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Research Article

A Pest Management Model with Stage Structure and Impulsive State Feedback Control

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A pest management model with stage structure and impulsive state feedback control is investigated. We get the sufficient condition for the existence of the order-1 periodic solution by differential equation geometry theory and successor function. Further, we obtain a new judgement method for the stability of the order-1 periodic solution of the semicontinuous systems by referencing the stability analysis for limit cycles of continuous systems, which is different from the previous method of analog of Poincarè criterion. Finally, we analyze numerically the theoretical results obtained.

1. Introduction

Banana leaves diseases are divided into epiphyte and virus. Banana bunchy top disease (i.e., Prawn banana, Green banana, Banana) is one of virus diseases, caused by Banana bunchy top virus. Banana farmers call it an incurable disease. Banana aphids are the major propagation medium of banana virus diseases. The development of banana aphids includes three stages: egg, nymph, and adult (winged form). Eggs do not carry and spread virus. Nymphs transmit virus to healthy plants only through short-distance crawling since genitalia and wings of nymphs are not fully developed yet, and therefore infected nymphs have slight infective power. After 4 instars, nymphs grow into adults which have fully developed genitalia and wings and can oviposit and transmit virus to healthy plants through migrating after piercing and sucking the virus of diseased plants, so infected adults have strong infective power. To avoid the outbreak of banana aphids, we will use ovicides to kill eggs or use insecticides to kill nymphs and adults.

In pest management, we spray pesticides only when pest density increases to a certain level called ET (economic threshold, i.e., pest population density at which control measures should be adopted to prevent an increasing pest population from reaching the economic injury level). ET is the index of pest density. Crop output will not decrease much when pest density is lower than ET; thus, we need not adopt any control measure. Once pest density rises to ET, some measures must be carried out to prevent EIL (economic injure tolerate level) from happening. To control pests, such a measure for spraying pesticides is always adopted when pest density arrives at a given ET.

Considering that immature pests cause a minor damage to crops, in this paper, we will spray insecticides when the density of immature pests increases to ET, which is a more effective preventive measure than we do when the density of mature pests increases to ET. Usually, insecticides have specificity; in other words, insecticides (such as 2000 to 2500 times dilution of acetamiprid 3% EC, 15000 times dilution of imidacloprid 70% WG, 1000 times dilution of omethoate 40% EC, and 2500 to 3000 times dilution of sumicidin 20% EC) can only kill nymphs and adults but cannot kill eggs. Therefore, a pest management model with stage structure and impulsive state feedback control is constructed as follows:

$$\frac{dx}{dt} = a_1 a_2 y - bx = ay - bx = P(x, y),$$
$$\frac{dy}{dt} = cx - dy = Q(x, y),$$
$$x < x^*,$$

$$\Delta x = -\alpha x,$$
$$\Delta y = -\beta y, \qquad x = x^{*}, \tag{1}$$

where $x(t), y(t)$ denote the proportions of immature pests (nymphs) and mature pests (adults) at time t, respectively, $a = a_1 a_2$ denotes the transformation rate from mature to immature pests, where a_1 denotes the birth rate of mature pests, a_2 denotes the transformation rate from eggs to immature pests, c denotes the transformation rate from immature to mature pests, b, d denote the death rate of immature and mature pests, respectively, a_1, a_2, a, b, c, d are positive constants, $0 < \alpha < 1$, $0 < \beta < 1$ are the ratios of killing immature and mature pests by spraying pesticides, respectively, and x^{*} denotes ET.

At present, for stage structure pest management model with impulsive effect, extinction and permanence have been proved by using Floquet theorem and comparison theorem [1–4]. For impulsive state feedback control systems, the sufficient condition for the existence and the orbitally asymptotically stability of the order-1 periodic solutions have been obtained by differential equation geometry theory, the method of successor function, and analog of Poincarè criterion [5–14]. However, for the pest management model with stage structure and impulsive state feedback control, almost no one investigates. In this paper, we try to obtain a new judgement method for the stability of the order-1 periodic solution by referencing the stability analysis of limit cycles for continuous systems. This is a superior method, by which the more perfect and simple conclusions than the others are obtained.

In the next section, we give some preliminaries. In Section 3, we get the sufficient condition for the existence of the order-1 periodic solution of system (1) by differential equation geometry theory and successor function. In Section 4, referencing the stability analysis of the limit cycles for continuous dynamic systems, we prove the order-1 periodic solution of system (1) is orbitally asymptotically stable under some conditions. In Section 5, we analyze numerically the theoretical results obtained.

2. Preliminaries

Definition 1 (see [15]). Suppose impulsive state differential equation

$$\frac{dx}{dt} = P(x, y), \qquad \frac{dy}{dt} = Q(x, y), \qquad (x, y) \notin M\{x, y\}, \tag{2}$$
$$\Delta x = \alpha(x, y), \qquad \Delta y = \beta(x, y), \qquad (x, y) \in M\{x, y\},$$

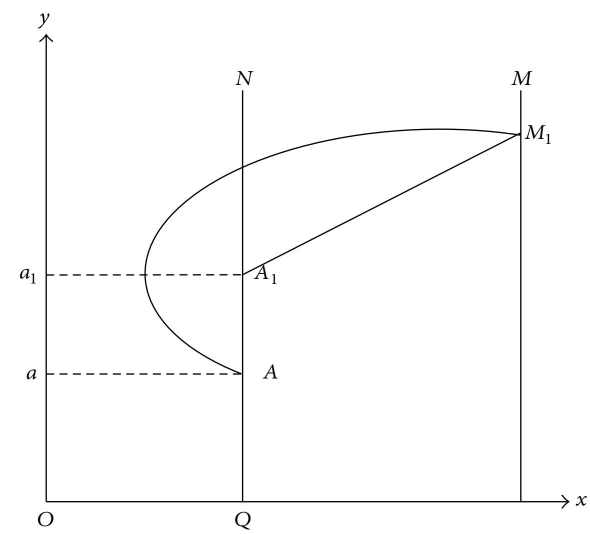

Figure 1: Successor function $f(A) = a_1 - a$.

whose solution mapping composes the system called semi-continuous dynamic system, denoted by (Ω, f, φ, M). Set initial point of mapping $p \in \Omega = R_2^{+} \setminus M\{x, y\}$; φ is a continuous mapping, $\varphi(M) = N$, and φ is called impulse mapping, where $M(x, y)$ and $N(x, y)$ are straight lines or curves on the plane $R_2^{+} = \{(x, y) \in R^2 : x \geq 0, y \geq 0\}$, $M\{x, y\}$ denotes impulse set, and $N\{x, y\}$ denotes phase set.

In system (1), impulse set $M = \{(x, y) \in R_2^{+} \mid x = x^{*}, y \geq 0\}$, impulse mapping φ: $(x, y) \in M \rightarrow ((1-\alpha)x^{*}, y) \in R_2^{+}$, phase set $N = \varphi(M) = \{(x, y) \in R_2^{+} \mid x = (1-\alpha)x^{*}, y \geq 0\}$. Therefore, system (1) composes a semicontinuous dynamic system (Ω, f, φ, M).

Definition 2. Let $f(P, t)$ be the semicontinuous dynamical system mapping described by system (2) at $\Omega \rightarrow \Omega$, and $f(P, t)$ is a mapping in itself. If there are a point P_1 in phase set N and a t_1 such that $f(P_1, t_1) = Q_1 \in M\{x, y\}$ (pulse mapping is $\varphi(Q_1) = \varphi(f(P_1, t_1)) = P_1 \in N$), then $f(P_1, t_1)$ is said to be the order-1 periodic solution.

