dynamical networks with switching topology and impulsive control. By using the Lyapunov functional method, impulsive control theory, and linear matrix inequality (LMI) approaches, some sufficient conditions are derived to guarantee the mean-square exponential synchronization of delay complex dynamical network with switch topology, which are independent of the network size and switch topology. Numerical simulations are given to illustrate the effectiveness of the obtained results in the end.

1. Introduction

Complex networks are everywhere in nature and our daily life, such as the Internet, ecosystems, social networks, World Wide Web, and neural networks. A complex network can be described by a set of nodes and edges interconnecting these nodes together. During the last two decades, complex networks have been focused on by scientists from various fields, such as mathematical, engineering, and social and economic science. There are many literatures concerning the collective behaviors of complex networks [1–11]. Among them, synchronization is the most interesting phenomenon [3–11], because the synchronization is a kind of typical collective behaviors exhibited in many natural systems.

Due to many complex systems may experience abrupt changes in their connection caused by some phenomena such as link failures, component failures or repairs, changing subsystem interconnections, and abrupt environmental disturbance, and so forth. Thus, the synchronization of complex network with switching topology has been studied in [12–17]. In [12], Yao et al. studied the synchronization of a general complex dynamical network with switching topology and the time-varying coupling is unknown but bounded. Wang et al. [13] investigated the synchronization issues of complex dynamical networks with switching topology. Yu et al. [14] explored the synchronization of switched linearly coupled neural networks with delay. Some sufficient conditions were given to guarantee the global synchronization. In [15], Liu et al. studied the local and global exponential synchronization of complex dynamical network with switching topology and time-varying coupling delays. In [16], some sufficient conditions were given to guarantee the synchronization of leader following issues with switching connective network and coupling delay. In [17], authors investigated the consensus problem in mean square for uncertain multiagent systems with stochastic measurement noises and symmetric or asymmetric time-varying delays.

Uncertainties commonly exist in the real world, such as stochastic forces on the physical systems and noisy measurements caused by environmental uncertainties. Thus, a stochastic behavior should be produced instead of a deterministic one [18]. In fact, signals transmitted between nodes

of complex networks are unavoidably subject to stochastic perturbations from environment, which may cause information contained in these signals to be lost [19]. Therefore, stochastic perturbations should be considered [18–23]. In [19–21], stochastic perturbations are all one-dimensional, which means that the signal transmitted by nodes is influenced by the same noise. In [18, 22], the authors considered stochastic synchronization of coupled neural networks, in which noise perturbations are vector forms. Vector-form perturbation means that different nodes are influenced by different noise, which is more practical in the real world. In [23], authors investigated the mean-square exponential synchronization of stochastic complex networks with Markovian switching and time-varying delays by using the pinning control method.

In many systems, the impulsive effects are common phenomena due to instantaneous perturbations at certain moments [24–28]. In the past several years, impulsive control strategies have been widely used to stabilize and synchronize coupled complex dynamical system, such as signal processing system, computer networks, automatic control systems, and telecommunications. In [24], Cai et al. investigated the robust impulsive synchronization of complex delayed dynamical networks. Yang et al. [25] studied the exponential synchronization of complex dynamical network with a coupling delay and impulsive control. Zhu et al. gave some global impulsive exponential synchronization criteria of time-delayed coupled chaotic systems [27]. In [28], some sufficient conditions were given to guarantee the consensus of nonlinear multiagent systems with switching topology.

Based on the above discussion, studying the synchronization problem of the complex dynamical network with switch topology and impulsive effects is very useful and meaningful. It should be pointed out that exponential synchronization of the coupling delay complex dynamical networks with switch topology and impulsive effects has received very little research attention.

In this paper, we investigate the problem of exponential synchronization of coupling delay complex dynamical network with switching topology and impulsive effects, basing on the Lyapunov theory and impulsive control theory and by assuming that there exists finite connective topology of the complex dynamical network, and the connective topology may be switched (or jumped) from one to another at different moments. For controlling, the synchronization state can be any weighting average of the network states. It means that each node of the dynamical network can contribute to synchronization of the network in its weight. Finally, some sufficient conditions are given to guarantee the synchronization of the complex dynamical network, which are independent of the network size and switch topology.

2. Model and Preliminaries

The switching complex dynamical networks investigated in this paper consist of N nodes, whose state is described as follows:

