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Research Article

Implicative Int-Soft Filters of R_0-Algebras

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The notion of implicative int-soft filters is introduced, and related properties are investigated. A relation between an int-soft filter and an implicative int-soft filter is discussed, and conditions for an int-soft filter to be an implicative int-soft filter are provided. Characterizations of an implicative int-soft filter are considered, and a new implicative int-soft filter from an old one is displayed. The extension property of an implicative int-soft filter is established.

1. Introduction

To solve complicated problems in economics, engineering, and the environment, we cannot successfully use classical methods because of various uncertainties typical for those problems. Uncertainties cannot be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as probability theory, theory of (intuitionistic) fuzzy sets, theory of vague sets, theory of interval mathematics, and theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [1]. Maji et al. [2] and Molodtsov [1] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory.

To overcome these difficulties, Molodtsov [1] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. Maji et al. [2] described the application of soft set theory to a decision making problem. Maji et al. [3] also studied several operations on the theory of soft sets. Chen et al. [4] presented a new definition of soft set parametrization reduction and compared this definition to the related concept of attributes reduction in rough set theory.

R_0-algebras, which are different from BL-algebras, have been introduced by Wang [5]. The filter theory in R_0-algebras is discussed in [7]. In [8], Jun et al. applied the notion of intersection-soft sets to the filter theory in R_0-algebras. They introduced the concept of strong int-soft filters in R_0-algebras and investigated related properties. They established characterizations of a strong int-soft filter and provided a condition for an int-soft filter to be strong. They also constructed an extension property of a strong int-soft filter.

In this paper, we introduce a new notion which is called an implicative int-soft filter and investigate related properties. We discuss a relation between an int-soft filter and an implicative int-soft filter. We provide conditions for an int-soft filter to be an implicative int-soft filter. We consider characterizations of an implicative int-soft filter and construct a new implicative int-soft filter from an old one. We establish the extension property of an implicative int-soft filter.

2. Preliminaries

Definition 1 (see [5]). Let L be a bounded distributive lattice with order-reversing involution $\neg$ and a binary operation $\rightarrow$. Then $(L, \wedge, \vee, \neg, \rightarrow)$ is called an R_0-*algebra* if it satisfies the following axioms:

(R1) $x \to y = \neg y \to \neg x$,
(R2) $1 \to x = x$,
(R3) $(y \to z) \wedge ((x \to y) \to (x \to z)) = y \to z$,
(R4) $x \to (y \to z) = y \to (x \to z)$,
(R5) $x \to (y \vee z) = (x \to y) \vee (x \to z)$,
(R6) $(x \to y) \vee ((x \to y) \to (\neg x \vee y)) = 1$.

Let L be an R_0-algebra. For any $x, y \in L$, we define $x \odot y = \neg(x \to \neg y)$ and $x \oplus y = \neg x \to y$. It is proven that $\odot$ and $\oplus$ are commutative and associative and $x \oplus y = \neg(\neg x \odot \neg y)$, and $(L, \wedge, \vee, \odot, \to, 0, 1)$ is a residuated lattice. In the following, let x^n denote $x \odot x \odot \cdots \odot x$, where x appears n times for $n \in \mathbb{N}$.

We refer the reader to the book [9] for further information regarding R_0-algebras.

Lemma 2 (see [7]). *Let L be an R_0-algebra. Then the following properties hold:*

$$(\forall x, y \in L) \quad (x \le y \Longleftrightarrow x \longrightarrow y = 1), \tag{1}$$

$$(\forall x, y \in L) \quad (x \le y \longrightarrow x), \tag{2}$$

$$(\forall x \in L) \quad (\neg x = x \longrightarrow 0), \tag{3}$$

$$(\forall x, y \in L) \quad ((x \longrightarrow y) \vee (y \longrightarrow x) = 1), \tag{4}$$

$$\begin{aligned}(\forall x, y \in L) \quad (&x \le y \Longrightarrow y \longrightarrow z \\ &\le x \longrightarrow z, z \longrightarrow x \le z \longrightarrow y),\end{aligned} \tag{5}$$

$$(\forall x, y \in L) \quad (((x \longrightarrow y) \longrightarrow y) \longrightarrow y = x \longrightarrow y), \tag{6}$$

$$\begin{aligned}(\forall x, y \in L) \quad (&x \vee y = ((x \longrightarrow y) \longrightarrow y) \\ &\wedge ((y \longrightarrow x) \longrightarrow x)),\end{aligned} \tag{7}$$

