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Research Article

Hopf Bifurcation Analysis of a Predator-Prey Biological Economic System with Nonselective Harvesting

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A modified predator-prey biological economic system with nonselective harvesting is investigated. An important mathematical feature of the system is that the economic profit on the predator-prey system is investigated from an economic perspective. By using the local parameterization method and Hopf bifurcation theorem, we analyze the Hopf bifurcation of the proposed system. In addition, the modified model enriches the database for the predator-prey biological economic system. Finally, numerical simulations illustrate the effectiveness of our results.

1. Introduction

At present, the increasingly serious problem of environmental degradation and resource shortage makes the analysis and modeling of biological systems more interesting. From the perspective of human needs, the exploitation of biological resources and harvest of population are usually practiced in the fields of wildlife, fishery, and forestry management. It is well known that one of dominant themes in both ecology and mathematical ecology is the dynamic relationship between predators and their prey because of its universal existence and importance in population dynamics. Many authors [1–12] have studied the dynamics of predator-prey models with harvesting and obtained various dynamic behaviors, such as permanence, extinction, stability of equilibrium, Hopf bifurcation, and limit cycle. Most of these discussions on biological models are based on normal systems governed by differential equations or difference equations.

In these years, it has been shown that harvesting has a strong impact on dynamic evolution of a population; see [8–16]. In fact, the harvesting should be a variable from real world view, because it may vary with seasonality, market demand, harvesting cost, and so on. On the other hand, economic profit is a very important factor for governments, merchants, and even every citizen, so it is necessary to research biological economic systems with economic profit, which can be described by differential-algebraic equations. In particular, according to the economic principle in [5], Zhang et al. [11–13] put forward a class of modified predator-prey systems, which are established by differential-algebraic equations. The advantages of the systems proposed in [11–13] are that these models investigated the interaction mechanism in the predator-prey ecosystem and offered a new cognitive perspective for the harvested predator-prey biological system. That is, the harvest effort on the predator-prey system can be realized from an economic perspective. However, to our knowledge, the systems in most of the articles on this subject are with just one capture harvesting, such as the system with predator harvesting or the system with prey harvesting; so far there have been no attempts in the study of the bifurcation of the predator-prey biological economic system with nonselective harvesting.

The aim of this paper is to investigate the Hopf bifurcation of a predator-prey biological economic system with nonselective harvesting by using bifurcation theory in [17, 18] and center manifold theory in [17–19].

The rest of the paper is arranged as follows: a predator-prey biological economic system with nonselective harvesting is established in Section 2. We investigate the Hopf bifurcation for this system in the closed positive cone R_+^3 in Section 3. Numerical simulations will be performed to illustrate the analytical results in Section 4. A brief discussion is given in Section 5.

2. Model

The basic model we consider is based on the following Lotka-Volterra predator-prey model with harvest:

$$\frac{d\tilde{x}}{d\tilde{t}} = r\tilde{x}\left(1 - \frac{\tilde{x}}{K}\right) - a\tilde{x}\tilde{y} - \tilde{E}\tilde{x},$$
$$\frac{d\tilde{y}}{d\tilde{t}} = -\tilde{d}\tilde{y} + \tilde{b}\tilde{x}\tilde{y} - \tilde{E}\tilde{y}, \tag{1}$$

where $\tilde{x}$ and $\tilde{y}$ denote prey and predator population densities at time $\tilde{t}$, respectively. $r > 0$, $\tilde{d} > 0$ are the intrinsic growth rate of prey and the death rate of predator in the absence of food, respectively. $K > 0$ is the carrying capacity of prey. $a > 0$ and $\tilde{b} > 0$ measure the effect of the interaction of the two populations. $\tilde{E}$ represents harvesting effort. $\tilde{E}\tilde{x}$ and $\tilde{E}\tilde{y}$ indicate that the harvests for prey and predator population are proportional to their densities at time $\tilde{t}$.

Based on the model system (1) and the economic theory of fishery resource proposed by Gordon [5] in 1954, a differential-algebraic model which consists of two differential equations and an algebraic equation can be established as follows:

$$\frac{d\tilde{x}}{d\tilde{t}} = r\tilde{x}\left(1 - \frac{\tilde{x}}{K}\right) - a\tilde{x}\tilde{y} - \tilde{E}\tilde{x},$$
$$\frac{d\tilde{y}}{d\tilde{t}} = -\tilde{d}\tilde{y} + \tilde{b}\tilde{x}\tilde{y} - \tilde{E}\tilde{y}, \tag{2}$$
$$0 = \tilde{E}\left(\tilde{p}_x\tilde{x} - \tilde{c}_x\right) + \tilde{E}\left(\tilde{p}_y\tilde{y} - \tilde{c}_y\right) - m,$$

where $\tilde{p}_x$ and $\tilde{p}_y$ are harvesting reward per unit harvesting effort for unit weight of prey and predator and $\tilde{c}_x$ and $\tilde{c}_y$ are harvesting cost per unit harvesting effort for prey and predator, respectively. $m > 0$ is the economic profit per unit harvesting effort.

For convenience, substituting these dimensionless variables in system (2),

$$x = \frac{\tilde{x}}{K}, \quad y = \frac{a\tilde{y}}{r}, \quad E = \frac{\tilde{E}}{r}, \quad t = r\tilde{t},$$
$$\mu = m, \quad d = \frac{\tilde{d}}{r}, \quad b = \frac{\tilde{b}K}{r}, \quad p_1 = rK\tilde{p}_x, \tag{3}$$
$$p_2 = \frac{r^2}{a}\tilde{p}_y, \quad c_1 = r\tilde{c}_x, \quad c_2 = r\tilde{c}_y,$$

and then obtain the following biological economic system expressed by differential-algebraic equation:

$$\frac{dx}{dt} = x\left(1 - x - y - E\right),$$
$$\frac{dy}{dt} = y\left(-d + bx - E\right), \tag{4}$$
$$0 = E\left(p_1 x - c_1\right) + E\left(p_2 y - c_2\right) - \mu.$$

In this paper, we mainly discuss the effects of economic profit on the dynamics of the system (4) in the region $R_+^3 = \{(x, y, E) \mid x > 0, y > 0, E > 0\}$.

For convenience, we let

$$f(\mu, X) = \begin{pmatrix} f_1(\mu, X) \\ f_2(\mu, X) \end{pmatrix} = \begin{pmatrix} x(1 - x - y - E) \\ y(-d + bx - E) \end{pmatrix},$$
$$g(\mu, X) = E\left(p_1 x + p_2 y - c_1 - c_2\right) - \mu, \tag{5}$$
$$X = (x, y, E)^T.$$

3. Hopf Bifurcation

In this section, we will present some analytical criteria for the Hopf bifurcation of the bioeconomic system (4). In order to obtain the criteria, we need the following preparations.

