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Research Article

Stabilization of the FO-BLDCM Chaotic System in the Sense of Lyapunov

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Based on an integer-order Brushless DC motors (IO-BLDCM) system, we give a fractional-order Brushless DC motors (FO-BLDCM) system in this paper. There exists a chaotic attractor for fractional-order $0.95 < q \leq 1$ in the FO-BLDCM system. Furthermore, using the Lyapunov direct method for fractional-order system, a control scheme is proposed to stabilize the FO-BLDCM chaotic system in the sense of Lyapunov. Numerical simulation shows that the control scheme in this paper is valid for the FO-BLDCM chaotic system.

1. Introduction

Brushless DC motors (BLDCM) system [1–4] has several advantages over brushed DC motors, like elimination of ionizing sparks, overall reduction of electromagnetic interference, reduced noise, longer lifetime, increased efficiency and reliability, and so forth; BLDCM has been widely applied in positioning and actuation systems, motion control systems, radio controlled cars, and industrial automation design. Recently, some results have shown that the chaotic motion can be presented in BLDCM system. However, the chaotic motion in BLDCM system is not acceptable in practical situations, because it can destroy the stable operation of the BLDCM system and can lead to system malfunction in practical applications. So, BLDCM system stability is usually a prerequisite of practical application. Up to now, in order to control the chaotic motion in BLDCM system, some schemes have been presented [2–4].

On the other hand, it has been recognized that many real-world physical systems can be described by fractional-order differential equations, such as the fractional-order telegraph system [5], the fractional-order heat conduction system [6], the fractional-order diffusion and superdiffusion system [7, 8], the fractional-order Chua system [9], the fractional-order Duffing system [10], the fractional-order cellular neural network [11], the fractional-order gyroscopes system [12], and the fractional-order microelectromechanical system [13]. Meanwhile, it is well known that many fractional-order systems exhibit chaotic behavior, such as the fractional Lorenz chaotic system [14], the fractional-order Chua chaotic system [9], the fractional-order Duffing chaotic system [10], the fractional-order Volta chaotic system [15], the fractional-order gyroscopes chaotic system [12], the fractional-order microelectromechanical chaotic system [13], and the fractional-order chaotic electronic circuit [16]. Furthermore, control of the fractional-order chaotic systems has been attracting more attention in recent years [17–20].

Motivated by the above considerations, a FO-BLDCM system under loading conditions is presented. By numerical calculation, we find that the FO-BLDCM system exhibits a chaotic attractor, and we obtain the largest Lyapunov exponent of the FO-BLDCM system. Furthermore, based on the Lyapunov direct method for fractional-order system [21–23], we propose a control scheme to stabilize the FO-BLDCM chaotic system. The result in this paper shows that the FO-BLDCM chaotic system can be stable in the sense of Lyapunov.

2. A FO-BLDCM Chaotic System

Ge and Chang [2] proposed a mathematical model for a BLDCM system under loading conditions and found that the BLDCM system creates chaotic behavior. Based on the mathematical model in [2], we present a FO-BLDCM system model which is described as follows:

$$\begin{aligned} {}_0^C D_t^q x_1 &= V_q - x_1 - x_2 x_3 + \rho x_3, \\ {}_0^C D_t^q x_2 &= V_d - \delta x_2 + x_1 x_3, \\ {}_0^C D_t^q x_3 &= -T_L + \sigma (x_1 - x_3) + \eta x_1 x_2, \end{aligned} \tag{1}$$

where $0 < q \leq 1$ is the fractional order and ${}_0^C D_t^q x_i = (1/\Gamma(1-q)) \int_0^t (dx_i(\tau)/(t-\tau)^q)$ $(i = 1, 2, 3)$. Constants V_q, V_d are related to the fictitious inductance on the quadrature-axis and direct-axis, respectively; constant T_L is related to the external load, Coulomb friction, cogging effect, and so forth; constants δ, η are related to the fictitious inductance on the quadrature-axis and direct-axis; constant σ is related to the inertia of rotator and the viscous damping coefficient; constant ρ is a free parameter. In this paper, we choose system parameters in FO-BLDCM system (1) as follows: $V_q = 0.168$, $V_d = 20.66$, $T_L = 0.53$, $\delta = 0.875$, $\eta = 0.26$, $\sigma = 4.56$, and $\rho = 60$.

By the improved version of the Adams-Bashforth-Moulton numerical algorithm [24] for fractional-order nonlinear system, the fractional-order system (1) can be discretized as follows:

$$\begin{aligned} x_1(n+1) &= x_1(0) + \frac{h^q}{\Gamma(q+2)} \Bigg[V_q - x_1^p(n+1) - x_1^p(n+1) x_3^p \\ &\quad + \rho y_3^p(n+1) \\ &\quad + \sum_{j=0}^{n} c_{j,n+1} \left(V_q - x_1(j) - x_1(j) x_3(j) + \rho x_3(j) \right) \Bigg], \\ x_2(n+1) &= x_2(0) + \frac{h^q}{\Gamma(q+2)} \Bigg[V_d - \delta x_2^p(n+1) \\ &\quad + x_1^p(n+1) x_3^p(n+1) \\ &\quad + \sum_{j=0}^{n} c_{j,n+1} \left(V_d - \delta x_2(j) + x_1(j) x_3(j) \right) \Bigg], \\ x_3(n+1) &= x_3(0) + \frac{h^q}{\Gamma(q+2)} \Bigg[-T_L \\ &\quad + \sigma \left(x_1^p(n+1) - x_3^p(n+1) \right) + \eta x_1^p(n+1) x_2^p(n+1) \\ &\quad + \sum_{j=0}^{n} c_{j,n+1} \left(-T_L + \sigma (x_1(j) - x_3(j)) + \eta x_1(j) x_2(j) \right) \Bigg], \end{aligned}$$

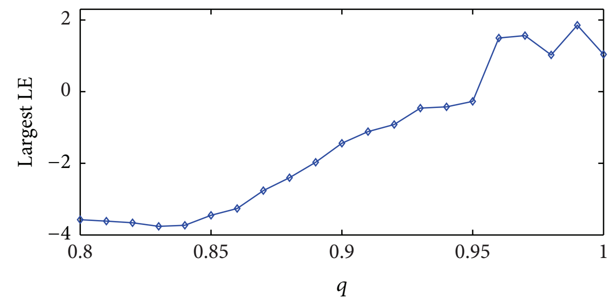

Figure 1: The largest Lyapunov exponent of FO-BLDCM system (1) with respect to the fractional-order q.

$$\begin{aligned} x_1^p(n+1) &= x_1(0) + \frac{1}{\Gamma(q)} \\ &\quad \cdot \sum_{j=0}^{n} b_{j,n+1} \left(V_q - x_1(j) - x_1(j) x_3(j) + \rho x_3(j) \right), \\ x_2^p(n+1) &= x_2(0) + \frac{1}{\Gamma(q)} \sum_{j=0}^{n} b_{j,n+1} \left(V_d - \delta x_2(j) + x_1(j) x_3(j) \right), \\ x_3^p(n+1) &= y_3(0) + \frac{1}{\Gamma(q)} \\ &\quad \cdot \sum_{j=0}^{n} b_{j,n+1} \left(-T_L + \sigma (x_1(j) - x_3(j)) + \eta x_1(j) x_2(j) \right), \\ c_{j,n+1} & \\ &= \begin{cases} n^{q+1} - (n-q)(n+1)^q, & j = 0 \\ (n-j+2)^{q+1} + (n-j)^{q+1} - 2(n-j+1)^{q+1}, & 1 \leq j \leq n \\ 1, & j = n+1, \end{cases} \\ b_{j,n+1} &= \frac{h^q}{q} \left[(n-j+1)^q - (n-j)^q \right], \quad 0 \leq j \leq n. \end{aligned} \tag{2}$$

The error is

$$\begin{aligned} \left| x_i(t_n) - x_i(n) \right| &= o\left(h^{\beta}\right), \\ \beta &= \min(2, 1+q), \end{aligned} \tag{3}$$

where h is step size.

Recently, Zhou and Huang [25] introduced one numerical method to calculate the largest Lyapunov exponent (LLE) for the fractional-order nonlinear system. Now, we can calculate the largest Lyapunov exponent (LLE) of FO-BLDCM system (1) for difference fractional-order q. Figure 1 displays the LLE of FO-BLDCM system (1) with respect to the fractional-order q.

According to Figure 1, the LLE of FO-BLDCM system (1) is larger than zero for $0.95 < q \leq 1$, which implies that chaotic behavior will emerge for $0.95 < q \leq 1$. Figure 2 shows a chaotic attractor of the FO-BLDCM system (1) for $q = 0.96$, in which the LLE is 1.4972. Conversely, the LLE of FO-BLDCM system (1) is less than zero for $q \leq 0.95$, which implies that FO-BLDCM system (1) is stable. For example, the LLE is −0.4246 for $q = 0.94$, and the evolution of state of x_1, x_2, and x_3 is shown in Figure 3.

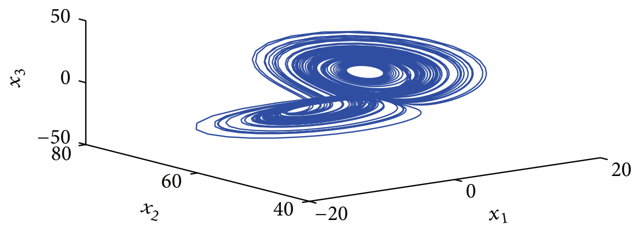

FIGURE 2: A chaotic attractor in FO-BLDCM system (1) for $q = 0.96$.

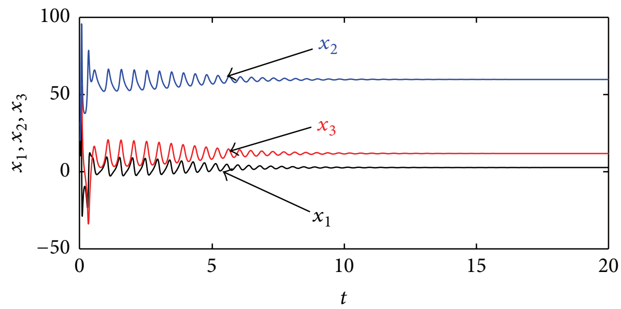

FIGURE 3: Evolution of the state of x_1, x_2, and x_3 in FO-BLDCM system (1) with $q = 0.94$.

3. Control of the FO-BLDCM Chaotic System

In this section, control of the FO-BLDCM chaotic system will be discussed.

Theorem 1. *Give the controlled FO-BLDCM system as*

$$\begin{aligned} {}_0^C D_t^q x_1 &= V_q - x_1 - x_2 x_3 + \rho x_3, \\ {}_0^C D_t^q x_2 &= V_d - \delta x_2 + x_1 x_3, \\ {}_0^C D_t^q x_3 &= -T_L + \sigma (x_1 - x_3) + \eta x_1 x_2 + u(x_1, x_2) \end{aligned} \quad (4)$$

and $u(x_1, x_2) = [m_1 - \rho - \sigma - (\eta - 1)\bar{x}_2](x_1 - \bar{x}_1) + (m_2 - \bar{x}_1 - \eta x_1)(x_2 - \bar{x}_2)$ *is a scalar controller,* $\bar{x}_i$ $(i = 1, 2, 3)$ *is the equilibrium point of FO-BLDCM system (1), and* m_i $(i = 1, 2)$ *is suitable constant. If* $m_1 = -2l_1 l_2$, $m_2 = -2l_3 l_4$, *and* l_i $(i = 1, 2, 3, 4)$ *satisfies* $|l_1| < 1$, $|l_3| < \sqrt{\delta}$, $l_2^2 + l_4^2 < \sigma$, *then the equilibrium point* $\bar{x}_i$ $(i = 1, 2, 3)$ *in controlled system (4) is stable in the sense of Lyapunov.*

Proof. First, let $y_i(t) = (x_i - \bar{x}_i)$ $(i = 1, 2, 3)$. Then, the controlled system (4) can be changed as

$$\begin{aligned} {}_0^C D_t^q y_1(t) &= -y_1(t) - \bar{x}_3 y_2(t) + (\rho - \bar{x}_2) y_3(t) \\ &\quad - y_2(t) y_3(t), \\ {}_0^C D_t^q y_2(t) &= \bar{x}_3 y_1(t) - \delta y_2(t) + \bar{x}_1 y_3(t) \\ &\quad + y_1(t) y_3(t), \\ {}_0^C D_t^q y_3(t) &= (m_1 - \rho + \bar{x}_2) y_1(t) + (m_2 - \bar{x}_1) y_2(t) \\ &\quad - \sigma y_3(t). \end{aligned} \quad (5)$$

Obviously, $y_i(t) = 0$ $(i = 1, 2, 3)$ is the origin of controlled system (5).

Second, let $y(t) = (y_1(t) \;\; y_2(t) \;\; y_3(t))^{\mathbf{T}}$, where $\mathbf{T}$ denotes the transpose for a matrix. Then, we have the following:

$$\begin{aligned} &0.5\, {}_0^C D_t^q [y(t)]^{\mathbf{T}} y(t) - [y(t)]^{\mathbf{T}} {}_0^C D_t^q y(t) \\ &= 0.5 \sum_{i=1}^{3} {}_0^C D_t^q [y_i(t)]^2 - \sum_{i=1}^{3} y_i(t)\, {}_0^C D_t^q y_i(t) \\ &= \sum_{i=1}^{3} \frac{1}{\Gamma(1-q)} \int_0^t \frac{y_i(\tau)}{(t-\tau)^q} dy_i(\tau) \\ &- \sum_{i=1}^{3} \frac{1}{\Gamma(1-q)} \int_0^t \frac{y_i(t)}{(t-\tau)^q} dy_i(\tau) \\ &= \sum_{i=1}^{3} \frac{1}{\Gamma(1-q)} \int_0^t \frac{[y_i(\tau) - y_i(t)]}{(t-\tau)^q} dy_i(\tau) \\ &= \sum_{i=1}^{3} \frac{1}{\Gamma(1-q)} \int_0^t \frac{0.5}{(t-\tau)^q} d[y_i(\tau) - y_i(t)]^2 \\ &= \sum_{i=1}^{3} \frac{0.5}{\Gamma(1-q)} \left\{ \left. \frac{[y_i(\tau) - y_i(t)]^2}{(t-\tau)^q} \right|_{\tau=t} \right. \\ &\left. - \frac{[y_i(0) - y_i(t)]^2}{t^q} - \int_0^t \frac{q[y_i(\tau) - y_i(t)]^2}{(t-\tau)^{q+1}} d\tau \right\}. \end{aligned} \quad (6)$$

Hence,

$$\begin{aligned} &0.5\, {}_0^C D_t^q [y(t)]^{\mathbf{T}} y(t) - [y(t)]^{\mathbf{T}} {}_0^C D_t^q y(t) \\ &= \sum_{i=1}^{3} \frac{0.5}{\Gamma(1-q)} \left\{ \left. \frac{[y_i(\tau) - y_i(t)]^2}{(t-\tau)^q} \right|_{\tau=t} \right. \\ &\left. - \frac{[y_i(0) - y_i(t)]^2}{t^q} - \int_0^t \frac{q[y_i(\tau) - y_i(t)]^2}{(t-\tau)^{q+1}} d\tau \right\}. \end{aligned} \quad (7)$$

Now, let us consider the first term in formula (7); we have

$$\begin{aligned} &\sum_{i=1}^{3} \frac{0.5}{\Gamma(1-q)} \left. \frac{[y_i(\tau) - y_i(t)]^2}{(t-\tau)^q} \right|_{\tau=t} \\ &= \sum_{i=1}^{3} \frac{0.5}{\Gamma(1-q)} \lim_{\tau \to t} \frac{[y_i(\tau) - y_i(t)]^2}{(t-\tau)^q} \\ &= \sum_{i=1}^{3} \frac{0.5}{\Gamma(1-q)} \lim_{\tau \to t} \frac{2[y_i(\tau) - y_i(t)][dy_i(\tau)/d\tau]}{q(t-\tau)^{q-1}} \\ &= \sum_{i=1}^{3} \frac{1}{q\Gamma(1-q)} \lim_{\tau \to t} (t-\tau)^{1-q} [y_i(\tau) - y_i(t)] \\ &\cdot \left[\frac{dy_i(\tau)}{d\tau} \right] = 0. \end{aligned} \quad (8)$$

According to (7)-(8), one can yield to

$$0.5\,{}_0^CD_t^q\left[y(t)\right]^{\mathrm{T}}y(t)-\left[y(t)\right]^{\mathrm{T}}{}_0^CD_t^qy(t)$$
$$=-\sum_{i=1}^{3}\frac{0.5}{\Gamma(1-q)}\left\{\frac{\left[y_i(0)-y_i(t)\right]^2}{t^q}\right. \tag{9}$$
$$\left.+\int_0^t\frac{q\left[y_i(\tau)-y_i(t)\right]^2}{(t-\tau)^{q+1}}d\tau\right\}\leq 0.$$

From inequality (9), one has

$$0.5\,{}_0^CD_t^q\left[y(t)\right]^{\mathrm{T}}y(t)\leq\left[y(t)\right]^{\mathrm{T}}{}_0^CD_t^qy(t). \tag{10}$$

Finally, consider a positive definite Lyapunov function as follows:

$$V(t)=0.5\left[y(t)\right]^{\mathrm{T}}y(t). \tag{11}$$

According to system (5) and inequality (10), we can obtain the fractional derivative of the Lyapunov function as follows:

$${}_0^CD_t^qV(t)\leq\left[y(t)\right]^{\mathrm{T}}{}_0^CD_t^qy(t)=\sum_{i=1}^{3}y_i(t)\,{}_0^CD_t^qy_i(t)$$
$$=-y_1^2(t)-\delta y_2^2(t)-\sigma y_3^2(t) \tag{12}$$
$$+m_1y_1(t)y_3(t)+m_2y_2(t)y_3(t).$$

Based on inequality (12), using the assumptions $m_1=-2l_1l_2$, $m_2=-2l_3l_4$, $|l_1|<1$, $|l_3|<\sqrt{\delta}$, and $l_2^2+l_4^2<\sigma$, we can obtain the following:

$${}_0^CD_t^qV(t)\leq-\left[l_1y_1(t)+l_2y_3(t)\right]^2$$
$$-\left[l_3y_2(t)+l_4y_3(t)\right]^2-\left(1-l_1^2\right)y_1^2(t)$$
$$-\left(\delta-l_3^2\right)y_2^2(t)-\left(\sigma-l_2^2-l_4^2\right)y_3^2(t) \tag{13}$$
$$\leq 0,\quad\forall y_i(t)\ \ (i=1,2,3).$$

So, according to the results in [22, 23], inequality (13) implies that the origin of controlled system (5) is stable in the sense of Lyapunov. Therefore, the equilibrium point $\overline{x}_i$ $(i=1,2,3)$ in controlled system (4) is stable in the sense of Lyapunov, which allows concluding the proof. □

Remark 2. Using the assumptions $m_1=-2l_1l_2$, $m_2=-2l_3l_4$, $|l_1|<1$, $|l_3|<\sqrt{\delta}$, and $l_2^2+l_4^2<\sigma$, we can obtain that the suitable constant m_i $(i=1,2)$ in Theorem 1 satisfies $m_1^2+\delta^{-1}m_2^2<4\sigma$.

Remark 3. According to controlled system (4), if controlled system (4) reached the equilibrium point $\overline{x}_i$ $(i=1,2,3)$, then the scalar controller $u(x_1,x_2)$ is removed.

Next, we apply Theorem 1 to stably control the equilibrium point $\overline{x}_i$ $(i=1,2,3)$. It is easy to obtain that the fractional-order BDCM chaotic system (1) with $q=0.96$ has three real equilibrium points: $p_1=(-2.6658, 59.7612, -11.8655)$, $p_2=(0.0481, 23.6112, -0.00329)$, and $p_3=(2.6926, 59.7852, 11.7551)$. Taking the control parameters $m_1=1$ and $m_2=2$ and initial conditions $x_1=10$, $x_2=20$, and $x_3=30$ for controlled fractional-order system (4), the simulation results are shown in Figures 4–6. Here, constant l_i $(i=1,2,3,4)$ is $l_1=1/2$, $l_2=-1$, $l_3=2$, and $l_4=-0.5$. Figure 4 shows the evolution of the states in system (4), where $(\overline{x}_1,\overline{x}_2,\overline{x}_3)=(-2.6658, 59.7612, -11.8655)$. This result indicates that system (4) can be stable in the equilibrium point p_1; Figure 5 shows the evolution of the states in system (4), where $(\overline{x}_1,\overline{x}_2,\overline{x}_3)=(0.0481, 23.6112, -0.00329)$. This result indicates that system (4) can be stable in the equilibrium point p_2. Figure 6 shows the evolution of the states in system (4), where $(\overline{x}_1,\overline{x}_2,\overline{x}_3)=(2.6926, 59.7852, 11.7551)$. This result indicates that system (4) can be stable in the equilibrium point p_3.

According to Figures 4–6, we can obtain that the proposed theorem is valid for FO-BLDCM chaotic system.

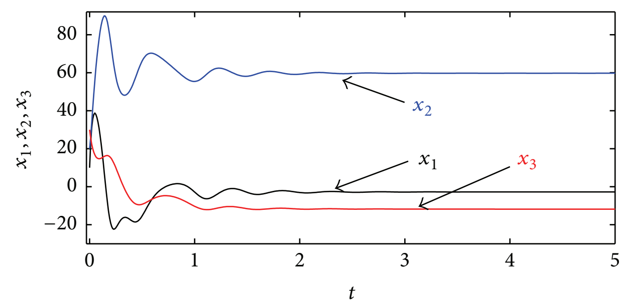

Figure 4: Evolution of the state of x_1, x_2, and x_3 in controlled system (4) with $q=0.96$ for equilibrium point p_1.

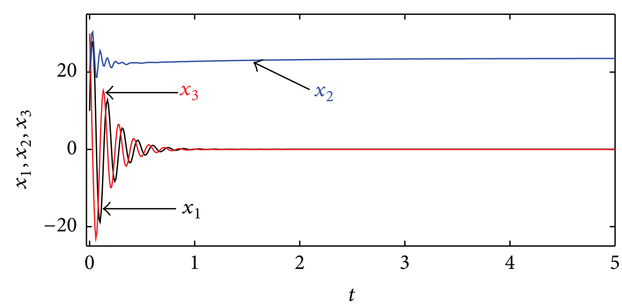

Figure 5: Evolution of the state of x_1, x_2, and x_3 in controlled system (4) with $q=0.96$ for equilibrium point p_2.

4. Conclusions

A FO-BLDCM system is proposed in this paper. The chaotic motion can be presented in the FO-BLDCM system for $0.95<q\leq 1$. The chaotic phase portraits for $q=0.96$ and the largest Lyapunov exponent with varying the fractional order are obtained by numerical calculation. Based on the Lyapunov direct method for fractional-order system, we proposed a control scheme to stabilize the FO-BLDCM chaotic system in the sense of Lyapunov. The simulation results show that the proposed scheme is effective.

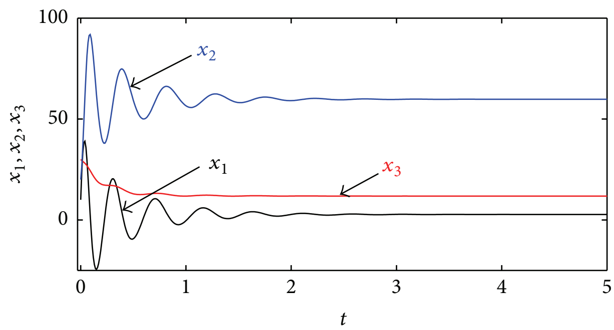

FIGURE 6: Evolution of the state of x_1, x_2, and x_3 in controlled system (4) with $q = 0.96$ for equilibrium point p_3.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] N. Hemati, "Strange attractors in brushless DC motors," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 41, no. 1, pp. 40–45, 1994.

[2] Z.-M. Ge and C.-M. Chang, "Chaos synchronization and parameters identification of single time scale brushless DC motors," *Chaos, Solitons and Fractals*, vol. 20, no. 4, pp. 883–903, 2004.

[3] Z. M. Ge, C. M. Chang, and Y. S. Chen, "Anti-control of chaos of single time scale brushless dc motors and chaos synchronization of different order systems," *Chaos, Solitons and Fractals*, vol. 27, no. 5, pp. 1298–1315, 2006.

[4] D. Q. Wei, L. Wan, X. S. Luo, S. Y. Zeng, and B. Zhang, "Global exponential stabilization for chaotic brushless DC motors with a single input," *Nonlinear Dynamics*, vol. 77, no. 1-2, pp. 209–212, 2014.

[5] L. Beghin and E. Orsingher, "The telegraph process stopped at stable-distributed times and its connection with the fractional telegraph equation," *Fractional Calculus and Applied Analysis*, vol. 6, no. 2, pp. 187–204, 2003.

[6] W. Zhang, X. Cai, and S. Holm, "Time-fractional heat equations and negative absolute temperatures," *Computers & Mathematics with Applications*, vol. 67, no. 1, pp. 164–171, 2014.

[7] R. Gorenflo, F. Mainardi, D. Moretti, G. Pagnini, and P. Paradisi, "Fractional diffusion: probability distributions and random walk models," *Physica A*, vol. 305, no. 1-2, pp. 106–112, 2002.

[8] C. P. Li, Z. G. Zhao, and Y. Q. Chen, "Numerical approximation of nonlinear fractional differential equations with subdiffusion and superdiffusion," *Computers & Mathematics with Applications*, vol. 62, no. 3, pp. 855–875, 2011.

[9] T. T. