

Research Article

Horizontal Coalition Stability Analysis of Supply Chain Entities Based on Sequential Game

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Aiming to find the effect of the same status entities' horizontal coordination on supply chain, this paper studied the coalition stability of dealers in a two-stage supply chain with one supplier and multiple dealers. First, a vertical sequential game model is built, where the supplier is leader and the multiple dealers are followers. In the second stage of the game, multiple dealers face two selections: playing Nash game with each other or developing a coalition. Then, according to the results acquired by comparing the dealers' profits which depend on their coalition situations, the criterion of coalition stability is developed. Finally, numerical simulation is used to verify the validity of the model, and some insights are obtained. For example, if the sensitivity coefficient T of the market price is fixed, dealers' coalition tends to be stable with the increasing of the substitution rate k in a reasonable range; the supplier's optimal wholesale price is constant with and without dealer's coalition, but dealers' coalition causes demand to decrease, which leads to the decrease of the supplier's profit too. The result of this paper provides an important reference for the formation of dealers' coalition in IT or automobile supply chain.

1. Introduction

With the development of market economy, many new-style supply chains come into being and become popular, such as automobile supply chain, IT supply chain, and franchise supply chain consisting of one supplier and multiple dealers. Like other supply chains, the entities in these supply chains compete with each other for maximizing their own profits. Taking an automobile supply chain consisting of one carmaker and several 4S dealers as an example, the competition among supply chain entities exists not only in different statuses of the supply chain, such as the competition between carmaker and 4S dealers, but also in the same status of the supply chain, such as the competition among multiple 4S dealers. Reasonable competition can improve the work efficiency of the supply chain, while irrational competition results in waste of resources, cost increase, price war, and so forth. Meanwhile, there is cooperation among entities of the supply chain, which also can be divided into two situations as the competition. Although full cooperation among entities can give the whole supply chain maximum return [1], all entities participating in the full cooperation are often hard to

be stable because the benefit and foresight of each entity are different. Balancing the competition and cooperation among the entities holds great significances: (1) coordinating the relationship among the entities can help the entities to acquire the optimal profits and guarantee the stability of supply chain; (2) coordinating the relationship among the entities could promote the core competitiveness of supply chain. Therefore, the coordination for the relationship among the supply chain entities is a meaningful issue.

The coordination of supply chain is always a focus in the study of supply chain and is involved in many fields of supply chain, such as lead time variation control [2], supply chain coordination strategies about errors [3], and assessment of contracts' coordinating power [4]. The coordination of entities' relationship in a supply chain mainly includes vertical coordination of entities in different statuses and horizontal coordination of entities in the same status. At present, numerous researches focus on vertical coordination. For example, Weng introduced a quantity discount as an incentive in the basis of the traditional newsboy model, in order to coordinate the relationship between manufacturers and consumers and guide consumers to buy the appropriate

number of products [5]. Chaharsooghi and Heydari proposed a coordination model to obtain the joint determination of order quantity and reorder point based on credit option in a two-stage supply chain [6]. Arkan and Hejazi designed a coordination mechanism based on a credit period to coordinate orders in a two-echelon supply chain and to achieve channel coordination and a win-win outcome [7]. Wu considered two channel policies for both competing supply chains, vertical integration (VI) and manufacturer's Stackelberg (MS), to achieve the supply chain coordination [8].

Another important coordination method among entities is horizontal coordination, namely, the coordination among the entities in the same status of a supply chain. Through the establishment of horizontal coalition and reduction of the internal competition in a supply chain, entities in the horizontal coalition can obtain more profits, and the horizontal coalition will have a greater say throughout the supply chain. And stability is the key of the horizontal coordination among entities in the supply chain and the increase of the supply chain profit. Therefore, it is necessary to analyze the stability of the horizontal coalition in a supply chain. Now, the research on the problem of horizontal coalition stability of supply chain has drawn more attention. Mahesh studied the formation of agents' dynamic coalition and the condition of the coalition stability in a competitive market [9]. Then Mahesh studied coalition stability in a supply chain consisting of multiple vendors and an assembler [10]. Krajewska et al. studied horizontal cooperation among freight carriers, analyzed the profit margins resulting from horizontal cooperation among freight carriers, and discussed the possibilities of sharing these profit margins fairly among the partners [11]. Karray investigated the effects of horizontal joint promotions (HJP), initiated by competing retailers on the supply chain's strategies and profits, and studied market conditions conducive for profitable joint retail promotions under different channel structures [12]. Seok and Nof discussed collaborative capacity sharing (CCS) among manufacturers at the same horizontal layer in supply networks which can be used to minimize manufacturers' lost sales as well as maximize their production capacity utilization in a long-term period against lumpy demand [13].