$$\begin{aligned} dx_i(t) = \Bigg\{ & f\left(t, x_i(t), x_i(t-\tau(t))\right) \\ & + \sum_{j=1}^{N} a_{ij}(r(t))\,\Sigma x_j(t) + \sum_{j=1}^{N} b_{ij}(r(t))\,\Sigma x_j\left(t-\tau_c(t)\right) \Bigg\} dt \\ & + \sigma_i\left(t, x(t), x(t-\tau(t)), x\left(t-\tau_c(t)\right), r(t)\right) dw_i(t), \\ & t \neq t_k, \\ \Delta x_i(t_k) = x_i(t_k^+) - x_i(t_k^-) = \epsilon_{ik} x_i(t_k^-), & \quad t = t_k, \\ & k \in Z^+, \quad i = 1, 2, \ldots, N, \end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in \mathbb{R}^n$ is the state vector of the ith node of the network, $f(t, x_i(t), x_i(t-\tau(t))) = [f_1(t, x_i(t), x_i(t-\tau(t))), f_2(t, x_i(t), x_i(t-\tau(t))), \ldots, f_n(t, x_i(t), x_i(t-\tau(t)))]^T$ is a continuous vector-valued function, $\Sigma = \mathrm{diag}(\varrho_1, \varrho_2, \ldots, \varrho_n)$ is an inner coupling of the networks that satisfies $\varrho_j > 0$, $j = 1, 2, \ldots, n$, and $r(t) : [0, \infty) \to \aleph = \{1, 2, \ldots, M\}$ is a switching signal, which is a piecewise constant function. Here, $A(r(t)) = [a_{ij}(r(t))] \in \mathbb{R}^{n\times n}$ and $B(r(t)) = [b_{ij}(r(t))] \in \mathbb{R}^{n\times n}$ are the outer-coupling matrices of the network at time t at nodes $r(t)$, $t-\tau_c(t)$, and $r(t)$, respectively, such that $a_{ij}(r(t)) \geq 0$ for $i \neq j$, $a_{ii}(r(t)) = -\sum_{j=i, j\neq i}^{N} a_{ij}(r(t))$, $b_{ij}(r(t)) \geq 0$ for $i \neq j$, and $b_{ii}(r(t)) = -\sum_{j=i, j\neq i}^{N} b_{ij}(r(t))$. $\tau(t)$ is the inner time-varying delay satisfying $\tau \geq \tau(t) \geq 0$ and $\tau_c(t)$ is the coupling time-varying delay satisfying $\tau_c \geq \tau_c(t) \geq 0$. Finally, $\sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t)), r(t)) = \sigma_i(t, x_1(t), \ldots, x_n(t), x_1(t-\tau(t)), \ldots, x_n(t-\tau(t)), x_1(t-\tau_c(t)), \ldots, x_n(t-\tau_c(t)), r(t)) \in \mathbb{R}^{n\times n}$ and $w_i(t) = (w_{i1}(t), w_{i2}(t), \ldots, w_{in}(t))^T \in \mathbb{R}^n$ is a bounded vector-form Weiner process, satisfying

$$\begin{gathered} \mathbb{E}w_{ij}(t) = 0, \qquad \mathbb{E}w_{ij}^2(t) = 1, \\ \mathbb{E}w_{ij}(t)\, w_{ij}(s) = 0 \; (s \neq t). \end{gathered} \tag{2}$$

ϵ_{ik} is the ith node impulsive gain at $t = t_k$. The discrete set $\{t_k\}$ satisfies $t_k = t_{k-1} + T$, $t_0 \geq 0$, $t_k \to +\infty$ as $k \to +\infty$, note $x(t_k^-) = \lim_{t\to t_k^-} x(t)$, and $x(t_k^+) = \lim_{t\to t_k^+} x(t) = x(t_k)$. In this paper, $A(r(t))$ is assumed to be irreducible in the sense that there are no isolated nodes.

The initial conditions associated with (1) are

$$x_i(s) = \xi_i(s), \qquad -\check{\tau} \leq s \leq 0, \quad i = 1, 2, \ldots, N, \tag{3}$$

where $\check{\tau} = \max\{\tau(t), \tau_c(t)\}$, $\xi_i \in C^b_{\mathcal{F}_0}([-\check{\tau}, 0], \mathbb{R}^n)$ with the norm $\|\xi_i\|^2 = \sup_{-\check{\tau}\leq s\leq 0} \xi_i(s)^T \xi_i(s)$ and our objective is to control system (1) so that it stays in the trajectory $s(t) \in \mathbb{R}^n$ of the system

$$ds(t) = f(t, s(t), s(t-\tau(t)))\, dt. \tag{4}$$

Remark 1. In general, the synchronization state $s(t)$ may be an equilibrium point, a periodic orbit, or a chaotic attractor.

The following assumptions will be used throughout this paper for establishing the synchronization conditions.

(H1) $\tau(t)$ and $\tau_c(t)$ are bounded and continuously differentiable functions such that $0 < \tau(t) \le \tau$, $\dot{\tau}(t) < \overline{\tau} < 1$, $0 < \tau_c(t) \le \tau_c$, and $\dot{\tau}_c(t) < \overline{\tau}_c < 1$. Let $\check{\tau} = \max\{\tau, \tau_c\}$ and $\check{\overline{\tau}} = \max\{\overline{\tau}, \overline{\tau}_c\}$.

(H2) Let $\sigma(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r) = \sigma(t, e_1(t), \ldots, e_N(t), e_1(t-\tau(t)), \ldots, e_N(t-\tau(t)), e_1(t-\tau_c(t)), \ldots, e_N(t-\tau_c(t)), r)$. Then there exist positive definite constant matrices Υ_{i1}^r, Υ_{i2}^r, and Υ_{i3}^r for $i = 1, 2, \ldots, N$ and $r = 1, 2, \ldots, M$ such that

$$\begin{aligned}
&\operatorname{Tr}\left[\sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r)^T\right.\\
&\quad\left.\times \sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r)\right]\\
&\le \sum_{j=1}^{N} e_j(t)^T \Upsilon_{i1}^r e_j(t) + \sum_{j=1}^{N} e_j(t-\tau(t))^T \Upsilon_{i2}^r e_j(t-\tau(t))\\
&\quad + \sum_{j=1}^{N} e_j(t-\tau_c(t))^T \Upsilon_{i3}^r e_j(t-\tau_c(t)).
\end{aligned} \tag{5}$$

(H3) $\sigma_i(t, s(t), \ldots, s(t), s(t-\tau(t)), \ldots, s(t-\tau(t)), s(t-\tau_c(t)), \ldots, s(t-\tau_c(t)), r(t)) = 0$.

