

Research Article

The Chaos Dynamic of Multiproduct Cournot Duopoly Game with Managerial Delegation

Fang Wu and Junhai Ma

College of Management and Economics, Tianjin University, Tianjin 300072, China

Correspondence should be addressed to Junhai Ma; mjhtju@aliyun.com

Received 19 December 2013; Revised 15 March 2014; Accepted 19 March 2014; Published 18 May 2014

Academic Editor: Mustafa Kulenović

Copyright © 2014 F. Wu and J. Ma. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Although oligopoly theory is generally concerned with the single-product firm, what is true in the real world is that most of the firms offer multiproducts rather than single products in order to obtain cost-saving advantages, cater for the diversity of consumer tastes, and provide a barrier to entry. We develop a dynamical multiproduct Cournot duopoly model in discrete time, where each firm has an owner who delegates the output decision to a manager. The principle of decision-making is bounded rational. And each firm has a nonlinear total cost function due to the multiproduct framework. The Cournot Nash equilibrium and the local stability are investigated. The tangential bifurcation and intermittent chaos are reported by numerical simulations. The results show that high output adjustment speed can lead to output fluctuations which are characterized by phases of low volatility with small output changes and phases of high volatility with large output changes. The intermittent route to chaos of Flip bifurcation and another intermittent route of Flip bifurcation which contains Hopf bifurcation can exist in the system. The study can improve our understanding of intermittent chaos frequently observed in oligopoly economy.

1. Introduction

Dynamic Cournot model is a typical nonlinear discrete time varying system which takes output as its decision variable. Some references investigated the nonlinear behaviors when some parameters are varied in oligarchic game, such as Flip and Hopf bifurcation. Wang and Ma [1] analyzed the Flip and Hopf bifurcations in a Cournot-Bertrand mixed model. Tramontana and Elsadany [2] proposed a three oligopoly market with heterogeneous expectations, analyzed double routes to chaos (Hopf and Flip bifurcation), and presented the differences of the two routes from the economic view. Besides the above, some researchers extended Cournot model to different forms according to the difference of the real economic environments. Ma and Pu [3] built a triopoly repeated game model based on heterogeneous expectations in which the market has quadratic inverse demand function and the firm possesses cubic total cost function. Xin et al. [4] built a nonlinear discrete output game model for two oligopolistic firms whose products are adnascent and investigated its complex dynamics. Ma and Tu [5] proposed a

new dynamical Cournot duopoly game model with delayed bounded rationality in electricity market. Guo and Ma [6] studied the game model and complexity of retailer collecting and selling in closed-loop supply chain. Fanti et al. [7] analyzed the dynamic of a nonlinear Cournot duopoly with managerial delegation.

However, all above studies are limited in single-product firm. Dynamic oligopoly models lack the case of multiproduct firm. In productive practice, firms usually produce multiproducts rather than single products to obtain cost-saving advantages, cater for the diversity of consumers' tastes, and provide a barrier to entry. For example, McDonalds can produce both hamburgers and French fries at a lower average cost than what it would cost two separate firms to produce the same goods.

At present, in most of the literature [1–6], producers are all based on the objective of maximum profit, but when ownership and management right are separate, managers are probably to manipulate the benefit for their personal purpose than just pure profit maximization. In order to make manager perform more aggressively in the market, the owner usually

designs an incentive contract to exert a high level of effort of manager; for example, revenue [8, 9], sales [10], market shares [11, 12], and relative profit [7, 13, 14] are usually used as an optimal incentive scheme in management practice.

In the model of multiproduct framework, the most important problem is the structure of the cost function. In this research, the model uses a more general cost function which implies the concepts about economies of scope and economies of scale.

In this study, we build a duopoly multiproduct output model which extends the models of [1–7], which consider single-product firm, and introduce the managerial delegation to the multiproduct model which modifies the models of [1–6]. Then the existence of tangential bifurcation and intermittent chaos are proved by using numerical simulation experiments.

The plan of the paper is as follows. Section 2 presents the multiproduct game model with managerial delegation. Section 3 investigates the Cournot-Nash equilibrium and its local stability. Section 4 analyzed the properties of output dynamic in the nonlinear model by numerical simulation. Section 5 sums up briefly.

2. The Model

2.1. Relevant Concepts. In this part, we will introduce some concepts which the model includes.

2.1.1. Economies of Scale and Diseconomies of Scale. Economies of scale are the cost advantages that enterprises obtain due to size, throughput, or scale of operation. Economies of scale occur when long-run average cost falls as output increases [15, 16].

Diseconomies of scale are the forces that cause larger firms and governments to produce goods and services at increased per-unit costs. Diseconomies of scale occur when long-run average cost rises as output increases. The concept is the opposite of economies of scale [15, 16].

2.1.2. Economies of Scope and Diseconomies of Scope. Economies of scope is the situation in which the joint cost of producing two or more products is less than the sum of the separate costs of producing the products. If the sum of producing two or more products by separate firms is less than producing two or more products jointly by the same firm, diseconomies of scope are said to exist [15].

2.2. The Model. Consider two firms $i = 1, 2$, each of which produces two products, a and b . The prices of the two products are given by

$$\begin{aligned} p_a &= l_a - (q_{1a} + q_{2a}) - k(q_{1b} + q_{2b}), \\ p_b &= l_b - (q_{1b} + q_{2b}) - k(q_{1a} + q_{2a}). \end{aligned} \quad (1)$$

In (1), $i = 1, 2$, $j = a, b$, where $l_j > 0$, q_{ij} is the output of product j produced by firm i . $k \in [-1, 1]$: $k \in [-1, 0)$ implies that products are complements; $k \in (0, 1)$ implies products are alternative; products a and b are homogeneous

when $k = 1$; and each firm is a monopolist when $k = 0$. The firms' products are perfect substitutes within each market (a or b) and are imperfect substitutes across markets.

The total cost functions of firms 1 and 2 are given as following:

$$\begin{aligned} C_1 &= c_{1a}q_{1a} + c_{1b}q_{1b} + d_{1a}q_{1a}^2 + d_{1b}q_{1b}^2 + h_1q_{1a}q_{1b} \\ &\quad - f_1q_{1a}^{1/2}q_{1b}^{1/2} + S_1 + s_{1a} + s_{1b}, \\ C_2 &= c_{2a}q_{2a} + c_{2b}q_{2b} + d_{2a}q_{2a}^2 + d_{2b}q_{2b}^2 + h_2q_{2a}q_{2b} \\ &\quad - f_2q_{2a}^{1/2}q_{2b}^{1/2} + S_2 + s_{2a} + s_{2b}. \end{aligned} \quad (2)$$

All parameters are assumed to be positive (see [15–17]). They are as follows:

d_{ij} : diseconomies of scale;

h_i : diseconomies of scope (see [17]);

f_i : unit economies of scope, firm i produces single product when $f_i = 0$ (see [17]);

S_i : nonproduct specific indivisible costs;

s_{ij} : indivisible cost.

Each firm has an owner who delegates the output decision to its manager, and each owner picks an incentive contract (the objective function of manager) for the manager to maximize firm's profit. The profits of firms are as (3) describe, and they are owners' objective functions:

$$\begin{aligned} \prod_1 &= p_a q_{1a} + p_b q_{1b} - C_1, \\ \prod_2 &= p_a q_{2a} + p_b q_{2b} - C_2. \end{aligned} \quad (3)$$

Manager i is told that his (her) salary includes a fixed salary and a bonus offered in the incentive contract; that is,

$$T_i + U_i W_i. \quad (4)$$

T_i and U_i are constants. We can see that if the control variable is q_{ij} , maximizing $T_i + U_i W_i$ and maximizing W_i are equivalent. So, we can consider W_i as the objective function of manager. The objective function of manager i can only belong to the following class (see [18]):

$$W_i = r_i \prod_i + (1 - r_i) F_i. \quad (5)$$

Owner i selects his (her) weighting coefficient r_i . At present, the forms of F_i mainly include the incentives for sales which is proposed by Vickers (see [10]), incentives for revenue (see [8, 9]), incentives for market shares (see [11, 12]), and incentives for relative profit (see [13, 14]). In this paper, we use relative profit as an incentive scheme. Thus, the managers' objective functions are as (6) describe

$$\begin{aligned} W_1 &= r_1 \prod_1 + (1 - r_1) \left(\prod_1 - \prod_2 \right), \\ W_2 &= r_2 \prod_2 + (1 - r_2) \left(\prod_2 - \prod_1 \right). \end{aligned} \quad (6)$$

In (6), $0 < r_i < 2$ ($i = 1, 2$) implies the attitude of the manager. r_i is exogenous variable which is exerted by owner as his contract before the game. If $r_i \in (0, 1]$, the owner is interested in competitive, and if $r_i \in (1, 2)$, the owner is interested in cooperative.

Two firms follow the bounded rational expectation which based on the local estimate of the marginal bonus as (7) describe:

$$\begin{aligned} q'_{1a} &= q_{1a} + \alpha_1 q_{1a} \frac{\partial W_1}{\partial q_{1a}}, \\ q'_{1b} &= q_{1b} + \alpha_2 q_{1b} \frac{\partial W_1}{\partial q_{1b}}, \\ q'_{2a} &= q_{2a} + \beta_1 q_{2a} \frac{\partial W_2}{\partial q_{2a}}, \\ q'_{2b} &= q_{2b} + \beta_2 q_{2b} \frac{\partial W_2}{\partial q_{2b}}. \end{aligned} \quad (7)$$

So, the dynamical system can be described as 4-dimension discrete difference equations as following form:

$$\begin{aligned} q'_{1a} &= q_{1a} + \alpha_1 q_{1a} \\ &\times \left[r_1 \left(-2q_{1a} + l_1 - q_{2a} - k(q_{1b} + q_{2b}) - kq_{1b} - c_{1a} \right. \right. \\ &\quad \left. \left. - 2d_{1a}q_{1a} - h_1q_{1b} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\ &\quad \times \left(-2q_{1a} + l_1 - k(q_{1b} + q_{2b}) - kq_{1b} - c_{1a} \right. \\ &\quad \left. \left. - 2d_{1a}q_{1a} - h_1q_{1b} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} + kq_{2b} \right) \right], \\ q'_{1b} &= q_{1b} + \alpha_2 q_{1b} \\ &\times \left[r_1 \left(-kq_{1a} - 2q_{1b} + l_2 - q_{2b} - k(q_{1a} + q_{2a}) - c_{1b} \right. \right. \\ &\quad \left. \left. - 2d_{1b}q_{1b} - h_1q_{1a} + \frac{f_1 q_{1a}^{1/2}}{2q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\ &\quad \times \left(-kq_{1a} - 2q_{1b} + l_2 - k(q_{1a} + q_{2a}) - c_{1b} \right. \\ &\quad \left. \left. - 2d_{1b}q_{1b} - h_1q_{1a} + \frac{f_1 q_{1a}^{1/2}}{2q_{1b}^{1/2}} + kq_{2a} \right) \right], \end{aligned}$$

$$\begin{aligned} q'_{2a} &= q_{2a} + \beta_1 q_{2a} \\ &\times \left[r_2 \left(-2q_{2a} + l_1 - q_{1a} - k(q_{1b} + q_{2b}) - kq_{2b} - c_{2a} \right. \right. \\ &\quad \left. \left. - 2d_{2a}q_{2a} - h_2q_{2b} + \frac{f_2}{2q_{2a}^{1/2}q_{2b}^{1/2}} \right) + (1 - r_2) \right. \\ &\quad \times \left(-2q_{2a} + l_1 - k(q_{1b} + q_{2b}) - kq_{2b} - c_{2a} \right. \\ &\quad \left. \left. - 2d_{2a}q_{2a} - h_2q_{2b} + \frac{f_2}{2q_{2a}^{1/2}q_{2b}^{1/2}} + kq_{1b} \right) \right], \\ q'_{2b} &= q_{2b} + \beta_2 q_{2b} \\ &\times \left[r_2 \left(-kq_{2a} - 2q_{2b} + l_2 - q_{1b} - k(q_{1a} + q_{2a}) - c_{2b} \right. \right. \\ &\quad \left. \left. - 2d_{2b}q_{2b} - h_2q_{2a} + \frac{f_2 q_{2a}^{1/2}}{2q_{2b}^{1/2}} \right) + (1 - r_2) \right. \\ &\quad \times \left(-kq_{2a} - 2q_{2b} + l_2 - k(q_{1a} + q_{2a}) - c_{2b} \right. \\ &\quad \left. \left. - 2d_{2b}q_{2b} - h_2q_{2a} + \frac{f_2 q_{2a}^{1/2}}{2q_{2b}^{1/2}} + kq_{1a} \right) \right]. \end{aligned} \quad (8)$$

q'_{ij} denotes the unit-time advancement of variable q_{ij} . $\alpha_1, \alpha_2, \beta_1,$ and β_2 are output adjustment parameters; for example, $\alpha_1 q_{1a}$ is the output speed of adjustment of q_{1a} .