As for coordination mechanism among entities, some researches adopted supply chain contract [14]. Cao et al. studied the coordination mechanism of supply chain which has one manufacturer and multiple retailers in the case of the presence of disturbance of the cost and demand of product and obtained the optimal contract to distribute the profits [15]. Pezeshki et al. established a dyadic supply chain model, examined the model under both full information and partial information updating situations, and proposed a coordinating contract for each case [16]. Sajadieh et al. developed a joint optimal policy to acquire more benefits for the cases with less unpredictable lead times, lower shortage prices, and no transportation cost [17]. Jiang et al. discussed three contract mechanisms, wholesale price (WP), pairwise revenue sharing (PRS), and spanning revenue sharing (SRS), to coordinate a three-stage supply chain with competing manufacturers [18].

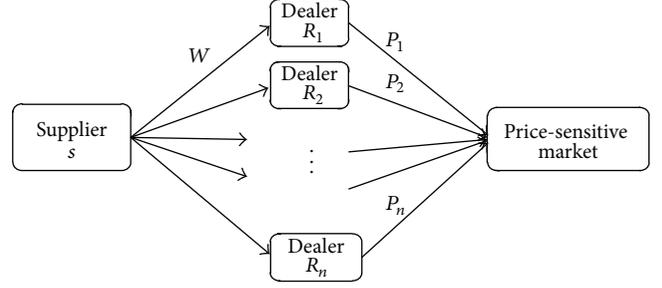


FIGURE 1: Supply chain structure.

Researches mentioned above mainly focus on theories or methods of supply chain vertical coordination, while the research on the horizontal coordination, especially the horizontal coalition of entities in an actual supply chain, is not enough. Furthermore, there are few studies about the distinguishing characteristics of stable supply chain coalition. Thus, the research on supply chain horizontal coalition is necessary. Based on these observations, this paper studies the horizontal coalition stability of a two-stage supply chain consisting of a supplier and multiple dealers by using the game theory and acquires supply chain coalition stability criterion by comparing the profits of dealers with and without coalition. First, the vertical sequential game model which consists of one supplier as a leader and multiple dealers as the followers is described. Second, each dealer can operate alone and compete with others or join the coalition which has a unified price, so the dealers will play Nash game or form a coalition with each other. Then, the criterion of coalition stability is acquired by comparing each dealer's benefit with coalition and without coalition. Only if all dealers' profits with coalition are more than those without coalition, the dealers' coalition will be stable. Finally, the main conclusions of this paper are verified by numerical simulation.

2. Model Description

The two-stage supply chain consisting of one supplier S and n dealers $(R_1 \cdots R_n)$ is shown in Figure 1. The supplier sells products to downstream dealers facing a price-sensitive market. The supplier manufactures the products at a marginal cost C and sells them to the dealers at a wholesale price W . Each dealer i runs at a marginal cost c_i and a retail price P_i ($i = 1, \dots, n$).

According to [19], the market demands of two differentiated products are $f_A = a - kp'_A + bp'_B$ and $f_B = a - kp'_B + bp'_A$. Based on these two functions, the demand function of dealer i can be acquired:

$$D_i = \begin{cases} a - TP_1 + k(P_2 + \cdots + P_n) & i = 1 \\ a - TP_i + k \sum_{j=1, j \neq i}^n P_j & 1 < i < n \\ a - TP_n + k \sum_{j=1}^{n-1} P_j & i = n. \end{cases} \quad (1)$$

In formula (1), parameter a denotes the saturation value of market demand of one dealer, so the total saturation value of market demand is na . Parameter T denotes market price sensitivity, and the price sensitivity becomes greater and greater with the increase of T . There is a positive correlation between T and the ratio of the saturation value a and dealer i 's retail price P_i . The increase of retail price leads to the decrease of market demand, so $T > 0$. Parameter k denotes the substitution rate between one dealer's product and other dealers' products. The larger the k is, the higher the substitution rate will be, and $0 < k < 1$.

From formula (1), the coefficient matrix of the demand function can be obtained:

$$M = \begin{bmatrix} -T & k & \cdots & k \\ k & -T & \ddots & \vdots \\ \vdots & \ddots & \ddots & k \\ k & \cdots & k & -T \end{bmatrix}. \quad (2)$$

If $b = [P_1, P_2, \dots, P_n]^T$, (1) can be simplified as

$$D = [D_1, D_2, \dots, D_n]^T = a + Mb. \quad (3)$$

Lemma 1. If $P_i / \sum_{j=1}^n P_j > k/(T+k)$, market demand of dealer i will be less than the saturation value of market demand of one dealer; in other words, $D_i < a$.

Proof. Equation (1) can be written as

$$D_i = a - TP_i + k \left(\sum_{j=1}^n P_j - P_i \right). \quad (4)$$

Making the left side of formula (4) be less than a , the following formula can be deduced:

$$k \sum_{j=1}^n P_j < (T+k) P_i. \quad (5)$$

Because $T+k > 0$ and $\sum_{j=1}^n P_j > 0$, the above formula can be transformed into

$$\frac{P_i}{\sum_{j=1}^n P_j} > \frac{k}{T+k}. \quad (6)$$

From the derivation process above, it is concluded that if $P_i / \sum_{j=1}^n P_j > k/(T+k)$, $D_i < a$.

The proof is completed. \square

Lemma 2. If $T/k > n-1$, the total market demand of all dealers' products D_A is less than the total saturation value of market demand na .