Now, we try to find all positive equilibrium points of the system (4). The positive equilibrium point $X_0 = (x_0, y_0, E_0)^T$ of system (4) satisfies the following equations:

$$1 - x - y - E = 0,$$
$$-d + bx - E = 0, \tag{6}$$
$$E\left(p_1 x - c_1\right) + E\left(p_2 y - c_2\right) - \mu = 0.$$

By computing, we can easily obtain that the system (4) has an equilibrium point $X_0 = (x_0, y_0, E_0)^T = ((1/b)E_0 + d/b, -((b+1)/b)E_0 + (b-d)/b, E_0)^T$, where E_0 satisfies the equation

$$\left(p_1 - p_2 - p_2 b\right)E^2 + \left(p_1 d + p_2 b - p_2 d - bc_1 - bc_2\right)E - b\mu = 0. \tag{7}$$

Obviously,

$$E_0 = \Big(\left(p_2 d + bc_1 + bc_2 - p_1 d - p_2 b\right) \pm \left(\left(p_1 d + p_2 b - p_2 d - bc_1 - bc_2\right)^2 + 4b\mu\left(p_1 - p_2 - p_2 b\right)\right)^{1/2}\Big) \cdot \left(2\left(p_1 - p_2 - p_2 b\right)\right)^{-1}. \tag{8}$$

The paper only concentrates on the positive equilibrium point of the system (4), since the biological meaning of the positive equilibrium point implies that the prey, the predator, and the harvest effort on prey all exist, which are relevant to our study. Therefore, throughout the paper, we assume that

$$b > d, \quad E_0 < \frac{b-d}{b+1}, \quad p_1 - p_2 - p_2 b < 0,$$
$$p_1 d + p_2 b - p_2 d - bc_1 - bc_2 > 0, \tag{9}$$
$$\left(p_1 d + p_2 b - p_2 d - bc_1 - bc_2\right)^2 + 4b\mu\left(p_1 - p_2 - p_2 b\right) > 0.$$

For the system (4), we consider the following local parameterization:

$$\overline{X} = \psi(\mu, Y) = \overline{X}_0(\mu) + U_0 Y + V_0 h(\mu, Y),$$
$$g(\mu, \psi(\mu, Y)) = 0, \tag{10}$$

where

$$U_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad V_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \qquad Y = (y_1, y_2)^T \in R^2, \tag{11}$$

and h is a continuous mapping from $R \times R^2$ into R which is smooth with respect to Y. And then, we can deduce that the parametric system of the system (4) takes the form of

$$\begin{aligned} \dot{y}_1 &= f_1(\mu, \psi(\mu, Y)), \\ \dot{y}_2 &= f_2(\mu, \psi(\mu, Y)). \end{aligned} \tag{12}$$

Therefore, the Jacobian matrix $A(\mu)$ of the system (12) at $Y = 0$ takes the form of

$$\begin{aligned} &\left.\begin{pmatrix} D_{y_1} f_1(\mu, \psi(\mu, Y)) & D_{y_2} f_1(\mu, \psi(\mu, Y)) \\ D_{y_1} f_2(\mu, \psi(\mu, Y)) & D_{y_2} f_2(\mu, \psi(\mu, Y)) \end{pmatrix}\right|_{Y=0} \\ &= \begin{pmatrix} D_{\overline{X}} f_1(\mu, \overline{X}_0(\mu)) \\ D_{\overline{X}} f_2(\mu, \overline{X}_0(\mu)) \end{pmatrix} \begin{pmatrix} D_{\overline{X}} g(\mu, \overline{X}_0(\mu)) \\ U_0^T \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ I_2 \end{pmatrix} \\ &= \begin{pmatrix} D_X f_1(\mu, \overline{X}_0(\mu)) & D_y f_1(\mu, \overline{X}_0(\mu)) \\ D_X f_2(\mu, \overline{X}_0(\mu)) & D_y f_2(\mu, \overline{X}_0(\mu)) \end{pmatrix} \\ &= \begin{pmatrix} -x_0 + \dfrac{p_1 E_0}{Z_0} x_0 & -x_0 + \dfrac{p_2 E_0}{Z_0} x_0 \\ b y_0 + \dfrac{p_1 E_0}{Z_0} y_0 & \dfrac{p_2 E_0}{Z_0} y_0 \end{pmatrix}, \end{aligned} \tag{13}$$

where $Z = p_1 x + p_2 y - c_1 - c_2$, $Z_0 = p_1 x_0 + p_2 y_0 - c_1 - c_2$. Therefore, the characteristic equation of the matrix $J(\mu)$ can be expressed as

$$\lambda^2 + a_1(\mu)\lambda + a_2(\mu) = 0, \tag{14}$$

where $a_{11} = -x_0 + p_1 E_0 x_0 / Z_0$, $a_{12} = -x_0 + p_2 E_0 x_0 / Z_0$, $a_{21} = b y_0 + p_1 E_0 y_0 / Z_0$, $a_{22} = p_2 E_0 y_0 / Z_0$, and $a_1(\mu) = -(a_{11} + a_{22}) = x_0 - p_1 E_0 x_0 / Z_0 - p_2 E_0 y_0 / Z_0$, $a_2(\mu) = a_{11} a_{22} - a_{12} a_{21} = b x_0 y_0 + (p_1 - p_2 - b p_2) E_0 x_0 y_0 / Z_0$.

Remark 1. The positive equilibrium point $\overline{X_0}$ of the system (4) corresponds to the equilibrium point $Y = 0$ of the parametric system (12). For this reason, $A(\mu)$ can be considered as Jacobian matrix of the system (4) at $\overline{X_0}$, which can be also determined by the method in [16].

In (14), letting $a_1(\mu) = 0$, we obtain the bifurcation value $\mu_0 = Z_0^2 x_0 / (p_1 x_0 + p_2 y_0)$. In fact, if we let $a_1^2(\mu) < 4 a_2(\mu)$, then (14) has a pair of conjugate complex roots:

$$\lambda_{1,2} = -\frac{1}{2} a_1(\mu) \pm i \sqrt{a_2(\mu) - \frac{a_1^2(\mu)}{4}} := \alpha(\mu) \pm i\omega(\mu). \tag{15}$$

By computing, we have

$$\begin{aligned} &\alpha(\mu_0) = 0, \qquad \alpha'(\mu_0) = \frac{p_1 x_0 + p_2 y_0}{2 Z_0^2} \neq 0, \\ &\omega(\mu_0) = \sqrt{b x_0 y_0 + \frac{(p_1 - p_2 - b p_2) E_0}{Z_0} x_0 y_0} > 0. \end{aligned} \tag{16}$$

Therefore, a phenomenon of Hopf bifurcation occurs at the bifurcation value μ_0.

