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Research Article

Existence of Positive Periodic Solutions for a Predator-Prey System of Holling Type IV Function Response with Mutual Interference and Impulsive Effects

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We investigate the existence of periodic solutions for a predator-prey system with Holling function response and mutual interference. Our model is more general than others since it has both Holling type IV function and impulsive effects. With some new analytical tricks and the continuation theorem in coincidence degree theory proposed by Gaines and Mawhin, we obtain a set of sufficient conditions on the existence of positive periodic solutions for such a system. In addition, in the remark, we point out some minor errors which appeared in the proof of theorems in some published papers with relevant predator-prey models. An example is given to illustrate our results.

1. Introduction

In recent years, many authors [1–7] have extensively considered different types of predator-prey system. One of the typical systems is the following system:

$$\begin{aligned} \dot{x}(t) &= xg(x) - \psi(x) y^m, \\ \dot{y}(t) &= y\left(-d + k\psi(x) y^{m-1} - q(y)\right), \end{aligned} \tag{1}$$

which was introduced by Hassell in 1975 (see [8] for more details). The character of (1) is that it has the mutual interference constant m $(0 < m < 1)$. When Hassell studied the capturing behavior between the hosts (some bees) and parasite (a kind of butterfly), he noted that the hosts had the tendency to leave each other when they met, which interfered the hosts capturing effects. He also found that the mutual interference would be stronger while the populations of the parasite became larger and therefore he introduced the concept of mutual interference constant m. From then on, many authors began to study some kinds of predator-prey systems with mutual interference; see [9–12] for more details. Recently, Wang and Zhu [13] investigated a Volterra model with mutual interference and a Holling II type functional response

$$\begin{aligned} \dot{x}(t) &= x(t)\left(r_1(t) - b_1(t)x(t)\right) - \frac{c_1(t)x(t)}{k^2 + x(t)} y^m(t), \\ \dot{y}(t) &= y(t)\left(-r_2(t) - b_2(t)y(t)\right) + \frac{c_2(t)x(t)}{k^2 + x(t)} y^m(t). \end{aligned} \tag{2}$$

And Wang et al. [14] discussed a Volterra model with mutual interference and a Holling III type functional response:

$$\begin{aligned} \dot{x}(t) &= x(t)\left(r_1(t) - b_1(t)x(t)\right) - \frac{c_1(t)x^2(t)}{k^2 + x^2(t)} y^m(t), \\ \dot{y}(t) &= y(t)\left(-r_2(t) - b_2(t)y(t)\right) + \frac{c_2(t)x^2(t)}{k^2 + x^2(t)} y^m(t), \end{aligned} \tag{3}$$

where

$$\frac{x^2}{k^2 + x^2} = p(x) \tag{4}$$

is the Holling III type predation function. We can easily get $\dot{p}(x) > 0$ for $x > 0$ which shows that the predation rate increases with the increasing prey population density. But some experiments and observations indicate that the nonmonotonic response occurs at a level: when the nutrient concentration reaches a high level, an inhibitory effect on the specific growth rate may occur. That means that the predation function $p(x)$ may not always increase. To describe such inhibitory effect, Andrews in 1968 (see [15] for more details) suggested another type of Holling function called Holling type IV function

$$p(x) = \frac{mx}{\alpha^2 + \omega x + \beta x^2}. \tag{5}$$

On the other hand, because of many natural and man-made factors, such as fire, drought, flooding, hunting, and harvesting, the intrinsic discipline of biological species usually undergoes some discrete changes of relatively short duration at some fixed times. More appropriate mathematical models for those situations are probably systems with impulsive effects. In recent years, many researchers have investigated several kinds of impulsive differential equations (see [16–27] and the references therein).

In this paper, we consider the following predator-prey system of Holling type IV function response with mutual interference and impulsive effects:

$$\begin{aligned}
\dot{x}(t) &= x(t)\left(r_1(t) - b_1(t)x(t)\right) \\
&\quad - \frac{c_1(t)x(t)}{\alpha^2 + \omega x(t) + \beta x^2(t)} y^m(t), \\
\dot{y}(t) &= y(t)\left(-r_2(t) - b_2(t)y(t)\right) \\
&\quad + \frac{c_2(t)x(t)}{\alpha^2 + \omega x(t) + \beta x^2(t)} y^m(t), \\
&\qquad t \neq t_k, \quad k = 1, 2, \ldots, \\
\Delta x(t_k) &= x(t_k^+) - x(t_k) = d_{1k}x(t_k), \quad k = 1, 2, \ldots, \\
\Delta y(t_k) &= y(t_k^+) - y(t_k) = d_{2k}y(t_k), \quad k = 1, 2, \ldots,
\end{aligned} \tag{6}$$

where $x(t)$ denotes the density of the prey population and $y(t)$ denotes the density of the predator population; $r_1(t)$ is the growth rate of the prey in the absence of predator; $r_2(t)$ is the death rate of predator in the absence of prey; $b_1(t)$ is the decay rate of the prey in the competition among the preys; $b_2(t)$ is the decay rate of the predator in the competition among the predators; $c_1(t)$ is the predation rate of predator, and $c_2(t)$ is the coefficient of transformation from preys to predators; $d_{1k}x(t_k)$ and $d_{2k}y(t_k)$ represent the populations $x(t)$ and $y(t)$ at t_k regular harvest pulse.

By use of the continuation theorem in coincidence degree theory and some new analytical tricks, we have derived sufficient conditions for the existence of positive periodic solutions of the general system (6). In proving the theorem, we have avoided the errors that exist in the existing articles. We also provide an example to illustrate our theorem.

2. Preliminaries

Definition 1. A function $z(t) = (x(t), y(t))^{\mathsf{T}} \in \mathsf{R}^2$ is said to be a T-periodic solution of system (6), if it satisfies the following conditions:

(i) $z(t)$ is a piecewise continuous map with first-class discontinuity points in $\{t_k\} \cap [0, T]$, and each discontinuity point is continuous on the left,

(ii) $z(t)$ satisfies system (6) in the interval $[0, T]$,

(iii) $z(t)$ satisfies $z(t + T) = z(t)$, $t \in [0, \infty)$.

Throughout this paper, the following assumptions hold.

$[A_1]$ $r_1(t), b_1(t), r_2(t), b_2(t)$, $c_1(t)$, and $c_2(t)$ are all continuous positive periodic functions with a common period $T > 0$.

$[A_2]$ $0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$, $t_k \neq T$ $(k = 1, 2, \ldots)$, α, ω, β are positive constants, $m \in (0, 1)$, and there exists a positive integer q, such that $t_{k+q} = t_k + T$, $d_{i(k+q)} = d_{ik} \in (-1, 0]$ $(i = 1, 2)$, $[0, T] \cap \{t_k\} = \{t_1, t_2, \ldots t_q\}$ for $k = 1, 2, \ldots$.

Let X and Y be two Banach spaces, $L : \operatorname{Dom} L \subset X \to Y$ is a linear map, and $N : X \to Y$ is a continuous map. If dim Ker L = codim Im $L < +\infty$ and Im $L \subset Y$ are closed, then we call operator L a Fredholm operator with index zero [28]. If L is a Fredholm operator with index zero and there exist continuous projects $P : X \to X$ and $Q : Y \to Y$ such that $\operatorname{Im} P = \operatorname{Ker} L$, $\operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q)$, then $L|_{\operatorname{Dom} L \cap \operatorname{Ker} P} : (I - P)X \to \operatorname{Im} L$ has an inverse function, which we set as K_p. Assume $\Omega \subset X$ is any open bounded set, if $QN(\overline{\Omega})$ is bounded and $K_p(I - Q)N(\overline{\Omega}) \subset X$ is relatively compact, then we say N is L-compact on $\overline{\Omega}$. Since Im Q is isomorphic to Ker L, there exists isomorphism J : ImQ $\to$ Ker L. Now we come to the continuation theorem [28, page 40].