Proof. The total market demand of all dealers' products is

$$D_A = D_1 + D_2 + \cdots + D_n = na - [T - (n-1)k] \sum_{i=1}^n P_i. \quad (7)$$

Only when $[T - (n-1)k] \sum_{i=1}^n P_i > 0$, can inequation $D_A < na$ be obtained. With $\sum_{i=1}^n P_i > 0$, if $T - (n-1)k > 0$, which is equal to $T/k > n-1$, the following conclusions can be obtained: $D_A < na$.

The proof is completed. \square

The supplier's profit function can be expressed as

$$\pi_S = D_A (W - C) = \left(na - [T - (n-1)k] \sum_{i=1}^n P_i \right) (W - C). \quad (8)$$

Each dealer i 's profit function is

$$\begin{aligned} \pi_{R_i} &= D_i (P_i - W - c_i) \\ &= \left[a - TP_i + k \left(\sum_{j=1}^n P_j - P_i \right) \right] (P_i - W - c_i). \end{aligned} \quad (9)$$

3. Vertical Sequential Game between the Supplier and Dealers

In the above supply chain model, the supplier is monopolistic because just one supplier sells products to multiple dealers. There is a vertical sequential game between supplier (the leader) and dealers (the followers) in the supply chain. The game is divided into two steps and backward induction is usually used to solve the sequential game. First, the supplier sets the wholesale price W to achieve its maximal profit based on the dealers' optimal reaction function of wholesale price $P_i(W)$. Then, each dealer i sets the retail price based on the wholesale price W and the optimal reaction function $P_i(W)$.

It is important to notice that each dealer considers not only the supplier's wholesale price W but also the impact of other dealers' retail prices when pricing its product. All dealers have the same status in the supply chain, and they can set their retail prices simultaneously. Now, the wholesale price W can be regarded as a market environment parameter. The pricing process of each dealer is affected by other dealers' pricing processes, and all dealers' pricing processes are a typical Nash game. Thus, a horizontal Nash game happens in the dealers' pricing process of the vertical sequential game.

Theorem 3. The profit function of each dealer i is a concave function of the price P_i .

Proof. From formula (9), the first-order partial derivative and second-order partial derivative of π_{R_i} can be obtained:

$$\frac{\partial \pi_{R_i}}{\partial P_i} = a - 2TP_i + k \left(\sum_{j=1}^n P_j - P_i \right) + T(W + c_i), \quad (10)$$

$$\frac{\partial^2 \pi_{R_i}}{\partial P_i^2} = -2T.$$

According to the definition in formula (1), $T > 0$, so $\partial^2 \pi_{R_i} / \partial P_i^2 = \partial^2 \pi_{R_2} / \partial P_2^2 = \cdots = \partial^2 \pi_{R_n} / \partial P_n^2 = -2T < 0$.

In other words, the second-order derivative of π_{R_i} is less than 0. Therefore, the profit function of each dealer i π_{R_i} is a concave function of P_i . \square

Theorem 4. *The necessary condition of the conclusion that the profit function of dealer i π_{R_i} can reach its maximum value at the stationary point is $a - k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) > 0$; the necessary condition of the conclusion that the profit function of dealer i π_{R_i} can reach its maximum value at the boundary of the profit curve is $a - k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) < 0$; the necessary condition of the conclusion that the profit function of dealer i π_{R_i} can reach its maximum value at the overlapping point of the stationary point and the lower boundary is $a - k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) = 0$.*

Proof. According to Lemma 1, an inequality constraint exists in the following optimization problem of each dealer i : $P_i / \sum_{j=1}^n P_j > k/T + k$. So, this profit optimization problem can be written as

$$\begin{aligned} \max_{P_i} \pi_{R_i}(P_i) \\ \text{s.t. } \frac{P_i}{\sum_{j=1, j \neq i}^n P_j} - \frac{k}{T} > 0 \\ P_i > 0. \end{aligned} \quad (11)$$

Using KKT conditions [20], the Lagrange function of the above optimization problem can be written as

$$\begin{aligned} L(P_i, \lambda) = & \left[a - TP_i + k \left(\sum_{j=1}^n P_j - P_i \right) \right] (P_i - W - c_i) \\ & + \lambda \left(\frac{P_i}{\sum_{j=1, j \neq i}^n P_j} - \frac{k}{T} \right) \quad (\lambda \geq 0). \end{aligned} \quad (12)$$

To acquire the optimal value, the following conditions must be satisfied:

$$\begin{aligned} P_i \frac{\partial L(P_i, \lambda)}{\partial P_i} &= 0, \\ \lambda \frac{\partial L(P_i, \lambda)}{\partial \lambda} &= 0. \end{aligned} \quad (13)$$

Because P_i is retail price and is always greater than 0, the equations above can be simplified as

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= 0, \\ \lambda \frac{\partial L(P_i, \lambda)}{\partial \lambda} &= 0. \end{aligned} \quad (14)$$