$$(\forall x \in L) \quad (x \odot \neg x = 0, x \oplus \neg x = 1), \tag{8}$$

$$(\forall x, y \in L) \quad (x \odot y \le x \wedge y, x \odot (x \longrightarrow y) \le x \wedge y), \tag{9}$$

$$(\forall x, y, z \in L) \quad ((x \odot y) \longrightarrow z = x \longrightarrow (y \to z)), \tag{10}$$

$$(\forall x, y \in L) \quad (x \le y \longrightarrow (x \odot y)), \tag{11}$$

$$(\forall x, y, z \in L) \quad (x \odot y \le z \Longleftrightarrow x \le y \to z), \tag{12}$$

$$(\forall x, y, z \in L) \quad (x \le y \Longrightarrow x \odot z \le y \odot z), \tag{13}$$

$$(\forall x, y, z \in L) \quad (x \longrightarrow y \le (y \longrightarrow z) \to (x \longrightarrow z)), \tag{14}$$

$$(\forall x, y, z \in L) \quad ((x \longrightarrow y) \odot (y \longrightarrow z) \le x \longrightarrow z). \tag{15}$$

Definition 3 (see [7]). A nonempty subset F of L is called a *filter* of L if it satisfies the following:

(i) $1 \in F$,
(ii) $(\forall x \in F)\ (\forall y \in L)\ (x \to y \in F \Rightarrow y \in F)$.

Definition 4 (see [10]). A subset F of L is called an *implicative filter* of L if it satisfies the following:

(i) $1 \in F$,
(ii) $(\forall x, y, z \in L)\ (x \to (y \to z) \in F, x \to y \in F \Rightarrow x \to z \in F)$.

Note that every implicative filter is a filter. The following is a characterization of filters.

Soft set theory was introduced by Molodtsov [1] and Çağman and Enginğlu [11].

In what follows, let U be an initial universe set and let E be a set of parameters. We say that the pair (U, E) is a *soft universe*. Let $\mathscr{P}(U)$ (resp., $\mathscr{P}(E)$) denote the power set of U (resp., E).

By analogy with fuzzy set theory, the notion of soft set is defined as follows.

Definition 5 (see [1, 11]). A soft set of E over U (a soft set of E for short) is

$$\begin{aligned}&\text{any function } f_A : E \to \mathscr{P}(U), \\ &\text{such that } f_A(x) = \emptyset \text{ if } x \notin A, \text{ for } A \in \mathscr{P}(E),\end{aligned} \tag{16}$$

or, equivalently, any set

$$\begin{aligned}\mathscr{F}_A := \{(x, f_A(x)) \mid x \in E, f_A(x) \in \mathscr{P}(U), \\ f_A(x) = \emptyset \text{ if } x \notin A\}\end{aligned} \tag{17}$$

for $A \in \mathscr{P}(E)$.

3. Implicative Int-Soft Filters

In what follows, denote by $S(U, L)$ the set of all soft sets of L over U, where L is an R_0-algebra unless otherwise specified.

Definition 6 (see [8]). A soft set $\mathscr{F}_L \in S(U, L)$ is called an *int-soft filter* of L if it satisfies

$$(\forall \gamma \in \mathscr{P}(U)) \quad (\mathscr{F}_L^\gamma \neq \emptyset \Longrightarrow \mathscr{F}_L^\gamma \text{ is a filter of } L), \tag{18}$$

where $\mathscr{F}_L^\gamma = \{x \in L \mid \gamma \subseteq f_L(x)\}$ which is called the γ-*inclusive set* of $\mathscr{F}_L$.

If $\mathscr{F}_L$ is an int-soft filter of L, every γ-inclusive set $\mathscr{F}_L^\gamma$ is called an *inclusive filter* of L.

Lemma 7 (see [8]). *Let $\mathscr{F}_L \in S(U, L)$. Then $\mathscr{F}_L$ is an int-soft filter of L if and only if the following assertions are valid:*

$$(\forall x \in L) \quad (f_L(x) \subseteq f_L(1)), \tag{19}$$

$$(\forall x, y \in L) \quad (f_L(x \longrightarrow y) \cap f_L(x) \subseteq f_L(y)). \tag{20}$$

Definition 8. Let $\mathscr{F}_L \in S(U, L)$. Then $\mathscr{F}_L$ is called an *implicative int-soft filter* of L if and only if it satisfies (19) and

$$\begin{aligned}(\forall x, y, z \in L) \quad (&f_L(x \longrightarrow z) \\ &\supseteq f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y)).\end{aligned} \tag{21}$$

Example 9. Let $L = \{0, a, b, c, d, 1\}$ be a set with the order $< a < b < c < d < 1, 0$ and the following Cayley tables:

x	$\neg x$
0	1
a	d
b	c
c	b
d	a
1	0

$\rightarrow$	0	a	b	c	d	1
0	1	1	1	1	1	1
a	d	1	1	1	1	1
b	c	c	1	1	1	1
c	b	b	b	1	1	1
d	a	a	b	c	1	1
1	0	a	b	c	d	1

Then $(L, \wedge, \vee, \neg, \rightarrow)$ is an R_0-algebra (see [10]), where $x \wedge y = \min\{x, y\}$ and $x \vee y = \max\{x, y\}$. Let $\mathscr{F}_L \in S(U, L)$ be given as follows:

$$\mathscr{F}_L = \{(0, \gamma_1), (a, \gamma_1), (b, \gamma_1), (c, \gamma_2), (d, \gamma_2), (1, \gamma_2)\}, \tag{22}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. Then $\mathscr{F}_L$ is an implicative int-soft filter of L.

We provide a relation between an int-soft filter and an implicative int-soft filter.

Theorem 10. *Every implicative int-soft filter is an int-soft filter.*

Proof. Let $\mathscr{F}_L$ be an implicative int-soft filter of L. If we take $x = 1$ in (21), then

$$\begin{aligned} f_L(z) &= f_L(1 \longrightarrow z) \\ &\supseteq f_L(1 \longrightarrow (y \longrightarrow z)) \cap f_L(1 \longrightarrow y) \\ &= f_L(y \longrightarrow z) \cap f_L(y) \end{aligned} \tag{23}$$

for all $y, z \in L$. Therefore, $\mathscr{F}_L$ is an int-soft filter of L. □

The following example shows that the converse of Theorem 10 is not true in general.

Example 11. Let $L = \{0, a, b, c, 1\}$ be a set with the order $0 < a < b < c < 1$ and the following Cayley tables:

x	$\neg x$
0	1
a	c
b	b
c	a
1	0

$\rightarrow$	0	a	b	c	1
0	1	1	1	1	1
a	c	1	1	1	1
b	b	b	1	1	1
c	a	a	b	1	1
1	0	a	b	c	1

Then $(L, \wedge, \vee, \neg, \rightarrow)$ is an R_0-algebra (see [10]), where $x \wedge y = \min\{x, y\}$ and $x \vee y = \max\{x, y\}$. Let $\mathscr{F}_L \in S(U, L)$ be given as follows:

$$\mathscr{F}_L = \{(0, \gamma_1), (a, \gamma_1), (b, \gamma_1), (c, \gamma_2), (1, \gamma_2)\}, \tag{24}$$

where γ_1 and γ_2 are subsets of U with $\gamma_1 \subsetneq \gamma_2$. Then $\mathscr{F}_L$ is an int-soft filter of L (see [8]). But it is not an implicative int-soft filter of L since

$$f_L(b \longrightarrow a) = \gamma_1 \supsetneq \gamma_2 = f_L(b \longrightarrow (b \longrightarrow a)) \cap f_L(b \longrightarrow b). \tag{25}$$

We provide conditions for an int-soft filter to be an implicative int-soft filter.

Theorem 12. *An int-soft filter $\mathscr{F}_L$ of L is implicative if and only if it satisfies*

$$f_L(x \longrightarrow z) \supseteq f_L(x \longrightarrow (\neg z \longrightarrow y)) \cap f_L(y \longrightarrow z) \tag{26}$$

for all $x, y, z \in L$.

Proof. Let $\mathscr{F}_L$ be an int-soft filter of L that satisfies condition (26). Using (R1), (R4), and (26), we have

$$\begin{aligned} f_L(x \longrightarrow z) &= f_L(\neg z \longrightarrow \neg x) \\ &\supseteq f_L(\neg z \longrightarrow (\neg\neg x \longrightarrow \neg y)) \cap f_L(\neg y \longrightarrow \neg x) \\ &= f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y) \end{aligned} \tag{27}$$

for all $x, y, z \in L$. Thus, $\mathscr{F}_L$ is an implicative int-soft filter of L.

Conversely, suppose that $\mathscr{F}_L$ is an implicative int-soft filter of L. Then

$$\begin{aligned} f_L(x \longrightarrow z) &= f_L(\neg z \longrightarrow \neg x) \\ &\supseteq f_L(\neg z \longrightarrow (\neg y \longrightarrow \neg x)) \cap f_L(\neg z \longrightarrow \neg y) \\ &= f_L(x \longrightarrow (\neg z \longrightarrow y)) \cap f_L(y \longrightarrow z) \end{aligned} \tag{28}$$

for all $x, y, z \in L$ by (R1), (R4), and (21). □

Lemma 13 (see [8]). *Every int-soft filter $\mathscr{F}_L$ is order preserving; that is,*

$$(\forall x, y \in L) \quad (x \leq y \Longrightarrow f_L(x) \subseteq f_L(y)), \tag{29}$$

$$\begin{aligned} (\forall x, y, z \in L) \quad & (f_L(x \longrightarrow (\neg z \longrightarrow y)) \\ & \cap f_L(y \longrightarrow z) \subseteq f_L(x \longrightarrow (\neg z \longrightarrow z))), \end{aligned} \tag{30}$$

$$(\forall x, y \in L) \quad (f_L(x \odot y) = f_L(x) \cap f_L(y) = f_L(x \wedge y)). \tag{31}$$

Proposition 14. *Every implicative int-soft filter $\mathcal{F}_L$ of L satisfies the following assertions:*

$$(\forall x, y \in L) \quad (f_L(x \longrightarrow y) = f_L(x \longrightarrow (\neg y \longrightarrow y))), \tag{32}$$

$$\begin{aligned}(\forall x, y, z \in L) \quad & (f_L(x \longrightarrow y) \\ & \supseteq f_L(z \longrightarrow (x \longrightarrow (\neg y \longrightarrow y))) \\ & \cap f_L(z))\end{aligned} \tag{33}$$

$$(\forall x, y \in L) \quad (f_L(x \longrightarrow y) = f_L(x \longrightarrow (x \longrightarrow y))), \tag{34}$$

$$\begin{aligned}(\forall x, y \in L) \quad & (f_L(x \longrightarrow y) \\ & \supseteq f_L(z \longrightarrow (x \longrightarrow (x \longrightarrow y))) \cap f_L(z)),\end{aligned} \tag{35}$$

$$(\forall x \in L) \ (\forall n \in \mathbb{N}) \quad (f_L(nx) = f_L(x)), \tag{36}$$

$$(\forall x \in L) \quad (f_L(x \vee \neg x) \supseteq f_L(1)). \tag{37}$$

Proof. If we put $z = y$ in (26), then

$$\begin{aligned} f_L(x \longrightarrow y) &\supseteq f_L(x \longrightarrow (\neg y \longrightarrow y)) \cap f_L(y \longrightarrow y) \\ &= f_L(x \longrightarrow (\neg y \longrightarrow y)) \cap f_L(1) \\ &= f_L(x \longrightarrow (\neg y \longrightarrow y)) \end{aligned} \tag{38}$$

for all $x, y \in L$. Since $x \to y \leq \neg y \to (x \to y)$ for all $x, y \in L$, it follows from (29) and (R4) that

$$\begin{aligned} f_L(x \longrightarrow y) &\subseteq f_L(\neg y \longrightarrow (x \longrightarrow y)) \\ &= f_L(x \longrightarrow (\neg y \longrightarrow y)) \end{aligned} \tag{39}$$

for all $x, y \in L$. Consequently, we get $f_L(x \to y) = f_L(x \to (\neg y \to y))$ for all $x, y \in L$. Equation (33) follows from (32) and (20).

If we put $z = y$ and $y = x$ in (21), then

$$\begin{aligned} f_L(x \longrightarrow y) &\supseteq f_L(x \longrightarrow (x \longrightarrow y)) \cap f_L(x \longrightarrow x) \\ &= f_L(x \longrightarrow (x \longrightarrow y)) \cap f_L(1) \\ &= f_L(x \longrightarrow (x \longrightarrow y)) \end{aligned} \tag{40}$$

for all $x, y \in L$. Since $x \to y \leq x \to (x \to y)$ for all $x, y \in L$, (29) implies that

$$f_L(x \longrightarrow y) \subseteq f_L(x \longrightarrow (x \longrightarrow y)). \tag{41}$$

Combining (40) and (41), we have $f_L(x \to y) = f_L(x \to (x \to y))$ for all $x, y \in L$.