3. The Nash Equilibrium and the Local Stability

In order to investigate the Cournot-Nash equilibrium and its local stability, the Jacobian matrix of system (8) is given as following form:

$$J = \begin{bmatrix} D_{11} & D_{12} & D_{13} & D_{14} \\ D_{21} & D_{22} & D_{23} & D_{24} \\ D_{31} & D_{32} & D_{33} & D_{34} \\ D_{41} & D_{42} & D_{43} & D_{44} \end{bmatrix}, \quad (9)$$

where

$$\begin{aligned} D_{11} &= 1 + \alpha_1 \\ &\times \left[r_1 \left(-2q_{1a} + l_1 - q_{2a} - k(q_{1b} + q_{2b}) - kq_{1b} - c_{1a} \right. \right. \\ &\quad \left. \left. - 2d_{1a}q_{1a} - h_1q_{1b} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\ &\quad \left. - 2d_{1a}q_{1a} - h_1q_{1b} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} + kq_{2b} \right], \end{aligned}$$

$$\begin{aligned}
& \times \left(-2q_{1a} + l_1 - k(q_{1b} + q_{2b}) - kq_{1b} - c_{1a} \right. \\
& \quad \left. -2d_{1a}q_{1a} - h_1q_{1b} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} + kq_{2b} \right) \Bigg] \\
& + \alpha_1q_{1a} \left[r_1 \left(-2 - 2d_{1a} - \frac{f_1}{4q_{1a}^{3/2}q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\
& \quad \left. \times \left(-2 - 2d_{1a} - \frac{f_1}{4q_{1a}^{3/2}q_{1b}^{1/2}} \right) \right], \\
D_{12} &= \alpha_1q_{1a} \left[r_1 \left(-2k - h_1 + \frac{f_1}{4q_{1a}^{1/2}q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\
& \quad \left. \times \left(-2k - h_1 + \frac{f_1}{4q_{1a}^{1/2}q_{1b}^{1/2}} \right) \right], \\
D_{13} &= -\alpha_1q_{1a}r_1, \\
D_{14} &= -\alpha_1q_{1a}r_1k, \\
D_{21} &= \alpha_2q_{1b} \left[r_1 \left(-2k - h_1 + \frac{f_1}{4q_{1a}^{1/2}q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\
& \quad \left. \times \left(-2k - h_1 + \frac{f_1}{4q_{1a}^{1/2}q_{1b}^{1/2}} \right) \right], \\
D_{22} &= 1 + \alpha_2 \\
& \times \left[r_1 \left(-kq_{1a} - 2q_{1b} + l_2 - q_{2b} - k(q_{1a} + q_{2a}) - c_{1b} \right. \right. \\
& \quad \left. \left. -2d_{1b}q_{1b} - h_1q_{1a} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} \right) + (1 - r_1) \right. \\
& \quad \times \left(-kq_{1a} - 2q_{1b} + l_2 - k(q_{1a} + q_{2a}) - c_{1b} \right. \\
& \quad \left. \left. -2d_{1b}q_{1b} - h_1q_{1a} + \frac{f_1}{2q_{1a}^{1/2}q_{1b}^{1/2}} + kq_{2a} \right) \right] \\
& + \alpha_2q_{1b} \left[r_1 \left(-2 - 2d_{1b} - \frac{f_1}{4q_{1a}^{1/2}q_{1b}^{3/2}} \right) + (1 - r_1) \right. \\
& \quad \left. \times \left(-2 - 2d_{1b} - \frac{f_1}{4q_{1a}^{1/2}q_{1b}^{3/2}} \right) \right], \\
D_{23} &= -\alpha_2q_{1b}r_1k, \\
D_{24} &= -\alpha_2q_{1b}r_1, \\
D_{31} &= -\beta_1q_{2a}r_2, \\
D_{32} &= -\beta_1q_{2a}r_2k,
\end{aligned}$$

$$\begin{aligned}
D_{33} &= 1 + \beta_1 \\
& \times \left[r_2 \left(-2q_{2a} + l_1 - q_{1a} - k(q_{1b} + q_{2b}) - kq_{2b} - c_{2a} \right. \right. \\
& \quad \left. \left. -2d_{2a}q_{2a} - h_2q_{2b} + \frac{f_2}{2q_{2a}^{1/2}q_{2b}^{1/2}} \right) + (1 - r_2) \right. \\
& \quad \times \left(-2q_{2a} + l_1 - k(q_{1b} + q_{2b}) - kq_{2b} - c_{2a} \right. \\
& \quad \left. \left. -2d_{2a}q_{2a} - h_2q_{2b} + \frac{f_2}{2q_{2a}^{1/2}q_{2b}^{1/2}} + kq_{1b} \right) \right] \\
& + \beta_1q_{2a} \left[r_2 \left(-2 - 2d_{2a} - \frac{f_2}{4q_{2a}^{3/2}q_{2b}^{1/2}} \right) + (1 - r_2) \right. \\
& \quad \left. \times \left(-2 - 2d_{2a} - \frac{f_2}{4q_{2a}^{3/2}q_{2b}^{1/2}} \right) \right], \\
D_{34} &= \beta_1q_{2a} \left[r_2 \left(-2k - h_2 + \frac{f_2}{4q_{2a}^{1/2}q_{2b}^{1/2}} \right) + (1 - r_2) \right. \\
& \quad \left. \times \left(-2k - h_2 + \frac{f_2}{4q_{2a}^{1/2}q_{2b}^{1/2}} \right) \right], \\
D_{41} &= -\beta_2q_{2b}r_2k, \\
D_{42} &= -\beta_2q_{2b}r_2, \\
D_{43} &= \beta_2q_{2b} \left[r_2 \left(-2k - h_2 + \frac{f_2}{4q_{2b}^{1/2}q_{2a}^{1/2}} \right) + (1 - r_2) \right. \\
& \quad \left. \times \left(-2k - h_2 + \frac{f_2}{4q_{2b}^{1/2}q_{2a}^{1/2}} \right) \right], \\
D_{44} &= 1 + \beta_2 \\
& \times \left[r_2 \left(-kq_{2a} - 2q_{2b} + l_2 - q_{1b} - k(q_{1a} + q_{2a}) - c_{2b} \right. \right. \\
& \quad \left. \left. -2d_{2b}q_{2b} - h_2q_{2a} + \frac{f_2}{2q_{2a}^{1/2}q_{2b}^{1/2}} \right) + (1 - r_2) \right. \\
& \quad \times \left(-kq_{2a} - 2q_{2b} + l_2 - k(q_{1a} + q_{2a}) - c_{2b} \right. \\
& \quad \left. \left. -2d_{2b}q_{2b} - h_2q_{2a} + \frac{f_2}{2q_{2a}^{1/2}q_{2b}^{1/2}} + kq_{1a} \right) \right] \\
& + \beta_2q_{2b} \left[r_2 \left(-2 - 2d_{2b} - \frac{f_2q_{2a}^{1/2}}{4q_{2b}^{3/2}} \right) + (1 - r_2) \right. \\
& \quad \left. \times \left(-2 - 2d_{2b} - \frac{f_2q_{2a}^{1/2}}{4q_{2b}^{3/2}} \right) \right].
\end{aligned}$$

