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Research Article

Global Behavior of $x_{n+1} = (\alpha + \beta x_{n-k})/(\gamma + x_n)$

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This paper aims to investigate the global stability of negative solutions of the difference equation $x_{n+1} = (\alpha + \beta x_{n-k})/(\gamma + x_n)$, $n = 0, 1, 2, \ldots$, where the initial conditions $x_{-k}, \ldots, x_0 \in (-\infty, 0)$, k is a positive integer, and the parameters β, $\gamma < 0$, $\alpha > 0$. By utilizing the invariant interval and periodic character of solutions, it is found that the unique negative equilibrium is globally asymptotically stable under some parameter conditions. Additionally, two examples are given to illustrate the main results in the end.

1. Introduction

The study of nonlinear difference equations has always attracted a considerable attention (see, e.g., [1–30] and the references cited therein). In particular, some references investigated the dynamical behavior of positive solutions of difference equations (see, e.g., [3–5, 11]), and some references examined the dynamical behavior of negative solutions of some difference equations (see, e.g., [6, 7]).

Gibbons et al. [4] studied the behavior of nonnegative solutions to the recursive sequence

$$x_{n+1} = \frac{\alpha + \beta x_{n-1}}{\gamma + x_n}, \quad n = 0, 1, \ldots, \tag{1}$$

with $\alpha, \beta, \gamma \geq 0$, and also presented an open problem, which had been solved by Stević in [8]. Kulenović and Ladas, in addition, considered (1) in their book [9].

Stević [10] considered the behavior of nonnegative solutions of the following second-order difference equation

$$x_{n+1} = \frac{\alpha + \beta x_{n-1}}{1 + g(x_n)}, \quad n = 0, 1, \ldots, \tag{2}$$

where $g : \mathbb{R}^+ \bigcup \{0\} \to \mathbb{R}$ is a nonnegative increasing mapping.

Douraki et al. [11] studied the qualitative behavior of positive solutions of the difference equation

$$x_{n+1} = \frac{p + q x_{n-k}}{1 + x_n}, \quad n = 0, 1, \ldots, \tag{3}$$

where the initial values $x_{-k}, \ldots, x_{-1}, x_0 \in (0, +\infty)$, k is a positive integer, and $p, q \geq 0$. Moreover, (3) is a special case of the following open problem (see also in [11]), which was proposed by Kulenović and Ladas in [9].

Open Problem (equation (6.97) in [9]). Assume that $p, q \in [0, \infty)$. Investigate the global behavior of positive solutions of (3).

Stević [12] considered the boundedness, oscillatory behavior, and global stability of nonnegative solutions of the difference equation

$$x_{n+1} = \frac{\alpha + \beta x_{n-k}}{f(x_n, \ldots, x_{n-k+1})}, \quad n = 0, 1, \ldots, \tag{4}$$

where $k \in \mathbb{N}$, α, $\beta \geq 0$, and $f : \mathbb{R}_+^k \to \mathbb{R}_+$ is a continuous function nondecreasing in each variable such that $f(0, \ldots, 0) > 0$.

It is worthwhile to note that the above mentioned references ([4, 10–12]), especially [4, 11], only discussed the dynamical behavior of positive solutions of difference equation. Furthermore, inspired by the above work and [6, 7],

the main goal of this paper is to study the global behavior of negative solutions of the difference equation

$$x_{n+1} = \frac{\alpha + \beta x_{n-k}}{\gamma + x_n}, \quad n = 0, 1, \ldots, \tag{5}$$

where k is a positive integer, $\alpha > 0$, $\beta, \gamma < 0$, and the initial conditions $x_{-k}, \ldots, x_0 \in (-\infty, 0)$.

In fact, it is easy to see that (5) is an extension of an open problem introduced by Kulenović and Ladas in [9] and also is a special case of (4) by a simple change. However, here we establish some results regarding the global stability, invariant interval, and periodic character of negative solutions of (5).

2. Linearized Stability and Period 2 Solutions

The aim of this section is to discuss the local stability of the unique negative equilibrium of (5). The period 2 solutions of (5), in addition, will be verified.

In this section, we need the following lemma.