Using (20) and (34), we have

$$\begin{aligned} f_L(x \longrightarrow y) &= f_L(x \longrightarrow (x \longrightarrow y)) \\ &\supseteq f_L(z \longrightarrow (x \longrightarrow (x \longrightarrow y))) \cap f_L(z) \end{aligned} \tag{42}$$

for all $x, y, z \in L$. The proof of (36) is by induction on n. For $n = 2$, if we use (34), then

$$\begin{aligned} f_L(2x) = f_L(x \oplus x) &= f_L(\neg x \longrightarrow x) \\ &= f_L(\neg x \longrightarrow (\neg x \longrightarrow 0)) \\ &= f_L(\neg x \longrightarrow 0) = f_L(x) \end{aligned} \tag{43}$$

for all $x \in L$. Suppose (36) holds for $n = k$; that is, $f_L(kx) = f_L(x)$ for all $x \in L$. It follows from (34) that

$$\begin{aligned} f_L((k+1)x) = f_L(x \oplus kx) &= f_L(\neg x \longrightarrow kx) \\ &= f_L(\neg x \longrightarrow (x \oplus (k-1)x)) \\ &= f_L(\neg x \longrightarrow (\neg x \longrightarrow (k-1)x)) \\ &= f_L(\neg x \longrightarrow (k-1)x) \\ &= f_L(x \oplus (k-1)x) \\ &= f_L(kx) = f_L(x) \end{aligned} \tag{44}$$

for all $x \in L$. Therefore, (36) is valid. Note that

$$\begin{aligned} 1 &= ((\neg x \longrightarrow x) \longrightarrow x) \longrightarrow ((\neg x \longrightarrow x) \longrightarrow x) \\ &= \neg x \longrightarrow (((\neg x \longrightarrow x) \longrightarrow x) \longrightarrow \neg(\neg x \longrightarrow x)) \end{aligned} \tag{45}$$

and $\neg x \to ((\neg x \to x) \to x) = 1$ for all $x \in L$. It follows from (21) and (R1) that

$$\begin{aligned} f_L(1) &= f_L(\neg x \longrightarrow (((\neg x \longrightarrow x) \longrightarrow x) \longrightarrow \neg(\neg x \longrightarrow x))) \\ &\quad \cap f_L(\neg x \longrightarrow ((\neg x \longrightarrow x) \longrightarrow x)) \\ &\subseteq f_L(\neg x \longrightarrow \neg(\neg x \longrightarrow x)) \\ &= f_L((\neg x \longrightarrow x) \longrightarrow x) \end{aligned} \tag{46}$$

for all $x \in L$. Similarly $f_L(1) \subseteq f_L((x \to \neg x) \to \neg x)$ for all $x \in L$. It follows from (31) that

$$\begin{aligned} f_L(1) &\subseteq f_L((x \longrightarrow \neg x) \longrightarrow \neg x) \cap f_L((\neg x \longrightarrow x) \longrightarrow x) \\ &= f_L(((x \longrightarrow \neg x) \longrightarrow \neg x) \wedge ((\neg x \longrightarrow x) \longrightarrow x)) \\ &= f_L(x \vee \neg x) \end{aligned} \tag{47}$$

for all $x \in L$. □

Theorem 15. *Let $\mathcal{F}_L$ be an int-soft filter of L. If $\mathcal{F}_L$ satisfies condition* (33)*, then $\mathcal{F}_L$ is an implicative int-soft filter of L.*

Proof. Let $\mathcal{F}_L$ be an int-soft filter of L which satisfies condition (33). If we take $z = 1$ and $y = z$ in (33) and use (30), then

$$\begin{aligned} f_L(x \longrightarrow z) &\supseteq f_L(1 \longrightarrow (x \longrightarrow (\neg z \longrightarrow z))) \cap f_L(1) \\ &= f_L(x \longrightarrow (\neg z \longrightarrow z)) \\ &\supseteq f_L(x \longrightarrow (\neg z \longrightarrow y)) \cap f_L(y \longrightarrow z) \end{aligned} \tag{48}$$

for all $x, y, z \in L$. It follows from Theorem 12 that $\mathcal{F}_L$ is an implicative int-soft filter of L. □