Definition 3 (see [15]). Suppose N is the phase set of system (1), M is the impulse set of system (1), and both N and M are straight lines (see Figure 1). The intersection point of N and x-axis is Q, the distance between point A ($A \in N$) and point Q is noted by a, M_1 denotes the intersection point of trajectory passing through point A and M, phase point of M_1 is A_1 ($A_1 \in N$), and the distance between A_1 and Q is noted by a_1. One defines subsequent point of A as A_1, and the successor function of A is $f(A) = a_1 - a$.

Remark 4. If $f(A) = 0$, the trajectory passing through point A is the order-1 periodic solution of the system.

Lemma 5 (see [15]). *Successor function $f(A)$ is continuous.*

According to Lemma 5, we can get the following lemma.

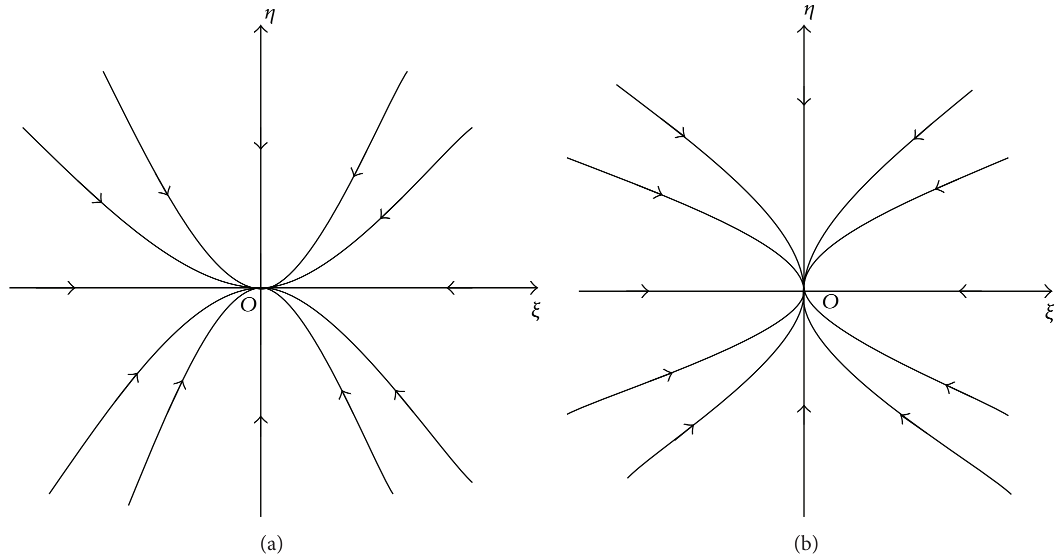

Figure 2: (a) $q > 0$ and $\lambda_1 > \lambda_2$. (b) $q > 0$ and $\lambda_1 < \lambda_2$.

Lemma 6 (see [15]). *Assume continuous dynamical system* (X, Ψ)*; if there exist two points* A, B *in the phase set such that successor function* $f(A) > 0$, $f(B) < 0$, *we can find a point* C *between* A *and* B *in the phase set satisfying* $f(C) = 0$. *So there must exist an order-1 periodic solution passing through point* C.

3. Existence of the Order-1 Periodic Solution

For system (1), if $\alpha = 0$, $\beta = 0$, that is, without impulse effects, there is a unique singular point $O(0,0)$ if and only if

$$\begin{vmatrix} -b & a \\ c & -d \end{vmatrix} = bd - ac \neq 0. \quad (3)$$

Taking the transform

$$\begin{pmatrix} x \\ y \end{pmatrix} = T \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \quad (4)$$

system (1) turns into

$$\frac{d}{dt}\begin{pmatrix} \xi \\ \eta \end{pmatrix} = T^{-1} A T \begin{pmatrix} \xi \\ \eta \end{pmatrix}, \quad A = \begin{bmatrix} -b & a \\ c & -d \end{bmatrix}. \quad (5)$$

There are four forms for Jordan standard of the two-dimensional matrix:

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}, \quad \begin{bmatrix} \lambda_1 & 0 \\ 1 & \lambda_1 \end{bmatrix}, \quad \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{bmatrix}, \quad \begin{bmatrix} u & v \\ -v & u \end{bmatrix}, \quad (6)$$

where λ_1, λ_2 are two real roots of characteristic equation

$$D(\lambda) = \begin{vmatrix} -b-\lambda & a \\ c & -d-\lambda \end{vmatrix} = \lambda^2 + p\lambda + q = 0, \quad (7)$$

where $p = b + d$, $q = bd - ac$, and obviously, $p > 0$, $p^2 - 4q > 0$.

By theory of stability, singular point $O(0,0)$ has the following two cases [16]:

(i) if $q > 0$, $O(0,0)$ is an asymptotically stable node; see Figures 2(a) and 2(b);

(ii) if $q < 0$, $O(0,0)$ is a saddle point; see Figures 3(a) and 3(b).

Theorem 7. *If* $q < 0$, *that is,* $bd - ac < 0$, *there exists a point* $C \in N$ *satisfying* $f(C) = 0$*; that is to say, there exists an order-1 periodic solution of system (1).*

Proof. If $q < 0$, that is, $bd - ac < 0$, for system (1), the impulse set M is straight line $x = x^*$, the phase set N is straight line $x = (1-\alpha)x^*$, and N, M intersect x-axis at G, F, respectively (see Figure 4). Denote isoclinic lines $dy/dt = 0$, $dx/dt = 0$ and boundary between the two isoclinic lines as L_1, L_2, L_3, where L_2 intersects N at B and L_3 intersects N, M at E, A, respectively.

Firstly, we analyze the existence of order-1 periodic solution of system (1) in the domain ΔOFA.

Suppose $C \in N$ is the phase point of A; then $C_y < E_y$, where C_y, E_y is the y coordinate of C, E, respectively. Choose

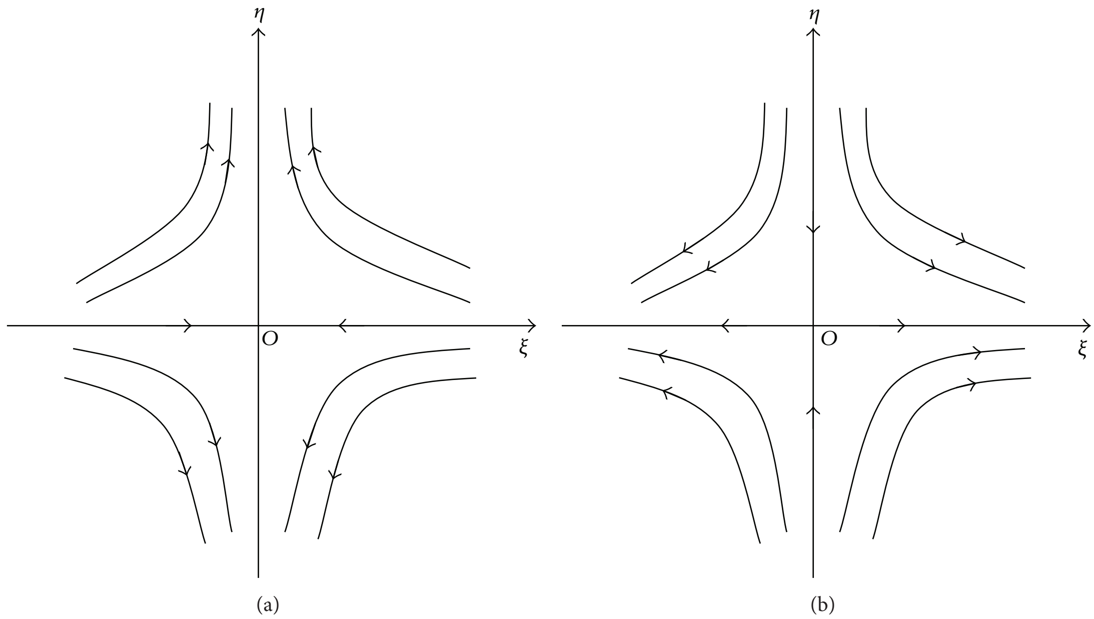

FIGURE 3: (a) $q < 0$ and $\lambda_1 < 0, \lambda_2 > 0$. (b) $q < 0$ and $\lambda_1 > 0, \lambda_2 < 0$.