Lemma 1 (see [3]). *Assume that $a, b \in \mathbb{R}$ and $k \in \{1, 2, \ldots\}$. Then*

$$|a| + |b| < 1 \tag{6}$$

is a sufficient condition for the asymptotic stability of the difference equation

$$x_{n+1} - a x_n + b x_{n-k} = 0, \quad n = 0, 1, \ldots. \tag{7}$$

Suppose in addition that one of the following two cases holds.

(a) *k is odd and $b < 0$.*

(b) *k is even and $ab < 0$.*

Then (6) *is also a necessary condition for the asymptotic stability of* (7).

Assume that $\overline{x}$ is an equilibrium of (5). Then it satisfies the equation $\overline{x}^2 + (\gamma - \beta)\overline{x} - \alpha = 0$, which implies that the unique negative equilibrium of (5) is

$$\overline{x} = \frac{\beta - \gamma - \sqrt{(\beta - \gamma)^2 + 4\alpha}}{2}. \tag{8}$$

Let

$$x_{n+1} = f(x_n, x_{n-1}, \ldots, x_{n-k}) = \frac{\alpha + \beta x_{n-k}}{\gamma + x_n}. \tag{9}$$

Then the linearized equation of (5) at $\overline{x}$ is

$$u_{n+1} = p_0 u_n + p_1 u_{n-1} + \cdots + p_k u_{n-k}, \tag{10}$$

where

$$p_i = \frac{\partial f}{\partial x_{n-i}} (\overline{x}, \overline{x}, \ldots, \overline{x}). \tag{11}$$

Straightforward calculations yield

$$u_{n+1} + \frac{\overline{x}}{\gamma + \overline{x}} u_n - \frac{\beta}{\gamma + \overline{x}} u_{n-k} = 0. \tag{12}$$

From here and Lemma 1, we obtain the following result.

Theorem 2. *The unique negative equilibrium $\overline{x}$ of* (5) *is locally asymptotically stable if $\gamma < \beta$.*

Proof. If $\gamma < \beta$, then

$$\left| \frac{\overline{x}}{\gamma + \overline{x}} \right| + \left| \frac{-\beta}{\gamma + \overline{x}} \right| = \frac{\beta + \overline{x}}{\gamma + \overline{x}} < 1. \tag{13}$$

It follows from Lemma 1 that $\overline{x}$ is locally asymptotically stable. Thus the proof is complete. □

Now, we will examine the existence of period 2 solutions of (5).

Theorem 3. *Let $\{x_n\}_{n=-k}^{\infty}$ be a negative solution of* (5). *Then the following statements hold.*

(a) *Assume that k is odd. Then,*

  (i) *equation* (5) *has a negative prime period 2 solution $\ldots, w, r, w, r, \ldots$ if and only if $\beta = \gamma$;*

  (ii) *if $\beta = \gamma$, then the values w and r of all negative prime period 2 solutions are given by $\{w, r \in (-\infty, 0) : wr = \alpha\}$;*

  (iii) *if $\beta \neq \gamma$, then* (5) *has no negative solutions with prime period 2.*

(b) *Assume that k is even. Then* (5) *has no negative solutions with prime period 2.*

Proof. Let $\{x_n\}_{n=-k}^{\infty}$ be a negative solution of (5).

(a) Assume that k is odd. Then,

  (i) suppose that $\ldots, w, r, w, r, \ldots$ is a negative prime period 2 solution. Then from (5), we have

$$w = \frac{\alpha + \beta w}{\gamma + r}, \qquad r = \frac{\alpha + \beta r}{\gamma + w}. \tag{14}$$

  From the above relations, we derive that

$$(w - r)(\beta - \gamma) = 0. \tag{15}$$

  Since $w \neq r$, then $\beta = \gamma$.
  The reverse part is clear by a simple computation. So it is omitted.

  (ii) Applying the relations (14), we get that $w(\gamma + r) = \alpha + \beta w$ and $r(\gamma + w) = \alpha + \beta r$; namely, $w(\gamma - \beta) + wr = \alpha$ and $r(\gamma - \beta) + wr = \alpha$. As $\beta = \gamma$, then $wr = \alpha$.

  (iii) If $\beta \neq \gamma$, then it follows from (15) that $w = r$.