Corollary 16. *Let $\mathcal{F}_L$ be an int-soft filter of L. If $\mathcal{F}_L$ satisfies condition* (32)*, then $\mathcal{F}_L$ is an implicative int-soft filter of L.*

Theorem 17. *If an int-soft filter $\mathcal{F}_L$ of L satisfies condition* (36)*, then it is implicative.*

Proof. Since $x \to y \leq x \to (\neg y \to y)$ for all $x, y \in L$, it follows from (29) that

$$f_L(x \longrightarrow y) \subseteq f_L(x \longrightarrow (\neg y \longrightarrow y)) \tag{49}$$

for all $x, y \in L$. Now condition (36) implies that

$$\begin{aligned} f_L(x \longrightarrow y) &= f_L(2(x \longrightarrow y)) \\ &= f_L(\neg(x \longrightarrow y) \longrightarrow (x \longrightarrow y)) \\ &= f_L(x \longrightarrow (\neg(x \longrightarrow y) \longrightarrow y)) \\ &\supseteq f_L(x \longrightarrow (\neg y \longrightarrow y)) \end{aligned} \tag{50}$$

for all $x, y \in L$. Combining (49) and (50) induces $f_L(x \to y) = f_L(x \to (\neg y \to y))$ for all $x, y \in L$. Therefore, $\mathcal{F}_L$ is an implicative int-soft filter of L by Corollary 16. □

Theorem 18. *If an int-soft filter $\mathcal{F}_L$ of L satisfies condition* (37)*, then it is implicative.*

Proof. Using (19), (20), (37), and (R4), we have

$$\begin{aligned} f_L(x \longrightarrow y) &\supseteq f_L((y \vee \neg y) \longrightarrow (x \longrightarrow y)) \cap f_L(y \vee \neg y) \\ &\supseteq f_L((y \vee \neg y) \longrightarrow (x \longrightarrow y)) \cap f_L(1) \\ &= f_L((y \vee \neg y) \longrightarrow (x \longrightarrow y)) \\ &= f_L(y \longrightarrow (x \longrightarrow y)) \\ &\cap f_L(\neg y \longrightarrow (x \longrightarrow y)) \\ &= f_L(x \longrightarrow (y \longrightarrow y)) \\ &\cap f_L(\neg y \longrightarrow (x \longrightarrow y)) \\ &= f_L(x \longrightarrow 1) \cap f_L(\neg y \longrightarrow (x \longrightarrow y)) \\ &= f_L(\neg y \longrightarrow (x \longrightarrow y)) \end{aligned} \tag{51}$$

for all $x, y \in L$. Since $x \to y \leq \neg y \to (x \to y)$ for all $x, y \in L$, it follows from (29) that $f_L(x \to y) \subseteq f_L(\neg y \to (x \to y))$ for all $x, y \in L$. Therefore,

$$f_L(x \longrightarrow y) = f_L(\neg y \longrightarrow (x \longrightarrow y)) \tag{52}$$

for all $x, y \in L$, and so $\mathcal{F}_L$ is an implicative int-soft filter of L by Corollary 16. □

Theorem 19. *Let $\mathcal{F}_L \in S(U, L)$ satisfy condition* (19) *and*

$$\begin{aligned} (\forall x, y, z \in L) \quad & (f_L(x) \\ & \supseteq f_L(z \longrightarrow ((x \longrightarrow y) \longrightarrow x)) \\ & \cap f_L(z)), \end{aligned} \tag{53}$$

$$\begin{aligned} (\forall x, y, z \in L) \quad & (f_L(x \longrightarrow z) \\ & \supseteq f_L(x \longrightarrow y) \cap f_L(y \longrightarrow z)). \end{aligned} \tag{54}$$

Then $\mathcal{F}_L$ is an implicative int-soft filter of L.