Formula (14) can be divided into three situations to be discussed, respectively, according to the different values of λ and $\partial L(P_i, \lambda)/\partial \lambda$: (1) $\lambda = 0, \partial L(P_i, \lambda)/\partial \lambda > 0$; (2) $\lambda > 0, \partial L(P_i, \lambda)/\partial \lambda = 0$; (3) $\lambda = 0, \partial L(P_i, \lambda)/\partial \lambda = 0$. \square

Situation 1 ($\lambda = 0, \partial L(P_i, \lambda)/\partial \lambda > 0$). Formula (14) can be transmitted into

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= 0, \\ \frac{\partial L(P_i, \lambda)}{\partial \lambda} &> 0, \\ \lambda &= 0. \end{aligned} \quad (15)$$

Putting formula (12) into formula (15), then

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= a - 2TP_i + k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) \\ &= \frac{\partial \pi_{R_i}}{\partial P_i} = 0 \\ \frac{P_i}{\sum_{j=1, j \neq i}^n P_j} - \frac{k}{T} &> 0. \end{aligned} \quad (16)$$

Now, the first-order derivative of dealer i 's profit function π_{R_i} is equal to 0. The max/min value of π_{R_i} can be obtained at stationary point. According to Theorem 3, the function curve of π_{R_i} is concave, so the maximum value of π_{R_i} can be obtained.

Put the equation of (16) into its inequality, and

$$a - k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) > 0. \quad (17)$$

Formula (17) is the necessary condition based on which the maximum value of π_{R_i} can be obtained at stationary point.

Situation 2 ($\lambda > 0, \partial L(P_i, \lambda)/\partial \lambda = 0$). Formula (14) can be transmitted into

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= 0, \\ \frac{\partial L(P_i, \lambda)}{\partial \lambda} &= 0, \\ \lambda &> 0. \end{aligned} \quad (18)$$

Put formula (12) into formula (18), and

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= \frac{\partial \pi_{R_i}}{\partial P_i} + \frac{\lambda}{\sum_{j=1, j \neq i}^n P_j} = 0, \\ \frac{P_i}{\sum_{j=1, j \neq i}^n P_j} - \frac{k}{T} &= 0. \end{aligned} \quad (19)$$

According to $\lambda > 0$ and the first equation of formula (19), $\partial \pi_{R_i}/\partial P_i < 0$. Therefore, the profit curve of dealer i is a monotone decreasing curve and the maximum value of π_{R_i} can be obtained on the lower boundary of P_i .

Put the second equation of (19) into the first, so

$$\frac{\lambda}{\sum_{j=1, j \neq i}^n P_j} = -a + k \sum_{j=1, j \neq i}^n P_j - T(W + c_i). \quad (20)$$

Because of $\lambda / \sum_{j=1, j \neq i}^n P_j > 0$, formula (20) can be rewritten as

$$a - k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) < 0. \quad (21)$$

Formula (21) is the necessary condition based on which the maximum value of π_{R_i} can be obtained on the curve boundary.

Situation 3 ($\lambda = 0, \partial L(P_i, \lambda) / \partial \lambda = 0$). Formula (14) can be transmitted into

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= 0, \\ \frac{\partial L(P_i, \lambda)}{\partial \lambda} &= 0, \\ \lambda &= 0. \end{aligned} \quad (22)$$

Putting formula (12) into formula (22), then

$$\begin{aligned} \frac{\partial L(P_i, \lambda)}{\partial P_i} &= a - 2TP_i + k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) = \frac{\partial \pi_{R_i}}{\partial P_i} = 0, \\ \frac{P_i}{\sum_{j=1, j \neq i}^n P_j} - \frac{k}{T} &= 0. \end{aligned} \quad (23)$$

The first-order derivative of dealer i 's profit function π_{R_i} is equal to 0 according to the first equation of formula (23), and the lower boundary of the profit curve is just the stationary point of it according to the second equation of (23).

Solving (23), the following formula can be obtained:

$$a - k \sum_{j=1, j \neq i}^n P_j + T(W + c_i) = 0. \quad (24)$$

Formula (24) is the necessary condition based on which the maximum value of π_{R_i} can be obtained at the overlapping point of the stationary point and the lower boundary. It is important to notice that the curve boundary mentioned above is the point which is unlimitedly close to the lower boundary of P_i because of the inequality condition of (9): $P_i / \sum_{j=1, j \neq i}^n P_j - k/T > 0$. Based on the above analysis, it can be seen that there are three kinds of optimal value: the stationary point, the lower boundary, the overlapping point of the stationary point, and the lower boundary. The last two cases are something impractical because the market demand of dealer iD_i will be equal to the saturation value of market demand a in these two cases. In fact, there is a great difference between D_i and a . Consequently, there will be a further study on the first case in this paper.