(b) Assume that k is even. Let $\ldots, w, r, w, r, \ldots (w \neq r)$ be a negative prime period 2 solution; then

$$w = \frac{\alpha + \beta r}{\gamma + r}, \qquad r = \frac{\alpha + \beta w}{\gamma + w}. \tag{16}$$

It follows from the above equations that $(w - r)(\beta + \gamma) = 0$. As $\beta + \gamma < 0$, then $w = r$, which contradicts the hypothesis that $w \neq r$. □

3. Invariant Interval

In this section, we will consider the invariant interval of negative solutions of (5).

Let

$$f(u,v)=\frac{\alpha+\beta v}{\gamma+u}. \tag{17}$$

Lemma 4. *The following statements are true.*

(a) *Assume that $\gamma\in(-\infty,(\beta-\sqrt{\beta^2+4\alpha})/2]$. Then,*

  (i) $\gamma<\beta<0$;

  (ii) *if $\beta<-\sqrt{\alpha/2}$, then $\beta\gamma>\alpha$ and $\overline{x}\in(\beta,0)\subset[\gamma,0]$.*

(b) *Assume that $f(u,v)$ is defined by (17) and $u,v\in(-\infty,0]$. Then $f(u,v)$ is nonincreasing in u and nondecreasing in v.*

Proof. The proofs of (a) and (b) are as follows:

(a) if $\gamma\in(-\infty,(\beta-\sqrt{\beta^2+4\alpha})/2]$, then

  (ii) $\gamma-\beta\le(\beta-\sqrt{\beta^2+4\alpha})/2-\beta=-(\beta+\sqrt{\beta^2+4\alpha})/2<0$, clearly, $\gamma<\beta<0$;

  (iii) the condition $\beta<-\sqrt{\alpha/2}$ implies that $\alpha/\beta^2<2$. Then

$$\begin{aligned}\frac{\left|\left(\beta-\sqrt{\beta^2+4\alpha}\right)/2\right|}{|\alpha/\beta|}&=\frac{\beta\left(\beta-\sqrt{\beta^2+4\alpha}\right)}{2\alpha}\\&=\frac{-2\beta}{\beta+\sqrt{\beta^2+4\alpha}}\\&=\frac{1}{\left((-1/2)+\sqrt{(1/4)+(\alpha/\beta^2)}\right)}\\&>\frac{1}{\left(-1/2+\sqrt{1/4+2}\right)}=1,\end{aligned} \tag{18}$$

which leads to $\gamma\le(\beta-\sqrt{\beta^2+4\alpha})/2<\alpha/\beta$. Note that $\gamma<\beta<0$; thus $\beta\gamma>\alpha$. Also

$$\overline{x}-\beta=\frac{2(\beta\gamma-\alpha)}{\sqrt{(\beta-\gamma)^2+4\alpha}-(\beta+\gamma)}>0. \tag{19}$$

Clearly, $\overline{x}\in(\beta,0)\subset[\gamma,0]$.

(b) Note that

$$\frac{\partial f}{\partial u}=-\frac{\alpha+\beta v}{(\gamma+u)^2}<0,\qquad \frac{\partial f}{\partial v}=\frac{\beta}{\gamma+u}>0. \tag{20}$$

So the result holds. □

Theorem 5. *Assume that $\gamma\in(-\infty,(\beta-\sqrt{\beta^2+4\alpha})/2]$. Then $[\gamma,0]$ is an invariant interval of (5).*

Proof. Suppose that $\{x_n\}_{n=-k}^{\infty}$ is a solution to (5) with initial conditions $x_{-k},\ldots,x_0\in[\gamma,0]$. Since $\gamma\in(-\infty,(\beta-\sqrt{\beta^2+4\alpha})/2]$, it follows by a direct computation that $\alpha+\beta\gamma-\gamma^2\le0$. By Lemma 4(b), we immediately get that the function $f(u,v)$ is nonincreasing in u and nondecreasing in v with $u,v\in(-\infty,0)$. Then

$$\begin{aligned}x_1&=\frac{\alpha+\beta x_{-k}}{\gamma+x_0}=f(x_0,x_{-k})\\&<f(x_0,0)<f(\gamma,0)=\frac{\alpha}{2\gamma}<0,\\x_1&=\frac{\alpha+\beta x_{-k}}{\gamma+x_0}=f(x_0,x_{-k})\\&>f(0,x_{-k})>f(0,\gamma)=\frac{\alpha+\beta\gamma}{\gamma}\ge\gamma,\end{aligned} \tag{21}$$

which implies that $x_1\in[\gamma,0]$. It follows by induction that $x_n\in[\gamma,0]$ for all $n\ge1$. Thus, the proof is complete. □