In order to calculate the Hopf bifurcation, according to [3, 17], when $\mu = \mu_0$, $\overline{X} = \overline{X_0}$, we need to lead the normal form of the system (4) as follows:

$$\begin{aligned} \dot{y}_1 = {} & \omega_0 y_2 + \frac{1}{2} a_{11}^1 y_1^2 + a_{12}^1 y_1 y_2 \\ & + \frac{1}{2} a_{22}^1 y_2^2 + \frac{1}{6} a_{111}^1 y_1^3 + \frac{1}{2} a_{112}^1 y_1^2 y_2 \\ & + \frac{1}{2} a_{122}^1 y_1 y_2^2 + \frac{1}{6} a_{222}^1 y_2^3 + o(|Y|^4), \\ \dot{y}_2 = {} & -\omega_0 y_1 + \frac{1}{2} a_{11}^2 y_1^2 + a_{12}^2 y_1 y_2 \\ & + \frac{1}{2} a_{22}^2 y_2^2 + \frac{1}{6} a_{111}^2 y_1^3 + \frac{1}{2} a_{112}^2 y_1^2 y_2 \\ & + \frac{1}{2} a_{122}^2 y_1 y_2^2 + \frac{1}{6} a_{222}^2 y_2^3 + o(|Y|^4), \end{aligned} \tag{17}$$

where $\omega_0 = \omega(\mu_0)$. And it can be proved that the parametric system (12) with $\mu = \mu_0$ and $\overline{X} = \overline{X}_0$ takes the form of

$$\begin{aligned} \dot{y}_1 = {} & f_{1y_1}(\mu_0, \overline{X}_0) y_1 + f_{1y_2}(\mu_0, \overline{X}_0) y_2 \\ & + \frac{1}{2} f_{1y_1y_1}(\mu_0, \overline{X}_0) y_1^2 + f_{1y_1y_2}(\mu_0, \overline{X}_0) y_1 y_2 \\ & + \frac{1}{2} f_{1y_2y_2}(\mu_0, \overline{X}_0) y_2^2 + \frac{1}{6} f_{1y_1y_1y_1}(\mu_0, \overline{X}_0) y_1^3 \\ & + \frac{1}{2} f_{1y_1y_1y_2}(\mu_0, \overline{X}_0) y_1^2 y_2 + \frac{1}{2} f_{1y_1y_2y_2}(\mu_0, \overline{X}_0) y_1 y_2^2 \\ & + \frac{1}{6} f_{1y_2y_2y_2}(\mu_0, \overline{X}_0) y_2^3 + o(|Y|^4), \end{aligned}$$

$$\begin{aligned}\dot{y}_2 = {} & f_{2y_1}\left(\mu_0,\overline{X}_0\right)y_1 + f_{2y_2}\left(\mu_0,\overline{X}_0\right)y_2\\ & + \frac{1}{2}f_{2y_1y_1}\left(\mu_0,\overline{X}_0\right)y_1^2 + f_{2y_1y_2}\left(\mu_0,\overline{X}_0\right)y_1y_2\\ & + \frac{1}{2}f_{2y_2y_2}\left(\mu_0,\overline{X}_0\right)y_2^2 + \frac{1}{6}f_{2y_1y_1y_1}\left(\mu_0,\overline{X}_0\right)y_1^3\\ & + \frac{1}{2}f_{2y_1y_1y_2}\left(\mu_0,\overline{X}_0\right)y_1^2y_2 + \frac{1}{2}f_{2y_1y_2y_2}\left(\mu_0,\overline{X}_0\right)y_1y_2^2\\ & + \frac{1}{6}f_{2y_2y_2y_2}\left(\mu_0,\overline{X}_0\right)y_2^3 + o\left(|Y|^4\right).\end{aligned} \tag{18}$$

In the following, we will calculate the coefficients of the above parametric system (18). By calculation, we derive

$$\begin{aligned} D_{\overline{X}}f_1\left(\mu,\overline{X}\right) &= \left(1-2x-y-E,-x,-x\right),\\ D_{\overline{X}}f_2\left(\mu,\overline{X}\right) &= \left(by,-d+bx-E,-y\right),\\ D_{\overline{X}}g\left(\mu,\overline{X}\right) &= \left(p_1E,p_2E,p_1x+p_2y-c_1-c_2\right),\\ D\psi\left(\mu,Y\right) &= \left(D_{y_1}\psi\left(\mu,Y\right),D_{y_2}\psi\left(\mu,Y\right)\right)\\ &= \begin{pmatrix} D_{\overline{X}}g\left(\mu,\overline{X}\right)\\ U_0^T\end{pmatrix}^{-1}\begin{pmatrix}0\\ I_2\end{pmatrix}\\ &= \begin{pmatrix} p_1E & p_2E & p_1x+p_2y-c_1-c_2\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}^{-1}\begin{pmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{pmatrix}\\ &= \begin{pmatrix} 1 & 0\\ 0 & 1\\ -\dfrac{p_1E}{Z} & -\dfrac{p_2E}{Z}\end{pmatrix}.\end{aligned} \tag{19}$$

Thus,

$$\begin{aligned} f_{1y_1}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_1\left(\mu,\overline{X}\right)D_{y_1}\psi\left(\mu,Y\right)\\ &= 1-2x-y-E+\frac{p_1E}{Z}x,\\ f_{1y_2}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_1\left(\mu,\overline{X}\right)D_{y_2}\psi\left(\mu,Y\right) = -x+\frac{p_2E}{Z}x,\\ f_{2y_1}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_2\left(\mu,\overline{X}\right)D_{y_1}\psi\left(\mu,Y\right) = by+\frac{p_1E}{Z}y,\\ f_{2y_2}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_2\left(\mu,\overline{X}\right)D_{y_2}\psi\left(\mu,Y\right)\\ &= -d+bx-E+\frac{p_2E}{Z}y.\end{aligned} \tag{20}$$

Substituting μ_0, $\overline{X}_0$ into (20),

$$\begin{aligned} f_{1y_1}\left(\mu_0,\overline{X}_0\right) &= -\frac{p_2E_0}{Z_0}y_0,\\ f_{1y_2}\left(\mu_0,\overline{X}_0\right) &= -x_0+\frac{p_2E_0}{Z_0}x_0,\\ f_{2y_1}\left(\mu_0,\overline{X}_0\right) &= by_0+\frac{p_1E_0}{Z_0}y_0,\\ f_{2y_2}\left(\mu_0,\overline{X}_0\right) &= \frac{p_2E_0}{Z_0}y_0.\end{aligned} \tag{21}$$

From (20), we have

$$\begin{aligned} D_{\overline{X}}f_{1y_1}\left(\mu,\overline{X}\right) &= \left(-2+\frac{p_1E\left(p_2y-c_1-c_2\right)}{Z^2},\right.\\ &\qquad \left.-1-\frac{p_1p_2Ex}{Z^2},-1+\frac{p_1x}{Z}\right),\\ D_{\overline{X}}f_{1y_2}\left(\mu,\overline{X}\right) &\\ &= \left(-1+\frac{p_2E\left(p_2y-c_1-c_2\right)}{Z^2},-\frac{p_2^2Ex}{Z^2},\frac{p_2x}{Z}\right),\\ D_{\overline{X}}f_{2y_1}\left(\mu,\overline{X}\right) &\\ &= \left(-\frac{p_1^2Ey}{Z^2},b+\frac{p_1E\left(p_1x-c_1-c_2\right)}{Z^2},\frac{p_1y}{Z}\right),\\ D_{\overline{X}}f_{2y_2}\left(\mu,\overline{X}\right) &\\ &= \left(b-\frac{p_1^2Ey}{Z^2},\frac{p_2E\left(p_1x-c_1-c_2\right)}{Z^2},-1+\frac{p_2y}{Z}\right).\end{aligned} \tag{22}$$