Define error state $e_i(t) = x_i(t) - s(t)$,

$$\begin{aligned}
de_i(t) = &\left\{ f(t, x_i(t), x_i(t-\tau(t))) - f(t, s(t), s(t-\tau(t)))\right.\\
&+ \sum_{j=1}^{N} a_{ij}(r(t)) \Sigma e_j(t) + \sum_{j=1}^{N} b_{ij}(r(t)) \Sigma e_j(t-\tau_c(t))\\
&\left. + u_i(t) \right\} dt\\
&+ \sigma_i(t, e(t), e(t-\tau(t)), e(t-\tau_c(t)), r(t))\, dw_i(t),\\
&\qquad t \ne t_k,\\
\Delta e_i(t_k) = &\, e_i(t_k^+) - e_i(t_k^-) = \epsilon_{ik} e_i(t_k^-), \quad t = t_k,\\
&k \in Z^+, \quad i = 1, 2, \ldots, N.
\end{aligned} \tag{6}$$

Definition 2. The complex network (1) is said to be exponentially synchronized in mean square if the trivial solution of system (6) is such that

$$\sum_{i=1}^{N} \mathbb{E}\left\| e_i(t, t_0, \xi_i) \right\|^2 \le K e^{-\kappa t}, \tag{7}$$

for some $K > 0$ and some $\kappa > 0$ for any initial data $\xi_i \in \mathscr{C}^b_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$.

Definition 3. A continuous function $f(t, x, y) : [0, +\infty] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is said to belong to the function class QUAD, denoted by $f \in \mathrm{QUAD}(P, \Delta, \eta, \theta)$ for some given matrix $\Sigma = \operatorname{diag}\{\varrho_1, \varrho_2, \ldots, \varrho_n\}$ if there exist a positive definite diagonal matrix $P = \operatorname{diag}\{p_1, p_2, \ldots, p_n\}$, a diagonal matrix $\Delta = \operatorname{diag}\{\delta_1, \delta_2, \ldots, \delta_n\}$, and constants $\eta > 0$, $\theta > 0$ such that $f(\cdot)$ satisfies the condition

$$\begin{aligned}
&(x-y)^T P\left(\left(f(t, x, z) - f(t, y, w)\right) - \Delta\Sigma(x-y)\right)\\
&\quad \le -\eta(x-y)^T(x-y) + \theta(z-w)^T(z-w)
\end{aligned} \tag{8}$$

for all $x, y, z, w \in \mathbb{R}^n$.

Remark 4. The function class QUAD includes almost all the well-known chaotic systems with or without delays such as the Lorenz system, the Rössler system, the Chen system, the delayed Chua's circuit, the logistic delayed differential system, the delayed Hopfield neural network, and the delayed CNNs. We shall simply write

$$\begin{gathered}
\check{p} = \max\{p_1, p_2, \ldots, p_n\}, \qquad \widehat{p} = \min\{p_1, p_2, \ldots, p_n\},\\
\check{\delta} = \max\{\delta_1, \delta_2, \ldots, \delta_n\}.
\end{gathered} \tag{9}$$

In order to derive the main results, it is necessary to propose the following lemmas.

Lemma 5 (see [16]). *The following linear matrix inequality*

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} > 0, \tag{10}$$

where $Q = Q^T$, $R = R^T$, is equivalent to one of the following conditions:

(i) $Q > 0$, $R - S^T Q^{-1} S > 0$.

(ii) $R > 0$, $Q - S^T R^{-1} S > 0$.

3. Main Result

In this section, we investigate the exponential stability condition of the error system (6). Some new criteria are presented for the exponential synchronization of network (1) based on the Lyapunov functional method, linear matrix inequality approach, and impulsive control theory.

Theorem 6. *Let assumptions (H1) and (H2) be true and let $f \in \mathrm{QUAD}(P, \Delta, \eta, \theta)$. If there exist positive constants α_r and β_r such that*

$$\begin{bmatrix} A(r)^s + \check{\delta} I_N - \alpha_r I_N & \dfrac{B(r)}{2} \\ \dfrac{B(r)^T}{2} & -\beta_r I_N \end{bmatrix} \le 0, \quad \text{for } r = 1, 2, \ldots, M, \tag{11}$$

$$\left(\gamma+\check{a}+\frac{b}{1-\overline{\tau}}e^{\gamma\tau}+\frac{\check{c}}{1-\overline{\tau}_c}e^{\gamma\tau_c}\right)\times\frac{(1+T\check{\tau})}{T}+\frac{2\ln|1+\epsilon|}{T}-\gamma\leq-\eta, \tag{12}$$