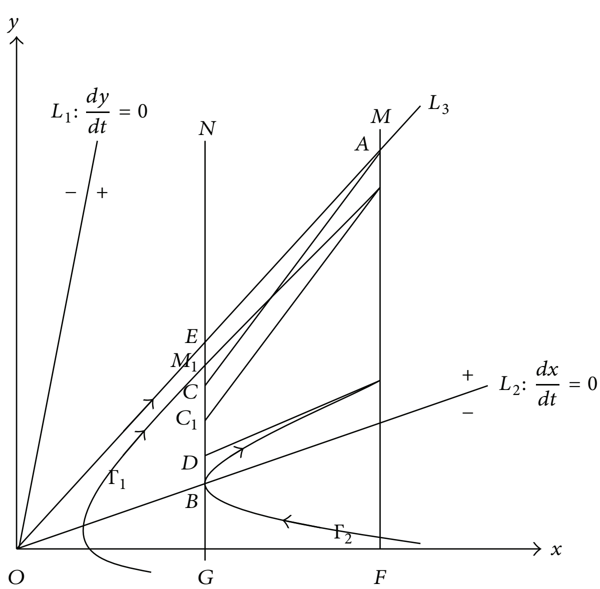

FIGURE 4: The existence of order-1 periodic solution of system (1) in the domain ΔOFA when $D_y > B_y$.

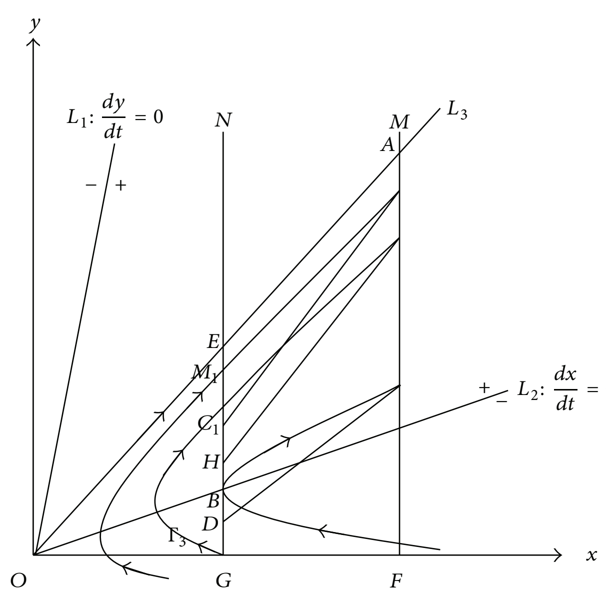

FIGURE 5: The existence of order-1 periodic solution of system (1) in the domain ΔOFA when $D_y < B_y$.

M_1 between E and C; the trajectory Γ_1 passing through M_1 intersects the phase set N at C_1 after impulse effect, and then C_1 is the subsequent point of M_1. Since distinct trajectories do not intersect, C_1 must be below C; we have $C_{1y} < C_y < M_{1y}$, where C_{1y}, M_{1y} is the y coordinate of C_1, M_1, respectively. Therefore, $f(M_1) = C_{1y} - M_{1y} < 0$.

Suppose the trajectory Γ_2 passing through B intersects the phase set N at D after impulse effect; then D is the subsequent point of B, and there are two cases.

Case 1. If D is above B, then $D_y > B_y$, where D_y, B_y is the y coordinate of D, B, respectively; we have $f(B) = D_y - B_y > 0$. By Lemma 6, there exists a point $C \in \overline{BM_1} \subset N$ such that $f(C) = 0$. Therefore, there exists an order-1 periodic solution of system (1) passing through C. The proof is completed.

Case 2. If D is below B, then $D_y < B_y$; we have $f(B) = D_y - B_y < 0$ (see Figure 5). In addition, the trajectory Γ_3 passing through G intersects the phase set N at H after impulse

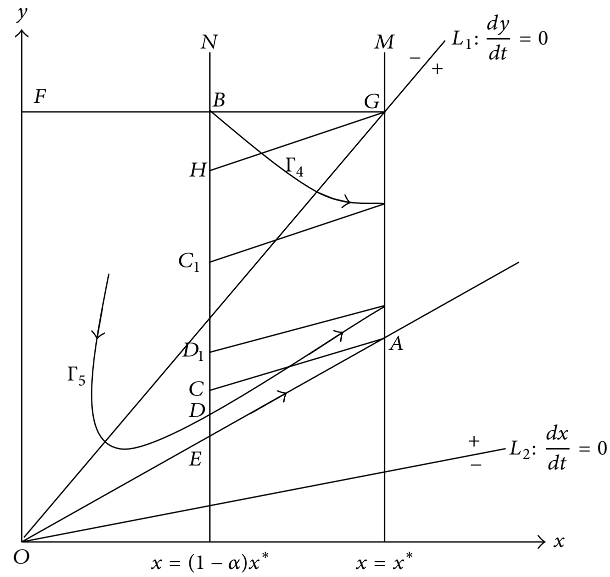

Figure 6: The existence of order-1 periodic solution of system (1) in the trapezoid $OAGF$.

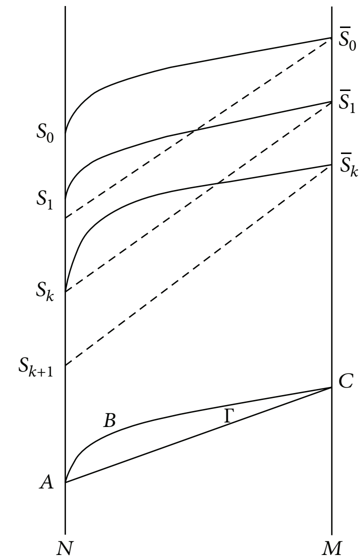

Figure 7: The order-1 periodic solution Γ of system (1) is stable.

effect; then H is the subsequent point of G, and H must be below C_1 and above G because distinct trajectories do not intersect. Therefore, we have $f(G) = H_y - G_y = H_y > 0$, where H_y, G_y is the y coordinate of H, G, respectively. By Lemma 6, there exists a point $C \in \overline{GB} \subset N$ such that $f(C) = 0$. The proof is completed.

Secondly, suppose L_3 intersects N, M at E, A, respectively, and L_1 intersects M at G. Draw a straight line which is perpendicular to N and y-axis; the foot points are B, F, respectively (see Figure 6). Let us analyze the existence of order-1 periodic solution of system (1) in the trapezoid $OAGF$.

Suppose C, H are the phase points of A, G, respectively. On the one hand, the trajectory Γ_4 passing through B intersects the phase set N at C_1 after impulse effect; then C_1 is the subsequent point of B, and C_1 must be below H because distinct trajectories do not intersect. We have $f(B) = C_{1y} - B_y < 0$, where C_{1y}, B_y is the y coordinate of C_1, B, respectively.

On the other hand, choose a point $D \in N$ between E and C. The trajectory Γ_5 passing through D intersects the phase set N at D_1 after impulse effect, and then D_1 is the subsequent point of D, and D_1 must be above C because distinct trajectories do not intersect. We have $f(D) = D_{1y} - D_y > 0$, where D_{1y}, D_y is the y coordinate of D_1, D, respectively.

