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Research Article

Fare Optimality Analysis of Urban Rail Transit under Various Objective Functions

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Urban rail transit fare strategies include fare structures and fare levels. We propose a rail transit line fare decision based on an operating plan that falls under elastic demand. Combined with the train operation plan, considering flat fare and distance-based fare, and based on the benefit analysis of both passenger flow and operating enterprises, we construct the objective functions and build an optimization model in terms of the operators' interests, the system's efficiency, system regulation goals, and the system costs. The solving algorithm based on the simulated annealing algorithm is established. Using as an example the Changsha Metro Line 2, we analyzed the optimized results of different models under the two fare structures system. Finally the recommendations of fare strategies are given.

1. Introduction

In the urban rail system, fare strategies include fare structures and fare levels. Fare structures are the relationship between the fare amount and the trains' travel distance, which includes the flat fare and the graduated fare (distance-based, section-based, and so on).

Savage [1] found from conducting a time-series analysis of the bus operations of the Chicago Transit Authority from 1953 to 2005 that Chicago could improve social welfare by reducing service frequencies and then by using the saved money to lower fares for a given budget constraint. Using the city of Haifa, Israel, as a case study, Sharaby and Shiftan [2] focused on evaluating the impact of fare integration on travel behavior and on transit ridership, which showed a significant increase in passenger demand and ticket sales when a simple fare system with free transfers, reducing fares for many passengers was adopted. They found that fare reduction was a significant factor in attracting transit users. Litman [3] studied transit price elasticity and cross-elasticity wherein he evaluated public transit benefits and costs (see also, [4]). Some studies [5–7] showed that fare systems, service levels, living standards, and travel demands would affect public transit ridership. Winston and Maheshri [8] found, through estimating the contribution of each US urban rail operation to social welfare based on the demand for and cost of its service, that with the exception of BART in the San Francisco Bay area, every system actually reduces welfare and was unable to become socially desirable even with optimal pricing or with a physical restructuring of its network.

Li et al. [9] developed a profit maximization model to optimize rail line length, number and the locations of stations, the headway, and the fares. Chien and Tsai [10] studied optimization of fare structure and service frequency for maximum profitability of a rail line, while peak period and off-peak period were considered, and they conducted sensitivity analyses of fares and headways. Borndörfer et al. [11] studied models for fare planning in public transport, and the study included some objectives such as the maximization of demand, revenue, profit, or social profit, and they proposed a nonlinear optimization approach based on a detailed discrete choice model of user behavior. Then they used the resulting models to compute and to compare optimized fare systems for the city of Potsdam, Germany. Lam and Zhou [12] presented a bilevel model to optimize the fare structure for transit networks with elastic demand under the assumption of fixed transit service frequency, where the upper-level problem seeks to maximize the operator's revenue, whereas the lower-level problem is a stochastic user equilibrium transit assignment model with capacity constraints. Zhou et

al. [13] also built a bilevel transit fare equilibrium model for a deregulated transit system, where the upper-level problem is to maximize the profit of each transit operator within an oligopolistic market for there exists a generalized Nash game between transit operators, and the lower-level problem is stochastic user equilibrium assignment model with elastic OD demand.

Obviously, the fare decision of urban rail transit systems is a multiple objective problem. Studies from various objective functions have shown the differences of fare strategies. In this paper, we present the models of fare strategy under some objective functions, including the two typical fare structures, flat fare (FF) and distance-based fare (DBF). Then we compare the optimal solutions of these models.

The remainder of this paper is organized as follows. In the next section, we analyze the fare decision problem, which includes the generalized travel costs of passengers and the operator's benefits. In Section 3 we discuss the constraints and objective functions and present the optimization models with various objective functions. In Section 4, we develop a solution algorithm based on a simulated annealing (SA) algorithm. In Section 5, the case of Changsha Metro Line 2 is used to illustrate the application of the proposed models and of the solution algorithm. In particular, we analyze and discuss the solutions of each fare structure under objective functions. Finally, the conclusions and a recommended fare policy are given in Section 6.

2. Problem Statements

Urban passenger flow typically exhibits an obvious characteristic of elastic demand that is affected by generalized travel costs determined by the train operation organization. So the fare of urban rail transit must be optimized comprehensively by combining with the train schedule.

For simplification, the following assumptions are made in this paper.