Developing the equation of (16), $\partial \pi_{R_i} / \partial P_i = 0$, linear equations with n variables can be obtained as

$$\begin{aligned} -2TP_1^* + k(P_2^* + \dots + P_n^*) &= -a - T(W + c_1) \\ kP_1^* - 2TP_2^* + k(P_3^* + \dots + P_n^*) &= -a - T(W + c_2) \\ &\vdots \\ k(P_1^* + \dots + P_{n-1}^*) - 2TP_n^* &= -a - T(W + c_n). \end{aligned} \quad (25)$$

Nash equilibrium existing in the pricing strategies of the dealers will be calculated by solving formula (25). Cramer's rule can be used to solve (25). The determinant of the equations' coefficients is

$$Q = \begin{vmatrix} -2T & k & \dots & k \\ k & -2T & \ddots & \vdots \\ \vdots & \ddots & \ddots & k \\ k & \dots & k & -2T \end{vmatrix}. \quad (26)$$

Making $b_1 = [-a - T(W + c_1), -a - T(W + c_2), \dots, -a - T(W + c_n)]^T$ and replacing the column i of Q with b_1 , Q_i can be obtained:

$$Q_i = \begin{vmatrix} \dots & b_1 & \dots \\ & \text{column } i & \end{vmatrix}. \quad (27)$$

According to Cramer's rule, the optimal retail price of dealer i is

$$P_i^* = \frac{Q_i}{Q} = f_i(W). \quad (28)$$

W is regarded as a market parameter in the process of solving Nash equilibrium, and the expression of each P_i^* contains W . In the vertical sequential game, P_i^* is a function of W . Based on the expression (28) of all retail prices, the supplier's optimal wholesale price W^* can be found.

Putting (25) and (28) into formula (8), then

$$\pi_S = \left[na - T \sum_{i=1}^n f_i(W) + (n-1)k \sum_{i=1}^n f_i(W) \right] (W - C). \quad (29)$$

Making $d\pi_S/dW = 0$, the supplier's optimal wholesale price W^* can be found by solving it. Then putting W^* back into (28), each dealer's optimal retail price P_i^* can be obtained.

4. Coalition Stability Concept

In Bertrand model, perfect market competition results in that the retail price of one product is equal to its marginal cost, so all manufacturers have no profit. The equilibrium solution of Bertrand model is impractical because no one will work for free. However, if the condition "perfect market competition" is replaced with "cooperation among manufacturers," everything will be different. On this occasion, all manufacturers can develop a joint pricing as long as they have

an agreement on benefit segmentation. Thus, the cutthroat price war in traditional Bertrand model can be avoided, and all manufacturers' profits are greater than zero.

Similar to Bertrand model, the supply chain studied in this paper also faces a price-sensitive market, but the dealers are not involved in perfect competition, and market sharing does not depend exactly on price. These dealers may develop a coalition for their common interests. Assuming that all dealers in the coalition have a unified retail price, an equilibrium point should be found as the unified price to balance the price and demand. Here, the unified pricing does not mean that the dealers can raise the retail price without restraint because the increase of the price will result in the decrease of market demand.

The establishment of a stable coalition needs the following condition: all dealers' profits with coalition are more than those without coalition. If any dealer suffers loss, the coalition will be considered as an unstable coalition.

Definition 5. Suppose that there are n dealers R_1, R_2, \dots, R_n who sell homogeneous products bought from a supplier. These dealers sell their products at prices P_1, P_2, \dots, P_n and get profits $\pi_{R_1}, \pi_{R_2}, \dots, \pi_{R_n}$, respectively. If they develop a coalition and make P as their unified price, they will get $\pi'_{R_1}, \pi'_{R_2}, \dots, \pi'_{R_n}$. Then, if $\pi_{R_1} < \pi'_{R_1}, \pi_{R_2} < \pi'_{R_2}, \dots, \pi_{R_n} < \pi'_{R_n}$ or, in other words, all dealers' profits with coalition are more than those without coalition, the coalition will be regarded as a stable coalition.

Inference 1. If dealers' coalition is stable, the profit allocation among all entities in the entire supply chain achieves Pareto optimality.

Proof. This inference can be proved in two cases. (1) If one or more dealers want more profits when dealers' coalition is stable, they can use two methods: unilateral price increase or unified price increase. However, these two approaches are not infeasible. A unilateral price increase leads to the collapse of the coalition, so each dealer's profit will return to its profit without coalition. According to Definition 5, each dealer's profit without coalition is less than that with coalition, so a unilateral price increase cannot benefit the dealers. Raising unified price is also not a good idea because the unified price calculated by using backward induction is the optimal solution of the sequential game and a positive growth of it will reduce all dealers' profits. In summary, the dealers have no way to increase their profits any more when their coalition is stable. (2) If the suppliers want more profits when dealers' coalition is stable, they can also use two methods: raising wholesale price or destroying dealers' coalition. The former is not available because the wholesale price is also calculated by using backward induction and changing it will lead to a loss of the supplier. The only way in which the suppliers can improve their profit is breaking up dealers' coalition. According to Definition 5, if dealers' coalition is broken, all their profits are less than the profits with coalition. The increase of the suppliers' profit results in the loss of the dealers, which is in accordance with Pareto optimality.

In a word, the profit allocation among all entities in the entire supply chain achieves Pareto optimality when dealers' coalition is stable. \square

5. Coalition Stability Analyses

According to Definition 5, judging the coalition stability needs a comparison of every dealer's profit with and without coalition. The latter has been studied above, so we need to analyze the former.