By Lemma 6, there exists a point $C \in \overline{BD} \subset N$ such that $f(C) = 0$. Therefore, there exists an order-1 periodic solution of system (1) passing through C. The proof is completed. □

4. Stability of the Order-1 Periodic Solution

Definition 8 (see [17]). On the positive half-trajectory of semicontinuous dynamic system denoted by $f(P, I^+)$, $I^+ = (0, +\infty)$, choose any time series $\{0 \le t_1 < t_2 < \cdots < t_n < \cdots\}$ such that $\lim_{t\to\infty} t_n = +\infty$. If Q is the limit point of point range $\{f(P, t_n)\}$, $n = 1, 2, \ldots$, One calls Q the ω limit point of point range $\{f(P, t_n)\}$, $n = 1, 2, \ldots$. The set Ω made up of all limit points of point range $\{f(P, t_n)\}$, $n = 1, 2, \ldots$, is called ω limit set.

Definition 9. Assume Γ is the order-1 periodic solution of semicontinuous dynamic system. If there exists a neighborhood $U(\Gamma)$ sufficiently small such that ω limit set of trajectory starting from any point $P \in U(\Gamma)$ is always Γ, the order-1 periodic solution Γ is stable. Otherwise, the order-1 periodic solution Γ is unstable.

In system (1), A is any point of the phase set N (see Figure 7); assume the single-closed curve consisting of curve $\widehat{ABC}$ and line segment $\overline{CA}$ is an order-1 periodic solution of system (1), denoted by Γ. Get point S_0 near A; there exists a point range:

$$\{S_1, S_2, \ldots, S_k, S_{k+1} \ldots\}, \tag{8}$$

where

$$S_1, S_2, \ldots, S_k, S_{k+1}, \ldots \tag{9}$$

are the subsequent points of $S_0, S_1, \ldots, S_{k-1}, S_k, \ldots$, respectively.

Establish coordinates at phase set and near A, the coordinate of A is 0. Let

$$s_0, s_1, \ldots, s_k, s_{k+1} \ldots \tag{10}$$

denote the coordinates of points

$$S_0, S_1, \ldots, S_k, S_{k+1}, \ldots, \tag{11}$$

respectively.

Proposition 10. *For any point S_0 near A, when $k \to \infty$, the point range*

$$S_0, S_1, \ldots, S_k, S_{k+1}, \ldots \longrightarrow A, \tag{12}$$

that is,

$$s_0, s_1, \ldots, s_k, s_{k+1}, \ldots \longrightarrow 0, \tag{13}$$

and then the order-1 periodic solution is stable (unidirectional).

Proposition 11 (königs). *Assume $\bar{s} = f(s)$ is a continuous transform from line segment N to itself; $S = 0$ is a fixed point under the transform. If the part near origin of curve $\bar{s} = f(s)$ on the plane $(s, \bar{s})$ lies in the interior of the domain*

$$\left|\frac{\bar{s}}{s}\right| \leq 1 - \varepsilon\,(\geq 1 + \varepsilon), \quad \varepsilon > 0, \tag{14}$$

the fixed point $S = 0$ is stable (unstable).

Proof. We prove firstly that the fixed point $S = 0$ is stable. Choose $\eta > 0$ sufficiently small such that for any point S in noncentral neighborhood $U^0(0; \eta)$ of the fixed point $S = 0$, $|s| \leq \eta$.

Let

$$\left|\frac{\bar{s}}{s}\right| \leq 1 - \varepsilon = \delta < 1, \tag{15}$$

and we have

$$|\bar{s}| \leq \delta\,|s| < |s|. \tag{16}$$

For any point range

$$\{S, S_1, S_2, \ldots, S_k, S_{k+1}, \ldots\}, \tag{17}$$

where $S, S_k \in U^0(0; \eta)$, $k = 1, 2, \ldots, n, \ldots$, we get sequence

$$\{|s|, |s_1|, |s_2|, \ldots, |s_k|, |s_{k+1}|, \ldots\}. \tag{18}$$

From Figure 7, we have $|\bar{s}_k| \leq \delta |s_k|, |s_{k+1}| = (1 - \beta)|\bar{s}_k|, \ldots$ $(k = 0, 1, 2, \ldots)$.

Let $\bar{s}_0 = \bar{s}$, $s_0 = s$; it is easy to deduce that

$$\begin{aligned} |s_n| &= (1 - \beta)\,|\bar{s}_{n-1}| \leq (1 - \beta)\,\delta\,|s_{n-1}| \leq \cdots \\ &\leq (1 - \beta)^n\,\delta^n\,|s|, \end{aligned} \tag{19}$$

and hence $|s_n| \to 0$ when $n \to \infty$. Upon that, the fixed point $S = 0$ is stable.

In the same way, we prove the fixed point $S = 0$ is unstable. The proof is completed. □

Corollary 12. *Assume the derivative of function $\bar{s} = f(s)$ at $S = 0$ exists; then $S = 0$ is stable when $|d\bar{s}/ds|_{S=0} < 1$.*

From Figure 8, assume the closed orbit consisting of the curve $\widehat{ABC}$ and line segment $\overline{CA}$ is the order-1 periodic solution of system (20), denoted by Γ, where $A \in N$, $C \in M$, N is the phase set, and M is impulse set. Draw normal line

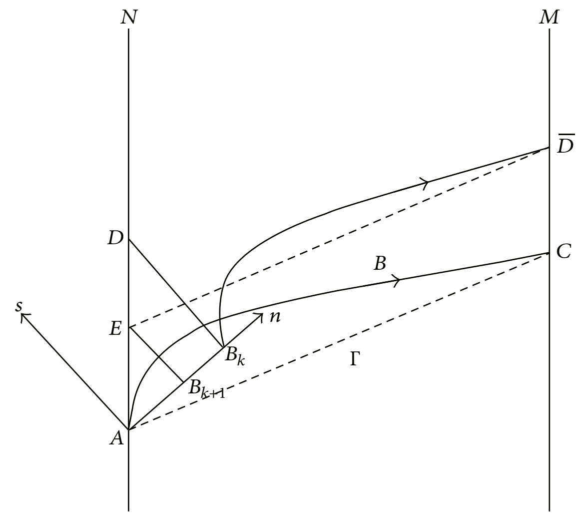

Figure 8: Establish coordinate system (s, n) on point A.

n passing through $A \in \Gamma$ and establish coordinate system (s, n) on point A. Choose any point $D \in N$ in small enough neighborhood of A. The trajectory starting from D intersects vertically n-axis at B_k and intersects impulse set M at $\overline{D}$. E denotes the phase point of $\overline{D}$, the trajectory passing through point E intersects vertically n-axis at B_{k+1} as t increases.