(A1) The research range is limited to an urban rail line for the independence of operational and fare policy of many urban rail transit lines.

(A2) The research time period is a travel time interval of passenger flow (e.g., the morning or evening peak hour).

(A3) The operation service of the urban rail line uses a long train route and an all-stop schedule. And every train has a uniform number of vehicles.

A transit line $l(N) = (S, E)$ is represented by an ordered sequence of stations $S = \{1, 2, \ldots, L_S\}$, and $1, 2, \ldots, L_S$ is arranged by the down direction. The sections are denoted by $E = \{e(i, j) \mid i, j \in S\}$, and the mileage of $e(i, j)$ is $w(e)$ and is also denoted by $w(i, j)$ or by w. Down and up directions are, respectively, denoted by $\varpi = 0$ and $\varpi = 1$.

Headway is denoted by H minutes within given research period T. The train speed is denoted by v and every train consists of m vehicles. The passenger load capacity and the maximum allowable load rate of each vehicle are V and λ, respectively.

The potential passenger demand and the actual passenger demand between stations i and j are denoted by Y_{ij} and Q_{ij}, respectively.

The fare of the two fare structures, f_{ij} $(i, j \in S, i \neq j)$, can be expressed as

$$f_{ij} = \begin{cases} f & \text{(Flat-fare)} \\ f_w^0 + f_w \left| w(i, j) \right| & \text{(Distance-based)}, \end{cases} \tag{1}$$

where f is the ticket price of the FF and f_w^0 and f_w are the fixed and variable components of the DBF, respectively.

2.1. Analysis of Generalized Travel Costs. Generalized travel costs of passengers are a weighted combination of fare spending, time costs, and congestion costs. Fare cost is decided by fare structure and its associated fare amounts. Time costs are weighted by wait time at stations and by in-vehicle time.

When arriving time follows a uniform distribution, the average wait time, t_w, can be calculated by

$$t_w = \frac{H}{2 \times 60}. \tag{2}$$

In-vehicle time of passengers comprises train operation time and stop time, t_0 (minutes), of each intermediate station. So in-vehicle time between i and j stations, $t_I(i, j)$, is

$$t_I(i, j) = \frac{w(i, j)}{v} + \left| j - i - 1 \right| t_0 \tag{3}$$

Congestion cost is related to the volume of passengers. During period T, passenger flow of section $e(k, k+1)$ of direction ϖ, denoted as $g(k, \varpi)$, is

$$g(k, \varpi) = \begin{cases} 0 \\ k = 1, \quad \varpi = 1 \\ 0 \\ k = L_S - 1, \quad \varpi = 0 \\ g(k-1, \varpi) + \sum_{j=k+1}^{H_S} Q_{kj} - \sum_{j=1}^{k-1} Q_{jk} \\ 2 \le k \le L_S - 1, \quad \varpi = 1 \\ g(k+1, \varpi) + \sum_{j=1}^{k-1} Q_{kj} - \sum_{j=k+1}^{H_S} Q_{jk} \\ 1 \le k \le L_S - 2, \quad \varpi = 0. \end{cases} \tag{4}$$

Therefore, the corresponding congestion cost $B_{k,k+1}$ can be given by

$$B_{k,k+1}(\varpi, H) = \begin{cases} 0 \\ g(k, \varpi) \le \dfrac{60TV}{H} \\ a_1 t_I(k, k+1) \left(\dfrac{H \cdot g(k, \varpi)}{60TV} \right)^{a_2} \\ \dfrac{60TV}{H} < g(k, \varpi) < \dfrac{60\lambda TV}{H}, \end{cases} \tag{5}$$

where a_1 and a_2 are dimensionless experience parameters.

As shown by the cost equations, except for fare expenses, the passengers' time and the congestion costs are determined by the train schedule. So passenger flow Q_{ij} within period T can be defined as a function of potential passenger Y_{ij} and the generalized travel costs, used and specified as

$$Q_{ij} = Y_{ij}\left[1 - a_w t_w - a_I t_I(i,j) - a_f f_{ij} - a_B B_{ij}\right], \quad (6)$$

where f_{ij} is the fare from station i to j, and a_w, a_I, a_f, and a_B are the parameters for the wait time, in-vehicle time, fare, and congestion cost, respectively. In order to ensure the nonnegativity of the passenger demand, the following condition should be satisfied:

$$0 \le 1 - a_w t_w - a_I t_I(i,j) - a_f f_{ij} - a_B B_{ij} \le 1. \quad (7)$$

2.2. The Operator's Benefit. The operator's benefit needs to consider both operating income and operating expenses. Operational income R is the product of passenger flow and its corresponding fare; that is,

$$R = \sum_{i=1}^{L_S}\sum_{j=1}^{L_S} f_{ij} Q_{ij}. \quad (8)$$

Operational cost consists of the following three components: train operating cost C_O, rail line cost C_L, and rail station cost C_S. It is represented as

$$C = C_O + C_L + C_S. \quad (9)$$

The train's operating cost is directly related to the fleet size F where F equals the turnaround time of passenger trains, Θ, divided by headway H. So the number of serviceable cars N within the period T can be formulated as

$$F = \frac{\Theta}{H/60}. \quad (10)$$

In addition to rervedents of vehicles above. the service cars, the number of reserved and maintenance cars is ε times the number of service cars (ε is about 25% generally). Vehicle operating costs accordingly comprise these three components of the vehicles described above. That is,

$$C_O = N c_U + \beta \varepsilon N c_U, \quad (11)$$

where c_U is average car operating costs per one hour and β $(0 < \beta < 1)$ is the cost parameter of reserved and maintenance cars.

Rail line maintenance costs can be expressed as

$$C_L = \gamma_0 w(1, L_S) + \frac{60\gamma_1}{H}, \quad (12)$$

where γ_0 is the fixed maintenance costs per kilometer line and γ_1 is the associated cost parameter related to headway H.

The total rail stations cost can be expressed as

$$C_S = \sum_{i\in S} C_S^i = \Lambda_0 L_S + \Lambda_1 \sum_{i\in S} Q_i, \quad (13)$$

where Λ_0 is the fixed maintenance cost of each station within period T, Λ_1 is the service cost for per passenger in a station, and Q_i (i.e., i $(i \in S)$) is the total volume of arrival and departure passengers. It can be given by

$$Q = \sum_{\varpi=0,1} Q_i = \sum_{j\ne i} Q_{ij} + \sum_{j\ne i} Q_{ji} \quad i, j \in S. \quad (14)$$

3. The Optimization Model

3.1. The Analysis of Constraints. The train frequency should satisfy the constraint of tracking interval time τ and the maximum headway τ_0 for maintaining service levels, and the train's load capacity is limited by the maximum allowable capacity rate λ.

Maximum passenger flow of section $(k_\varpi^*, k_\varpi^* + 1)$ for the direction ϖ, $Q(k_\varpi^*, \varpi)$, can be given by

$$Q(k_\varpi^*, \varpi) = \max_k \{g(k, \varpi) \mid 1 \le k \le L_S - 1\}. \quad (15)$$

Then, headway H should satisfy the constraint as

$$\tau \le H \le \min\left\{\frac{60T\lambda V}{Q(k_\varpi^*, \varpi)}, \tau_0\right\}. \quad (16)$$

Meanwhile, the fare of the whole line should be set for an upper bound limited in operational costs and economic level. The fare of upper bound for all fare structures is $\overline{f}$, and f_w^0 is the upper bound of fixed components of DBF. That is,

$$0 \le f_{ij} \le \overline{f} \quad \forall i, j \in S,\ i \ne j \quad (17)$$

$$0 \le f_w^0 \le \overline{f}_w^0. \quad (18)$$

3.2. Objective Functions and Optimization Models. For the problem of urban rail transit route fares, the selection of the optimization objective functions needs to consider the operators' benefits, the passengers' benefits, and the social service function. In many literatures, the problem has been studied as a multiobjective program. In the following, the different models will be established according to different optimization objectives in order to compare the effect of these objective functions.