where

$$\begin{aligned}
a_r&=\frac{\lambda_{\min}\left(-2\eta I_n+\check{p}\sum_{i=1}^{N}\Upsilon_{i1}^{r}+2\alpha_r P\Sigma\right)}{\check{p}},\quad \check{a}=\max_{r\in S}a_r,\\
b_r&=\frac{\lambda_{\max}\left(\sum_{i=1}^{N}P\Upsilon_{i2}^{r}+2\theta I_N\right)}{\widehat{p}},\quad \check{b}=\max_{r\in S}b_r,\\
c_r&=\frac{\lambda_{\max}\left(\sum_{i=1}^{N}P\Upsilon_{i3}^{r}+2\beta_r P\Sigma\right)}{\widehat{p}},\quad \check{c}=\max_{r\in S}c_r,\\
|1+\epsilon|&=\max_{i=1,2,\ldots,N,k\in Z^+}\left|1+\epsilon_{ik}\right|,
\end{aligned} \tag{13}$$

then the solutions $x_1(t), x_1(t), \ldots, x_N(t)$ *of system* (6) *are globally and exponentially stable.*

Proof. Define the Lyapunov-Krasovskii function

$$V(t,e(t))=\frac{1}{2}\sum_{i=1}^{N}e_i(t)^T P e_i(t) \tag{14}$$

and let $\tilde{e}^k(t)=(e_{1k}(t),e_{2k}(t),\ldots,e_{Nk}(t))^T$, $k=1,2,\ldots,n$. For $r(t)=r$, we have

$$\begin{aligned}
&\mathscr{L}V(t,e(t),r)\\
&=\sum_{i=1}^{N}e_i(t)^T P\Bigg\{f\left(t,x_i(t),x_i(t-\tau(t))\right)\\
&\qquad-f\left(t,s(t),s(t-\tau(t))\right)+\sum_{j=1}^{N}a_{ij}(r)\Sigma e_j(t)\\
&\qquad+\sum_{j=1}^{N}b_{ij}(r)\Sigma e_j\left(t-\tau_c(t)\right)\Bigg\}\\
&\quad+\frac{1}{2}\sum_{i=1}^{N}\mathrm{Tr}\Big\{\sigma_i\left(t,x(t),x(t-\tau(t)),x\left(t-\tau_c(t)\right),r\right)^T\\
&\qquad\times P\sigma_i\left(t,x(t),x(t-\tau(t)),x\left(t-\tau_c(t)\right),r\right)\Big\}\\
&=\sum_{i=1}^{N}e_i(t)^T P\left\{f\left(t,x_i(t),x_i(t-\tau(t))\right)\right.\\
&\qquad\left.-f\left(t,s(t),s(t-\tau(t))\right)-\Delta\Sigma e_i(t)\right\}\\
&\quad+\sum_{i=1}^{N}e_i(t)^T P\Delta\Sigma e_i(t)+\sum_{i=1}^{N}\sum_{j=1}^{N}a_{ij}(r)e_i(t)^T P\Sigma e_j(t)\\
&\quad+\sum_{i=1}^{N}\sum_{j=1}^{N}b_{ij}(r)e_i(t)^T P\Sigma e_j\left(t-\tau_c(t)\right)\\
&\quad+\frac{1}{2}\sum_{i=1}^{N}\mathrm{Tr}\Big\{\sigma_i\left(t,x(t),x(t-\tau(t)),x\left(t-\tau_c(t)\right),r\right)^T\\
&\qquad\times P\sigma_i\left(t,x(t),x(t-\tau(t)),x\left(t-\tau_c(t)\right),r\right)\Big\}\\
&\leq\Bigg\{-\eta\sum_{i=1}^{N}e_i(t)^T e_i(t)+\theta\sum_{i=1}^{N}e_i(t-\tau(t))^T e_i(t-\tau(t))\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\delta_k\tilde{e}^k(t)^T\tilde{e}^k(t)+\sum_{k=1}^{n}p_k\varrho_k\tilde{e}^k(t)^T A(r)\tilde{e}^k(t)\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\tilde{e}^k(t)^T B(r)\tilde{e}^k\left(t-\tau_c(t)\right)\\
&\quad+\frac{1}{2}\check{p}\sum_{j=1}^{N}\Bigg[\sum_{i=1}^{N}e_i(t)^T\Upsilon_{j1}^{r}e_i(t)\\
&\qquad+\sum_{i=1}^{N}e_i(t-\tau(t))^T\Upsilon_{j2}^{r}e_i(t-\tau(t))\\
&\qquad+\sum_{i=1}^{N}e_i\left(t-\tau_c(t)\right)^T\Upsilon_{j3}^{r}e_i\left(t-\tau_c(t)\right)\Bigg]\Bigg\}\\
&=\Bigg\{-\eta\sum_{i=1}^{N}e_i(t)^T e_i(t)+\theta\sum_{i=1}^{N}e_i(t-\tau(t))^T e_i(t-\tau(t))\\
&\quad+\frac{1}{2}\check{p}\sum_{i=1}^{N}\sum_{j=1}^{N}e_i\left(t-\tau_c(t)\right)^T\Upsilon_{j3}^{r}e_i\left(t-\tau_c(t)\right)\Bigg\}\\
&\quad+\Bigg\{\sum_{k=1}^{n}p_k\varrho_k\tilde{e}^k(t)^T A(r)\tilde{e}^k(t)\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\tilde{e}^k(t)^T B(r)\tilde{e}^k\left(t-\tau_c(t)\right)\\
&\quad+\frac{1}{2}\check{p}\Bigg[\sum_{i=1}^{N}\sum_{j=1}^{N}e_i(t)^T\Upsilon_{j1}^{r}e_i(t)\\
&\qquad+\sum_{i=1}^{N}\sum_{j=1}^{N}e_i(t-\tau(t))^T\Upsilon_{j2}^{r}e_i(t-\tau(t))\Bigg]\\
&\quad+\sum_{k=1}^{n}p_k\varrho_k\delta_k\tilde{e}^k(t)^T\tilde{e}^k(t)\Bigg\}
\end{aligned}$$