In our system, multiproduct associated with the assumption of managerial delegation makes the computation of stability very dauntingly huge. The stability of this model is difficult to be analyzed in a general case. In order to get the Cournot Nash equilibrium point and find its local stable region, we prefer to set a value to each parameter:

$$\begin{aligned} c_{1a} &= 0.15, & c_{1b} &= 0.1, & c_{2a} &= 0.2, & c_{2b} &= 0.1, \\ l_1 &= 6, & l_2 &= 6, & d_1 &= 0.5, & d_2 &= 0.6, \\ d_{1a} &= 0.85, & d_{1b} &= 0.9, & d_{2a} &= 0.95, & d_{2b} &= 0.97, \\ h_1 &= 0.5, & h_2 &= 0.5, & f_1 &= 0.8, & f_2 &= 0.7, \\ k &= 0.2; & r_1 &= 1.6, & r_2 &= 1.6. \end{aligned} \quad (11)$$

The initial values are chosen as $(q_{1a}(0), q_{1b}(0), q_{2a}(0), q_{2b}(0)) = (0.5, 0.5, 0.5, 0.5)$.

The Nash equilibrium of Cournot game is a state of output decision in which it is impossible to make any one player better off without making at least one player worse off. By computing $\partial \Pi_i / \partial q_{ij} = 0$, we can get the only positive Cournot Nash equilibrium point $q_e = [q_{1a}(0.9926), q_{1b}(0.9667), q_{2a}(0.8818), q_{2b}(0.9103)]$ which is stable and has practical implications in economics (see [1, 3–5]).

The characteristic equation of Jacobian matrix is as follows:

$$f(\lambda) = \lambda^4 + A_1\lambda^3 + A_2\lambda^2 + A_3\lambda + A_0 = 0, \quad (12)$$

where

$$A_1 = -4 + 3.87\alpha_1 + 3.876\alpha_2 + 3.617\beta_1 + 3.759\beta_2,$$

$$\begin{aligned} A_2 &= -10.85\beta_1 - 11.61\alpha_1 + 13.6\beta_1\beta_2 + 15\alpha_1\alpha_2 \\ &+ 11.76\alpha_1\beta_1 - 11.63\alpha_2 + 6 + 14.02\alpha_2\beta_1 \\ &+ 14.55\alpha_1\beta_2 - 11.28\beta_2 + 12.32\alpha_2\beta_2, \end{aligned}$$

$$\begin{aligned} A_3 &= 44.19\beta_1\beta_2\alpha_1 - 23.51\alpha_1\beta_1 + 45.57\alpha_1\alpha_2\beta_1 \\ &+ 47.67\alpha_1\alpha_2\beta_2 - 29.09\alpha_1\beta_2 - 4 + 11.61\alpha_1 + 11.63\alpha_2 \\ &- 30\alpha_1\alpha_2 + 44.55\alpha_2\beta_1\beta_2 + 11.28\beta_2 - 24.64\alpha_2\beta_2 \\ &- 27.19\beta_1\beta_2 - 28.04\alpha_2\beta_1 + 10.85\beta_1, \end{aligned}$$

$$\begin{aligned} A_0 &= 1 + 13.6\beta_1\beta_2 + 14.55\alpha_1\beta_1 - 44.19\beta_1\beta_2\alpha_1 \\ &- 45.57\alpha_1\alpha_2\beta_1 + 12.32\alpha_2\beta_2 - 3.617\beta_1 - 3.87\alpha_1 \\ &- 3.759\beta_2 + 14.02\alpha_2\beta_1 + 144.7\alpha_1\alpha_2\beta_1\beta_2 \\ &- 44.55\alpha_2\beta_1\beta_2 - 3.876\alpha_2 + 15\alpha_1\alpha_2 + 11.76\alpha_1\beta_1 \\ &- 47.67\alpha_1\alpha_2\beta_2. \end{aligned} \quad (13)$$

According to Jury conditions [19], the sufficient and necessary conditions that q_e is locally stable are

$$\begin{aligned} 1 + A_1 + A_2 + A_3 + A_0 &> 0, \\ 1 - A_1 + A_2 - A_3 + A_0 &> 0, \\ 1 - A_0^2 &> 0, \\ (1 - A_0^2)^2 - (A_3 - A_1A_0)^2 &> 0, \\ \left[(1 - A_0^2)^2 - (A_3 - A_1A_0)^2 \right]^2 \\ - \left[A_2(1 - A_0)^3 - (A_1 - A_3A_0)(A_3 - A_1A_0) \right]^2 &> 0. \end{aligned} \quad (14)$$

Therefore, the scope surrounded by $(\alpha_1, \alpha_2, \beta_1, \beta_2)$ which can satisfy the five conditions in (14) is the local region of q_e . The output will reach the Cournot Nash equilibrium point q_e after rounds of games with random initial output within the local stable region.

Considering $\beta_2 = 0.34$, the local stable region at Cournot Nash equilibrium consisting of $(\alpha_1, \alpha_2, \beta_1)$ is shown in Figure 1. Similarly, we can fix other parameters and get the stable region at coordinate planes $(\alpha_1, \alpha_2, \beta_2)$, $(\alpha_1, \beta_1, \beta_2)$, and $(\alpha_2, \beta_1, \beta_2)$. Here no discussions are made.

4. The Numerical Simulation

In this section, two intermittent routes to chaos are investigated in the system by numerical simulation.

4.1. Flip Bifurcation (Period-Doubling Bifurcation) and Its Intermittent Route to Chaos. We first restrict our experiment to the following set of parameters: $k = 0.2$, $r = 1.6$, $\alpha_2 = \beta_1 = \beta_2 = 0.34$.