3.2.1. Analysis of the Operator's Benefit. The direct benefit of urban rail transit operators is the train's operating profit, which can be expressed as the operation revenue (ticket income) minus the operation costs. The objective function of

the operator's direct operation benefit during the study period T can be expressed as

$$\begin{aligned}\operatorname{Max}\psi_1 &= P = R - C \\ &= \sum_{\varpi=0,1}\sum_{i=1}^{L_{S_s}}\sum_{j=1}^{L_{S_s}} f_{ij}Y_{ij} \\ &\quad\times\left(1-\frac{1}{2}a_w\frac{H}{60}-a_I\right. \\ &\quad\times\left(\frac{w(i,j)}{v}+|j-i-1|t_0\right) \\ &\quad\left.-a_f f_{ij}-a_B B_{ij}(\varpi,H,f_{ij})\right) \\ &\quad-\left(m(1+\varepsilon\beta)c_U\frac{60\Theta}{H}+\gamma_0 w(1,L_s)\right. \\ &\quad\left.+\frac{60\gamma_1}{H}+\Lambda_0 L_s+\Lambda_1\sum_{i\in S}Q_i(H,f_{ij})\right).\end{aligned} \tag{19}$$

3.2.2. Objective Function of System Benefits. Urban rail transit has the dual nature of public welfare and profitability. The overall benefit of an urban rail transit system relates both to the operator and to the passengers. The operator's benefit can be measured in its fare income. Passenger benefits E_p include generalized travel cost and travel benefits; namely,

$$E_p = \sigma Q_{ij} - Q_{ij}\left(f_{ij}+\frac{a_w}{a_f}t_w+\frac{a_I}{a_f}t_I(i,j)+\frac{a_B}{a_f}B_{ij}\right). \tag{20}$$

It should be pointed out that the value of σ can be understood as the net profit value of rail transit in terms of service and efficiency compared with other transportation models.

Therefore, the objective function of maximum system benefits can be expressed as follows:

$$\operatorname{Max}\psi_2 = \psi_1 + E_p. \tag{21}$$

3.2.3. Objective Function of Urban Rail Transit Supervision. Urban rail transit regulators pay attention to the sustainable operation capacity and the traffic service functions of the urban rail transit system. Thus, the operator's benefit and the volume of passenger turnover T_m provided by rail transit can be used as the supervision objectives. Among them, T_m can be expressed as

$$\begin{aligned}T_m &= \sum_{\varpi=0,1}\sum_{i=1}^{L_s}\sum_{j=1}^{L_s} w(i,j)Y_{ij} \\ &\quad\times\left(1-\frac{1}{2}a_w\frac{H}{60}-a_I\right. \\ &\quad\times\left(\frac{w(i,j)}{v}+|j-i-1|t_0\right) \\ &\quad\left.-a_f f_{ij}-a_B B_{ij}(\varpi,H,f_{ij})\right).\end{aligned} \tag{22}$$

In order to make the two objectives of the operation supervision into a single objective function, the relative weight factor φ of passenger turnover is introduced, and then the objective function can be expressed as

$$\begin{aligned}\operatorname{Max}\psi_3 &= \psi_1 + \varphi T_m \\ &= \sum_{\varpi=0,1}\sum_{i=1}^{L_S}\sum_{j=1}^{L_S}\left(f_{ij}+\varphi w(i,j)\right)Y_{ij} \\ &\quad\times\left(1-\frac{1}{2}a_w\frac{H}{60}-a_I\right. \\ &\quad\times\left(\frac{w(i,j)}{v}+|j-i-1|t_0\right) \\ &\quad\left.-a_f f_{ij}-a_B B_{ij}(\varpi,H,f_{ij})\right).\end{aligned} \tag{23}$$

3.2.4. Analysis of the Minimum System Cost Objectives. The cost minimization of a rail transit system can also be used as an optimization objective of fare strategies. The objective function is the sum of two part costs, operators' costs, and passengers' generalized travel costs; namely,

$$\operatorname{Min}\psi_4 = C + Q_{ij}\left(f_{ij}+\frac{a_w}{a_f}t_w+\frac{a_I}{a_f}t_I(i,j)+\frac{a_B}{a_f}B_{ij}\right). \tag{24}$$

Based on the above analysis, models (M1) through (M4) consist of one of the objective functions (19) through (24), respectively, and all the constraints (7), (16), (17), and (18).

4. Solution Algorithm Based on Simulated Annealing

Each of the above models (M1) through (M4) is a nonlinear optimization problem. Therefore, we have designed a SA-based general solution algorithm, for every fare structure in the models. The models (M1) through (M4) can be solved in a uniform solution structure.

In the algorithm, the generation algorithm of the feasible solution and the calculation method for travel OD matrix are each described here.

Algorithm 1. Generation algorithm of the feasible solution.

