Appendix

Proof of Lemma 1.

Retailer r 's reaction function given wholesale prices w_i can be derived from the First Order Conditions (FOC) of Eq.(3) .

According to equation (3) we can get:

$$\frac{\partial \Pi_r}{\partial p_i} = \frac{1-\theta-2p_i+2\theta p_{3-i}+w_i-\theta w_{3-i}}{1-\theta^2}=0\ ,\quad i=1,2\ . \tag{4}$$

For which the Hessian matrix is negative due to $\frac{\partial \Pi_r}{\partial p_1{}^2}=-\frac{2}{1-\theta^2}<0$ and $\frac{\partial^2\Pi_r}{\partial p_1{}^2}\frac{\partial^2\Pi_r}{\partial p_2{}^2}-\frac{\partial^2\Pi_r}{\partial p_1\partial p_2}\frac{\partial^2\Pi_r}{\partial p_2\partial p_1}=\frac{4}{(1-\theta^2)}>0$ for any [0, 1), satisfying the second order condition for the maximum.

From Eq. (4), retailer r 's reaction functions can be derived:

$$p_i(w_i, w_{3-i})=\frac{1+w_i}{2},\quad i=1,2\ . \tag{5}$$

By the reaction function (5), the manufacturer's wholesale price can be reached from the following FOC of the manufacturers i 's profit maximization problems.

$$\frac{\partial \Pi_i}{\partial w_i} = \frac{1-\theta-2w_i+\theta w_{3-i}}{2-2\theta^2}=0\ \ i=1,2\ . \tag{6}$$

By solving Eq. (6) = 0results into the following whole prices

$$w_i^{MA*}=\frac{1-\theta}{2-\theta}\ \ i=1,2\ ,$$

and the corresponding retail prices can obtain fromEq. (5) :

$$p_i^{MA*}=\frac{3-2\theta}{2(2-\theta)}\ \ i=1,2\ .$$

Furthermore, it is easy to get the optimal profits of manufacturer i as follows:

$$\Pi_1^{MA*}=\Pi_2^{MA*}=\frac{1-\theta}{2(2-\theta)^2(1+\theta)}\ .$$

Proof of Lemma 2.

According to equation (3) we can get:

$$\frac{\partial \Pi_r}{\partial p_i} = \frac{1-\theta-2p_i+2\theta p_{3-i}+\theta p_{o1}+w_i-\theta w_{3-i}}{1-\theta^2}=0\,,\ \ i=1,2\,. \tag{7}$$

It is easily established that the second order Hessian matrix of this problem is negative definite due to $\frac{\partial \Pi_r}{\partial p_1{}^2}=-\frac{2}{1-\theta^2}<0$ and $\frac{\partial^2 \Pi_r}{\partial p_1{}^2}\frac{\partial^2 \Pi_r}{\partial p_2{}^2}-\frac{\partial^2 \Pi_r}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_r}{\partial p_2 \partial p_1}=\frac{4}{(1-\theta^2)}>0$ for any [0, 1), satisfying the second order condition for the maximum.

By Eq. (7) we get: the reflect functionretailersof sales can be:

$$p_i(w_i, w_{3-i})=\frac{1-\theta+\theta p_{o1}+w_i-\theta w_i}{2(1-\theta)}\,,\ \ i=1,2\,. \tag{8}$$

By Eq. (8) the manufacturer's wholesale price Nash equation, we can get the following equation to allow maximize manufacturer *i*'s profit.

$$\frac{\partial \Pi_1}{\partial w_1} = \frac{1-\theta+2\theta p_{o1}-2w_1+\theta w_2}{2-2\theta^2}=0\,, \tag{9}$$

$$\frac{\partial \Pi_2}{\partial w_2} = \frac{1-\theta+\theta p_{o1}+\theta w_1-2w_2}{2-2\theta^2}=0\,, \tag{10}$$

$$\frac{\partial \Pi_1}{\partial p_{o1}} = \frac{4(\theta-1+\theta^2)p_{o1}+(1-\theta)(2\eta_1+2\theta-2\eta_1\theta+2\theta w_1+\theta w_2)}{2(1-\theta)^2(1+\theta)}=0\,. \tag{11}$$

By coupling the equation (9), (10), (11) = 0, we can get the wholesale price and the price of direct sales channels of m_1 :

$$w_1^{MB*}=\frac{(1-\theta)(8-(4-8\eta_1)\theta-(3+6\eta_1)\theta^2-(3+2\eta_1)\theta^3)}{2(8-8\theta-15\theta^2+5\theta^3+4\theta^4)}\,,$$

$$w_2^{MB*}=\frac{(1-\theta)(4-2(3-\eta_1)\theta+(1-2\eta_1)\theta^2)}{8-16\theta+\theta^2+4\theta^3} \text{ and}$$

$$p_{o1}^{MB*}=\frac{(1-\theta)(2+\theta)(7\theta-5\theta^2+2\eta_1(2-3\theta+\theta^2))}{2(8-8\theta-15\theta^2+5\theta^3+4\theta^4)}\,.$$