4. Global Stability

Recall that (5) has a unique negative equilibrium $\overline{x}$, which is locally asymptotically stable if $\gamma<\beta$ by Theorem 2. In this section, we will show that (a) $\overline{x}$ is globally asymptotically stable under the conditions $\gamma\in(-\infty,(\beta-\sqrt{\beta^2+4\alpha})/2]$ and $\beta<-\sqrt{\alpha/2}$; (b) every negative solution converges to $\overline{x}$ when $\beta\gamma=\alpha$.

Lemma 6 (see [3]). *Consider the difference equation*

$$y_{n+1}=f(y_n,y_{n-k}),\quad n=0,1,\ldots, \tag{22}$$

where $k\in\{1,2,\ldots\}$. Let $I=[a,b]$ be some interval of real numbers and assume that

$$f:[a,b]\times[a,b]\longrightarrow[a,b] \tag{23}$$

is a continuous function satisfying the following properties:

(a) *$f(u,v)$ is nonincreasing in u and nondecreasing in v;*

(b) *if $(m,M)\in[a,b]$ is a solution of the system*

$$m=f(M,m),\qquad M=f(m,M), \tag{24}$$

  then $m=M$.

Then (22) has a unique equilibrium $\overline{y}$ and every solution of (22) converges to $\overline{y}$.

Theorem 7. *Assume that $\gamma\in(-\infty,(\beta-\sqrt{\beta^2+4\alpha})/2]$ and $\beta<-\sqrt{\alpha/2}$. Then the following statements are true:*

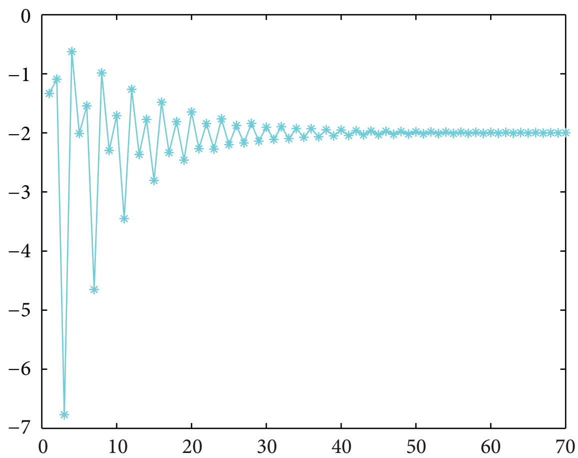

FIGURE 1: Evolution of x_{n+1} for (1) with parameters and initial condition given in Example 1.

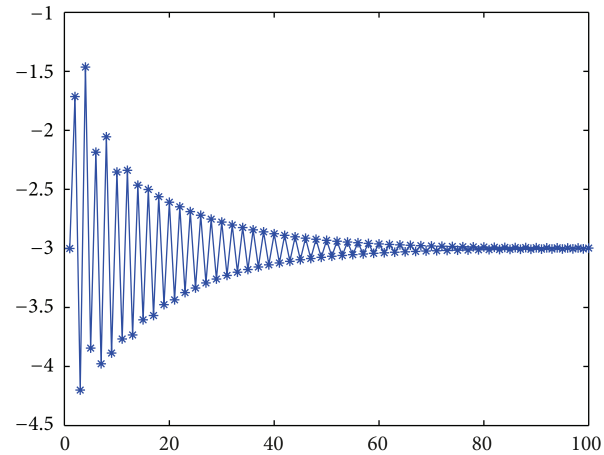

FIGURE 2: Evolution of x_{n+1} for (1) with parameters and initial condition given in Example 2.