All the dealers' profits with coalition can also be analyzed in the framework of the sequential game mentioned above, but Nash game will disappear when all dealers' prices become unified. Because all the dealers enjoy the same retail price P now, according to (8), the profit function of the supplier can be written as

$$\pi_{SC} = n[a - TP + (n - 1)kP](W_C - C). \quad (30)$$

If the supplier's wholesale price W is fixed, single dealer's profit function in (9) cannot be used to solve the optimal unified price P^* because every dealer will have their own optimal retail price in response to W and the unified price cannot be solved in that case. All dealers should be regarded as a whole in the process of coalition optimization and its profit function is

$$\pi_{RC} = [a - TP + (n - 1)kP] \left(nP - nW_C - \sum_{i=1}^n c_i \right). \quad (31)$$

Theorem 6. If $T > (n - 1)k$, the total profit function of all dealers π_{RC} is a concave function of P .

Proof. The first-order and second-order derivatives of formula (31) are

$$\begin{aligned} \frac{d\pi_{RC}}{dP} &= -[T - (n - 1)k] \left(nP - nW_C - \sum_{i=1}^n c_i \right) \\ &\quad + n[a - TP + (n - 1)P] \\ &= -2n[T - (n - 1)k]P + n[T - (n - 1)k]W_C \\ &\quad + [T - (n - 1)k] \sum_{i=1}^n c_i + na, \\ \frac{d^2\pi_{RC}}{dP^2} &= -2n[T - (n - 1)k]. \end{aligned} \quad (32)$$

If $T > (n - 1)k$, $d^2\pi_{RC}/dP^2 < 0$ can be deduced. The second-order derivative of π_{RC} is less than zero, so π_{RC} is a concave function of P .

Making $d\pi_{RC}/dP = 0$, then

$$\begin{aligned} -2n[T - (n - 1)k]P + n[T - (n - 1)k]W_C \\ + [T - (n - 1)k] \sum_{i=1}^n c_i + na = 0. \end{aligned} \quad (33)$$

With further derivation, the optimal price P^* can be obtained as

$$P^* = \frac{n[T - (n - 1)k]W_C + na + [T - (n - 1)k] \sum_{i=1}^n c_i}{2n[T - (n - 1)k]} \quad (34)$$

Putting formula (34) into (30), the supplier's profit can be obtained:

$$\pi_{SC} = \frac{1}{2} \left\{ na - n[T - (n - 1)k]W_C - [T - (n - 1)k] \sum_{i=1}^n c_i \right\} (W_C - C). \quad (35)$$

□

Theorem 7. *If the condition $T > (n - 1)k$ in Theorem 6 is satisfied, the supplier's profit function π_{SC} is a concave function of W .*

Proof. The first-order and second-order derivatives of W in formula (35) are

$$\begin{aligned} \frac{d\pi_{SC}}{dW_C} &= \frac{na}{2} + \frac{n[T - (n - 1)k]C}{2} - n[T - (n - 1)k]W_C \\ &\quad - \frac{[T - (n - 1)k]}{2} \sum_{i=1}^n c_i, \\ \frac{d^2\pi_{SC}}{dW_C^2} &= -n[T - (n - 1)k]. \end{aligned} \quad (36)$$

According to the condition $T > (n - 1)k$ in Theorem 6, $d^2\pi_{SC}/dW_C^2 < 0$ can be deduced. The second-order derivative of π_{SC} is less than zero, so π_{SC} is a concave function of W .

Making $d\pi_{SC}/dW_C = 0$, then

$$W_C^* = \frac{a}{2[T - (n - 1)k]} + \frac{C}{2} - \frac{1}{2n} \sum_{i=1}^n c_i. \quad (37)$$

Putting formula (37) into (34), then

$$P^* = \frac{3a}{4[T - (n - 1)k]} + \frac{C}{4} + \frac{1}{4n} \sum_{i=1}^n c_i. \quad (38)$$

Formulas (37) and (38) are the equilibrium solutions of the supplier and dealers in the vertical game with coalition. Putting these two formulas into (35), the supplier's profit can be obtained:

$$\begin{aligned} \pi_{SC} &= \left\{ \frac{na}{4} - \frac{n[T - (n - 1)k]C}{4} - \frac{T - (n - 1)k}{4} \sum_{i=1}^n c_i \right\} \\ &\quad \times \left(\frac{a}{2[T - (n - 1)k]} - \frac{C}{2} - \frac{1}{2n} \sum_{i=1}^n c_i \right). \end{aligned} \quad (39)$$

Making all the retail prices of (9) be equal to unified price P , the profit function of each dealer j can be written as

$$\pi_{R_j} = \{a - [T - (n - 1)k]P\} (P - W - c_j). \quad (40)$$

Putting formulas (37) and (38) into (40), the profit of each dealer j is

$$\begin{aligned} \pi_{R_j} &= \left\{ \frac{a}{4} - \frac{[T - (n - 1)k]C}{4} - \frac{[T - (n - 1)k]}{4n} \sum_{i=1}^n c_i \right\} \\ &\quad \times \left(\frac{a}{4[T - (n - 1)k]} - \frac{C}{4} + \frac{3}{4n} \sum_{i=1}^n c_i - c_j \right). \end{aligned} \quad (41)$$