Assume rectangular coordinate of A is $(\varphi(s), \psi(s))$; then for B_k, there is the relation between its rectangular coordinates (x, y) and curvilinear coordinates (s, n):

$$\begin{aligned} x &= \varphi(s) - n\psi'(s), \\ y &= \psi(s) + n\varphi'(s), \end{aligned} \tag{20}$$

where

$$\begin{aligned} \varphi'(s) &= \left.\frac{dx}{ds}\right|_A = \frac{P_0}{\sqrt{P_0^2 + Q_0^2}}, \\ \psi'(s) &= \left.\frac{dy}{ds}\right|_A = \frac{Q_0}{\sqrt{P_0^2 + Q_0^2}}, \end{aligned} \tag{21}$$

where P_0, Q_0 denote the values of P, Q at the point A, respectively; we have

$$\begin{aligned} P_0 &= P(\varphi(s), \psi(s)), \\ Q_0 &= Q(\varphi(s), \psi(s)). \end{aligned} \tag{22}$$

From (20), it is easy that we have

$$\begin{aligned} \frac{dy}{dx} &= \frac{\psi'(s) + \varphi'(s)\,(dn/ds) + n\varphi''(s)}{\varphi'(s) - \psi'(s)\,(dn/ds) - n\psi''(s)} \\ &= \frac{Q(\varphi(s) - n\psi'(s), \psi(s) + n\varphi'(s))}{P(\varphi(s) - n\psi'(s), \psi(s) + n\varphi'(s))}, \end{aligned} \tag{23}$$

and hence

$$\frac{dn}{ds} = \frac{Q\varphi' - P\psi' - n\left(P\varphi'' + Q\psi''\right)}{P\varphi' + Q\psi'} = F(s, n). \tag{24}$$

Since there is a zero solution $n = 0$ for (24), when there exist continuous partial derivatives for functions P, Q, there exists the continuous partial derivative of $F(s, n)$ with respect to n also; (24) is written as

$$\frac{dn}{ds} = F_n'(s,n)\Big|_{n=0} n + o(n). \tag{25}$$

In order to calculate

$$\frac{dn}{ds} = F_n'(s,n)\Big|_{n=0}, \tag{26}$$

we first get

$$\begin{aligned}\varphi''(s) &= -\frac{Q_0}{P_0^2+Q_0^2}\left[P_0^2Q_{x0} + P_0Q_0\left(Q_{y0} - P_{x0}\right) - Q_0^2P_{y0}\right],\\ \psi''(s) &= \frac{P_0}{P_0^2+Q_0^2}\left[P_0^2Q_{x0} + P_0Q_0\left(Q_{y0} - P_{x0}\right) - Q_0^2P_{y0}\right],\end{aligned} \tag{27}$$

where $P_{y0}, P_{x0}, Q_{y0}, Q_{x0}$ denote partial derivatives of P, Q when $n = 0$, respectively. Since $P = P_0, Q = Q_0$ when $n = 0$, it is easy to know $P_0\varphi'' + Q_0\psi'' = 0$. By (24) and (27), we have

$$\begin{aligned}F_n'(s,n)\Big|_{n=0} &= \frac{P_0^2Q_{y0} - P_0Q_0\left(P_{y0} + Q_{x0}\right) + Q_0^2P_{x0}}{\left(P_0^2+Q_0^2\right)^{3/2}}\\ &= H(s),\end{aligned} \tag{28}$$

where $H(s)$ denotes the curvature of orthogonal trajectory at A for system (1). Therefore, the approximate equation of (25) is

$$\frac{dn}{ds} = H(s)\,n, \tag{29}$$

whose solution is

$$n = n_0 e^{\int_0^s H(s')ds'}, \quad n_0 = n(0). \tag{30}$$

Theorem 13. *Assume h is the length of curve $\widehat{ABC}$ which is a section of the order-1 periodic solution Γ of system (1). The order-1 periodic solution Γ is stable when*

$$\int_0^h H(s)\,ds < 0. \tag{31}$$

Proof. Let us investigate trajectory $B_k\overline{D}EB_{k+1}$ (see Figure 8). In the coordinate system (s, n), the ordinate of B_k is denoted by n_0 and the ordinate of $\overline{D}$ is denoted by n. From (30), we have

$$|n(h)| < |n_0| \tag{32}$$

when $\int_0^h H(s)ds < 0$, where h is the length of curve $\widehat{ABC}$. By Propositions 10 and 11, the order-1 periodic solution Γ is stable. □

Corollary 14 (see Diliberto [18]). *If the integral along the order-1 periodic solution Γ satisfies $H(s) < 0$, the order-1 periodic solution Γ is stable.*

Let $ds = \sqrt{P_0^2 + Q_0^2}dt$; the left of (31) can be rewritten as

$$\begin{aligned}\int_0^h H(s)\,ds &= \int_0^T \frac{1}{P_0^2+Q_0^2}\left[P_0^2Q_{y0} - P_0Q_0\left(P_{y0}+Q_{x0}\right) + Q_0^2P_{x0}\right]dt = \int_0^T \left[P_{x0} + Q_{y0} - \frac{P_0^2P_{x0} + P_0Q_0\left(P_{y0}+Q_{x0}\right) + Q_0^2Q_{y0}}{P_0^2+Q_0^2}\right]dt\\ &= \int_0^T \left(P_{x0}+Q_{y0}\right)dt - \int_0^T \frac{1}{2}\frac{1}{P_0^2+Q_0^2}\frac{d}{dt}\left(P_0^2 + Q_0^2\right)dt = \int_0^T \left(P_{x0}+Q_{y0}\right)dt - \int_0^T \frac{1}{2}\frac{d}{dt}\left[\ln\left(P_0^2+Q_0^2\right)\right]dt;\end{aligned} \tag{33}$$

that is,

$$\int_0^h H(s)\,ds = \int_0^T \left(P_{x0}+Q_{y0}\right)dt - \int_0^T \frac{1}{2}\frac{d}{dt}\left[\ln\left(P_0^2+Q_0^2\right)\right]dt < 0. \tag{34}$$

Consider the integral along the periodic solution Γ' of continuous system

$$J_{\Gamma'} = \int_0^T \frac{1}{2}\frac{d}{dt}\left[\ln\left(P_0^2+Q_0^2\right)\right]dt = 0, \tag{35}$$

and we suppose the integral along the order-1 periodic solution Γ of semicontinuous system has the same result.

Denote $F(x, y) = ((1/2)(d/dt))[\ln(P_0^2 + Q_0^2)]$.

Lemma 15. *If function $F(x, y)$ is continuous and differentiable, the integral along the order-1 periodic solution of system (1) satisfies*

$$\int_0^T \frac{dF(x,y)}{dt}dt = 0, \tag{36}$$

where period of the order-1 periodic solution is T.

Proof. Let Γ be an order-1 periodic solution of system (1) (see Figure 9); $S_1(t)$ denotes the curve of system (1) from $A(x_1, y_1)$ to $B(x_2, y_2)$, $S_1(t) = A$ when $t = 0$, and $S_1(t) = B$ when $t = T$. $S_2(t)$ denotes line segment $\overline{BA}$.

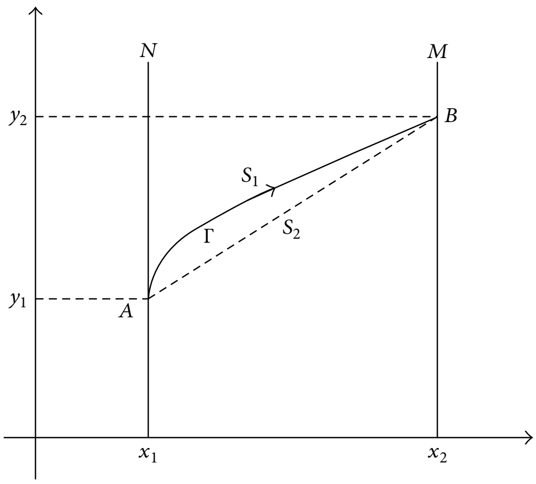

FIGURE 9: Γ is the order-1 periodic solution of system (1).

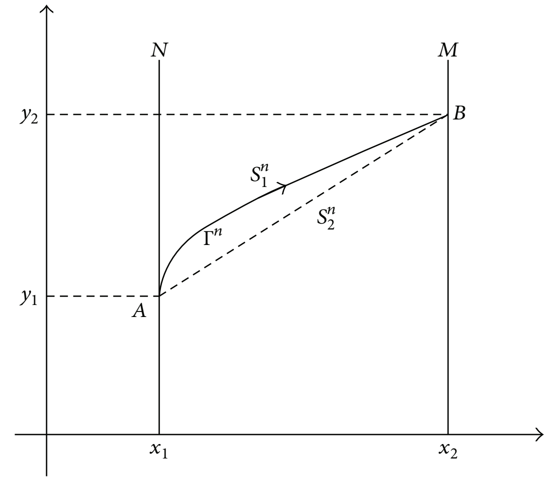

FIGURE 10: Γ^n is the order-1 periodic solution of system (37).

Take the transform $\tau = ((n-1)/n)t$; system (1) can be written as

$$\begin{aligned} \frac{dx}{d\tau} &= ay - bx = P(x,y), \\ \frac{dy}{d\tau} &= cx - dy = Q(x,y), \end{aligned} \qquad x < x^*, \qquad (37)$$
$$\begin{aligned} \Delta x &= -\alpha x, \\ \Delta y &= -\beta y, \end{aligned} \qquad x = x^*,$$

where the trajectory Γ^n of system (37) is similar to system (1) except for time variable.

Let $S_1^n(\tau)$ denote the curve of system (37) from $A(\overline{x}, y_1)$ to $B(\overline{x}, y_2)$ (see Figure 10); $S_1^n(\tau) = A$ when $\tau = 0$ and $S_1^n(\tau) = B$ when $\tau = ((n-1)/n)T$. $S_2^n(\tau)$ denotes line segment $\overline{BA}$; the parameter equation of $S_2^n(\tau)$ is

$$\begin{aligned} x &= \frac{(x_1 - x_2)n}{T}\tau + x_2, \\ y &= \frac{(y_1 - y_2)n}{T}\tau + y_2, \end{aligned} \qquad (38)$$

where $S_2^n(\tau) = B$ when $\tau = 0$ and $S_2^n(\tau) = A$ when $\tau = T/n$.

Obviously, system (37) $\to$ system (1); that is, $\Gamma^n \to \Gamma$, when $n \to \infty$, and thus we have

$$\begin{aligned} \Gamma \oint_0^T \frac{dF(x,y)}{dt}dt &= S_1 \oint_0^{((n-1)/n)T} \frac{dF(x,y)}{dt}dt + S_2 \oint_{((n-1)/n)T}^{T} \frac{dF(x,y)}{dt}dt \\ &= S_1 \oint_0^{((n-1)/n)T} \frac{dF(x,y)}{dt}dt + S_2 \oint_0^{T/n} \frac{dF(x,y)}{dt}dt \longrightarrow 0 \\ &\qquad (n \longrightarrow \infty); \end{aligned} \qquad (39)$$

that is, the integral along the order-1 periodic solution Γ of system (1) satisfies

$$\int_0^T \frac{dF(x,y)}{dt}dt = 0. \qquad (40)$$

The proof is completed. □

According to (34), we have the following theorem.

Theorem 16. *If the integral along the order-1 periodic solution Γ of system (1) satisfies*

$$\int_0^T \left(P_{x0} + Q_{y0}\right)dt < 0, \qquad (41)$$

Γ *is stable.*

Theorem 17. *The order-1 periodic solution of system (1) is stable.*

Proof. Since

$$\frac{\partial P(x,y)}{\partial x} + \frac{\partial Q(x,y)}{\partial y} = -b - d < 0, \qquad (42)$$

by Theorem 16, the order-1 periodic solution of system (1) is stable. The proof is completed. □

5. Numerical Analysis and Discussion

Without impulse effects, there is an equilibrium point $O(0,0)$ for system (1). If $q > 0$, $O(0,0)$ is an asymptotically stable

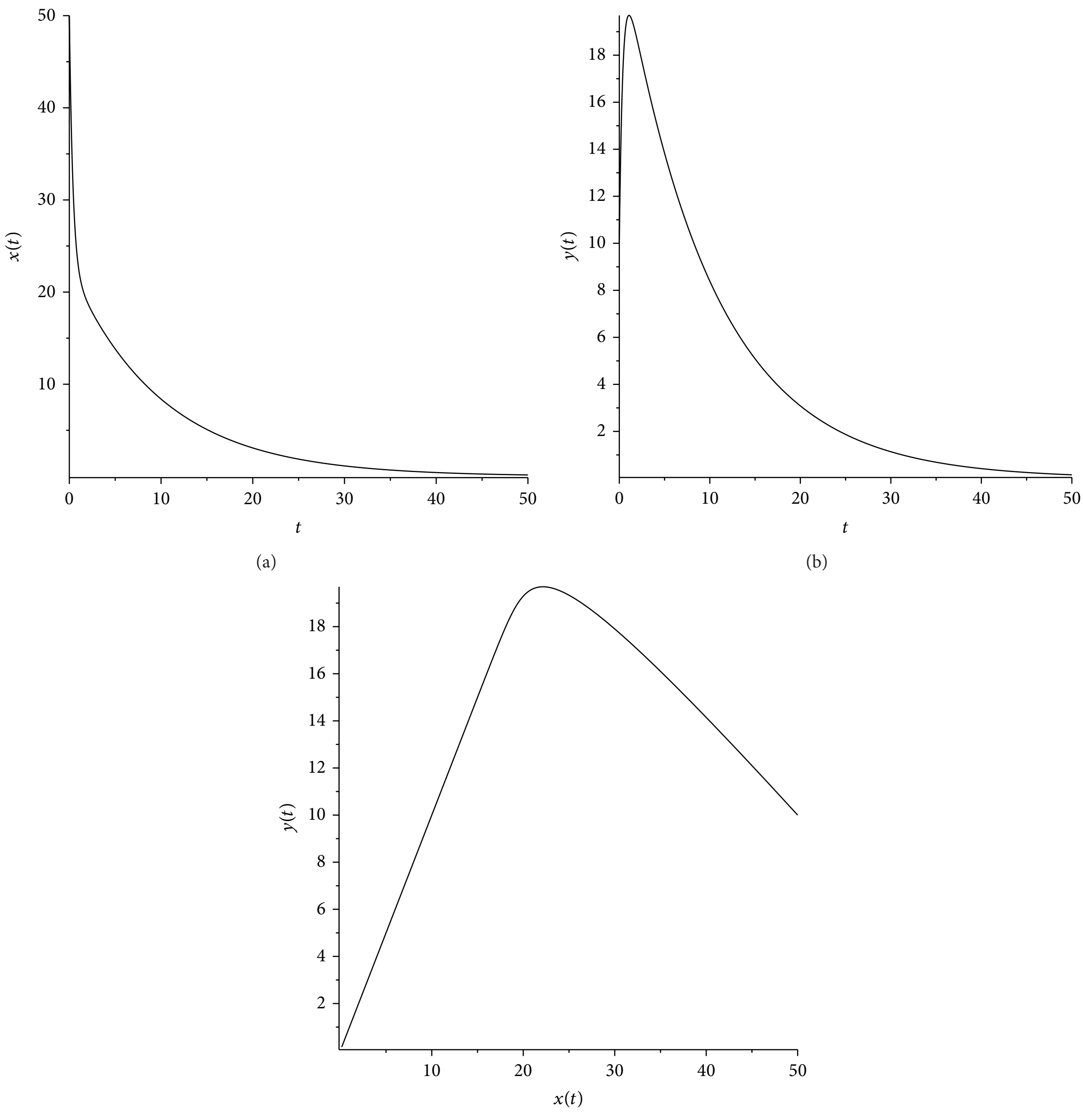

Figure 11: Time series and phase portrait of system (1) with $a = 1.7, b = 1.8, c = 0.8, d = 0.9, x^* = 50, \alpha = 0, \beta = 0, (x_0, y_0) = (50, 10)$.

node; if $q < 0$, $O(0, 0)$ is a saddle point. According to Theorem 7, if $q < 0$, that is, $bd - ac < 0$, there exists an order-1 periodic solution of system (1).