$$
\begin{aligned}
&= \left\{ \sum_{i=1}^{N} e_i(t)^T \left(-\eta I_N + \frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^{r} + \alpha_r P\Sigma \right) e_i(t) \right. \\
&\quad + \sum_{i=1}^{N} e_i(t-\tau(t))^T \left(\theta + \frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j2}^{r} \right) e_i(t-\tau(t)) \\
&\quad + \sum_{i=1}^{N} e_i(t-\tau_c(t))^T \left(\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j3}^{r} + \beta_r P\Sigma \right) \\
&\quad \left. \times e_i(t-\tau_c(t)) \right\} \\
&\quad + \left\{ \sum_{k=1}^{n} p_k \varrho_k \tilde{e}^k(t)^T \left[A(r) + \left(\check{\delta} - \alpha_r\right) I_N \right] \tilde{e}^k(t) \right. \\
&\quad + \sum_{k=1}^{n} p_k \varrho_k \tilde{e}^k(t)^T B(r) \tilde{e}^k(t-\tau_c(t)) \\
&\quad \left. - \sum_{k=1}^{n} p_k \varrho_k \tilde{e}^k(t-\tau_c(t))^T \beta_r \tilde{e}^k(t-\tau_c(t)) \right\} \\
&\le \left\{ \sum_{i=1}^{N} e_i(t)^T \left(-\eta I_N + \frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j1}^{r} + \alpha_r P\Sigma \right) e_i(t) \right. \\
&\quad + \sum_{i=1}^{N} e_i(t-\tau(t))^T \left(\theta I_N + \frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j2}^{r} \right) e_i(t-\tau(t)) \\
&\quad + \sum_{i=1}^{N} e_i(t-\tau_c(t))^T \left(\frac{1}{2}\check{p}\sum_{j=1}^{N}\Upsilon_{j3}^{r} + \beta_r P\Sigma \right) \\
&\quad \left. \times e_i(t-\tau_c(t)) \right\}.
\end{aligned} \tag{15}
$$

Then we have

$$
\mathscr{L}V(t, e(t)) \le a_r V(t) + b_r V(t-\tau(t)) + c_r V(t-\tau_c(t)). \tag{16}
$$

So we have

$$
\mathscr{L}V(t) \le \check{a}V(t) + \check{b}V(t-\tau(t)) + \check{c}V(t-\tau_c(t)). \tag{17}
$$

Define

$$
W(t) = e^{\gamma t}V(t) \tag{18}
$$

and use (17) to compute the operator

$$
\begin{aligned}
\mathscr{L}W(t) &= e^{\gamma t}\left[\gamma V(t) + \mathscr{L}V(t)\right] \\
&\le e^{\gamma t}\left[\gamma V(t) + \check{a}V(t) + \check{b}V(t-\tau(t)) \right. \\
&\quad \left. + \check{c}V(t-\tau_c(t))\right],
\end{aligned} \tag{19}
$$

which after applying the generalized Itô's formula, gives

$$
e^{\gamma t}\mathbb{E}V(t) = e^{\gamma t_0}\mathbb{E}V(t_0) + \mathbb{E}\int_{t_0}^{t} \mathscr{L}W(s)\,ds \tag{20}
$$

for any $t_k > t > t_0 > t_{k-1} \ge 0$. Hence we have

$$
\begin{aligned}
&e^{\gamma t}\mathbb{E}V(t) \\
&\le e^{\gamma t_0}\mathbb{E}V(t_0) \\
&\quad + \int_{t_0}^{t} e^{\gamma s}\left[\gamma V(s) + \check{a}V(s) \right. \\
&\qquad \left. + \check{b}V(s-\tau(s)) + \check{c}V(s-\tau_c(s))\right] ds \\
&\le e^{\gamma t_0}\mathbb{E}V(t_0) + (\gamma + \check{a})\int_{t_0}^{t} e^{\gamma s}\mathbb{E}V(s)\,ds \\
&\quad + \check{b}e^{\gamma\tau}\int_{t_0}^{t} e^{\gamma(s-\tau(s))}\mathbb{E}V(s-\tau(s))\,ds \\
&\quad + \check{c}e^{\gamma\tau_c}\int_{t_0}^{t} e^{\gamma(s-\tau_c(s))}\mathbb{E}V(s-\tau_c(s))\,ds.
\end{aligned} \tag{21}
$$