From Figures 2(a) and 2(b), the model has a unique equilibrium with $q_e = [q_{1a}(0.9926), q_{1b}(0.9667), q_{2a}(0.8818), q_{2b}(0.9103)]$. Starting from a small value of α_1 , the system is in the stability region. When α_1 increases, a loss of stability may occur via a Flip bifurcation (Period-doubling bifurcation). When the $\alpha_1 \in (0.2083, 0.5124)$, the output dynamic becomes unstable and chaotic behavior of output waves occurs after period 2, period 4, and period 8. If α_1 is further increased ($\alpha_1 > 0.5124$), the outputs become regular again. However as α_1 is further increased ($\alpha_1 > 0.53$), the outputs become once again chaotic. The abrupt change from chaos to period state is called tangential bifurcation and it is a kind of special bifurcation which is a route to chaos. The intermittent chaos is caused by tangential bifurcation in the market. Tangential bifurcation and intermittent chaos are all dynamic behaviors which are caused by the collision between chaotic attractor and periodic orbits. In the nonlinear dynamic of intermittent chaos, stable periodic state and chaotic state happen stochastically.

Figure 3 shows the enlarged drawing of the tangential bifurcation and the values of the maximum Lyapunov exponent with $\alpha_1 \in [0.5, 0.55]$. The system is in chaos if the

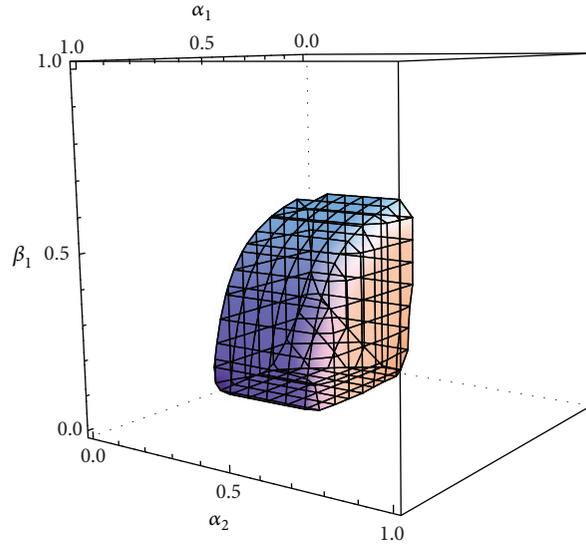


FIGURE 1: The local stable region with $\beta_2 = 0.34$.

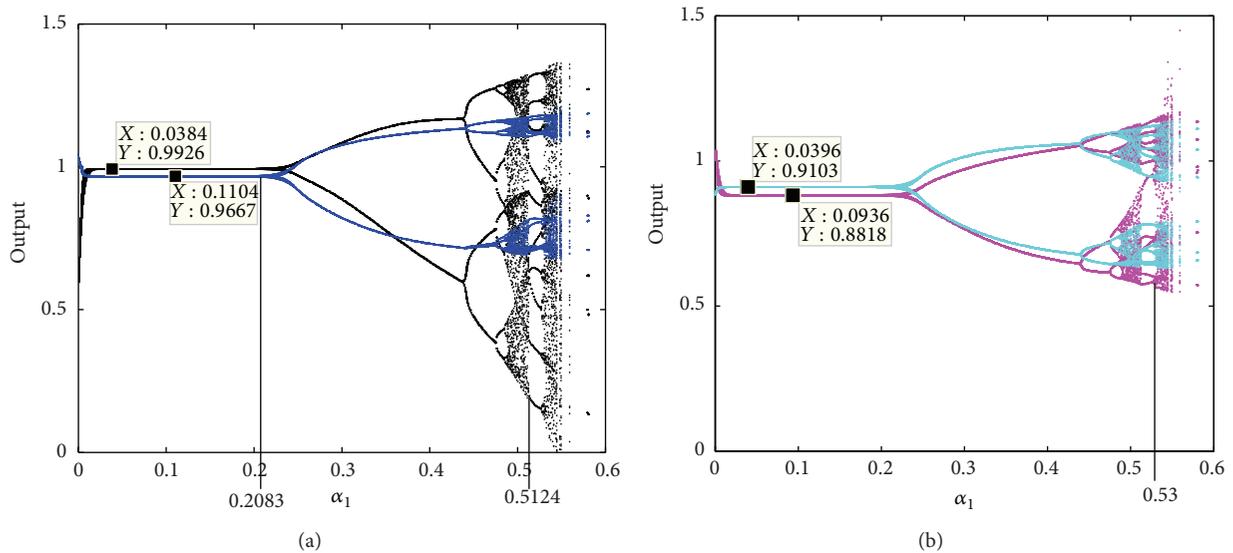


FIGURE 2: The bifurcation diagram with $k = 0.2, r = 1.6, \alpha_2 = \beta_1 = \beta_2 = 0.34$.

maximal Lyapunov exponent is bigger than zero, and the system is in steady period state if the maximal Lyapunov exponent is smaller than zero. From the change of the maximal Lyapunov exponent, the market is in chaotic state before tangential bifurcation. With the increase of α_1 , the market evolves into period state. Specially, in the period window, with the increase of α_1 , the periodic orbit which is caused by tangential bifurcation also shows bifurcation, and the structure of this bifurcation has a self-similar construction with the main bifurcation which evolves into chaos via Flip bifurcation. The system will show similar characteristics with changing α_2, β_1 , or β_2 , and not in detail.

Let us analyze closely the characteristics of chaos that are observed in the parameter interval ($0.5 < \alpha_1 < 0.55$).

Figure 4(a) is time series curves of q_{1a} when $\alpha_1 = 0.512$ which is slightly smaller than the tangential bifurcation point $\alpha_1 = 0.5124$ and shows apparently the characteristic of intermittent chaos, namely, a long laminar phase (see [20, 21]), where the output fluctuations behave regularly, and interrupted from time to time by chaotic bursts. Figure 4(b) is the time series with $\alpha_1 = 0.52$ which is in period orbit. And Figures 5(a) and 5(b) are their phase portraits, respectively.

Above investigation shows that output fluctuations are characterized by phases of low volatility with small output changes and phases of high volatility, with large output changes. One possible reason for this phenomenon is the intermittency of the output fluctuations which is caused by rapid output adjustment speed.