According to Definition 5, coalition stability can be judged by the comparison between all dealers' profits with coalition and without coalition. □

6. Numerical Simulations

In the numerical simulations, a two-stage supply chain consists of one supplier and 10 dealers; namely, $n = 10$ is employed. The supplier's marginal cost C is set to 1000, and ten dealers' marginal sale costs are $c_1 = 110$, $c_2 = 120$, $c_3 = 130$, $c_4 = 140$, $c_5 = 150$, $c_6 = 160$, $c_7 = 170$, $c_8 = 180$, $c_9 = 190$, and $c_{10} = 200$. The saturated market demand of each dealer is 2000, in other words, $a = 2000$. With T , k changing, the dealers' retail prices and profits, the supplier's wholesale price and profit, and market demand with and without coalition can be calculated. Partial data is showed in Table 1. Subscript "N" means "without coalition," and subscript "C" means "with coalition." D_C denotes the total market demand of 10 dealers, and each dealer's market demand is 1/10 of D_C because they set the same retail price.

Analyzing the data shown in Table 1, the following conclusions can be summarized.

- (1) If $(T, k) = (1, 0.03)$, $(1, 0.05)$, $(1, 0.1)$, and $(2, 0.12)$, all dealers' profits with coalition are greater than those without coalition, which accords with Definition 5, so the coalition is stable in these cases. However, if $(T, k) = (1, 0.1)$, the supplier's cost $C = 1000$ and its wholesale price $W = 10423$. The wholesale price of the supplier is about ten times as much as its cost, and the ratio of wholesale to cost will become greater with the increase of T . This case accords with a luxury sale situation, and the price-sensitive model in this paper is unsuitable for it because the consumers of luxury may not be price-sensitive.
- (2) Simulation results show that the supplier's optimal wholesale price is constant with and without coalition. It is clear that the profit of the supplier depends on two factors: wholesale price W and total market demand D_A . If the supplier increases the wholesale price W , it will lead to the increase of the unified price P , which makes the total market demand D_A decrease and the supplier's profit reduce. There is no motivation for the supplier to change its wholesale price, and the wholesale price is kept constant with and without coalition.
- (3) Dealers' coalition leads to a loss of the supplier. If dealers' coalition is stable, the unified price P will be higher than all dealers' retail prices they set

TABLE 1: The variety of decision variables and profits influenced by T, k .

	(T, k)					
	(1, 0.01)	(1, 0.03)	(1, 0.05)	(1, 0.1)	(2, 0.1)	(2, 0.12)
P_1	1902.4	2259.5	2814	11413	1582.3	1803.1
P_2	1907.4	2264.5	2818.9	11417	1587.2	1808
P_3	1912.4	2269.4	2823.7	11422	1592.1	1812.8
P_4	1917.4	2274.4	2828.6	11427	1596.9	1817.7
P_5	1922.3	2279.2	2833.5	11432	1601.8	1822.5
P_6	1927.3	2284.2	2838.4	11436	1606.7	1827.4
P_7	1932.3	2289.1	2843.2	11441	1611.6	1832.3
P_8	1937.3	2294	2848.1	11446	1616.4	1837.1
P_9	1942.2	2299	2853	11451	1621.3	1842
P_{10}	1947.2	2303.9	2857.9	11456	1626.2	1846.8
P	1937.1	2343.5	3016	15289	1652.4	1919.2
W_N	1521.4	1792.4	2040.7	10423	1331.6	1509.5
W_C	1521.4	1792.4	2040.7	10423	1331.6	1509.5
π_{SN}	1295200	2649300	5462004	80712280	780310	163550
π_{SC}	1237000	2291600	4233100	44392000	604740	119390
π_{RN}						
π_{R1}	73456	127580	214640	774690	39599	67469
π_{R2}	70758	123980	209920	765490	36768	63741
π_{R3}	68110	120430	205250	756350	34043	60120
π_{R4}	65512	116940	200640	747270	31422	56604
π_{R5}	62965	113490	196070	738240	28907	53194
π_{R6}	60469	110100	191560	729270	26496	49891
π_{R7}	58023	106760	187110	720350	24190	46693
π_{R8}	55627	103470	182700	711480	21990	43601
π_{R9}	53282	100230	178350	702680	19894	40615
π_{R10}	50988	97040	174050	693920	17903	37735
π_{RC}						
π_{R1}	72524	127600	227010	2240800	38444	70241
π_{R2}	70151	124700	223590	2236100	36620	67898
π_{R3}	67779	121810	220180	2221400	34796	65554
π_{R4}	65407	118920	216770	2226700	32973	63211
π_{R5}	63034	116030	213360	2221900	31149	60867
π_{R6}	60662	113130	209950	2217200	29325	58524
π_{R7}	58289	110240	206530	2212500	27501	56180
π_{R8}	955917	107350	203120	2207800	25678	53837
π_{R9}	53545	104460	199710	2203100	23854	51493
π_{R10}	51172	101570	196300	2198400	22030	49150
D_N						
D_1	271	357	463	880	281	367
D_2	266	352	458	875	271	357
D_3	261	347	453	870	261	347
D_4	256	342	448	864	251	336
D_5	251	337	443	859	240	326
D_6	246	332	438	854	230	316
D_7	241	327	433	849	220	306
D_8	236	322	427	843	210	295
D_9	231	317	422	838	199	285
D_{10}	226	312	417	833	189	275
D_A	2484	3343	4402	8566	2353	3210
D_C	2370	2890	3410	4710	1820	2340