Lemma 2 (see [28], Continuation Theorem). *Let X and Y be both Banach spaces, let $L : \operatorname{Dom} L \subset X \to Y$ be a Fredholm operator with index zero, let $\Omega \subset X$ be an open bounded set, and let $N : \overline{\Omega} \to Y$ be L-compact on $\overline{\Omega}$. If all the following conditions hold:*

$[C_1]$ *$Lx \neq \lambda Nx$ for each $\lambda \in (0, 1)$, $x \in \partial\Omega \cap \operatorname{Dom} L$,*

$[C_2]$ *for each $x \in \partial\Omega \cap \operatorname{Ker} L$, $QNx \neq 0$,*

$[C_3]$ $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$,

then the equation $Lx = Nx$ has at least one solution on $\overline{\Omega} \cap \operatorname{Dom} L$.

For the convenience, we denote

$$\begin{aligned}PC(\mathrm{R},\mathrm{R}) = \{\psi : \mathrm{R} \to \mathrm{R} \mid \psi \text{ is continuous at}\\ t \neq t_k,\ \psi(t_k^+), \psi(t_k^-) \text{ exist},\\ \psi(t_k) = \psi(t_k^-),\ k = 1,2,\ldots\},\\ \widetilde{C}(\mathrm{R},\mathrm{R}) := \{\psi : \mathrm{R} \to \mathrm{R} \mid \psi \text{ is continuous at}\\ t \neq t_k,\ \psi(t_k^+), \psi(t_k^-) \text{ exist},\ k = 1,2,\ldots\},\\ \overline{f} = \frac{1}{T}\int_0^T f(t)\,dt, \qquad f^L = \min_{t\in[0,T]} f(t),\\ f^M = \max_{t\in[0,T]} f(t).\end{aligned} \tag{7}$$

3. Existence of Positive Periodic Solutions

Theorem 3. *Besides (A_1) and (A_2), if there hold the following conditions:*

$$\overline{r}_1 \geqslant \max\left\{-\frac{1}{T}\sum_{k=1}^{q}\ln(1+d_{1k}), -\sum_{k=1}^{q}\ln(1+d_{1k})\right\}, \tag{8}$$

then system (6) has at least one positive T-periodic solution.

Proof. Suppose $(x(t), y(t)) \in \mathrm{R}^2$ is an arbitrary positive solution of system (6).

Let $x(t) = e^{u(t)}$, $y(t) = e^{v(t)}$, it follows from (6) that we have

$$\begin{aligned}\dot{u}(t) &= r_1(t) - b_1(t)e^{u(t)} - \frac{c_1(t)}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{mv(t)},\\ \dot{v}(t) &= -r_2(t) - b_2(t)e^{v(t)} + \frac{c_2(t)e^{u(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{(m-1)v(t)},\\ &\qquad t \neq t_k, \quad k = 1,2,\ldots,\\ \Delta u(t_k) &= \ln(1+d_{1k}), \quad k = 1,2,\ldots,\\ \Delta v(t_k) &= \ln(1+d_{2k}), \quad k = 1,2,\ldots.\end{aligned} \tag{9}$$

It is easy to see that if system (9) has one T-periodic solution $(u^*(t), v^*(t))^{\mathsf{T}}$, then $(x^*(t), y^*(t))^{\mathsf{T}} = (\exp[u^*(t)], \exp[v^*(t)])^{\mathsf{T}}$ is a positive T-periodic solution of (6). Therefore, we need only to prove that (9) has one T-periodic solution.

To apply Lemma 2, we take

$$\begin{aligned}X = \{z(t) = (u(t), v(t))^{\mathsf{T}} \mid u(t) \in PC(\mathrm{R},\mathrm{R}),\\ v(t) \in PC(\mathrm{R},\mathrm{R}),\ z(t+\mathrm{T}) = z(t)\}\end{aligned} \tag{10}$$

with the norm

$$\|z(t)\|_X = \left\|(u(t), v(t))^{\mathsf{T}}\right\|_X = \sup_{t\in[0,T]}|u(t)| + \sup_{t\in[0,T]}|v(t)|, \tag{11}$$

and let

$$\begin{aligned}\widetilde{X} = \{z(t) = (u(t), v(t))^{\mathsf{T}} \mid u(t) \in \widetilde{C}(\mathrm{R},\mathrm{R}),\\ v(t) \in \widetilde{C}(\mathrm{R},\mathrm{R}),\ z(t+T) = z(t) \text{ for}\\ t \neq t_k,\ k = 1,2,\ldots\}\end{aligned} \tag{12}$$

with the norm

$$\begin{aligned}\|z(t)\|_{\widetilde{X}} &= \left\|(u(t), v(t))^{\mathsf{T}}\right\|_{\widetilde{X}}\\ &= \sup_{t\in[0,T]\setminus\{t_1,t_2,\ldots,t_q\}}|u(t)| + \sup_{t\in[0,T]\setminus\{t_1,t_2,\ldots,t_q\}}|v(t)|,\\ Y &= \widetilde{X} \times \overbrace{\mathrm{R}^2 \times \mathrm{R}^2 \times \cdots \times \mathrm{R}^2}^{q} = \widetilde{X} \times \mathrm{R}^{2q}\end{aligned} \tag{13}$$

be equipped with the norm

$$\|\phi\|_Y = \|z\|_{\widetilde{X}} + \sum_{i=1}^{q}|\nu_i|_2, \quad \text{for } \phi = (z, \nu_1, \ldots, \nu_q) \in Y, \tag{14}$$

where $|\cdot|_2$ denotes the Euclidean norm of R^2. Then $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ are both Banach spaces.

Let

$$\mathrm{Dom}\,L = \{z(t) \in X \mid \text{for } t \neq t_k,\ \dot{z}(t) \in \widetilde{X}\} \subset X, \tag{15}$$

and define operators L and N as follows, respectively:

$$\begin{aligned}L(z(t)) &= L\begin{pmatrix}u(t)\\ v(t)\end{pmatrix} = \left(\begin{pmatrix}\dot{u}(t), & \text{for } t \neq t_k\\ \dot{v}(t), & \text{for } t \neq t_k\end{pmatrix}, \left\{\begin{pmatrix}\Delta u(t_k)\\ \Delta v(t_k)\end{pmatrix}\right\}_{k=1}^{q}\right),\\ N(z(t)) &= N\begin{pmatrix}u(t)\\ v(t)\end{pmatrix}\\ &= \left(\begin{pmatrix}r_1(t) - b_1(t)e^{u(t)} - \dfrac{c_1(t)}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{mv(t)}\\ -r_2(t) - b_2(t)e^{v(t)} + \dfrac{c_2(t)e^{u(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{(m-1)v(t)}\end{pmatrix},\right.\\ &\qquad \left.\left\{\begin{pmatrix}\ln(1+d_{1k})\\ \ln(1+d_{2k})\end{pmatrix}\right\}_{k=1}^{q}\right);\end{aligned} \tag{16}$$

then

$$\operatorname{Ker} L = \mathbb{R}^2,$$

$$\operatorname{Im} L = \left\{ y(t) = \left(\begin{pmatrix} f(t) \\ g(t) \end{pmatrix}, \left\{ \begin{pmatrix} a_k \\ e_k \end{pmatrix} \right\}_{k=1}^{q} \right) \in Y \mid \begin{pmatrix} \int_0^T f(t)\,dt + \sum_{k=1}^{q} a_k = 0 \\ \int_0^T g(t)\,dt + \sum_{k=1}^{q} e_k = 0 \end{pmatrix} \right\} \tag{17}$$

is closed in Y, and

$$\dim \operatorname{Ker} L = 2 = \operatorname{codim} \operatorname{Im} L. \tag{18}$$