Step 1. Select randomly $H \in [\tau,\tau_0]$ and $f_{ij} \in [0,\overline{f}]$ (FF: $f \in [0,\overline{f}]$ and DBF: $f_w^0 \in [0,\overline{f_w^0}]$, $f_w \in [0,(\overline{f}-f_w^0)/w(1,L_S)]$)

under the constraints of (16) or (17) or (18) and set initial congestion cost matrix B as a zero matrix.

Step 2. Judge whether (H, f_{ij}) satisfies $0 \leq 1 - a_w t_w - a_I t_I(i, j) - a_f f_{ij} \leq 1$. If it is not satisfied, return to Step 1.

Step 3. If (H, f_{ij}) is a feasible solution, then calculate the corresponding OD matrix Q by Algorithm 2; meanwhile, calculate the section of maximum passenger flow, $Q(k^*_{\varpi}, \varpi)$.

Step 4. If all the elements in Q satisfy the nonnegative constraints (7), then (H, f_{ij}) is a feasible solution. Otherwise, return to Step 1.

Algorithm 2. Calculation method for OD matrix.

Step 1. Set $m = 0$ to represent the number of iterations and the initial congestion costs matrix B^0 is a zero matrix. Calculate OD matrix Q^m with the solution (H, f_{ij}) and B^m by (5).

Step 2. Calculate the passenger flow, $g(\varpi, k)^m$, of each section based on (4). Then update congestion costs $B^{m+1} = B^m$ and calculate Q^{m+1} by (6).

Step 3. If Q^{m+1} and Q^m are sufficiently close (e.g., $\max_{i,j\in S, i\neq j}\{|Q^{m+1}(i, j) - Q^m(i, j)| \leq 1\}$), then take Q^{m+1} as the corresponding OD matrix of the solution (H, f_{ij}) and terminate this algorithm. Otherwise, let $m = m + 1$ and return to Step 2.

In order to realize a satisfied convergence speed, we adopted a very fast generation mechanism of new solution (VFSA; see [14]) which uses a quasi-Cauchy distribution method, depending on the temperature. It is expressed as

$$m_i^* = m_i + y_i (B_i - A_i)$$
$$y_i = \Gamma \left[\left(1 + \frac{1}{\Gamma}\right)^{|2u-1|} - 1 \right] \operatorname{sgn}(u - 0.5), \tag{25}$$

where m_i, $m_i^* \in [A_i, B_i]$ are the element I of current solutions and neighboring solutions, respectively; u is a random number in the range of $[0, 1]$; Γ is the current temperature.

Based on the above analysis, the general GA under every fare strategy is as described below.

Algorithm 3. The general SA algorithm for all fare strategies.

Step 1. Initialization. Generate the initial feasible solution (H, f_{ij}) under the initial temperature Γ_0 and then calculate $\Psi(H, f_{ij})$. Let $k = 0$ be the current running times of the outer cycle, let $n = 0$ be current running times of the inner cycle, and let $\Gamma = \Gamma_0$ be the current temperature. Set $\Gamma_{\min}$ as the minimum temperature of the outer cycle and Q as the number of iterations at each temperature.

Step 2. Construction of neighborhood. Generate neighborhood solutions (H^*, f_{ij}^*) by VFSA, and calculate the corresponding $\Psi(H^*, f_{ij}^*)$.

Table 1: Baseline values for parameters.

Symbol	Value	Unit	Symbol	Value	Unit
a_1	0.15	—	V	210	Passengers
a_2	4	—	λ	1.5	—
a_B	0.49	1/h	ε	25%	—
a_f	0.1	1/h	β	24%	—
a_I	0.05	1/h	σ	0.8	—
a_w	1.2	1/h	γ_0	3800	¥/km
c_U	325	¥/h-veh	γ_0	525	¥h
m	6	Vehicle	Λ_0	4200	¥/km
t_0	1/120	h	Λ_1	0.5	¥
τ_0	25	min	τ	2	min
v	40	km/h			

Step 3. Metropolis sampling. When $\Psi(H^*, f_{ij}^*) > \Psi(H, f)$, then let $(H, f) = (H^*, f_{ij}^*)$; otherwise, if $\exp(\Delta\Psi/\Gamma) >$ rand (i.e., rand is a random number in $(0, 1)$ and $\Delta\Psi$ is the difference between these current and optimal solutions), then let $(H, f) = (H^*, f_{ij}^*)$. Then set $n = n + 1$.