According to (19) and (22), we obtain

$$\begin{aligned} f_{1y_1y_1}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_{1y_1}\left(\mu,\overline{X}\right)D_{y_1}\psi\left(\mu,Y\right)\\ &= -2+\frac{2p_1E\left(p_2y-c_1-c_2\right)}{Z^2},\\ f_{1y_1y_2}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_{1y_1}\left(\mu,\overline{X}\right)D_{y_2}\psi\left(\mu,Y\right)\\ &= -1+\frac{p_2E\left(p_2y-p_1x-c_1-c_2\right)}{Z^2},\\ f_{1y_2y_2}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_{1y_2}\left(\mu,\overline{X}\right)D_{y_2}\psi\left(\mu,Y\right) = -\frac{2p_2^2Ex}{Z^2},\\ f_{2y_1y_1}\left(\mu,\overline{X}\right) &= D_{\overline{X}}f_{2y_1}\left(\mu,\overline{X}\right)D_{y_1}\psi\left(\mu,Y\right) = -\frac{2p_1^2Ey}{Z^2},\end{aligned}$$

$$\begin{aligned} f_{2y_1y_2}(\mu,\overline{X}) &= D_{\overline{X}}f_{2y_1}(\mu,\overline{X})\,D_{y_2}\psi(\mu,Y) \\ &= b+\frac{p_1E(p_1x-p_2y-c_1-c_2)}{Z^2}, \\ f_{2y_2y_2}(\mu,\overline{X}) &= D_{\overline{X}}f_{2y_2}(\mu,\overline{X})\,D_{y_2}\psi(\mu,Y) \\ &= \frac{2p_2E(p_1x-c_1-c_2)}{Z^2}. \end{aligned} \tag{23}$$

Substituting μ_0, $\overline{X}_0$ into (23),

$$\begin{aligned} f_{1y_1y_1}(\mu_0,\overline{X}_0) &= -2+\frac{2p_1E_0(p_2y_0-c_1-c_2)}{Z_0^2}, \\ f_{1y_2y_2}(\mu_0,\overline{X}_0) &= -\frac{2p_2^2E_0x_0}{Z_0^2}, \\ f_{1y_1y_2}(\mu_0,\overline{X}_0) &= -1+\frac{p_2E_0(p_2y_0-p_1x_0-c_1-c_2)}{Z_0^2}, \\ f_{2y_1y_1}(\mu_0,\overline{X}_0) &= -\frac{2p_1^2E_0y_0}{Z_0^2}, \\ f_{2y_1y_2}(\mu_0,\overline{X}_0) &= b+\frac{p_1E_0(p_1x_0-p_2y_0-c_1-c_2)}{Z_0^2}, \\ f_{2y_2y_2}(\mu_0,\overline{X}_0) &= \frac{2p_2E_0(p_1x_0-c_1-c_2)}{Z_0^2}. \end{aligned} \tag{24}$$

By (23) we get

$$\begin{aligned} D_{\overline{X}}f_{1y_1y_1}(\mu,\overline{X}) &= \left(-\frac{4p_1^2E(p_2y-c_1-c_2)}{Z^3},\right. \\ &\quad \frac{2p_1p_2E(p_1x-p_2y+c_1+c_2)}{Z^3}, \\ &\quad \left.\frac{2p_1(p_2y-c_1-c_2)}{Z^2}\right), \\ D_{\overline{X}}f_{1y_1y_2}(\mu,\overline{X}) &= \left(\frac{p_1p_2E(p_1x-3p_2y+3c_1+3c_2)}{Z^3},\right. \\ &\quad \frac{p_2^2E(3p_1x-p_2y+c_1+c_2)}{Z^3}, \\ &\quad \left.\frac{p_2(p_2y-p_1x-c_1-c_2)}{Z^2}\right), \\ D_{\overline{X}}f_{1y_2y_2}(\mu,\overline{X}) &= \left(-\frac{2p_2^2E(p_2y-p_1x-c_1-c_2)}{Z^3},\right. \\ &\quad \left.\frac{4p_2^3Ex}{Z^3},-\frac{2p_2^2x}{Z^2}\right), \\ D_{\overline{X}}f_{2y_1y_1}(\mu,\overline{X}) &= \left(\frac{4p_1^3Ey}{Z^3},-\frac{2p_1^2E(p_1x-p_2y-c_1-c_2)}{Z^3},-\frac{2p_1^2y}{Z^2}\right), \\ D_{\overline{X}}f_{2y_1y_2}(\mu,\overline{X}) &= \left(\frac{p_1^2E(3p_2y-p_1x+c_1+c_2)}{Z^3},\right. \\ &\quad \frac{p_1p_2E(p_2y-3p_1x+3c_1+3c_2)}{Z^3}, \\ &\quad \left.\frac{p_1(p_1x-p_2y-c_1-c_2)}{Z^2}\right), \\ D_{\overline{X}}f_{2y_2y_2}(\mu,\overline{X}) &= \left(\frac{2p_1p_2E(p_2y-p_1x+c_1+c_2)}{Z^3},\right. \\ &\quad -\frac{4p_2^2E(p_1x-c_1-c_2)}{Z^3}, \\ &\quad \left.\frac{2p_2(p_1x-c_1-c_2)}{Z^2}\right). \end{aligned} \tag{25}$$

Substituting μ_0, $\overline{X}_0$ into (19) and (25), it is easy to compute that

$$\begin{aligned} f_{1y_1y_1y_1}(\mu_0,\overline{X}_0) &= D_{\overline{X}}f_{1y_1y_1}(\mu_0,\overline{X}_0)\,D_{y_1}\psi(\mu_0,Y_0) \\ &= -\frac{6p_1^2E_0(p_2y_0-c_1-c_2)}{Z_0^3}, \\ f_{1y_1y_1y_2}(\mu_0,\overline{X}_0) &= D_{\overline{X}}f_{1y_1y_1}(\mu_0,\overline{X}_0)\,D_{y_2}\psi(\mu_0,Y_0) \\ &= \frac{2p_1p_2E_0(p_1x_0-2p_2y_0+2c_1+2c_2)}{Z_0^3}, \\ f_{1y_1y_2y_2}(\mu_0,\overline{X}_0) &= D_{\overline{X}}f_{1y_1y_2}(\mu_0,\overline{X}_0)\,D_{y_2}\psi(\mu_0,Y_0) \\ &= \frac{2p_2^2E_0(2p_1x_0-p_2y_0+c_1+c_2)}{Z_0^3}, \\ f_{1y_2y_2y_2}(\mu_0,\overline{X}_0) &= D_{\overline{X}}f_{1y_2y_2}(\mu_0,\overline{X}_0)\,D_{y_2}\psi(\mu_0,Y_0) \\ &= \frac{6p_2^3E_0x_0}{Z_0^3}, \\ f_{2y_1y_1y_1}(\mu_0,\overline{X}_0) &= D_{\overline{X}}f_{2y_1y_1}(\mu_0,\overline{X}_0)\,D_{y_1}\psi(\mu_0,Y_0) \\ &= \frac{6p_1^3E_0y_0}{Z_0^3}, \\ f_{2y_1y_1y_2}(\mu_0,\overline{X}_0) &= D_{\overline{X}}f_{2y_1y_1}(\mu_0,\overline{X}_0)\,D_{y_2}\psi(\mu_0,Y_0) \\ &= \frac{2p_1^2E_0(2p_2y_0-p_1x_0+c_1+c_2)}{Z_0^3}, \end{aligned}$$

$$
\begin{aligned}
f_{2y_1y_2y_2}\left(\mu_0,\overline{X}_0\right) &= D_{\overline{X}} f_{2y_1y_2}\left(\mu_0,\overline{X}_0\right) D_{y_2}\psi\left(\mu_0,Y_0\right)\\
&= \frac{2p_1p_2E_0\left(p_2y_0-2p_1x_0+2c_1+2c_2\right)}{Z_0^3},\\
f_{2y_2y_2y_2}\left(\mu_0,\overline{X}_0\right) &= D_{\overline{X}} f_{2y_2y_2}\left(\mu_0,\overline{X}_0\right) D_{y_2}\psi\left(\mu_0,Y_0\right)\\
&= -\frac{6p_2^2E_0\left(p_1x_0-c_1-c_2\right)}{Z_0^3}.
\end{aligned}
\tag{26}
$$