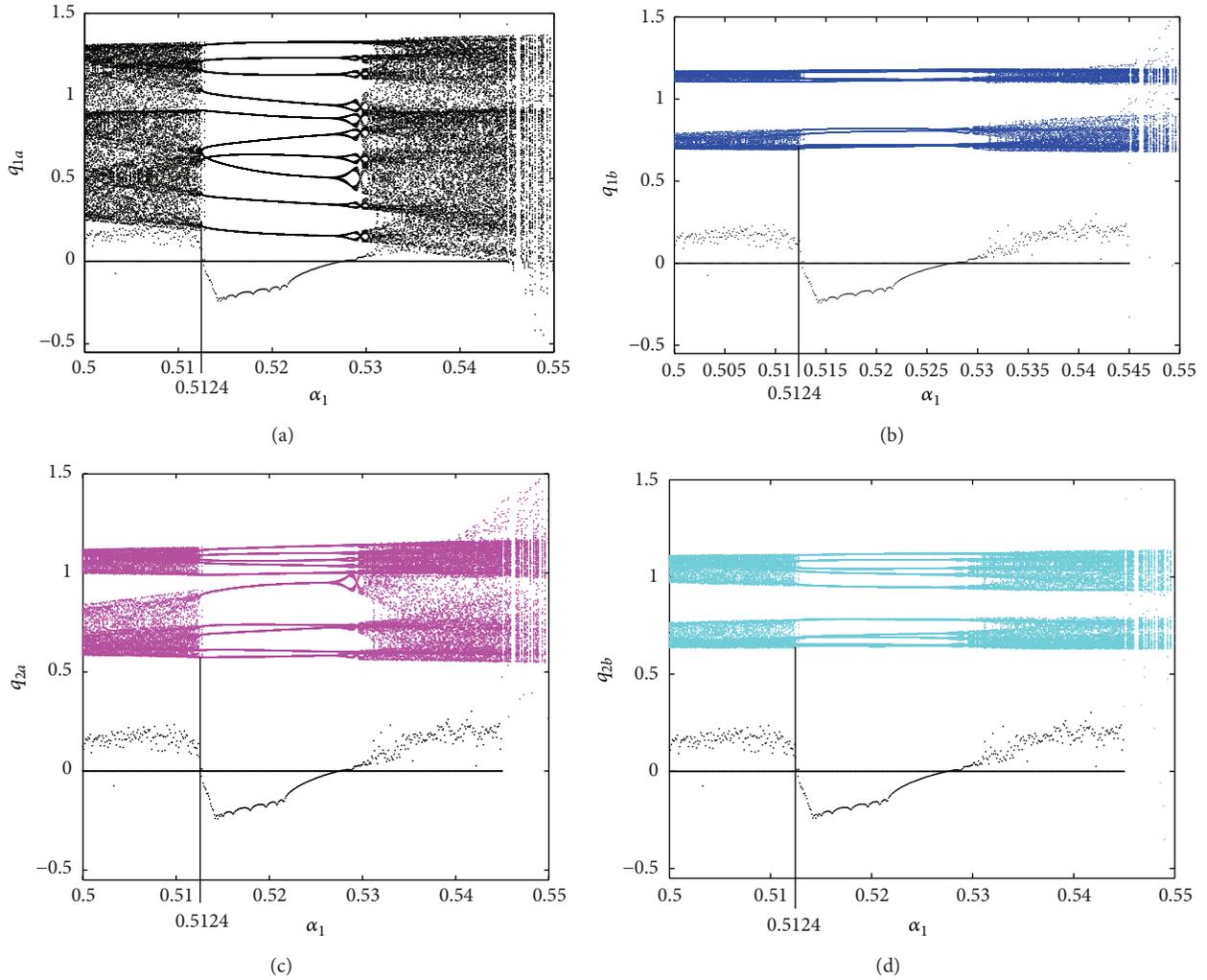


FIGURE 3: The enlarged drawing of tangential bifurcation and the maximum Lyapunov exponent.

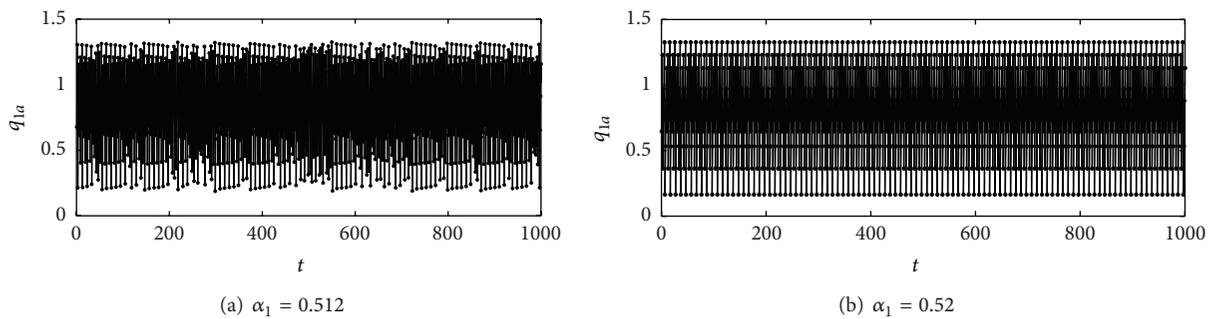


FIGURE 4: The time series of q_{1a} .

4.2. *The Flip Bifurcation (Period-Doubling Bifurcation) Which Contains Hopf Bifurcation and Its Intermittent Route to Chaos.* Figure 6 is the bifurcation diagram when $k = -0.2$, $r = 1.5$, $\alpha_2 = 0.35$, $\beta_1 = \beta_2 = 0.348$ with $\alpha_1 \in [0.3, 0.45]$ and its maximum Lyapunov exponent. When $\alpha_1 \in [0.3, 0.3535]$, the system enters into chaos via period 2 orbit and Hopf

bifurcation. When $\alpha_1 > 0.3535$, the tangential bifurcation occurs, and system enters into period 2 once again. With the increase of α_1 , system is in chaos when $\alpha_1 > 0.366$. When $\alpha_1 \in (0.38, 0.388)$, the system shows period doubling (Flip) orbit again, then entering the chaotic state when $\alpha_1 > 0.388$.