(i) *the unique negative equilibrium* $\bar{x} = (\beta - \gamma - \sqrt{(\beta-\gamma)^2 + 4\alpha})/2$ *of* (5) *is a global attractor with a basin* $S = [\gamma, 0]^{k+1}$;

(ii) *the unique negative equilibrium* $\bar{x} = (\beta - \gamma - \sqrt{(\beta-\gamma)^2 + 4\alpha})/2$ *of* (5) *is globally asymptotically stable.*

Proof. Suppose that $\{x_n\}_{n=-k}^{\infty}$ is a solution to (5) with initial values $x_{-k}, \ldots, x_0 \in S$. By Lemma 4(b), we immediately get that the function $f(u,v)$ is continuous and nonincreasing in u and nondecreasing in v on the invariant interval $[\gamma, 0]$. Furthermore, let $m, M \in I$ be a solution of the system $m = f(m, M)$, $M = f(M, m)$; then by Lemma 4(a)(i) and (ii), we have $\gamma < \beta < 0$ and $\bar{x} \in (\beta, 0) \subset [\gamma, 0]$, and by Theorem 3, we obtain $m = M$. Finally, it follows by Lemma 6 that $\lim_{n\to\infty} x_n = \bar{x}$. Hence, the proof is complete. □

Lemma 8. *Let* $\beta\gamma = \alpha$. *Then* (5) *has no nontrivial negative periodic solutions of (not necessarily prime) period* k.

Proof. If $\beta\gamma = \alpha$, then substituting (8), it becomes that $\bar{x} = \beta$. Suppose that $\{x_n\}_{n=-k}^{\infty}$ is a negative solution to (5) satisfying $x_n = x_{n-k}$ for all $n \geq 0$; then

$$x_{n+1} = \frac{\alpha + \beta x_n}{\gamma + x_n}. \tag{25}$$

Simplifying the above equation, it follows that $(\gamma + x_n)(x_{n+1} - \beta) = 0$. Clearly, $x_{n+1} = \beta = \bar{x}$ for all $n \geq 0$. The proof is complete. □

Theorem 9. *Assume that* $\beta\gamma = \alpha$. *Then every negative solution to* (5) *converges to the unique negative equilibrium* $\bar{x}$.

Proof. Assume that $\beta\gamma = \alpha$. It follows from Lemma 8 that $\bar{x} = \beta$. Then

$$x_{n+1} - \bar{x} = \frac{\alpha + \beta x_{n-k}}{\gamma + x_n} - \beta = \frac{\beta(x_{n-k} - x_n)}{\gamma + x_n}. \tag{26}$$

Case 1. If $x_{n-k} - x_n < 0$, then $x_{n+1} < \bar{x}$, so $x_{n-k} < x_n < \bar{x}$.

Case 2. If $x_{n-k} - x_n \geq 0$, then $x_{n+1} \geq \bar{x}$, so $\bar{x} \leq x_n \leq x_{n-k}$.

Next, we consider Case 1 (Case 2 is similar and thus it is omitted). If $x_{n-k} < x_n < \bar{x}$ for all $n \geq 0$, then it is clear that for $i \in \{0, 1, \ldots, k-1\}$ there exists α_i such that

$$\lim_{m\to\infty} x_{mk+i} = \alpha_i. \tag{27}$$

But then $\alpha_0, \alpha_1, \ldots, \alpha_{k-1}$ is a periodic solution of (not necessarily prime) period k. By Lemma 8 the result holds. □

5. Examples

To illustrate the main results in Section 4, here we present two examples.

Example 1. Consider (1) with $\alpha = 8, \beta = -4, \gamma = -6$, and $k = 3$. Then $\bar{x} = -2$. As $\gamma \in (-\infty, (\beta - \sqrt{\beta^2 + 4\alpha})/2]$ and $\beta < -\sqrt{\alpha}/2$, it follows from Theorem 7(ii) that the unique negative equilibrium is globally asymptotically stable. For the initial conditions $x_{-i} = -10$, $i = 0, 1, 2, 3$, Figure 1 exhibits how x_{n+1} evolves with n.

Example 2. Consider (1) with $\alpha = 12, \beta = -3, \gamma = -4$, and $k = 3$. Then $\bar{x} = -3$. As $\beta\gamma = \alpha$, it follows from Theorem 9 that the unique negative equilibrium is globally attractive. For the initial conditions $x_{-i} = -4$, $i = 0, 1, 2, 3$, Figure 2 exhibits how x_{n+1} evolves with n.

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