Proof. Using (R4) and (6), we have

$$\begin{aligned} &((x \longrightarrow z) \longrightarrow z) \longrightarrow (x \longrightarrow z) \\ &\quad = x \longrightarrow (((x \longrightarrow z) \longrightarrow z) \longrightarrow z) = x \longrightarrow (x \longrightarrow z) \end{aligned} \tag{55}$$

for all $x, z \in L$. It follows from (R2), (R4), (19), (53), and (54) that

$$\begin{aligned} f_L(x \longrightarrow z) &\supseteq f_L(1 \longrightarrow (((x \longrightarrow z) \longrightarrow z) \longrightarrow (x \longrightarrow z))) \\ &\cap f_L(1) \\ &= f_L(((x \longrightarrow z) \longrightarrow z) \longrightarrow (x \longrightarrow z)) \\ &= f_L(x \longrightarrow (x \longrightarrow z)) \\ &\supseteq f_L(y \longrightarrow (x \longrightarrow z)) \cap f_L(x \longrightarrow y) \\ &= f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y) \end{aligned} \tag{56}$$

for all $x, y, z \in L$. Thus, $\mathcal{F}_L$ is an implicative int-soft filter of L. □

Corollary 20. *Every int-soft filter satisfying condition* (53) *is an implicative int-soft filter.*

Proof. Let $\mathcal{F}_L$ be an int-soft filter of L that satisfies condition (53). Since $\mathcal{F}_L$ satisfies two conditions (19) and (54) (see [8]), we know that $\mathcal{F}_L$ is an implicative int-soft filter of L. □

Theorem 21. *A soft set $\mathcal{F}_L$ of L is an implicative int-soft filter of L if and only if the nonempty γ-inclusive set $\mathcal{F}_L^\gamma$ is an implicative filter of L for all $\gamma \in \mathcal{P}(U)$.*

The implicative filters $\mathcal{F}_L^\gamma$ in Theorem 21 are called *inclusive implicative filters* of L.

Proof. Assume that $\mathcal{F}_L$ is an implicative int-soft filter of L. Let $\gamma \in \mathcal{P}(U)$ be such that $\mathcal{F}_L^\gamma \neq \emptyset$. Then $\mathcal{F}_L$ is an int-soft filter of L (see Theorem 10), and so $\mathcal{F}_L^\gamma$ is a filter of L. Let $x, y, z \in L$ be such that $x \to (y \to z) \in \mathcal{F}_L^\gamma$ and $x \to y \in \mathcal{F}_L^\gamma$. Then $f_L(x \to (y \to z)) \supseteq \gamma$ and $f_L(x \to y) \supseteq \gamma$. It follows from (21) that

$$f_L(x \longrightarrow z) \supseteq f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y) \supseteq \gamma, \tag{57}$$

and so that $x \to z \in \mathscr{F}_L^\gamma$. Therefore, $\mathscr{F}_L^\gamma$ is an implicative filter of L for all $\gamma \in \mathscr{P}(U)$.

Conversely, suppose that the nonempty γ-inclusive set $\mathscr{F}_L^\gamma$ is an implicative filter of L for all $\gamma \in \mathscr{P}(U)$. Then $\mathscr{F}_L^\gamma$ is a filter of L, and so $\mathscr{F}_L$ is an int-soft filter of L. For every $x, y, z \in L$, let $\gamma = f_L(x \to (y \to z)) \cap f_L(x \to y)$. Then $x \to (y \to z) \in \mathscr{F}_L^\gamma$ and $x \to y \in \mathscr{F}_L^\gamma$, which imply that $x \to z \in \mathscr{F}_L^\gamma$. Hence,

$$f_L(x \longrightarrow z) \supseteq \gamma = f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y). \tag{58}$$

Thus, $\mathscr{F}_L$ is an implicative int-soft filter of L. □

Theorem 22 (extension property). *Let $\mathscr{F}_L$ and $\mathscr{G}_L$ be two int-soft filters of L such that $f_L(1) = g_L(1)$ and $f_L(x) \subseteq g_L(x)$ for all $x \in L$. If $\mathscr{F}_L$ is implicative, then so is $\mathscr{G}_L$.*

Proof. Assume that $\mathscr{F}_L$ is implicative. Then $f_L(x \vee \neg x) \supseteq f_L(1)$ by (37). It follows from the hypothesis that

$$g_L(x \vee \neg x) \supseteq f_L(x \vee \neg x) \supseteq f_L(1) = g_L(1) \tag{59}$$

for all $x \in L$. Therefore, $\mathscr{G}_L$ is an implicative int-soft filter of L by Theorem 18. □

We finally make a new implicative int-soft filter from an old one.