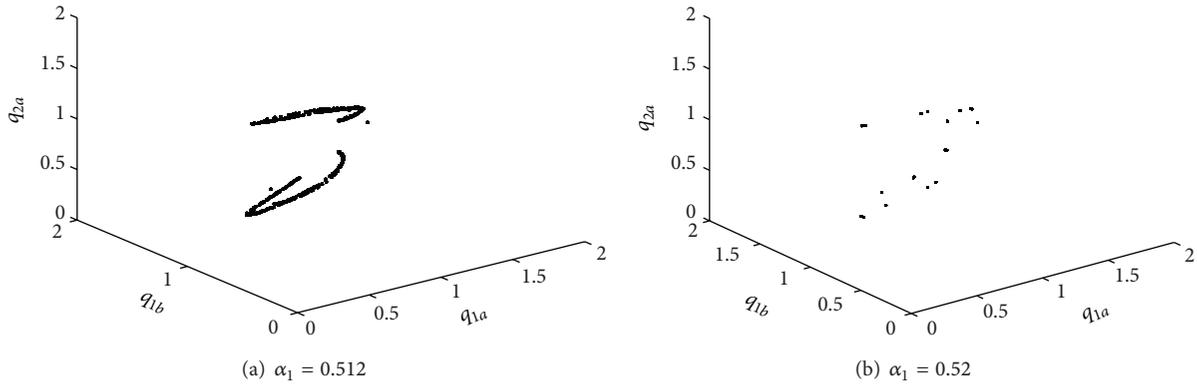


FIGURE 5: The 3-D phase portraits.

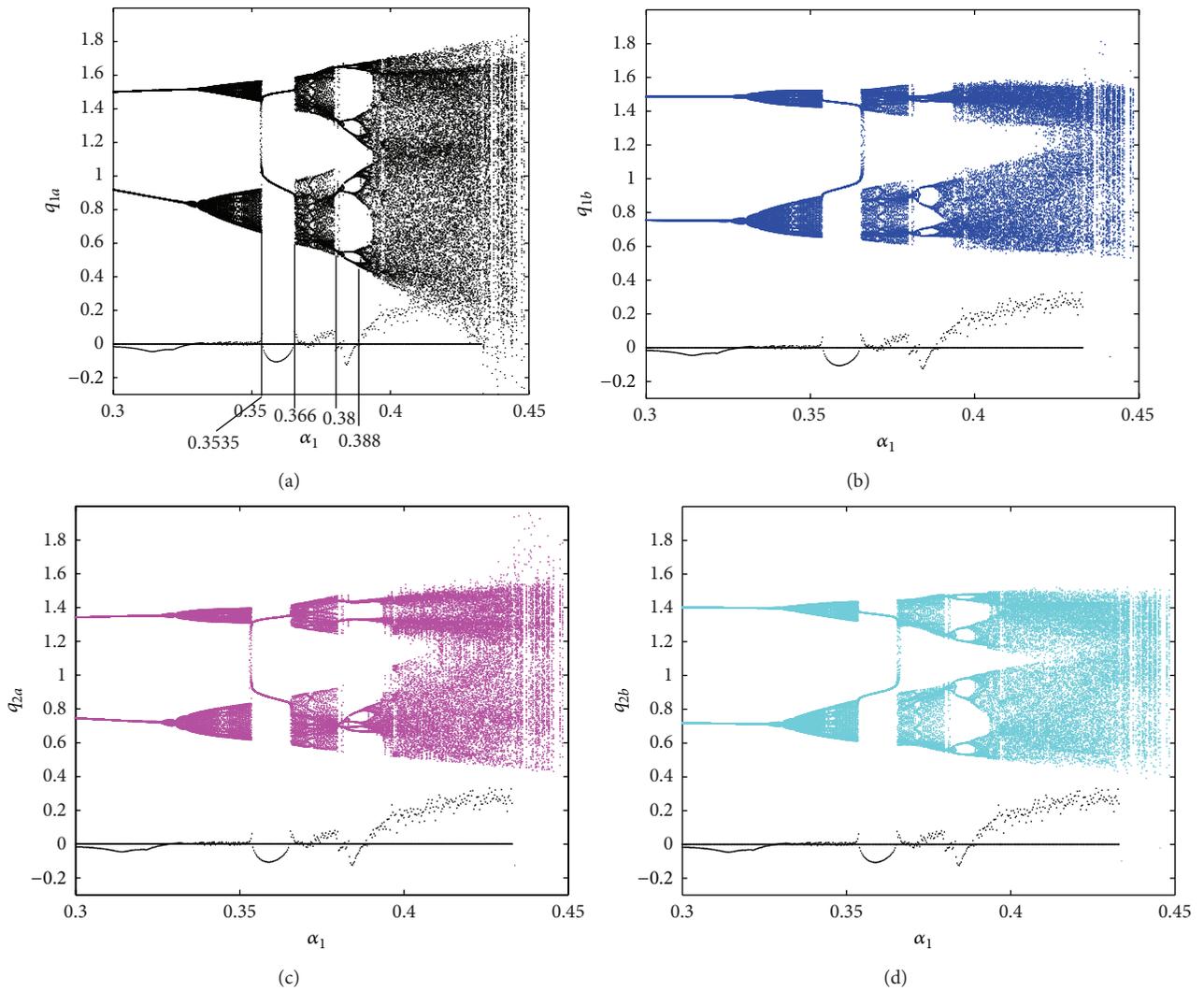


FIGURE 6: The bifurcation with $k = -0.2, r = 1.5, \alpha_2 = 0.35, \beta_1 = \beta_2 = 0.348$.

We take a value of $\alpha_1 = 0.353$ which is slightly smaller than 0.3535 (the tangential bifurcation point), the time series of q_{1a} exhibits nearly periodic laminar phases [20, 21] that are randomly interrupted by chaos bursts as Figure 7(a)

describes, and Figure 8(a) is its phase portrait. Figure 7(b) shows time series of a period 2 orbit for $\alpha_1 = 0.354$ which is slightly bigger than $\alpha_1 = 0.3535$, and Figure 8(b) shows its phase portrait.

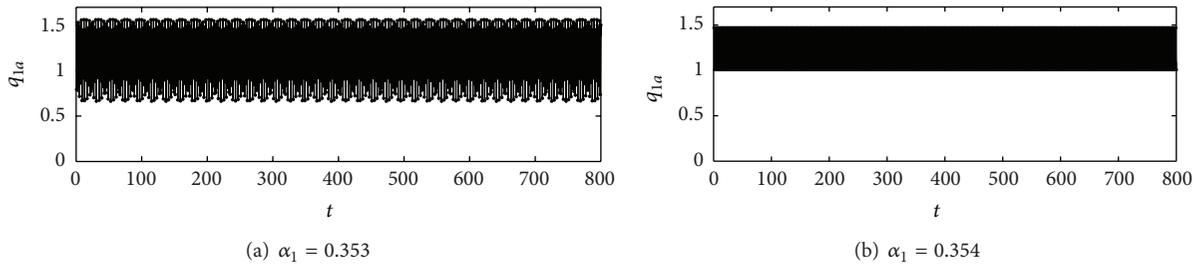


FIGURE 7: The time series of q_{1a} .

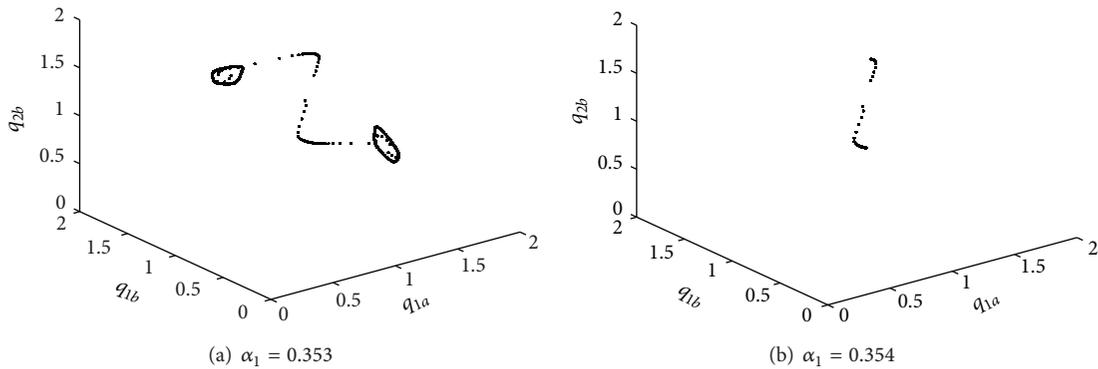


FIGURE 8: The phase portraits.

5. Conclusion

In this paper we propose a multiproduct Cournot duopoly model with relative profit delegation. The Cournot Nash equilibrium point and the local stability are analyzed. In particular, we show that the chaotic behavior is characterized by intermittency which is caused by tangential bifurcation. This means periodic state and unstable chaotic state appear alternately in the market. In real world, oligopoly market has been repeatedly disturbed by a series of severe output fluctuations which results tremendous economic losses of firms and the whole market. Our study suggests that the inappropriate output adjustment speed might be one reason that the output market becomes more vulnerable. The research provides the reference for enterprises and policy makers.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

Acknowledgments

The authors thank the reviewers for their careful reading and providing some pertinent suggestions. The research was supported by the National Natural Science Foundation of China (no. 61273231), Doctoral Fund of Ministry of Education of China (Grant no. 20130032110073), and supported by Tianjin university innovation fund.

References

- [1] H. W. Wang and J. H. Ma, "Complexity analysis of a Cournot-Bertrand duopoly game model with limited information," *Discrete Dynamics in Nature and Society*, vol. 2013, Article ID 287371, 6 pages, 2013.
- [2] F. Tramontana and A. E. A. Elsadany, "Heterogeneous triopoly game with isoelastic demand function," *Nonlinear Dynamics*, vol. 68, no. 1-2, pp. 187–193, 2012.
- [3] J. H. Ma and X. S. Pu, "Complex dynamics in nonlinear triopoly market with different expectations," *Discrete Dynamics in Nature and Society*, vol. 2011, Article ID 902014, 12 pages, 2011.
- [4] B. G. Xin, J. H. Ma, and Q. Gao, "Complex dynamics of an adnascent-type game model," *Discrete Dynamics in Nature and Society*, vol. 2008, Article ID 467972, 12 pages, 2008.
- [5] J. H. Ma and H. L. Tu, "Complexity of a Duopoly game in the electricity market with delayed bounded rationality," *Discrete Dynamics in Nature and Society*, vol. 2012, Article ID 698270, 13 pages, 2012.
- [6] Y. H. Guo and J. H. Ma, "Research on game model and complexity of retailer collecting and selling in closed-loop supply chain," *Applied Mathematical Modelling*, vol. 37, no. 7, pp. 5047–5058, 2013.
- [7] L. Fanti, L. Gori, and M. Sodini, "Nonlinear dynamics in a Cournot duopoly with relative profit delegation," *Chaos, Solitons & Fractals*, vol. 45, no. 12, pp. 1469–1478, 2012.
- [8] C. Fershtman and K. L. Judd, "Equilibrium incentives in oligopoly," *American Economic Review*, vol. 77, no. 5, pp. 927–940, 1987.

- [9] S. D. Sklivas, "The strategic choice of managerial incentives," *RAND Journal of Economics*, vol. 18, no. 3, pp. 452–458, 1987.
- [10] J. Vickers, "Delegation and the theory of the firm," *Economic Journal*, vol. 95, supplement, pp. 138–147, 1985.
- [11] T. Jansen, A. van Lier, and A. van Witteloostuijn, "A note on strategic delegation: the market share case," *International Journal of Industrial Organization*, vol. 25, no. 3, pp. 531–539, 2007.
- [12] R. A. Ritz, "Strategic incentives for market share," *International Journal of Industrial Organization*, vol. 26, no. 2, pp. 586–597, 2008.
- [13] V. S. Fumas, "Relative performance evaluation of management. The effects on industrial competition and risk sharing," *International Journal of Industrial Organization*, vol. 10, no. 3, pp. 473–489, 1992.
- [14] N. Miller and A. Pazgal, "Relative performance as a strategic commitment mechanism," *Managerial and Decision Economics*, vol. 23, no. 2, pp. 51–68, 2002.
- [15] S. C. Maurice and C. R. Thomas, *Managerial Economics*, China Machine Press, Beijing, China, 2003.
- [16] T. Dubiel-Teleszynski, "Nonlinear dynamics in a heterogeneous duopoly game with adjusting players and diseconomies of scale," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 1, pp. 296–308, 2011.
- [17] D. Hanly and K. C. K. Cheung, "Market structure of multi-product firms under free entry," *Economics Letters*, vol. 61, no. 2, pp. 159–163, 1998.
- [18] K. Basu, "Stackelberg equilibrium in oligopoly: an explanation based on managerial incentives," *Economics Letters*, vol. 49, no. 4, pp. 459–464, 1995.
- [19] T. Puu, *Attractors, Bifurcations and Chaos: Nonlinear Phenomena in Economics*, Springer, Berlin, Germany, 2000.
- [20] T. Kaizoji, "Intermittent chaos in a model of financial markets with heterogeneous agents," *Chaos, Solitons and Fractals*, vol. 20, no. 2, pp. 323–327, 2004.
- [21] R. A. Miranda, E. L. Rempel, A. C.-L. Chian, and F. A. Borotto, "Intermittent chaos in nonlinear wave-wave interactions in space plasmas," *Journal of Atmospheric and Solar-Terrestrial Physics*, vol. 67, no. 17-18, pp. 1852–1858, 2005.



Hindawi

Submit your manuscripts at
<http://www.hindawi.com>