Step 4. Test of the termination criterion of the inner cycle. If $n = N$, terminate the inner cycle and let $k = k + 1$; otherwise return to Step 2.

Step 5. Cooling schedule. Calculate the temperature $\Gamma(k)$.

Step 6. Test of the termination criterion of the outer cycle. When $\Gamma(k) \leq \Gamma_{\min}$, terminate this algorithm and output the optimal solution output; otherwise, return to Step 2.

5. Numerical Studies in Changsha Metro Line 2

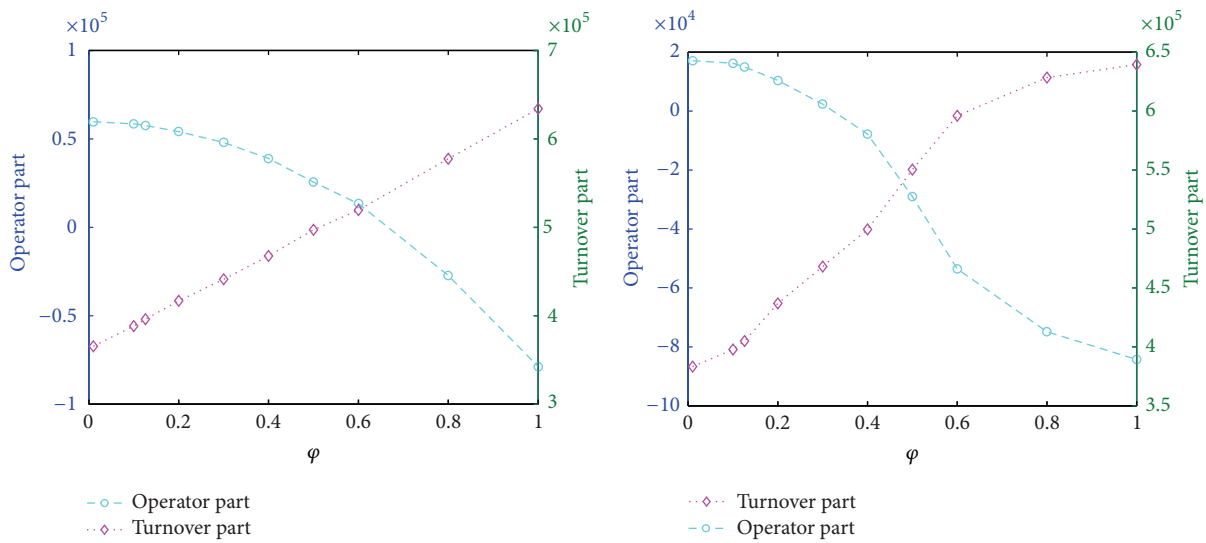
The first phase project of Metro Line 2 in Changsha, China, of which the total length is 21.36 km and includes 19 stations, is planned to be completed in 2015. In this section, the morning peak hour (7:00-8:00) of 2016 is chosen as the study period.

The baseline values for parameters are shown in Table 1. $\overline{f}$ and $\overline{f_w^0}$ are limited to 9.0 and 2.0¥. Take $K = 100$ and $\alpha = 0.99$ of the SA algorithm.

In the model of M3, because the objective function of M3 is divided into two parts, the operator part ψ_1 and the turnover volume part T_m, the factor φ needs to be demarcated. For $\varphi \in [0, 1]$, different values of φ are tested. The change in these two parts of the objective function for various φ is shown as in Figure 1. A balance can be made between these two parts when $\varphi = 0.55$ is taken as a reasonable value. Table 2 shows the various results of the models from M1 to M4 under the FF structure and under the DBF structure.

5.1. Comparison and Analysis on the Results of Models

5.1.1. Results Analysis of Two Fare Structures under Each Model. When only the objective function values are taken



(a) FF structure (b) DBF structure

Figure 1: Relationship between objective function value and φ in M3.

Table 2: Optimization results of different models.