By changing variable $s - \tau(s) = u$, we have

$$
\begin{aligned}
&\int_{t_0}^{t} e^{\gamma(s-\tau(s))}\mathbb{E}V(s-\tau(s))\,ds \\
&\quad = \int_{t_0-\tau(t_0)}^{t-\tau(t)} e^{\gamma u}\mathbb{E}V(u)\frac{du}{1-\dot{\tau}(t)} \\
&\quad \le \int_{t_0-\tau}^{t} e^{\gamma u}\mathbb{E}V(u)\frac{du}{1-\overline{\tau}} \\
&\quad \le \frac{1}{1-\overline{\tau}}\int_{t_0-\tau}^{t} e^{\gamma u}\mathbb{E}V(u)\,du.
\end{aligned} \tag{22}
$$

Similarly, we have

$$
\int_{t_0}^{t} e^{\gamma(s-\tau_c(s))}\mathbb{E}V(s-\tau(s))\,ds \le \frac{1}{1-\overline{\tau}_c}\int_{t_0-\tau_c}^{t} e^{\gamma u}\mathbb{E}V(u)\,du. \tag{23}
$$

Substituting (22) and (23) into (21), we get

$$
\begin{aligned}
e^{\gamma t}\mathbb{E}V(t) &\le e^{\gamma t_0}\mathbb{E}V(t_0) \\
&\quad + \left(\gamma + \check{a} + \frac{\check{b}e^{\gamma\tau}}{1-\overline{\tau}} + \frac{\check{c}e^{\gamma\tau_c}}{1-\overline{\tau}_c}\right)\int_{t_0-\tilde{\tau}}^{t} e^{\gamma u}\mathbb{E}V(u)\,du.
\end{aligned} \tag{24}
$$

By using Gronwall's inequality, we get

$$
e^{\gamma t}\mathbb{E}V(t) \le e^{\gamma t_0}\mathbb{E}V(t_0)\, e^{(\gamma+\check{a}+(\check{b}/(1-\overline{\tau}))e^{\gamma\tau}+(\check{c}/(1-\overline{\tau}_c))e^{\gamma\tau_c})(t-t_0+\tilde{\tau})}. \tag{25}
$$

On the other hand, from the construction of $V(t)$, we have

$$
V(t_k) \le (1+\epsilon_k)^2 V(t_k^-), \tag{26}
$$

where $|1+\epsilon_k| = \max_{i=1,2,\ldots,N}|1+\epsilon_{ik}|$.

According to (8)–(11), for any $t \in [t_{k-1}, t_k)$, we get

$$\begin{aligned} & e^{\gamma t}\mathbb{E}V(t) \\ & \le e^{\gamma t_{k-1}}\mathbb{E}V(t_{k-1})\, e^{(\gamma+\check{a}+(b/(1-\overline{\tau}))e^{\gamma\tau}+(\check{c}/(1-\overline{\tau}_c))e^{\gamma\tau_c})(t-t_{k-1}+\check{\tau})} \\ & \le e^{\gamma t_{k-1}}\mathbb{E}V(t_{k-1}^-) \\ & \quad \times e^{(\gamma+\check{a}+(\check{b}/(1-\overline{\tau}))e^{\gamma\tau}+(\check{c}/(1-\overline{\tau}_c))e^{\gamma\tau_c})(t-t_{k-1}+\check{\tau})+2\ln|1+\epsilon_{k-1}|} \\ & \le \cdots \le e^{\gamma t_0}\mathbb{E}V(t_0) \\ & \quad \times e^{(\gamma+\check{a}+(\check{b}/(1-\overline{\tau}))e^{\gamma\tau}+(\check{c}/(1-\overline{\tau}_c))e^{\gamma\tau_c})(t-t_0+k\check{\tau})+\sum_{v=1}^{k-1}2\ln|1+\epsilon_v|}. \end{aligned} \tag{27}$$

Let $|1+\epsilon| = \max_{v\in Z^+}|1+\epsilon_v|$. Because of $k = \lfloor (t-t_0)/T \rfloor$, we have

$$\begin{aligned} & \mathbb{E}V(t) \\ & \le \mathbb{E}V(t_0) \\ & \quad \times e^{(\gamma+\check{a}+(\check{b}/(1-\overline{\tau}))e^{\gamma\tau}+(\check{c}/(1-\overline{\tau}_c))e^{\gamma\tau_c})(t-t_0+k\check{\tau})+2(k-1)\ln|1+\epsilon_v|-(t-t_0)\gamma} \\ & \le \mathbb{E}V(t_0) \\ & \quad \times e^{(\gamma+\check{a}+(\check{b}/(1-\overline{\tau}))e^{\gamma\tau}+(\check{c}/(1-\overline{\tau}_c))e^{\gamma\tau_c})(1+T)\check{\tau}/T(t-t_0)+(2(t-t_0)/T-4)\ln|1+\epsilon_v|-(t-t_0)\gamma}. \end{aligned} \tag{28}$$

Using Condition (12) of Theorem 6, we get $\mathbb{E}V(t) \le \mathbb{E}V(t_0)e^{-\eta(t-t_0)}$. Hence, $\mathbb{E}||e_i(t)|| \le (V(t_0)/\check{p})^{1/2}e^{-(\eta/2)(t-t_0)}$. The proof of Theorem 6 is completed. □

Remark 7. Theorem 6 provides a sufficient condition for exponential synchronization of coupling delay switched stochastic dynamical networks with impulsive effects. If the time T is sufficiently small and the impulsive gains ϵ_{ik}, then exponential synchronization of the network (1) could be achieved.