According to (18), (21), (24), and (26), we obtain the parametric system (4), which takes the form of

$$
\begin{aligned}
\dot{y}_1 &= a_{11}y_1 + a_{12}y_2 + a_{13}y_1^2\\
&\quad + a_{14}y_1y_2 + a_{15}y_2^2 + a_{16}y_1^3\\
&\quad + a_{17}y_1^2y_2 + a_{18}y_1y_2^2 + a_{19}y_2^3 + o\left(|Y|^4\right),\\
\dot{y}_2 &= a_{21}y_1 + a_{22}y_2 + a_{23}y_1^2\\
&\quad + a_{24}y_1y_2 + a_{25}y_2^2 + a_{26}y_1^3\\
&\quad + a_{27}y_1^2y_2 + a_{28}y_1y_2^2 + a_{29}y_2^3 + o\left(|Y|^4\right),
\end{aligned}
\tag{27}
$$

where $a_{11} = -(p_2E_0/Z_0)y_0$, $a_{12} = -x_0 + (p_2E_0/Z_0)x_0$, $a_{13} = -1 + p_1E_0(p_2y_0 - c_1 - c_2)/Z_0^2$, $a_{14} = -1 + p_2E_0(p_2y_0 - p_1x_0 - c_1 - c_2)/Z_0^2$, $a_{15} = -p_2^2E_0x_0/Z_0^2$, $a_{16} = -p_1^2E_0(p_2y_0 - c_1 - c_2)/Z_0^3$, $a_{17} = p_1p_2E_0(p_1x_0 - 2p_2y_0 + 2c_1 + 2c_2)/Z_0^3$, $a_{18} = p_2^2E_0(2p_1x_0 - p_2y_0 + c_1 + c_2)/Z_0^3$, $a_{19} = p_2^3E_0x_0/Z_0^3$, $a_{21} = by_0 + (p_1E_0/Z_0)y_0$, $a_{22} = (p_2E_0/Z_0)y_0$, $a_{23} = -p_1^2E_0y_0/Z_0^2$, $a_{24} = b + p_1E_0(p_1x_0 - p_2y_0 - c_1 - c_2)/Z_0^2$, $a_{25} = p_2E_0(p_1x_0 - c_1 - c_2)/Z_0^2$, $a_{26} = p_1^3E_0y_0/Z_0^3$, $a_{27} = p_1^2E_0(2p_2y_0 - p_1x_0 + c_1 + c_2)/Z_0^3$, $a_{28} = p_1p_2E_0(p_2y_0 - 2p_1x_0 + 2c_1 + 2c_2)/Z_0^3$, and $a_{29} = -p_2^2E_0(p_1x_0 - c_1 - c_2)/Z_0^3$.

Compared with the normal form (17), we should normalize the parametric system (27) with the following nonsingular linear transformation:

$$
\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = p\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \qquad U := \left(u_1, u_2\right)^T, \tag{28}
$$

where $p = \left(\begin{smallmatrix} -a_{12} & 0 \\ a_{11} & -\omega_0 \end{smallmatrix}\right)$. For convenience, we use Y instead of U. Thus, the normal form of system (4) takes the form of

$$
\begin{aligned}
\dot{y}_1 &= \omega_0y_2 + b_{11}y_1^2 + b_{12}y_1y_2\\
&\quad + b_{13}y_2^2 + b_{14}y_1^3 + b_{15}y_1^2y_2\\
&\quad + b_{16}y_1y_2^2 + b_{17}y_2^3 + o\left(|Y|^4\right),\\
\dot{y}_2 &= -\omega_0y_1 + b_{21}y_1^2 + b_{22}y_1y_2\\
&\quad + b_{23}y_2^2 + b_{24}y_1^3 + b_{25}y_1^2y_2\\
&\quad + b_{26}y_1y_2^2 + b_{27}y_2^3 + o\left(|Y|^4\right),
\end{aligned}
\tag{29}
$$

where $b_{11} = -(a_{13}a_{12}^2 + a_{15}a_{11}^2 - a_{11}a_{12}a_{14})/a_{12}$, $b_{12} = -(a_{12}a_{14}\omega_0 - 2a_{11}a_{15}\omega_0)/a_{12}$, $b_{13} = -a_{15}\omega_0^2/a_{12}$, $b_{14} = -(a_{11}a_{17}a_{12}^2 + a_{19}a_{11}^3 - a_{16}a_{12}^3 - a_{12}a_{18}a_{11}^2)/a_{12}$, $b_{15} = -(2a_{11}a_{12}a_{18}\omega_0 - a_{17}a_{12}^2\omega_0 - 3a_{19}a_{11}^2\omega_0)/a_{12}$, $b_{16} = -(3a_{11}a_{19}\omega_0^2 - a_{12}a_{18}\omega^2)/a_{12}$, $b_{17} = a_{19}\omega_0^3/a_{12}$, $b_{21} = -(a_{23}a_{12}^2 + a_{25}a_{11}^2 - a_{11}a_{12}a_{24} - a_{11}b_{11})/\omega_0$, $b_{22} = -(a_{12}a_{24}\omega_0 - 2a_{11}a_{25}\omega_0 - a_{11}b_{12})/\omega_0$, $b_{23} = -(a_{25}\omega_0^2 - a_{11}b_{13})/\omega_0$, $b_{24} = (a_{26}a_{12}^3 + a_{12}a_{28}a_{11}^2 + a_{11}b_{14} - a_{11}a_{27}a_{12}^2 - a_{29}a_{11}^3)/\omega_0$, $b_{25} = (a_{27}a_{12}^2\omega_0 + 3a_{29}a_{11}^2\omega_0 + a_{11}b_{15} - 2a_{11}a_{12}a_{28}\omega_0)/\omega_0$, $b_{26} = -(3a_{11}a_{29}\omega_0^2 - a_{12}a_{28}\omega^2 - a_{11}b_{16})/\omega_0$, and $b_{27} = (a_{29}\omega_0^3 - a_{11}b_{17})/\omega_0$.

Summarizing the previous results, we arrive at the following theorem.

Theorem 2. *For the system (4), there exist a positive constant ε and two small enough neighborhoods of the positive equilibrium point $X_0(\mu)$: O and P, where $0 < \varepsilon \ll 1$, $O \subset P$.*

(1) If $\sigma_0 > 0$, that is,

$$
\begin{aligned}
&\frac{4b_{11}b_{21}}{\omega_0} + \frac{2b_{22}b_{23}}{\omega_0} + \frac{2b_{11}b_{22}}{\omega_0} + 6b_{14}\\
&\quad + 2b_{16} + 2b_{25} + 6b_{27}\\
&\quad > \frac{2b_{11}b_{12}}{\omega_0} + \frac{4b_{21}b_{23}}{\omega_0} + \frac{2b_{11}b_{12}}{\omega_0},
\end{aligned}
\tag{30}
$$

then,

(i) when $\mu_0 < \mu < \mu_0 + \varepsilon$, $X_0(\mu)$ repels all the points in P, and $X_0(\mu)$ is unstable;

(ii) when $\mu_0 - \varepsilon < \mu < \mu_0$, there exists at least one periodic solution in $\overline{O}$, one of them repels all the points in $\overline{O} \setminus \{X_0(\mu)\}$, and there also exists one (may be the same one) that repels all the points in $P \setminus \overline{O}$, and $X_0(\mu)$ is locally asymptotically stable.

(2) If $\sigma_0 < 0$, that is,

$$
\begin{aligned}
&\frac{4b_{11}b_{21}}{\omega_0} + \frac{2b_{22}b_{23}}{\omega_0} + \frac{2b_{11}b_{22}}{\omega_0} + 6b_{14}\\
&\quad + 2b_{16} + 2b_{25} + 6b_{27} < \frac{2b_{11}b_{12}}{\omega_0}\\
&\quad + \frac{4b_{21}b_{23}}{\omega_0} + \frac{2b_{11}b_{12}}{\omega_0},
\end{aligned}
\tag{31}
$$

then,

(i) when $\mu_0 - \varepsilon < \mu < \mu_0$, $X_0(\mu)$ absorbs all the points in P, and $X_0(\mu)$ is locally asymptotically stable;

(ii) when $\mu_0 < \mu < \mu_0 + \varepsilon$, there exists at least one periodic solution in $\overline{O}$, one of them absorbs all the points in $\overline{O} \setminus \{X_0(\mu)\}$, and there also exists one (may be the same one) that absorbs all the points in $P \setminus \overline{O}$, and $X_0(\mu)$ is unstable.

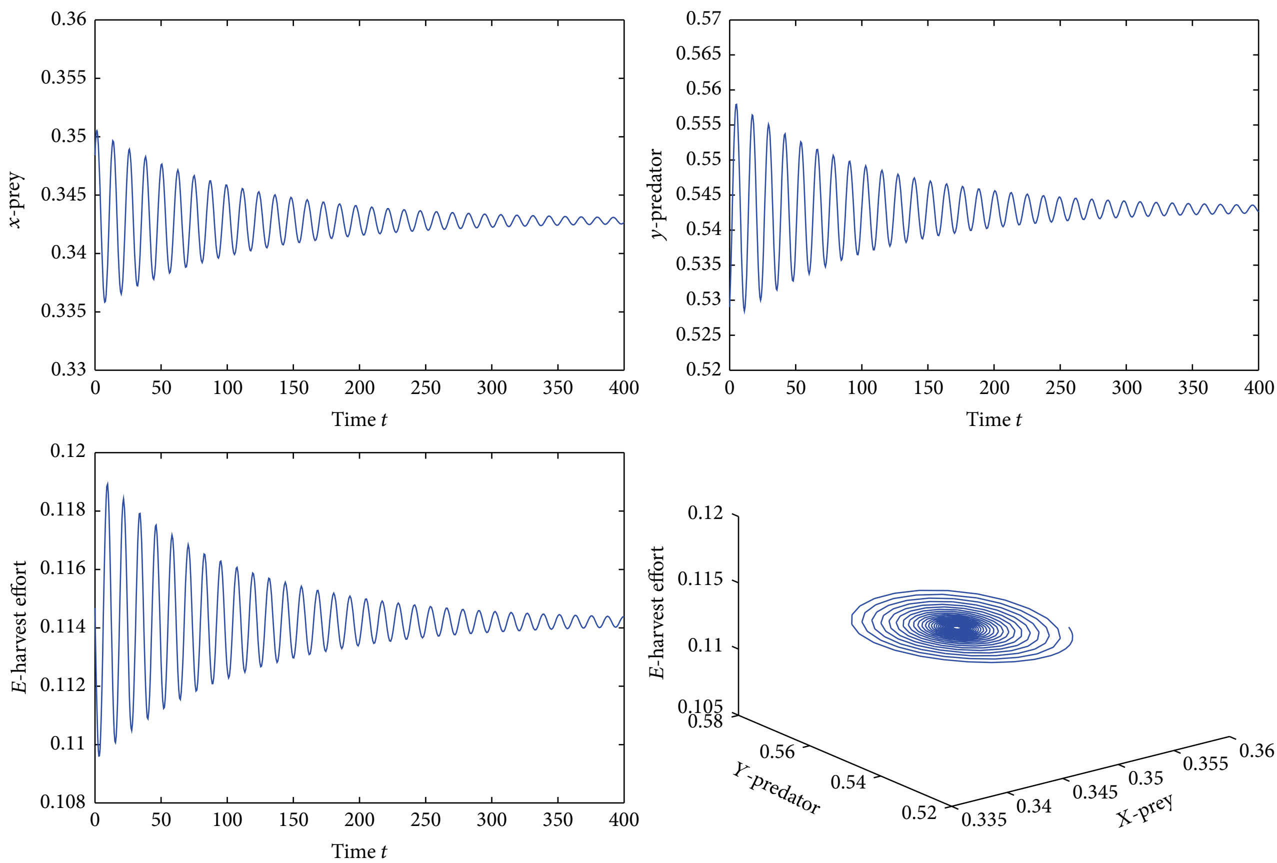

FIGURE 1: When $\mu = 0.155 < \mu_0$, the positive equilibrium point $X_0(\mu)$ is locally asymptotically stable.

Proof. From (17) and (29), we can obtain that

$$a^1_{11} = 2b_{11}, \quad a^1_{12} = b_{12}, \quad a^1_{22} = 2b_{13},$$
$$a^1_{111} = 6b_{14}, \quad a^1_{122} = 2b_{16}, \quad a^2_{11} = 2b_{21},$$
$$a^2_{12} = b_{22}, \quad a^2_{22} = 2b_{23}, \quad a^2_{112} = 2b_{25}, \quad a^2_{222} = 6b_{27}. \tag{32}$$

According to the Hopf bifurcation theorem in [1, 14], next we will judge the sign of the value σ_0 which is defined as follows:

$$\begin{aligned} 16\sigma_0 :=& \left\{a^1_{11}\left(a^2_{11} - a^1_{12}\right) + a^2_{22}\left(a^2_{12} - a^1_{22}\right) + \left(a^2_{11}a^2_{12} - a^1_{12}a^1_{22}\right)\right\} \cdot (\omega_0)^{-1} \\ &+ \left(a^1_{111} + a^1_{122} + a^2_{112} + a^2_{222}\right) \\ =& \left\{\frac{4b_{11}b_{21}}{\omega_0} + \frac{2b_{22}b_{23}}{\omega_0} + \frac{2b_{11}b_{22}}{\omega_0} + 6b_{14} + 2b_{16} + 2b_{25} + 6b_{27}\right\} \\ &- \left\{\frac{2b_{11}b_{12}}{\omega_0} + \frac{4b_{21}b_{23}}{\omega_0} + \frac{2b_{11}b_{12}}{\omega_0}\right\}. \end{aligned} \tag{33}$$

In what follows, there are two cases that ought to be discussed. That is, $\sigma_0 > 0$ and $\sigma_0 < 0$. And then the following process is similar to the proof of the Hopf bifurcation theorem in [1, 14], so the process is omitted here. □

Remark 3. The local stability of X_0 is equivalent to the local stability of $\overline{x}_0$.