It follows that L is a Fredholm mapping of index zero, and it is easy to know that P and Q are both continuous projectors such that

$$\operatorname{Im} P = \operatorname{Ker} L, \qquad \operatorname{Im} L = \operatorname{Ker} Q = \operatorname{Im}(I - Q), \tag{19}$$

where P and Q are defined by

$$P \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} = \frac{1}{T} \begin{pmatrix} \int_0^T u(t)\,dt + \sum_{k=1}^{q} a_k \\ \int_0^T v(t)\,dt + \sum_{k=1}^{q} e_k \end{pmatrix}, \tag{20}$$

where $\{\binom{a_k}{e_k}\}_{k=1}^{q}$ are arbitrary constant vector groups, if $\binom{u(t)}{v(t)} \in X \setminus \operatorname{Dom} L$, and $\{\binom{a_k}{e_k}\}_{k=1}^{q}$ satisfy

$$\int_0^{t_1} \dot{u}(t)\,dt + \sum_{k=2}^{k=q} \int_{t_{k-1}}^{t_k} \dot{u}(t)\,dt + \int_{t_q}^{T} \dot{u}(t)\,dt + \sum_{k=1}^{q} a_k = 0,$$

$$\int_0^{t_1} \dot{v}(t)\,dt + \sum_{k=2}^{k=q} \int_{t_{k-1}}^{t_k} \dot{v}(t)\,dt + \int_{t_q}^{T} \dot{v}(t)\,dt + \sum_{k=1}^{q} e_k = 0,$$

$$\text{if } \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \in \operatorname{Dom} L,$$

$$\begin{aligned} & Q\left(\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}, \left\{ \begin{pmatrix} a_k \\ e_k \end{pmatrix} \right\}_{k=1}^{q} \right) \\ & = \left(\frac{1}{T} \begin{pmatrix} \int_0^T u(t)\,dt + \sum_{k=1}^{q} a_k \\ \int_0^T v(t)\,dt + \sum_{k=1}^{q} e_k \end{pmatrix}, \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}_{k=1}^{q} \right), \quad \text{where} \\ & \left(\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}, \left\{ \begin{pmatrix} a_k \\ e_k \end{pmatrix} \right\}_{k=1}^{q} \right) \in Y. \end{aligned} \tag{21}$$

From L is a Fredholm operator with index zero, we get that L has a unique inverse. We define $K_p : \operatorname{Im} L \to \operatorname{Ker} P \cap \operatorname{Dom} L$ as the generalized inverse to L, that is,

$$\begin{aligned} & K_p \left(\begin{pmatrix} u(t) \\ v(t) \end{pmatrix}, \left\{ \begin{pmatrix} a_k \\ e_k \end{pmatrix} \right\}_{k=1}^{q} \right) \\ & = \begin{pmatrix} \int_0^t u(s)\,ds + \sum_{0<t_k<t} a_k - \frac{1}{T} \int_0^T \int_0^t u(s)\,ds\,dt - \sum_{k=1}^{q} a_k \\ \int_0^t v(s)\,ds + \sum_{0<t_k<t} e_k - \frac{1}{T} \int_0^T \int_0^t v(s)\,ds\,dt - \sum_{k=1}^{q} e_k \end{pmatrix}. \end{aligned} \tag{22}$$

Then by simply calculating we obtain

$$\begin{aligned} & QN \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \\ & = \left(\begin{pmatrix} \frac{1}{T} \int_0^T \left(r_1(t) - b_1(t) e^{u(t)} - \frac{c_1(t)}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}} e^{mv(t)} \right) dt + \frac{1}{T} \sum_{k=1}^{q} \ln(1 + d_{1k}) \\ \frac{1}{T} \int_0^T \left(-r_2(t) - b_2(t) e^{v(t)} + \frac{c_2(t) e^{u(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}} e^{(m-1)v(t)} \right) dt + \frac{1}{T} \sum_{k=1}^{q} \ln(1 + d_{2k}) \end{pmatrix}, \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}_{k=1}^{q} \right), \\ & K_p (I - Q) N \begin{pmatrix} u(t) \\ v(t) \end{pmatrix} \\ & = \begin{pmatrix} \int_0^t \left(r_1(s) - b_1(s) e^{u(s)} - \frac{c_1(s)}{\alpha^2 + \omega e^{u(s)} + \beta e^{2u(s)}} e^{mv(s)} \right) ds + \sum_{0<t_k<t} \ln(1 + d_{1k}) \\ \int_0^t \left(-r_2(s) - b_2(s) e^{v(s)} + \frac{c_2(s) e^{u(s)}}{\alpha^2 + \omega e^{u(s)} + \beta e^{2u(s)}} e^{(m-1)v(s)} \right) ds + \sum_{0<t_k<t} \ln(1 + d_{2k}) \end{pmatrix} \end{aligned}$$

$$
\begin{aligned}
&-\begin{pmatrix} \frac{1}{T}\int_0^T\int_0^t \left(r_1(s)-b_1(s)e^{u(s)}-\frac{c_1(s)}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{mv(s)}\right)ds\,dt+\sum_{k=1}^{q}\ln\left(1+d_{1k}\right) \\ \frac{1}{T}\int_0^T\int_0^t \left(-r_2(s)-b_2(s)e^{v(s)}+\frac{c_2(s)e^{u(s)}}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{(m-1)v(s)}\right)ds\,dt+\sum_{k=1}^{q}\ln\left(1+d_{2k}\right) \end{pmatrix} \\
&-\begin{pmatrix} \frac{1}{T}\int_0^t\int_0^T \left(r_1(s)-b_1(s)e^{u(s)}-\frac{c_1(s)}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{mv(s)}\right)ds\,ds \\ \frac{1}{T}\int_0^t\int_0^T \left(-r_2(s)-b_2(s)e^{v(s)}+\frac{c_2(s)e^{u(s)}}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{(m-1)v(s)}\right)ds\,ds \end{pmatrix} \\
&-\frac{t}{T}\begin{pmatrix} \sum_{k=1}^{q}\ln\left(1+d_{1k}\right) \\ \sum_{k=1}^{q}\ln\left(1+d_{2k}\right) \end{pmatrix} \\
&+\begin{pmatrix} \frac{1}{T^2}\int_0^T\int_0^t\int_0^T \left(r_1(s)-b_1(s)e^{u(s)}-\frac{c_1(s)}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{mv(s)}\right)ds\,ds\,dt \\ \frac{1}{T^2}\int_0^T\int_0^t\int_0^T \left(-r_2(s)-b_2(s)e^{v(s)}+\frac{c_2(s)e^{u(s)}}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{(m-1)v(s)}\right)ds\,ds\,dt \end{pmatrix} \\
&+\frac{1}{2}\begin{pmatrix} \sum_{k=1}^{q}\ln\left(1+d_{1k}\right) \\ \sum_{k=1}^{q}\ln\left(1+d_{2k}\right) \end{pmatrix} \\
&=\begin{pmatrix} \int_0^t \left(r_1(s)-b_1(s)e^{u(s)}-\frac{c_1(s)}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{mv(s)}\right)ds+\sum_{0<t_k<t}^{q}\ln\left(1+d_{1k}\right) \\ \int_0^t \left(-r_2(s)-b_2(s)e^{v(s)}+\frac{c_2(s)e^{u(s)}}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{(m-1)v(s)}\right)ds+\sum_{0<t_k<t}^{q}\ln\left(1+d_{2k}\right) \end{pmatrix} \\
&-\begin{pmatrix} \frac{1}{T}\int_0^T\int_0^t \left(r_1(s)-b_1(s)e^{u(s)}-\frac{c_1(s)}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{mv(s)}\right)ds\,dt+\sum_{k=1}^{q}\ln\left(1+d_{1k}\right) \\ \frac{1}{T}\int_0^T\int_0^t \left(-r_2(s)-b_2(s)e^{v(s)}+\frac{c_2(s)e^{u(s)}}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{(m-1)v(s)}\right)ds\,dt+\sum_{k=1}^{q}\ln\left(1+d_{2k}\right) \end{pmatrix} \\
&+\left(\frac{1}{2}-\frac{t}{T}\right)\begin{pmatrix} \int_0^T \left(r_1(s)-b_1(s)e^{u(s)}-\frac{c_1(s)}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{mv(s)}\right)ds \\ \int_0^T \left(-r_2(s)-b_2(s)e^{v(s)}+\frac{c_2(s)e^{u(s)}}{\alpha^2+\omega e^{u(s)}+\beta e^{2u(s)}}e^{(m-1)v(s)}\right)ds \end{pmatrix} \\
&+\left(\frac{1}{2}-\frac{t}{T}\right)\begin{pmatrix} \sum_{k=1}^{q}\ln\left(1+d_{1k}\right) \\ \sum_{k=1}^{q}\ln\left(1+d_{2k}\right) \end{pmatrix}.
\end{aligned}
\tag{23}
$$