If the switching signal $\sigma(t) \equiv 1$, then the network (1) has only one coupling matrix G. Suppose G is irreducible and $\xi^T = (\xi_1, \xi_2, \ldots, \xi_N)$ is the left eigenvector of coupling matrix G corresponding to eigenvalue 0. Let impulsive gains are $\epsilon_{ik} = b_k$. By the proof of Theorem 6, we can derive the exponential synchronization criteria of the network (1) with only one topology, which is given as follows.

Corollary 8. *Let assumptions (H1) and (H2) be true and let $f \in$ QUAD $(P, \Delta, \eta, \theta)$. If there exist positive constants α and β such that*

$$\begin{gathered} \begin{bmatrix} A^s + \check{\delta}I_N - \alpha I_N & \dfrac{B}{2} \\ \dfrac{B^T}{2} & -\beta I_N \end{bmatrix} \le 0, \\ \left(\gamma + a + \frac{b}{1-\overline{\tau}}e^{\gamma\tau} + \frac{c}{1-\overline{\tau}_c}e^{\gamma\tau_c}\right) \\ \times \frac{(1+T\check{\tau})}{T} + \frac{2\ln|1+\epsilon|}{T} - \gamma \le -\eta, \end{gathered} \tag{29}$$

where

$$\begin{aligned} a &= \frac{\lambda_{\min}\left(-2\eta I_n + \check{p}\sum_{i=1}^N \Upsilon_{i1} + 2\alpha P\Sigma\right)}{\check{p}}, \\ b &= \frac{\lambda_{\max}\left(\sum_{i=1}^N P\Upsilon_{i2} + 2\theta I_N\right)}{\widehat{p}}, \\ c &= \frac{\lambda_{\max}\left(\sum_{i=1}^N P\Upsilon_{i3} + 2\beta P\Sigma\right)}{\widehat{p}}, \\ |1+\epsilon| &= \max_{i=1,2,\ldots,N,k\in Z^+}|1+\epsilon_{ik}|. \end{aligned} \tag{30}$$

Then the solutions $x_1(t), x_1(t), \ldots, x_N(t)$ of system (6) are globally and exponentially stable.

Let impulsive gains $\epsilon_{ik} = b$, and choose the synchronization state $s(t) = (1/N)\sum_{i=1}^N x_i(t)$. By the proof of Theorem 6, we can derive the exponential synchronization criteria of the network (1) with the fixed impulsive gain, which is given as follows.

4. Numerical Simulation

In this section, we give two numerical simulations to illustrate the feasibility and effectiveness of the theoretical results presented in the previous sections.

Consider a three-order Chua's circuit described as follows:

$$\dot{x}(t) = f(x(t)), \tag{31}$$

where $x(t) = (x_1(t), x_2(t), x_3(t))^T$ and function $f(x(t))$ was chosen below:

$$f(x(t)) = \begin{bmatrix} m[x_2 - h(x_1)] \\ x_1 - x_2 + x_3 \\ -nx_2 \end{bmatrix}, \tag{32}$$

where $h(x_1) = (2/7)x_1 - (3/14)[|x_1+1| - |x_1-1|]$, $m = 9$, and $n = 14(2/7)$.

Consider a network model consists of 5 nodes and 2-connected topology. Each node in the network is a three-order Chua's circuit described by

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + \sum_{j=1}^5 g_{ij}^{\sigma(t)}\Gamma x_j(t-\tau) \\ &\quad + \sigma_i(t, x(t), x(t-\tau(t)), \\ &\qquad x(t-\tau_c(t)), r(t))\, dw_i(t), \\ &\qquad t \ne t_k, \\ \Delta x_i(t_k) &= x_i(t_k) - x_i(t_k^-) = \epsilon_{ik}x_i(t_k^-), \\ &\qquad t = t_k, \quad i = 1, 2, \ldots, 5, \end{aligned} \tag{33}$$

where $\tau = 1$, $\Gamma = I_3$, and $\sigma_i(t, x(t), x(t-\tau(t)), x(t-\tau_c(t))) = 0.1 \times \mathrm{diag}\{x_{i1}(t) - x_{i+1,1}(t), x_{i2}(t) - x_{i+1,2}(t)\}$.