Theorem 23. *For any $\mathscr{F}_L \in S(U, L)$, let $\mathscr{F}_L^*$ be a soft set of L over U defined by*

$$f_L^* : L \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} f_L(x), & \text{if } x \in \mathscr{F}_L^\gamma, \\ \delta, & \text{otherwise}, \end{cases} \tag{60}$$

where γ and δ are subsets of U with $\delta \subsetneq f_L(x)$. If $\mathscr{F}_L$ is an implicative int-soft filter of L, then so is $\mathscr{F}_L^*$.

Proof. Assume that $\mathscr{F}_L$ is an implicative int-soft filter of L. Then $\mathscr{F}_L^\gamma$ is an implicative filter of L for all $\gamma \subseteq U$ with $\mathscr{F}_L^\gamma \neq \emptyset$. Hence, $1 \in \mathscr{F}_L^\gamma$, and so

$$f_L^*(1) = f_L(1) \supseteq f_L(x) \supseteq f_L^*(x) \tag{61}$$

for all $x \in L$. Let $x, y, z \in L$. If $x \to (y \to z) \in \mathscr{F}_L^\gamma$ and $x \to y \in \mathscr{F}_L^\gamma$, then $x \to z \in \mathscr{F}_L^\gamma$. Hence,

$$\begin{aligned} f_L^*(x \longrightarrow z) &= f_L(x \longrightarrow z) \\ &\supseteq f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y) \\ &= f_L^*(x \longrightarrow (y \longrightarrow z)) \cap f_L^*(x \longrightarrow y). \end{aligned} \tag{62}$$

If $x \to (y \to z) \notin \mathscr{F}_L^\gamma$ or $x \to y \notin \mathscr{F}_L^\gamma$, then $f_L^*(x \to (y \to z)) = \delta$ or $f_L^*(x \to y) = \delta$. Thus,

$$f_L^*(x \longrightarrow z) \supseteq \delta = f_L^*(x \longrightarrow (y \longrightarrow z)) \cap f_L^*(x \longrightarrow y). \tag{63}$$

Therefore, $\mathscr{F}_L^*$ is an implicative int-soft filter of L. □

Theorem 24. *For any implicative filter F of L, there exists an implicative int-soft filter of L such that its inclusive implicative filter is F.*

Proof. Let $\mathscr{F}_L$ be a soft set of L over U in which f_L is given by

$$f_L : L \longrightarrow \mathscr{P}(U), \qquad x \longmapsto \begin{cases} \gamma, & \text{if } x \in F, \\ \emptyset, & \text{otherwise}, \end{cases} \tag{64}$$

where γ is a nonempty subset of U. Since $\in F$, we have $f_L(1) = \gamma \supseteq f_L(x)$ for all $x \in L$. For every $x, y, z \in L$, if $x \to (y \to z) \in F$ and $x \to y \in F$, then $x \to z \in F$. Hence,

$$f_L(x \longrightarrow z) = \gamma = f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y). \tag{65}$$

If $x \to (y \to z) \notin F$ or $x \to y \notin F$, then $f_L(x \to (y \to z)) = \emptyset$ or $f_L(x \to y) = \emptyset$. Thus,

$$f_L(x \longrightarrow z) \supseteq \emptyset = f_L(x \longrightarrow (y \longrightarrow z)) \cap f_L(x \longrightarrow y). \tag{66}$$

Therefore, $\mathscr{F}_L$ is an implicative int-soft filter of L. Obviously, $F = \mathscr{F}_L^\gamma$. □

4. Conclusion

In [8], Jun et al. have applied the notion of intersection-soft sets to the filter theory in R_0-algebras. They have introduced the concept of strong int-soft filters in R_0-algebras and investigated related properties. They have established characterizations of a strong int-soft filter and provided a condition for an int-soft filter to be strong. They also have constructed an extension property of a strong int-soft filter.

In this paper, we have introduced a new notion which is called an implicative int-soft filter and investigated related properties. We have discussed a relation between an int-soft filter and an implicative int-soft filter. We have provided conditions for an int-soft filter to be an implicative int-soft filter. We have considered characterizations of an implicative int-soft filter and constructed a new implicative int-soft filter from an old one. We also have established the extension property of an implicative int-soft filter.

Work is ongoing. Some important issues for future work are (1) to develop strategies for obtaining more valuable results, (2) to apply these notions and results for studying related notions in other algebraic structures with applications in soft set theory, and (3) to study the notions of the Boolean int-soft filters.

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