By the Lebesqgue convergence theorem, QN and $K_p(I-Q)N$ are both continuous. From the Arzela-Ascoli Theorem, we can get that $K_p(I-Q)N(\overline{\Omega})$ is relatively compact and $QN(\overline{\Omega})$ is bounded for any open set $\Omega \subset X$. So N is L-compact on $\overline{\Omega}$ for any open bounded set Ω.

Now we consider the operator equation $Lz = \lambda Nz$, $\lambda \in (0,1)$, that is,

$$\begin{aligned}
\dot{u}(t) &= \lambda\left(r_1(t) - b_1(t)e^{u(t)} - \frac{c_1(t)}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{mv(t)}\right),\\
\dot{v}(t) &= \lambda\left(-r_2(t) - b_2(t)e^{v(t)} + \frac{c_2(t)e^{u(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{(m-1)v(t)}\right),\\
&\qquad t \neq t_k, \quad k = 1,2,\ldots,\\
\Delta u(t_k) &= \lambda \ln(1 + d_{1k}), \quad k = 1,2,\ldots,\\
\Delta v(t_k) &= \lambda \ln(1 + d_{2k}), \quad k = 1,2,\ldots.
\end{aligned} \tag{24}$$

Integrating (24) over the interval $[0,T]$ leads to

$$\begin{aligned}
\int_0^T b_1(t)e^{u(t)}dt &= \int_0^T r_1(t)\,dt - \int_0^T \frac{c_1(t)}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{mv(t)}dt + \sum_{k=1}^{q}\ln(1 + d_{1k}),\\
\int_0^T b_2(t)e^{v(t)}dt &= -\int_0^T r_2(t)\,dt + \int_0^T \frac{c_2(t)e^{u(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}e^{(m-1)v(t)}dt + \sum_{k=1}^{q}\ln(1 + d_{2k}).
\end{aligned} \tag{25}$$

From the first equation of (25), we have

$$b_1^L \int_0^T e^{u(t)}dt \leqslant \int_0^T b_1(t)e^{u(t)}dt \leqslant \int_0^T r_1(t)\,dt = T\cdot \bar{r}_1. \tag{26}$$

So we get

$$\int_0^T e^{u(t)}dt \leqslant \frac{T\cdot \bar{r}_1}{b_1^L}. \tag{27}$$

Multiplying the first equation of (24) by $e^{u(t)}$ and integrating over $[0,T]$, we obtain

$$\begin{aligned}
0 &\leqslant \sum_{k=1}^{q} e^{u(t_k)}\left[1 - (1 + d_{1k})^{\lambda}\right]\\
&= \lambda\int_0^T\left[r_1(t)e^{u(t)} - b_1(t)e^{2u(t)} - \frac{c_1(t)e^{u(t)}e^{mv(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}\right]dt.
\end{aligned} \tag{28}$$

From (28) and the integral mean value theorem, there exists a $\zeta_1 \in [0,T]$ such that

$$\begin{aligned}
&\lambda\int_0^T\left[r_1(t)e^{u(t)} - b_1(t)e^{2u(t)}\right]dt - \sum_{k=1}^{q} e^{u(t_k)}\left[1 - (1 + d_{1k})^{\lambda}\right]\\
&\quad= \int_0^T \frac{\lambda c_1(t)e^{u(t)}e^{mv(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}dt\\
&\quad= c_1(\zeta_1)\int_0^T \frac{\lambda e^{u(t)}e^{mv(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}dt.
\end{aligned} \tag{29}$$

Similarly, multiplying the second equation of (24) by $e^{v(t)}$ and integrating over $[0,T]$, we obtain

$$\begin{aligned}
0 &\leqslant \sum_{k=1}^{q} e^{v(t_k)}\left[1 - (1 + d_{2k})^{\lambda}\right]\\
&= \lambda\int_0^T\left[-r_2(t)e^{v(t)} - b_2(t)e^{2v(t)} + \frac{c_2(t)e^{u(t)}e^{mv(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}\right]dt;
\end{aligned} \tag{30}$$

from (30) and the integral mean value theorem, there exists a $\zeta_2 \in [0,T]$ such that

$$\begin{aligned}
&\lambda\int_0^T\left[r_2(t)e^{v(t)} + b_2(t)e^{2v(t)}\right]dt + \sum_{k=1}^{q} e^{v(t_k)}\left[1 - (1 + d_{2k})^{\lambda}\right]\\
&\quad= \int_0^T \frac{\lambda c_2(t)e^{u(t)}e^{mv(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}dt\\
&\quad= c_2(\zeta_2)\int_0^T \frac{\lambda e^{u(t)}e^{mv(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}}dt.
\end{aligned} \tag{31}$$

From (29) and (31), we have

$$\begin{aligned}
&\lambda\int_0^T\left[r_2(t)e^{v(t)} + b_2(t)e^{2v(t)}\right]dt + \sum_{k=1}^{q} e^{v(t_k)}\left[1 - (1 + d_{2k})^{\lambda}\right]\\
&\quad= \frac{c_2(\zeta_2)}{c_1(\zeta_1)}\left[\lambda\int_0^T\left(r_1(t)e^{u(t)} - b_1(t)e^{2u(t)}\right)dt - \sum_{k=1}^{q} e^{u(t_k)}\left[1 - (1 + d_{1k})^{\lambda}\right]\right].
\end{aligned} \tag{32}$$

From (32), we get

$$\int_0^T r_2(t)e^{v(t)}dt \leqslant \frac{c_2^M}{c_1^L}\int_0^T r_1(t)e^{u(t)}dt, \tag{33}$$

which yields

$$r_2^L \int_0^T e^{v(t)} dt \leqslant \frac{c_2^M r_1^M}{c_1^L} \int_0^T e^{u(t)} dt. \tag{34}$$

From (27) and (34), we obtain

$$\int_0^T e^{v(t)} dt \leqslant \frac{c_2^M r_1^M T \bar{r}_1}{c_1^L r_2^L b_1^L}. \tag{35}$$

Set

$$A := \max \left\{ \frac{\bar{r}_1}{b_1^L}, \frac{c_2^M r_1^M \bar{r}_1}{c_1^L r_2^L b_1^L} \right\}, \tag{36}$$

and then, from (27), (35), and (36), we get

$$\int_0^T e^{u(t)} dt \leqslant T \cdot A, \qquad \int_0^T e^{v(t)} dt \leqslant T \cdot A. \tag{37}$$

Since $(u(t), v(t))^{\mathrm{T}} \in X$, there exist $\xi, \xi_1, \eta, \eta_1 \in [0, T]$ such that

$$\begin{aligned} u(\xi) &= \max_{t \in [0,T]} u(t), \qquad u(\xi_1^+) = \min_{t \in [0,T]} u(t), \\ v(\eta) &= \max_{t \in [0,T]} v(t), \qquad v(\eta_1^+) = \min_{t \in [0,T]} v(t). \end{aligned} \tag{38}$$

From (37) and (38), we see that

$$u(\xi_1^+) \leqslant \ln A, \qquad v(\eta_1^+) \leqslant \ln A. \tag{39}$$

On the other hand, it follows from (24) that

$$\begin{aligned} \int_0^T |\dot{u}(t)| dt &\leqslant 2 \int_0^T r_1(t) dt \\ &\quad + \sum_{k=1}^q \ln(1 + d_{1k}) \\ &\leqslant 2 \int_0^T r_1(t) dt = 2T \cdot \bar{r}_1, \end{aligned} \tag{40}$$

$$\begin{aligned} \int_0^T |\dot{v}(t)| dt &\leqslant 2 \int_0^T r_2(t) dt \\ &\quad + 2 \int_0^T b_2(t) e^{v(t)} dt - \sum_{k=1}^q \ln(1 + d_{2k}) \\ &\leqslant 2T \cdot \bar{r}_2 + 2b_2^M T \cdot A \\ &\quad - \sum_{k=1}^q \ln(1 + d_{2k}) := S_1. \end{aligned} \tag{41}$$

Thus, from (39)–(41), we have

$$\begin{aligned} u(t) &= \begin{cases} u(\xi_1^+) + \int_{\xi_1}^t \dot{u}(s) ds + \sum_{\xi_1 < t_k < t} \lambda \ln(1 + d_{1k}), \\ \qquad t \in (\xi_1, T] \\ u(\xi_1) + \int_{\xi_1}^t \dot{u}(s) ds - \sum_{t \leqslant t_k < \xi_1} \lambda \ln(1 + d_{1k}), \\ \qquad t \in [0, \xi_1] \end{cases} \\ &\leqslant \begin{cases} u(\xi_1^+) + \int_{\xi_1}^t \dot{u}(s) ds - 2\sum_{k=1}^q \ln(1 + d_{1k}), \\ \qquad t \in (\xi_1, T] \\ u(\xi_1^+) + \int_{\xi_1}^t \dot{u}(s) ds - 2\sum_{k=1}^q \ln(1 + d_{1k}), \\ \qquad t \in [0, \xi_1] \end{cases} \\ &\leqslant u(\xi_1^+) + \int_0^T |\dot{u}(s)| ds - 2\sum_{k=1}^q \ln(1 + d_{1k}) \\ &\leqslant \ln A + 2T \cdot \bar{r}_1 - 2\sum_{k=1}^q \ln(1 + d_{1k}) := S_2, \end{aligned} \tag{42}$$

$$\begin{aligned} v(t) &\leqslant v(\eta_1^+) + \int_0^T |\dot{v}(s)| ds - 2\sum_{k=1}^q \ln(1 + d_{2k}) \\ &\leqslant \ln A + 2T \cdot \bar{r}_2 + 2b_2^M T \cdot A - 3\sum_{k=1}^q \ln(1 + d_{2k}) \\ &:= S_3. \end{aligned} \tag{43}$$

Meanwhile, the first equation of (25) implies

$$T \cdot \bar{r}_1 \leqslant b_1^M \int_0^T e^{u(t)} dt + \frac{c_1^M}{\alpha^2} \int_0^T e^{mv(t)} dt - \sum_{k=1}^q \ln(1 + d_{1k}). \tag{44}$$

In view of $\int_0^T e^{mv(t)} dt \leqslant T^{1-m} (\int_0^T e^{v(t)} dt)^m$ and (34), we have from (44) that

$$\begin{aligned} T \cdot \bar{r}_1 &\leqslant b_1^M \int_0^T e^{u(t)} dt \\ &\quad + \frac{c_1^M T^{1-m}}{\alpha^2} \left(\frac{c_2^M r_1^M}{r_2^L c_1^L} \right)^m \left(\int_0^T e^{u(t)} dt \right)^m \\ &\quad - \sum_{k=1}^q \ln(1 + d_{1k}). \end{aligned} \tag{45}$$

If $0 < \int_0^T e^{u(t)} dt < 1$, then it follows from (45) that

$$\begin{aligned} \int_0^T e^{u(t)} dt &\geqslant \left(\frac{T \cdot \bar{r}_1 + \sum_{k=1}^q \ln(1 + d_{1k})}{b_1^M + (c_1^M T^{1-m}/\alpha^2)(c_2^M r_1^M / r_2^L c_1^L)^m} \right)^{1/m} \\ &:= S_4, \end{aligned} \tag{46}$$

so we have

$$\int_0^T e^{u(t)}dt \geqslant \min\{1, S_4\} := S_5. \tag{47}$$

From (38) and (47), we have

$$u(\xi) \geqslant \ln \frac{S_5}{T}. \tag{48}$$

This, together with (40), leads to

$$\begin{aligned} u(t) &\geqslant u(\xi) - \int_0^T |\dot{u}(t)|\,dt + 2\sum_{k=1}^{q} \ln(1 + d_{1k}) \\ &\geqslant \ln \frac{S_5}{T} - 2T \cdot \bar{r}_1 + 2\sum_{k=1}^{q} \ln(1 + d_{1k}) \\ &:= S_6. \end{aligned} \tag{49}$$

Let $H_1 = \max\{|S_2|, |S_6|\}$; then from (42) and (49), we have

$$\max_{t\in[0,T]} |u(t)| \leqslant H_1. \tag{50}$$

Let

$$\begin{aligned} \theta = \min \Bigg\{ x \in \{1, 2, \ldots,\} \mid \frac{C_2^L S_5 x}{\alpha^2 + \omega e^{S_2} + \beta e^{2S_2}} - \bar{r}_2 T \\ + \sum_{k=1}^{q} \ln(1 + d_{2k}) > 0 \Bigg\}. \end{aligned} \tag{51}$$

Case 1. If $0 < e^{(1-m)v(t)} < \theta^{-1}$, for any $t \in [0, T]$, then it follows from the second equation of (25) that

$$\begin{aligned} &\bar{r}_2 T - \sum_{k=1}^{q} \ln(1 + d_{2k}) + \int_0^T b_2(t)\, e^{v(t)} dt \\ &\quad = \int_0^T \frac{c_2(t)\, e^{u(t)}}{\alpha^2 + \omega e^{u(t)} + \beta e^{2u(t)}} e^{(m-1)v(t)} dt \\ &\quad \geqslant \frac{C_2^L S_5 \theta}{\alpha^2 + \omega e^{S_2} + \beta e^{2S_2}}, \end{aligned} \tag{52}$$

and from (38) we get

$$\begin{aligned} &e^{v(\eta)}T \\ &\geqslant \int_0^T e^{v(t)} dt \\ &\geqslant \frac{\left[\left(C_2^L S_5 \theta / \left(\alpha^2 + \omega e^{S_2} + \beta e^{2S_2}\right)\right) - \bar{r}_2 T + \sum_{k=1}^{q} \ln(1 + d_{2k})\right]}{b_2^M} \\ &:= S_7, \end{aligned} \tag{53}$$

that is,

$$v(\eta) \geqslant \ln \frac{S_7}{T}. \tag{54}$$

Similar to (49), from (41) and (54) we obtain

$$\begin{aligned} v(t) &\geqslant v(\eta) - \int_0^T |\dot{v}(t)|\,dt + 2\sum_{k=1}^{q} \ln(1 + d_{2k}) \\ &\geqslant \ln \frac{S_7}{T} - S_1 + 2\sum_{k=1}^{q} \ln(1 + d_{2k}) \\ &:= S_8. \end{aligned} \tag{55}$$