4. Numerical Simulations

In this section, we present some numerical simulations to illustrate our theoretical analysis. On the basis of condition (9), the coefficients of the system (4) are chosen as follows:

$$p_1 = 6.5, \quad p_2 = 3, \quad c_1 = 1.5, \quad c_2 = 1, \quad b = 1.5, \quad d = 0.4; \tag{34}$$

then the system (4) becomes

$$\begin{aligned} \frac{dx}{dt} &= x\left(1 - x - y - E\right), \\ \frac{dy}{dt} &= y\left(-0.4 + 1.5x - E\right), \\ 0 &= E\left(6.5x - 1.5\right) + E\left(3y - 1\right) - \mu. \end{aligned} \tag{35}$$

Clearly, the system (35) has a positive equilibrium point $X_0 = (x_0, y_0, E_0) = (0.3484, 0.5290, 0.1226)$, and the bifurcation value $\mu_0 = 0.1657$.

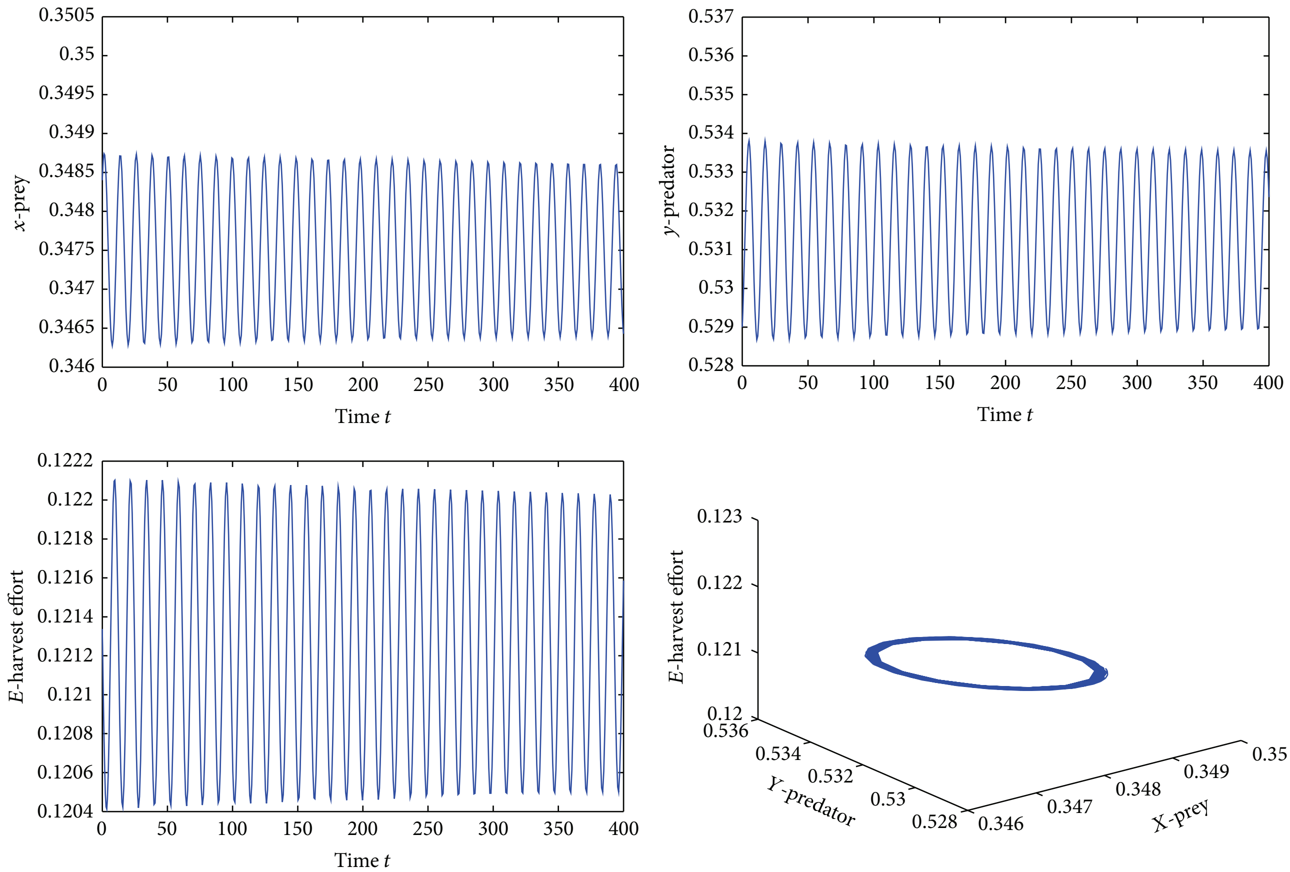

Figure 2: Periodic solutions bifurcating from $X_0(\mu)$ when $\mu = 0.164 < \mu_0$.

In Theorem 2, we let $\varepsilon = 0.015$, and then, by Theorem 2, the positive equilibrium point $X_0(\mu)$ of the system (35) is locally asymptotically stable when $\mu = 0.155 < \mu_0$, which has been illustrated in Figure 1; the periodic solution occurs from $X_0(\mu)$ when $\mu = 0.164 < \mu_0$, which has been illustrated in Figure 2; the positive equilibrium point $X_0(\mu)$ of the system (35) is unstable when $\mu = 0.171 > \mu_0$, which has been illustrated in Figure 3.

5. Discussion

In this paper, we investigate the effects of the varying economic profit on the dynamics of the bioeconomic system (4). According to Theorem 2, we can see that if the fishermen's pursuit of economic profit μ is equal to or larger than the bifurcation value μ_0, then the status of preys, the predators, and the harvest effort will be unstable. Clearly, this is harmful to the predator-prey biological economic system. Therefore, in order to ensure the continuable and healthy development of the biological economic system as well as maintain the ideal income from the harvest effort, the fishermen ought to guarantee that their positive economic profit μ is less than the bifurcation value μ_0.

As we know, harvesting has a strong impact on the dynamic evolution of a population. And many works have been done for the predator-prey system with harvesting; see [7–9, 11–16, 20]. Particularly, a class of biological economic systems is proposed in [8, 11–13, 15, 20]. Some scholars analyze the system with predator harvesting and some scholars analyze the system with prey harvesting. Compared with the above researches, the main contribution of this paper lies in the following aspect. The predator-prey system we consider incorporates nonselective harvesting. In fact, predator harvest and prey harvest can also bring economic benefits for us, so the system with nonselective harvesting that we investigate is more realistic. And the analysis result in this paper will be more scientific. So the modified model in our paper enriches the database for the predator-prey biological economic system.

In addition, stage structure, time delays, diffusion effects, and disease effects may be incorporated into our bioeconomic system, which would make the bioeconomic system exhibit much more complicated dynamics.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

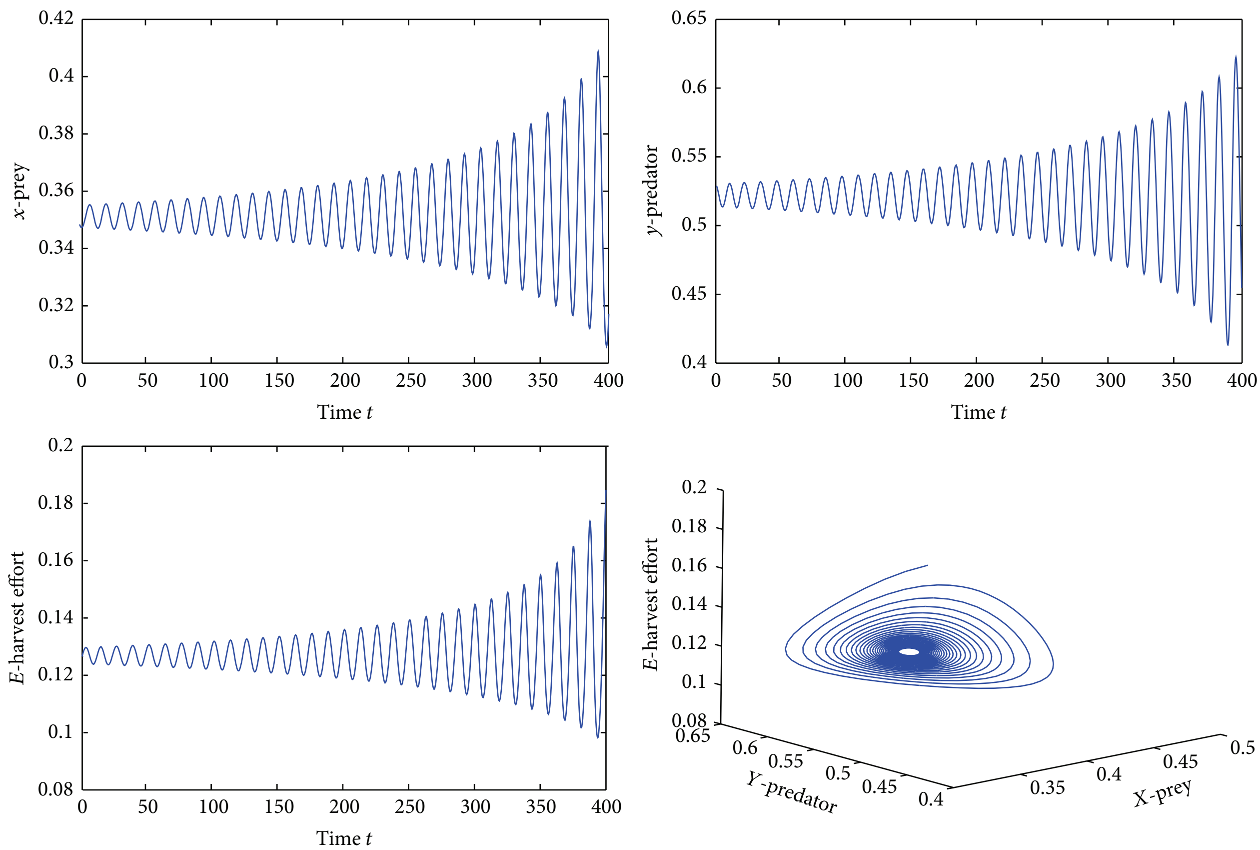

FIGURE 3: When $\mu = 0.171 > \mu_0$, the positive equilibrium point $X_0(\mu)$ is unstable.

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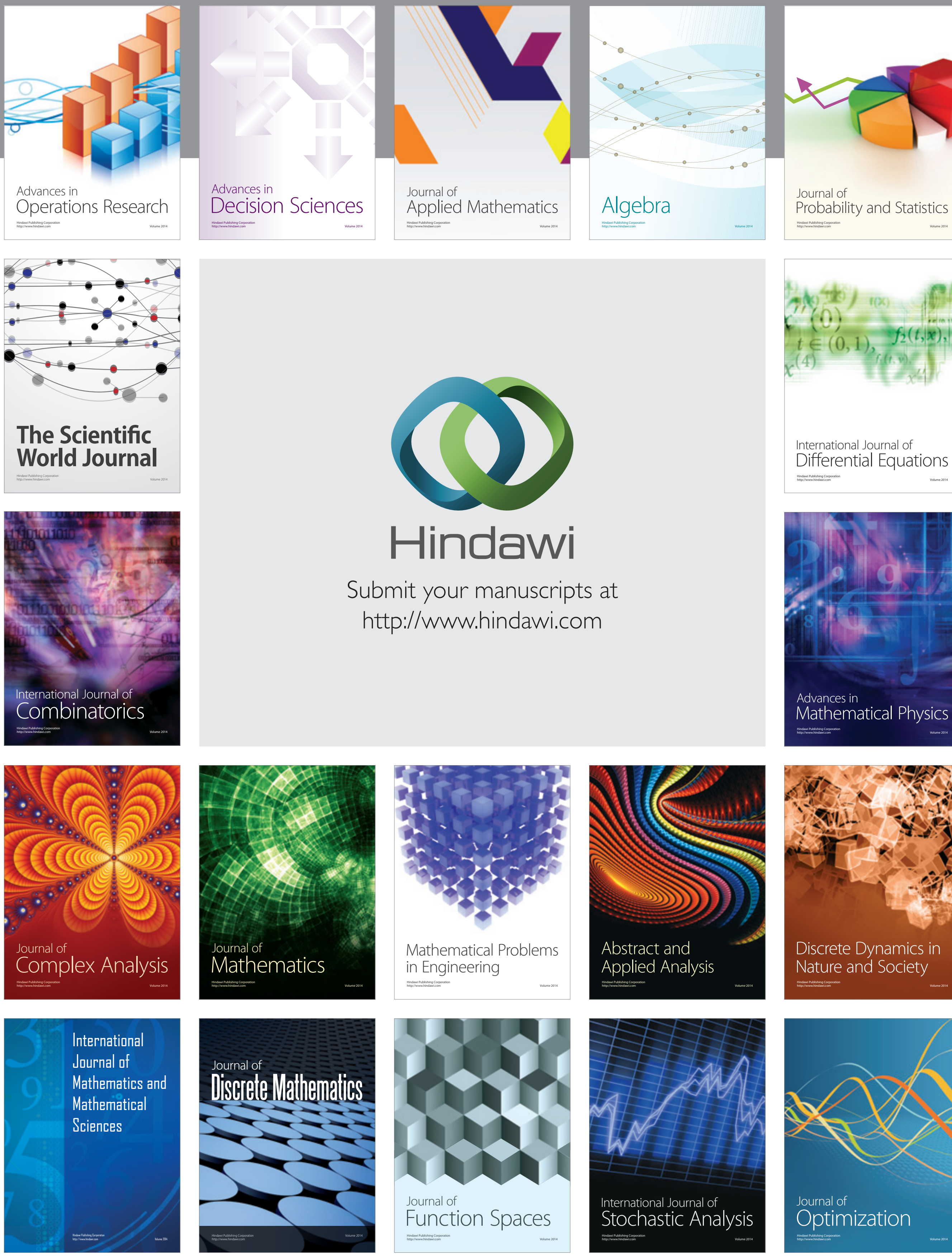
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