Therefore, the price of two products sold by retailer R can be obtained from (8):

$$p_1^{MB*}=\frac{24-4(7-4\eta_1)\theta-(15+22\eta_1)\theta^2+(7+2\eta_1)\theta^3+(6+4\eta_1)\theta^4}{4(8-8\theta-15\theta^2+5\theta^3+4\theta^4)} \text{ and}$$

$$p_2^{MB*}=\frac{24-4(7-3\eta_1)\theta-2(11+6\eta_1)\theta^2+(19-6\eta_1)\theta^3+(1+6\eta_1)\theta^4}{4(8-8\theta-15\theta^2+5\theta^3+4\theta^4)}.$$

Manufacturers optimal profit function can be expressed as:

$$\Pi_1^{MB*}=\frac{\begin{array}{c}64-192\theta+392\theta^2-560\theta^3+111\theta^4+250\theta^5-81\theta^6-20\theta^7+4{\eta_1}^2(1-\theta)^2\\ (32-64\theta+32\theta^3-5\theta^4-4\theta^5)+4\eta_1\theta(112-352\theta+264\theta^2+93\theta^3-136\theta^4+7\theta^5+12\theta^6)\end{array}}{8(1+\theta)^2(8-16\theta+\theta^2+4\theta^3)^2}$$

$$\Pi_2^{MB*}=\frac{(1-\theta)(4-2(3-\eta_1)\theta+(1-2\eta_1)\theta^2)^2}{2(1+\theta)(8-16\theta+\theta^2+4\theta^3)^2}.$$

Proof of Lemma 3.

Here we omit the proof process due to the symmetry with Lemma 2.

Proof of Proposition 1.

When $\eta_1=\eta_2=\eta$, under channel MD

$$w_1^{MD*}=w_2^{MD*}=\frac{2(1-\theta)^2(2-(1-3\eta)\theta)}{8-16\theta-11\theta^2+13\theta^3},$$

$$p_1^{MA*}=p_2^{MA*}=\frac{12-2(13-7\eta)\theta+(11-24\eta)\theta^2+(1+10\eta)\theta^3}{16-32\theta-22\theta^2+26\theta^3},$$

$$D_1^{MA*}=D_2^{MA*}=\frac{4-2(5-\eta)\theta+(5-2\eta)\theta^2}{16-32\theta-22\theta^2+26\theta^3}$$

$$\Pi_r^{MD*}=\frac{(4-2(5-\eta)\theta+(5-2\eta)\theta^2)^2}{2(1+\theta)(8-24\theta+13\theta^2)^2},$$

$$\Pi_1^{MD*}=\Pi_2^{MD*}=\frac{\begin{array}{c}(8-40\theta+111\theta^2-178\theta^3+132\theta^4-35\theta^5+2\eta^2(1-\theta)^2\\ (8-24\theta+23\theta^2-8\theta^3)+2\eta\theta(32-132\theta+193\theta^2-120\theta^3+27\theta^4))\end{array}}{(1+\theta)^2(8-24\theta+13\theta^2)^2}.$$

To make sure that $D_i^*, D_{oi}^*, w_i^*, p_i^*, p_{oi}^*$ is not negative, we can get that w_i^{MD*} has the tightest constraints. To make sure $w_i^{MD*}\geq 0$, which $\frac{2(1-\theta)^2(2-(1-3\eta)\theta)}{8-16\theta-11\theta^2+13\theta^3}\geq 0$, we can get $\theta<\frac{2}{13}(6-\sqrt{10})$.

Proof of Proposition 2 and 3

When $\eta_1=\eta_2=\eta$, to make sure that $D_i^*, D_{oi}^*, w_i^*, p_i^*, p_{oi}^*$ is not negative, we can get that $p_{oi}^{MB*}\geq 0$ has the tightest constraints

$$\frac{(1-\theta)(8-(4-8\eta)\theta-(3+6\eta)\theta^2-(3+2\eta)\theta^3)}{2(8-8\theta-15\theta^2+5\theta^3+4\theta^4)} \geq 0 \text{ and } \frac{(1-\theta)(4-2(3-\eta)\theta+(1-2\eta)\theta^2)}{8-16\theta+\theta^2+4\theta^3} \geq 0.$$

We can get real root $\theta_1 < \frac{113}{200}$.