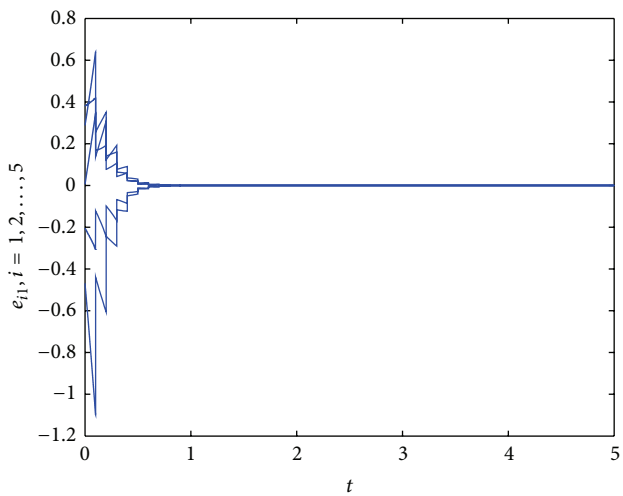

FIGURE 1: Time evolution of synchronization errors $e_{i1}(t)$, $(i = 1, 2, \ldots, 5)$ of the asymmetric coupled network (21) with random initial values.

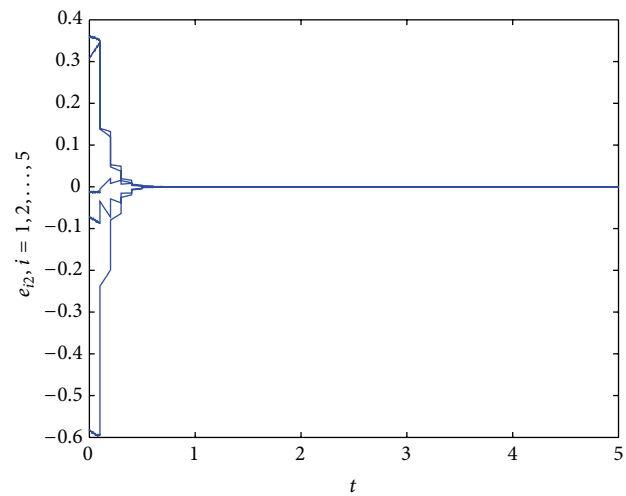

FIGURE 2: Time evolution of synchronization errors $e_{i2}(t)$, $(i = 1, 2, \ldots, 5)$ of the asymmetric coupled network (21) with random initial values.

The coupling matrices are as follows:

$$G^1 = \begin{bmatrix} -3 & 2 & 1 & 0 & 0 \\ 0 & -2 & 2 & 0 & 0 \\ 1 & 0 & -3 & 2 & 0 \\ 0 & 0 & 0 & -2 & 2 \\ 2 & 0 & 0 & 0 & -3 \end{bmatrix},$$
$$G^2 = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 0 & -2 & 1 \\ 1 & 1 & 0 & 0 & -2 \end{bmatrix}. \tag{34}$$

If we choose $P = I_3$ and $\Delta = 10I_3$, then function $f(x)$ satisfies the condition of function class QUAD(Δ, P), where $\alpha = 0.6218$. Let $\beta = 23.7$, $Q = 4.1I_3$, $\epsilon_{ik} = -0.8$, $T = 0.1$, $\eta = -3.488$, $a = 4.1$. $a_1 = 80.1500$, $a_2 = 62.3271$, $b = 6.5300$, $c_1 = 0.0307$, $c_2 = 80.0300$, $\alpha_1 = 40.0000$, $\beta_1 = 0.0002$, $\alpha_2 = 15.6568$, $\beta_2 = 20.0000$, and the synchronization state $s(t) = 0.2x_1 + 0.3x_2 + 0.1x_3 + 0.3x_4 + 0.1x_5$; then all the conditions in Theorem 6 are satisfied (by using the Matlab LMI toolbox). The switch time is $t = 0.2$ s. The simulation results are given in Figures 1–3. It can be seen clearly from Figures 1, 2, and 3 that all states of the asymmetric coupled network (21) tend to the synchronization state $s(t)$.

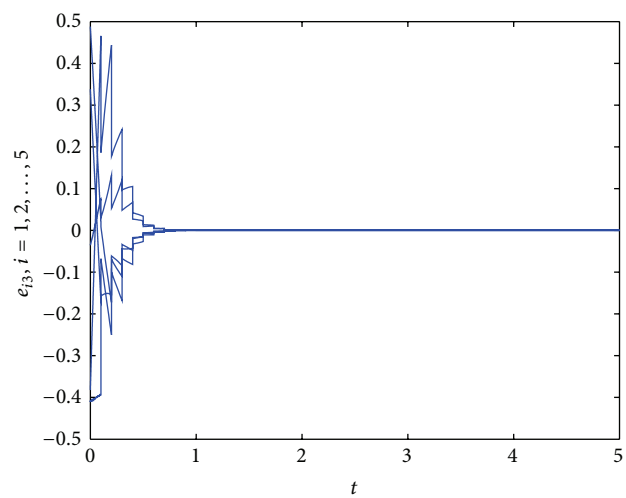

FIGURE 3: Time evolution of synchronization errors $e_{i3}(t)$, $(i = 1, 2, \ldots, 5)$ of the asymmetric coupled network (21) with random initial values.

5. Conclusion

In this paper, the exponential synchronization of the coupling delay stochastic complex networks with switch topology and impulsive effects has been investigated. Based on the Lyapunov stability theory, LMI, and impulsive control theory, some simple, yet generic, criteria for exponential synchronization have been derived. It has shown that criteria can provide an effective control scheme to synchronize for a given coupled delay, the network size, and switch topology. Furthermore, the effectiveness of the presented method has been verified by numerical simulations.

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