Fare structure	Models							
	M1		M2		M3		M4	
	FF	DBF	FF	DBF	FF	DBF	FF	DBF
Fare (¥)	5.05	$2.00 + 0.28w$	8.45	$1.98 + 0.27w$	3.44	$2.00 + 0.05w$	8.45	$0.35w$
Headway (min)	5.27	5.02	11.96	5.20	3.72	3.73	11.90	5.27
Demand (10^3 pass.)	57.35	74.04	3.30	75.07	80.78	94.94	3.40	92.75
Objective function value (10^3)	59.79	16.83	−181.63	−218.48	299.34	274.62	213.26	296.83
Fare rate (¥/pass.-km)	0.81	0.66	1.68	0.64	0.55	0.38	1.67	0.35
Maximum section demand (10^3 pass)	14.34	16.52	0.72	16.85	20.25	23.30	0.75	20.50
Turnover volume (10^4 pass.-km)	35.97	38.89	1.66	39.91	50.85	58.32	1.72	48.49
Average riding distance (km)	6.27	5.25	5.03	5.32	6.29	6.14	5.06	5.23
Maximum passenger density of section (10^3 pass.-km)	2.69	3.47	0.16	3.52	3.78	4.44	0.16	4.34
Maximum section load rate (%)	94.85	109.23	11.47	111.42	94.56	108.80	9.87	135.57

into consideration, the results of the FF structure are always better than those of the DBF structure. Under Model M1, the objective function value of the FF structure is far better than that of the DBF structure. But for Models M2 to M4, the difference of the objective function value is smaller. The objective function value of Model M2 is negative. This is because the costs of passengers and of the operators are both taken into consideration, especially the effect of the passengers' generalized travel costs.

The volume of passenger demand and the maximum section load rate between these two fare structures varies enormously in these models. The DBF structure is more attractive to the demand as the fare rate of the FF structure is always higher than that of the DBF structure. Taking the M1 for example, the headway of these two structures is almost equal, but the difference between the objective functions is extraordinarily notable because the difference of the fare rate is significant, 0.15¥/passenger-km.

For the following parameters, the parameter value of the DBF structure is always better than that of the FF structure: the total passenger volume, the turnover volume, the maximum section load rate, the line load, and the maximum section-passenger volume. It is clear that the FF structure has an inhibitory effect on passenger flow volume, especially for the short-distance passengers. Meanwhile, it is indicated that the DBF structure can make full use of the transportation capacity.

The parameter values of average riding distance under Models M2 and M4 are nearly equal. For Models M2 and M4, the average riding distances are shorter than those of the other models, and the average riding distance of the FF structure is shorter than that of the DBF structure. But in

Models M1 and M3, the average riding distances of the FF structure are longer than those of Models M2 and M4.

5.1.2. Comparison of Fare under Different Models. For the fare of the FF structure, Models M1 and M3 are both lower; the fares are ¥5.05 and ¥3.44, respectively. Their average riding distances are similar, while, the lower fare in Model M3 would attract more passengers than would Model M1. In Models M2 and M4, the fares are both ¥8.45, but the values of the two parameters, the maximum section load rate, and passenger demand, are not reasonable. This is because the FF structure cannot correctly reflect the relationship between the riding distance and the passengers' and operators' costs and benefits.

For the DBF structure, the fare of M1 almost equals the fare of M2, in which the former is $2.00 + 0.28w$ and the latter is $1.98 + 0.27w$. But for Model M3, the results of the DBF tend to be the FF structure because $f_w = 0.05$ is near to zero. For Model M4, $f_w^0 = 0$. In general, the full fares of the whole Changsha Metro Line 2 in Models M1, M2, and M4 are all about 7.5¥, and the full fare is 3.07¥ in Model M3.

5.1.3. Comparison of Train Operation Schedules in Optimal Strategies. Model M1 under the two fare structures and Models M2 and M4 under the DBF structure have similar headways, at about 5.2 minutes.

But the headways of Models M2 and M4 on their own two fare structures are greatly different. Take M2 for example: the headway on the FF structure is 11.96 minutes, and it is 5.2 minutes on the DBF structure. This is because the fare for the DBF structure is fairer than the fare for the FF structure. The DBF structure has an encouraging effect on passenger trips. However, the inhibition effect of the FF structure on passenger volume is more obvious. So the headways in the FF structures of Models M2 and M4 reach a very high level.

For Model M3, the headways on the two fare structures are both about 3.7 minutes. They are lower than the headways of the other models, a difference that reflects the fact that the regulator encourages the operator to provide a high level of service to attract a greater volume of passengers.