Case 2. There exists $\gamma \in [0, T]$ such that $e^{(1-m)v(\gamma)} \geqslant \theta^{-1}$, that is,

$$v(\gamma) \geqslant \frac{1}{m-1} \ln \theta. \tag{56}$$

From (41) and (56) we obtain

$$\begin{aligned} v(t) &\geqslant v(\gamma) - \int_0^T |\dot{v}(t)|\,dt + 2\sum_{k=1}^{q} \ln(1 + d_{2k}) \\ &\geqslant \frac{1}{m-1} \ln \theta - S_1 + 2\sum_{k=1}^{q} \ln(1 + d_{2k}) \\ &:= S_9. \end{aligned} \tag{57}$$

So we have

$$v(t) \geqslant \min\{S_8, S_9\} := S_{10}. \tag{58}$$

Let $H_2 = \max\{|S_3|, |S_{10}|\}$, then from (43) and (58), we get

$$\max_{t\in[0,T]} |v(t)| \leqslant H_2. \tag{59}$$

Clearly, H_1 and H_2 are independent of λ. Denote $H = H_1 + H_2 + H_0$, where H_0 is taken sufficiently large such that each solution $(u^*, v^*)^{\top}$ (if the system has at least one solution) of the following system of algebraic equations:

$$\begin{aligned} \bar{b}_1 e^{u} - \bar{r}_1 + \frac{\bar{c}_1 e^{mv}}{\alpha^2 + \omega e^{u} + \beta e^{2u}} &= \sum_{k=1}^{q} \ln(1 + d_{1k}), \\ \bar{b}_2 e^{v} + \bar{r}_2 - \frac{\bar{c}_2 e^{\alpha+(m-1)\beta}}{\alpha^2 + \omega e^{u} + \beta e^{2u}} &= \sum_{k=1}^{q} \ln(1 + d_{2k}), \end{aligned} \tag{60}$$

satisfies $|u^*| + |v^*| < H_0$ and

$$\max\{|S_{11}|, |S_{12}|\} + \max\{|S_{13}|, |S_{14}|\} < H_0, \tag{61}$$

where

$$S_{11} = \ln \frac{\bar{r}_1}{\bar{b}_1},$$
$$S_{12} = \min \left\{ 0, \ln \left(\frac{\bar{r}_1 + \sum_{k=1}^{q} \ln(1 + d_{1k})}{\bar{b}_1 + (\bar{c}_1/\alpha^2)} \right) \right\},$$
$$S_{13} = \min \left\{ 0, \frac{1}{1-m} \cdot \ln \left(\bar{c}_2 e^{S_{12}} \cdot \left(\left(\bar{b}_2 + \bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k}) \right) \cdot \left(\alpha^2 + \omega e^{S_{11}} + \beta e^{S_{11}} \right) \right)^{-1} \right) \right\},$$
$$S_{14} = \frac{1}{m-1} \ln \left(-\frac{\bar{b}_1 \alpha^2 \sum_{k=1}^{q} \ln(1 + d_{2k})}{\bar{c}_2 \bar{r}_1} \right). \tag{62}$$

Let $\Omega = \{z \in X : \|z\| < H\}$; then Ω satisfies condition $[C_1]$ in Lemma 2. If $z \in \partial\Omega \cap \operatorname{Ker} L = \partial\Omega \cap \mathrm{R}^2$, then z is a constant vector in R^2 with $\|z\| = H$. So

$$\begin{aligned} &QNz \\ &= QN \begin{pmatrix} u \\ v \end{pmatrix} \\ &= \left(\begin{pmatrix} \bar{b}_1 e^u - \bar{r}_1 + \dfrac{\bar{c}_1 e^{mv}}{\alpha^2 + \omega e^u + \beta e^{2u}} - \sum_{k=1}^{q} \ln(1 + d_{1k}) \\ \bar{b}_2 e^v + \bar{r}_2 - \dfrac{\bar{c}_2 e^{u+(m-1)v}}{\alpha^2 + \omega e^u + \beta e^{2u}} - \sum_{k=1}^{q} \ln(1 + d_{2k}) \end{pmatrix}, \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}_{k=1}^{q} \right) \neq 0, \end{aligned} \tag{63}$$

which shows that condition $[C_2]$ in Lemma 2 is satisfied. Finally, we prove that condition $[C_3]$ in Lemma 2 is satisfied. The isomorphism J of $\operatorname{Im} Q$ onto $\operatorname{Ker} L$ can be defined by

$$\begin{gathered} J : \operatorname{Im} Q \longrightarrow \operatorname{Ker} L, \\ \left(\begin{pmatrix} l_1 \\ l_2 \end{pmatrix}, \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}_{k=1}^{q} \right) \longmapsto \begin{pmatrix} l_1 \\ l_2 \end{pmatrix}. \end{gathered} \tag{64}$$

For $z \in \Omega \cap \operatorname{Ker} L$, we have

$$\begin{aligned} &JQNz \\ &= JQN \begin{pmatrix} u \\ v \end{pmatrix} \\ &= \begin{pmatrix} \bar{b}_1 e^u - \bar{r}_1 + \dfrac{\bar{c}_1 e^{mv}}{\alpha^2 + \omega e^u + \beta e^{2u}} - \sum_{k=1}^{q} \ln(1 + d_{1k}) \\ \bar{b}_2 e^v + \bar{r}_2 - \dfrac{\bar{c}_2 e^{u+(m-1)v}}{\alpha^2 + \omega e^u + \beta e^{2u}} - \sum_{k=1}^{q} \ln(1 + d_{2k}) \end{pmatrix}. \end{aligned} \tag{65}$$

Denote $\phi : \operatorname{Dom} L \times [0,1] \rightarrow X$ as the form

$$\phi(u, v, \mu) = \begin{pmatrix} \bar{b}_1 e^u - \bar{r}_1 - \sum_{k=1}^{q} \ln(1 + d_{1k}) \\ -\dfrac{\bar{c}_2 e^{u+(m-1)v}}{\alpha^2 + \omega e^u + \beta e^{2u}} - \sum_{k=1}^{q} \ln(1 + d_{2k}) \end{pmatrix} + \mu \begin{pmatrix} \dfrac{\bar{c}_1 e^{mv}}{\alpha^2 + \omega e^u + \beta e^{2u}} \\ \bar{b}_2 e^v + \bar{r}_2 \end{pmatrix}, \tag{66}$$

where μ is a parameter. We will show that when $(u, v)^{\mathsf{T}} \in \partial\Omega \cap \operatorname{Ker} L$, $\phi(u, v, \mu) \neq 0$ for any $\mu \in [0,1]$. Assume the conclusion is not true; that is, there is a constant vector $(u, v)^{\mathsf{T}}$ with $|u| + |v| = H$ satisfying $\phi(u, v, \mu) = 0$, that is,

$$\bar{b}_1 e^u - \bar{r}_1 - \sum_{k=1}^{q} \ln(1 + d_{1k}) + \frac{\mu \bar{c}_1 e^{mv}}{\alpha^2 + \omega e^u + \beta e^{2u}} = 0, \tag{67}$$

$$\mu \bar{b}_2 e^v + \mu \bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k}) - \frac{\bar{c}_2 e^{u+(m-1)v}}{\alpha^2 + \omega e^u + \beta e^{2u}} = 0. \tag{68}$$

By (67) we easily see

$$u \leqslant \ln \frac{\bar{r}_1}{\bar{b}_1} = S_{11}, \tag{69}$$

and we also get

$$\begin{aligned} \bar{r}_1 &\leqslant \bar{b}_1 e^u - \sum_{k=1}^{q} \ln(1 + d_{1k}) + \frac{\bar{c}_1 e^{mv}}{\alpha^2 + \omega e^u + \beta e^{2u}} \\ &\leqslant \bar{b}_1 e^u - \sum_{k=1}^{q} \ln(1 + d_{1k}) + \frac{\bar{c}_1 e^{mv}}{\alpha^2}. \end{aligned} \tag{70}$$

Case 1. If $mv \leqslant u$, from (70) and $[D_2]$, we have $u \geqslant \ln((\bar{r}_1 + \sum_{k=1}^{q} \ln(1 + d_{1k}))/(\bar{b}_1 + (\bar{c}_1/\alpha^2)))$.

Case 2. If $mv > u$, there exists a $n^* \in N$, such that $u < mv < n^* u$; from (70), we have

$$\bar{r}_1 \leqslant \bar{b}_1 e^u - \sum_{k=1}^{q} \ln(1 + d_{1k}) + \frac{\bar{c}_1 e^{n^* u}}{\alpha^2}. \tag{71}$$

Case 2.1. If $0 < e^u < 1$, from (71) and $[D_2]$, we obtain $u \geqslant \ln((\bar{r}_1 + \sum_{k=1}^{q} \ln(1 + d_{1k}))/(\bar{b}_1 + (\bar{c}_1/\alpha^2)))$.

Case 2.2. If $e^u \geqslant 1$, we have $u \geqslant 0$.

So we have

$$u \geqslant \min\left\{0, \ln\left(\frac{\bar{r}_1 + \sum_{k=1}^{q} \ln(1 + d_{1k})}{\bar{b}_1 + (\bar{c}_1/\alpha^2)}\right)\right\} = S_{12}. \quad (72)$$

Then from (69) and (72), we get $|u| \leqslant \max\{|S_{11}|, |S_{12}|\}$.

From (68), we have

$$\frac{\bar{c}_2 e^{u+(m-1)v}}{\alpha^2 + \omega e^u + \beta e^{2u}} \leqslant \bar{b}_2 e^v + \bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k}), \quad (73)$$

and from (69) and (72), we have

$$\frac{\bar{c}_2 e^{S_{12}}}{\alpha^2 + \omega e^{S_{11}} + \beta e^{2S_{11}}} \leqslant \bar{b}_2 e^{(2-m)v} + \left(\bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k})\right) e^{(1-m)v}. \quad (74)$$

Case 1. If $v > 0$, then we get the lower bounds of v.

Case 2. If $v < 0$, then we see $e^{(1-m)v} \geqslant e^{(2-m)v}$, which together with (74) yields

$$\frac{\bar{c}_2 e^{S_{12}}}{\alpha^2 + \omega e^{S_{11}} + \beta e^{S_{11}}} \leqslant \left(\bar{b}_2 + \bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k})\right) e^{(1-m)v}, \quad (75)$$

which implies

$$v \geqslant \frac{1}{1-m} \cdot \ln\left(\bar{c}_2 e^{S_{12}} \cdot \left(\left(\bar{b}_2 + \bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k})\right) \cdot (\alpha^2 + \omega e^{S_{11}} + \beta e^{S_{11}})\right)^{-1}\right). \quad (76)$$

So we get

$$v \geqslant \min\left\{0, \frac{1}{1-m} \cdot \ln\left(\bar{c}_2 e^{S_{12}} \cdot \left(\left(\bar{b}_2 + \bar{r}_2 - \sum_{k=1}^{q} \ln(1 + d_{2k})\right) \cdot \left(\alpha^2 + \omega e^{S_{11}} + \beta e^{S_{11}}\right)\right)^{-1}\right)\right\} = S_{13}. \quad (77)$$

From (68) and (69), we obtain

$$\frac{\bar{c}_2 \bar{r}_1 e^{(m-1)v}}{\bar{b}_1 \alpha^2} = \frac{\bar{c}_2 e^{S_{11}} e^{(m-1)v}}{\alpha^2} \geqslant \frac{\bar{c}_2 e^{u+(m-1)v}}{\alpha^2 + \omega e^u + \beta e^{2u}} \geqslant -\sum_{k=1}^{q} \ln(1 + d_{2k}), \quad (78)$$

and then we have

$$v \leqslant \frac{1}{m-1} \ln\left(-\frac{\bar{b}_1 \alpha^2 \sum_{k=1}^{q} \ln(1 + d_{2k})}{\bar{c}_2 \bar{r}_1}\right) = S_{14}. \quad (79)$$

Then from (77) and (79), we get $|v| \leqslant \max\{|S_{13}|, |S_{14}|\}$.

Therefore,

$$|u| + |v| \leqslant \max\{|S_{11}|, |S_{12}|\} + \max\{|S_{13}|, |S_{14}|\} < H_0 < H, \quad (80)$$

which leads to a contradiction. Using the property of topological degree, we have

$$\begin{aligned} &\deg\left\{JQN(u, v)^{\top}, \Omega \cap \ker L, (0, 0)^{\top}\right\} \\ &\quad = \deg\left\{\phi(u, v, 1), \Omega \cap \ker L, (0, 0)^{\top}\right\} \\ &\quad = \deg\left\{\phi(u, v, 0), \Omega \cap \ker L, (0, 0)^{\top}\right\}. \end{aligned} \quad (81)$$

By $[D_1]$ and $[D_2]$, we see that the following system of algebraic equation

$$\begin{aligned} &\bar{b}_1 e^u - \bar{r}_1 - \sum_{k=1}^{q} \ln(1 + d_{1k}) = 0, \\ &-\frac{\bar{c}_2 e^{u+(m-1)v}}{k^2 + \omega e^u + \beta e^{2u}} - \sum_{k=1}^{q} \ln(1 + d_{2k}) = 0, \end{aligned} \quad (82)$$

has a unique solution in R^2. Thus, a standard and direct calculation shows that

$$\deg\left\{JQN(u, v)^{\top}, \Omega \cap \ker L, (0, 0)^{\top}\right\} = -1. \quad (83)$$

Obviously, the open set Ω satisfies all conditions in Lemma 2, and therefore we claim that system (9) has at least one T-periodic solution on $\overline{\Omega} \cap \operatorname{Dom} L$; that is, system (6) has at least one positive periodic solution. Thus we complete the proof. □

Remark 4. Our model (6) is more general than those in [24, 25] since there are different types of Holling functions. The results in [24, 25] do not give the decision on existence of positive solution to (6).

Remark 5. In our proof, by new tricks, we avoid the errors that existed in [23–25]. In the proof of Theorem 2.1 [23, page 230], the authors stated that: "let $L : \operatorname{Dom} L \subset X \to Y$, $u \to (u', \Delta u(t_1), \dots, \Delta u(t_q))$,

$$Nu = \left(\begin{bmatrix} r_1(t) - D_1(t) - a_{11}(t) e^{u_1(t)} - a_{13}(t) e^{u_3(t)} + D_1(t) e^{u_2(t)-u_1(t)} \\ r_2(t) - D_2(t) - a_{22}(t) e^{u_2(t)} + D_2(t) e^{u_1(t)-u_2(t)} \\ -r_3(t) + a_{31}(t) e^{u_1(t-\tau_1)} - a_{33}(t) e^{u_3(t-\tau_2)} \end{bmatrix}, \left\{ \begin{bmatrix} \ln(1+b_{1k}) \\ \ln(1+b_{2k}) \\ \ln(1+b_{3k}) \end{bmatrix} \right\}_{k=1}^{q} \right), \tag{84}$$

where

$$\begin{gathered} X = \left\{ (u_1(t), u_2(t), u_3(t))^{\mathsf{T}} \right. \\ \left. \in PC(\mathsf{R}, \mathsf{R}^3) : u_i(t+T) = u_i(t),\ i = 1,2,3 \right\}, \\ Y = X \times \mathsf{R}^{3q}, \end{gathered} \tag{85}$$

$PC(\mathsf{R}, \mathsf{R}^3) = \{x : \mathsf{R} \to \mathsf{R}^3 \mid x$ be continuous at $t \neq t_k$, $x(t_k^+)$, $x(t_k^-)$ exist and $x(t_k^-) = x(t_k)$, $k = 1, 2, \dots,\}$." From this, we can derive that $u' \in X$; that is, $u' = (u_1'(t), u_2'(t), u_3'(t))^{\mathsf{T}} \in PC(\mathsf{R}, \mathsf{R}^3)$ and $u_i'(t+T) = u_i'(t)$, $i = 1, 2, 3$. From the definition of $PC(\mathsf{R}, \mathsf{R}^3)$, we obtain that $(u_1'(t_k), u_2'(t_k), u_3'(t_k))$ exists, and then we can deduce that $u_1(t), u_2(t)$ and $u_3(t)$ are continuous at $t = t_k$. So we have $\Delta u(t_1) = 0, \dots, \Delta u(t_q) = 0$ for any $u \in \operatorname{Dom} L$. So we get $Lu \neq Nu$, for any $u \in \operatorname{Dom} L$, which contradicts the conclusion of Theorem 2.1 [23]. Errors similarly reappear in proving Theorem 2.4 [24, page 3393] and Theorem 2.1 [25, page 1047].

4. An Illustrative Example

The following illustrative example demonstrates the effectiveness of our main result.

Example 1. Consider the following predator-prey system of Holling type IV function response with mutual interference and impulsive effects

$$\begin{aligned} \dot{x}(t) &= x(t)\left((6+\sin t) - (2-\sin t)\, x(t)\right) \\ &\quad - \frac{(0.7+0.2\sin t)\, x(t)}{9+2x(t)+3x^2(t)} y^{0.5}(t), \\ \dot{y}(t) &= y(t)\left(-(0.03+0.02\sin t) - (0.3-0.1\sin t)\, y(t)\right) \\ &\quad + \frac{(0.3+0.5\sin t)\, x(t)}{9+2x(t)+3x^2(t)} y^{0.5}(t), \\ &\qquad t \neq t_k, \quad k = 1, 2, \dots, \\ \Delta x(t_k) &= d_{1k} x(t_k), \quad k = 1, 2, \dots, \\ \Delta y(t_k) &= d_{2k} y(t_k), \quad k = 1, 2, \dots. \end{aligned} \tag{86}$$

We fix the parameters $d_{1k} = -0.5$, $d_{2k} = -0.3$, $t_{k+3} = t_k + 2\pi$, $[0, 2\pi] \cap \{t_k\} = \{t_1, t_2, t_3\}$. By an easy calculation, we obtain $\overline{r}_1 = 6$, $\sum_{k=1}^{3} \ln(1 + d_{1k}) = -3\ln 2$.

Therefore we have

$$\begin{aligned} \overline{r}_1 = 6 > -3\ln 2 &= \sum_{k=1}^{3} \ln(1+d_{1k}), \\ T\overline{r}_1 = 12\pi > -3\ln 2 &= \sum_{k=1}^{3} \ln(1+d_{1k}). \end{aligned} \tag{87}$$

Thus, by Theorem 3, system (86) has at least one positive 2π-periodic solution.

5. Conclusion

In this work, we have considered a more general predator-prey model with Holling type IV function response and the impulsive effect. By use of the continuation theorem in coincidence degree theory and new anaytical tricks, we have provided the sufficient conditions to ensure the existence of the positive solution to this model. We also point out minor errors in some papers on relevant models. In the future, investigation on the convergence of the positive solutions will be probably very interesting and significant since $(0, 0)$ is obviously the solution of (6) and that zero solution stands for the extinction of the species.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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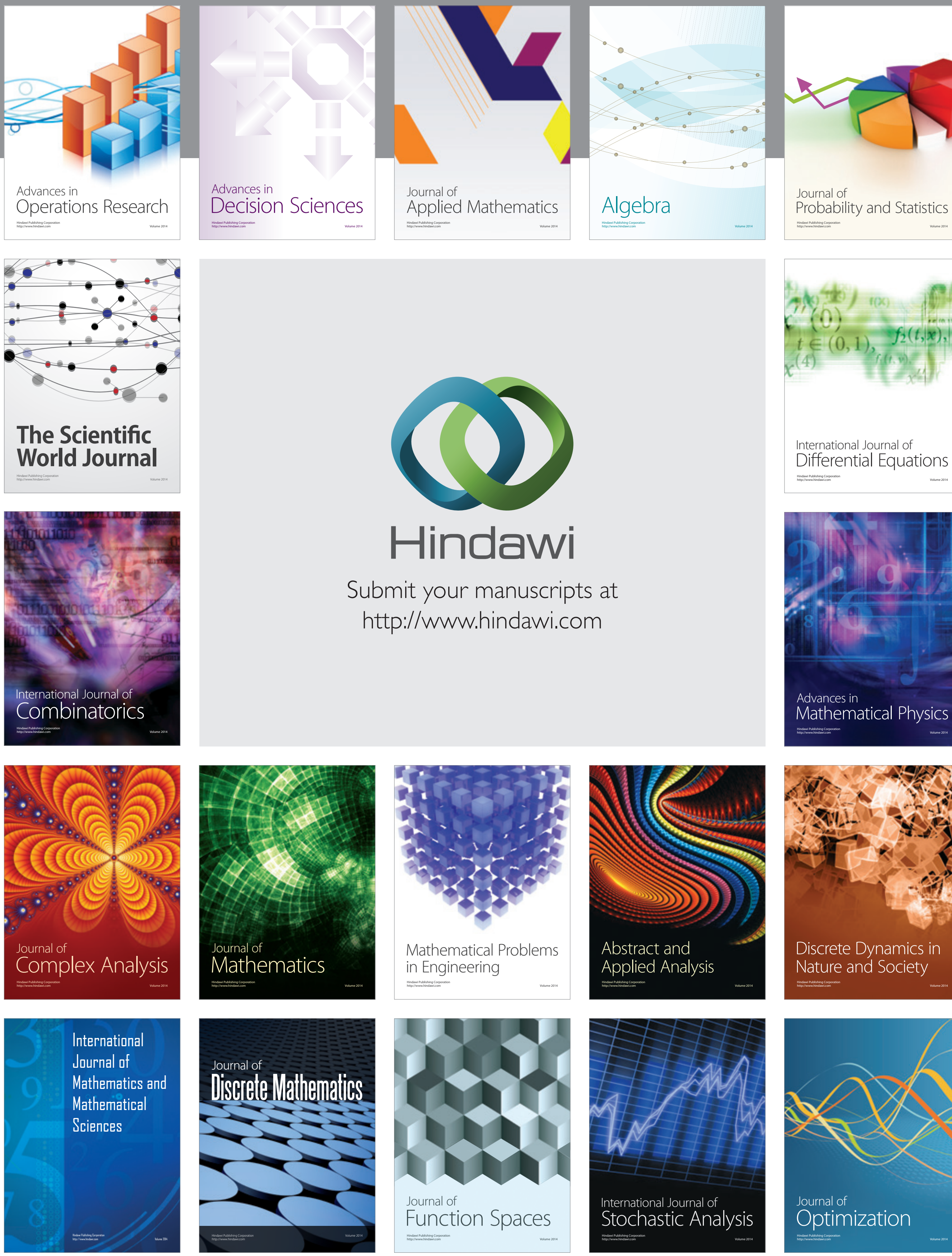
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