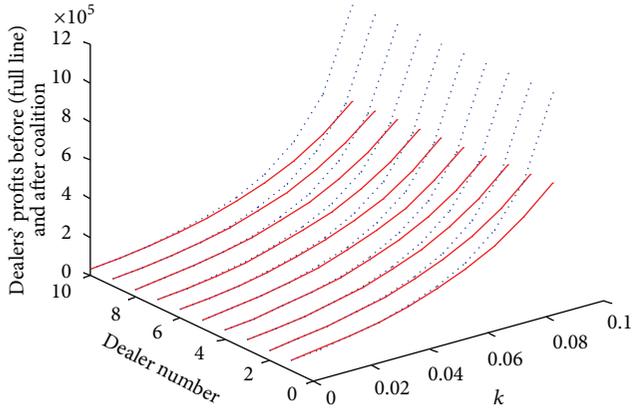


FIGURE 2: Dealers' profits with and without coalition ($T = 1$).

without coalition. The increase of retail price results in the decrease of demand, and the wholesale price is constant with and without coalition, so the supplier's profit with coalition is less than without coalition. In other words, dealers' coalition has a negative impact on the supplier.

Making $T = 1, 0 < k < 0.09$ and $T = 2, 0 < k < 0.2$, the ratio of wholesale price to cost is less than ten in these two value ranges. Moreover, market situations can be described by these two value ranges adequately and completely, so the profit curves of the dealers with and without coalition are drawn as shown in Figures 2 and 3.

In Figures 2 and 3, solid lines and dotted lines are intertwined at first. The dealers' profits with coalition (dotted lines) are gradually higher than those without coalition (solid lines), and the coalition becomes stable with the increase of k . If market price sensitivity T is fixed, the bigger the substitution rate k is (in the above ranges), the more stable the dealers' coalition is. Simulation results show that point $k = 0.03$ is a breakthrough point from stable coalition to unstable one if $T = 1$. In other words, all dealers' profits with coalition are more than those without coalition when $k \geq 0.03$. Similarly, $k = 0.11$ if $T = 2, k = 0.209$ if $T = 3, k = 0.314$ if $T = 4$, and $k = 0.423$ if $T = 5$ are also the breakthrough points. Obviously, the critical point k increases with the increase of T .

7. Conclusions

Horizontal coalition stability of a two-stage supply chain, which consists of one supplier and multiple dealers, is studied in this paper. Vertical sequential game and horizontal Nash game are used to analyze the profits of the supplier and dealers with and without coalition. If the profits of the supplier and dealers in different coalition situations (different values of T and k) are known, the coalition stability can be judged by the criterion that the profits of all dealers with coalition should be more than those without coalition. Unified pricing is a coordination mechanism for dealers, and the following conclusions can be obtained from numerical simulations: (1) the profits of the supplier and dealers cannot increase at

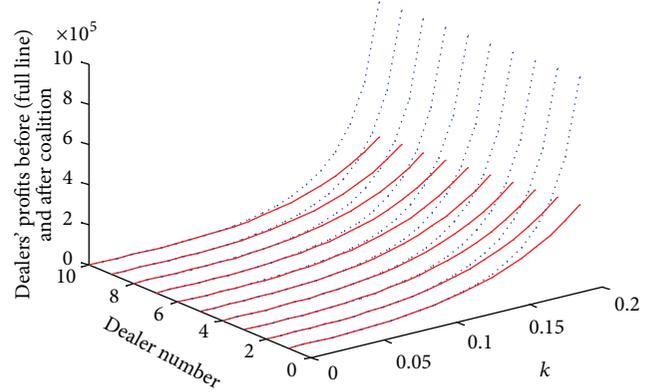


FIGURE 3: Dealers' profits with and without coalition ($T = 2$).

the same time. If the coalition is stable, the increase of dealers' profits results in the loss of the supplier; (2) through the analysis on vertical sequential game, the supplier's optimal wholesale price is constant with and without coalition. If the dealers' coalition is stable, the unified price of dealers with coalition will be higher than all dealers' retail prices without coalition; (3) if market price sensitivity T is fixed, the increase of the substitution rate k in a reasonable range will lead to the increase of the difference between each dealer's profit without coalition and with coalition, and the dealers will be more likely to ally with each other.

The conclusions mentioned above provide an important reference to horizontal coalition decision making of entities in the same status in supply chain. For example, these conclusions can help 4S dealers to decide when and how to form a coalition based on different market situations to acquire the optimal profits.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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