5.2. The Parameter Analysis of the Optimization Model. For Model M2, the effect of different parameters σ on the optimization results is shown in Figure 2. When $\sigma \in [1.173, 1.174]$, the fare and the headway have undergone a drastic change. $\sigma \leq 1.73$ and $\sigma \geq 1.73$ represent two competition cases of multiple urban traffic modes: the other urban traffic modes can have an obvious substitution effect on rail transit and vice versa. The former reflected a market-oriented strategy of fare and operation; the latter showed a welfare-oriented strategy of fare and operation. There is a significant difference between the two cases.

At the same time, in the case of M1, the spacing distribution of passenger flow is shown in Figure 3. As we can see, whether FF or DBF, the riding distance of 60% of the passengers is mainly concentrated in 0 to 6 km. Through a comprehensive comparison of the optimization results of every model, it can be found that the average distances in the DBF are less than those in the FF, which indicates that the inhibition effect of the FF system on short-distance passengers is more obvious.

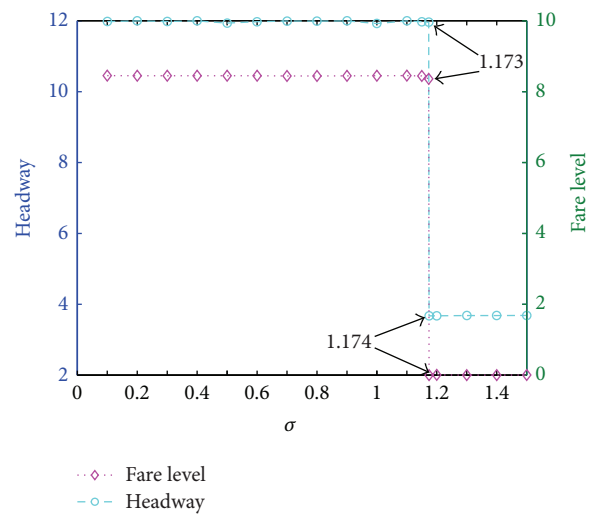


Figure 2: Effect of σ in model M2.

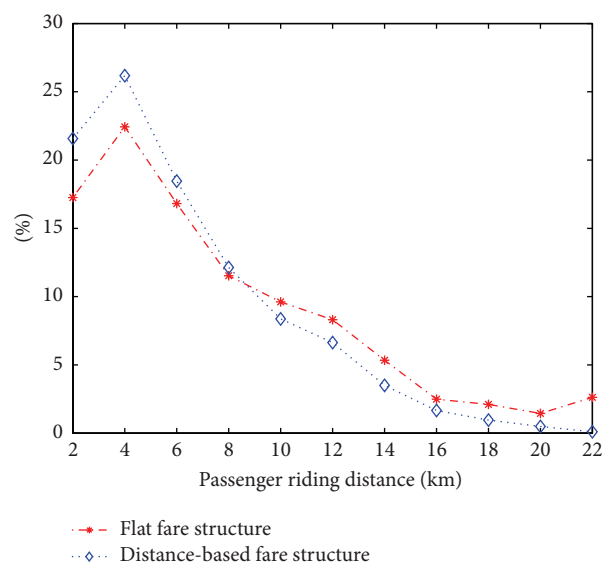


Figure 3: Distribution of passenger riding distance.

6. Conclusions

In this paper, for the two typical fare structures (FF and DBF), we present the models of fare decision under different objective functions, respectively, and build a solution algorithm based on GA algorithm. From the comparison and the analysis for the optimization results of multiple models, the following conclusions for Changsha Metro Line 2 can be drawn.

In general, the optimization result of M1 is relatively reasonable in the FF structure, and the fare should be taken as 5.05¥. The fare level in the DBF structure is $2 + 0.25w$ (for M1-M2) or $0 + 0.35w$ (for M4), and passenger fares on the whole

travel are about 7¥. However, rail transit operation regulators tend toward the lower fares scheme (3.44¥ in the FF structure or in the DBF structure $2 + 0.05w$; the full fare is 3.07¥).

The FF structure has an obvious inhibitory effect on short-range passenger flow, which, generally, is not recommended. The flat fare will be under consideration only when the government would like to supply the enough financial subsidy or economic support so that the low price policy is available.

Considering the fact that the passenger percentage of riding distance from 0 to 6 km accounts for roughly 60% of the total demand, the DBF structure or some of its simplified structures had better be taken to attract more short-haul passengers.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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