To verify the theoretical results obtained in this paper, we choose q as the parameter and analyze numerically the following cases.

Case 1. Let $a = 1.7, b = 1.8, c = 0.8, d = 0.9, x^* = 50, \alpha = 0, \beta = 0, x_0 = 50, y_0 = 10$; we have $q = 0.26$ (see Figure 11). According to the above discussion, $O(0, 0)$ is an asymptotically stable node when $q > 0$. It implies that $x(t), y(t)$ tend to be extinct as t increases without any control measures.

Case 2. Choose α as the control parameter; let $a = 1.7, b = 1.5, c = 0.8, d = 0.6, x^* = 50, \alpha = 0.1, \beta = 0.3, x_0 = 40, y_0 = 35$; we have $q = -0.46$, the impulse set $M = \{(x, y) \in R_2^+ \mid x = 50, y \geq 0\}$, and the phase set $N = \{(x, y) \in R_2^+ \mid x = (1 - \alpha)x^* = 45, y \geq 0\}$. According to Theorem 7, if $q < 0$, that is, $bd - ac < 0$, there exists an order-1 periodic solution (see Figure 12). We can observe that there exists an order-1 periodic solution of system (1) which lies between the phase set and the impulse set (i.e., between 45 and 50).

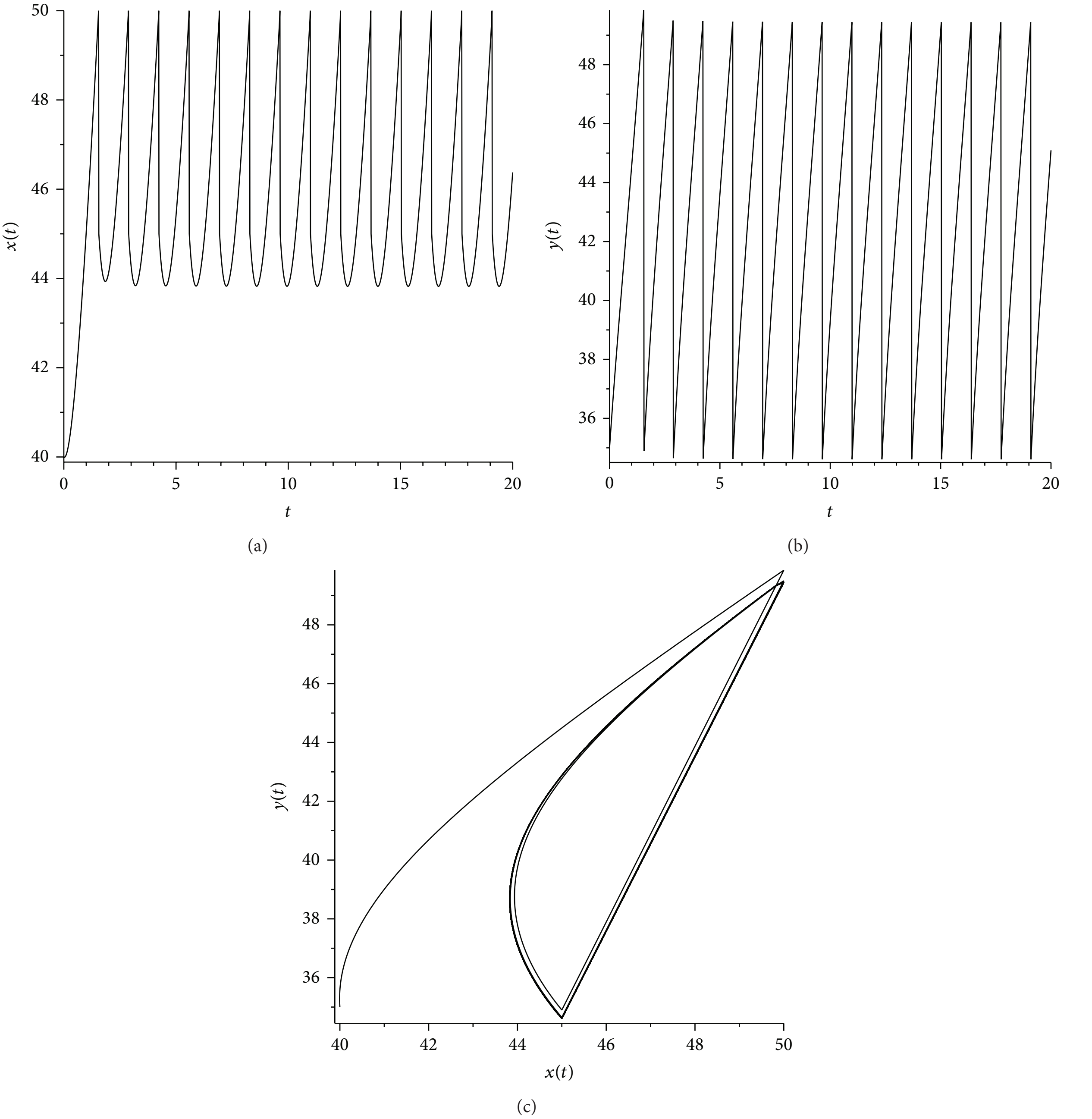

Figure 12: Time series and phase portrait of system (1) with $a = 1.7, b = 1.5, c = 0.8, d = 0.6, x^* = 50, \alpha = 0.1, \beta = 0.3, (x_0, y_0) = (40, 35)$.

Changing $\alpha = 0.8, x_0 = 5, y_0 = 40$, the rest of the parameters are the same as Figure 12, and we obtain Figure 13. From the phase portrait of Figure 13, we can observe that there exists an order-1 periodic solution of system (1) which lies between the phase set and the impulse set (i.e., between 10 and 50).

Figures 12 and 13 give the time series and phase portraits when $\alpha < \beta$ ($\alpha = 0.1, \beta = 0.3$) and $\alpha > \beta$ ($\alpha = 0.8, \beta = 0.3$), respectively, and show different positions of the periodic solution under different parameter values and different initial values. Furthermore, the phase portrait of Figure 12 indicates that the mature pests always keep increasing, but Figure 13 indicates that the mature pests firstly decrease and then begin to increase. Therefore, the control parameter α ($\alpha > \beta$ or $\alpha < \beta$) can result in different change in density of mature pests and different efficiencies of killing mature pests by spraying pesticides which will give a conclusion theoretically to the researchers in killing mature pests. Researchers should give suitable control parameter α and appropriate initial values in order to obtain a steady and optimal control. In fact, immature pests (nymphs) are more easily to be killed by pesticides than mature pests (adults); thus, Figure 13 is more feasible than Figure 12.

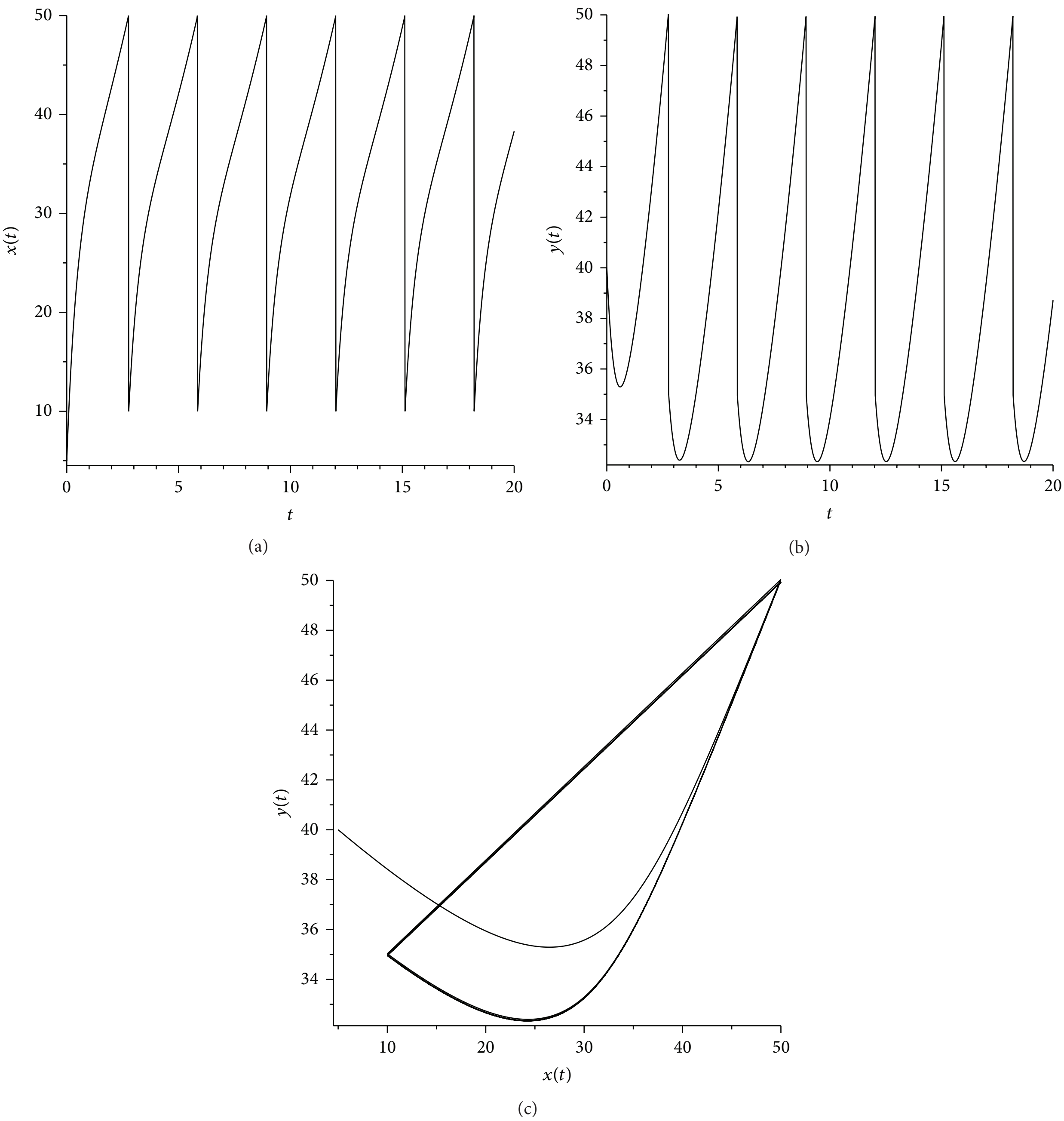

FIGURE 13: Time series and phase portrait of system (1) with $a = 1.7, b = 1.5, c = 0.8, d = 0.6, x^* = 50, \alpha = 0.8, \beta = 0.3, (x_0, y_0) = (5, 40)$.

Time series portraits of Figures 12 and 13 show that the order-1 periodic solution of system (1) is stable, and it is consistent with Theorem 17. The numerical analysis illustrates that we can achieve the aim of controlling immature and mature pests by impulsively spraying pesticides when immature pests density increases to x^*.

According to the obtained conclusions, we can predict the cycle time without repeated measurements, which can save a lot of labor and material resources. Obviously, the model with impulsive state feedback control is closer to the reality than the periodic impulsive model where there is no density dependence.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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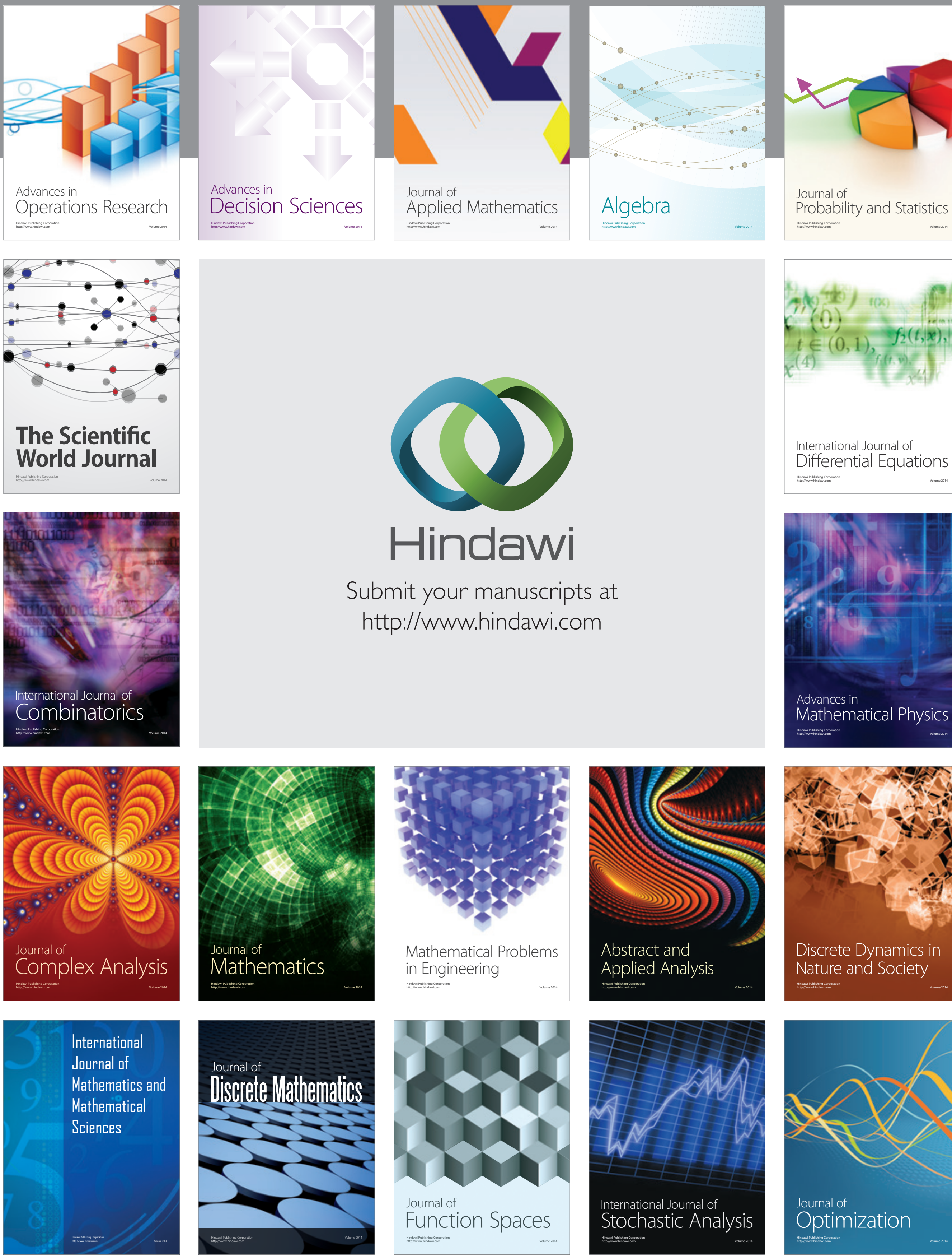
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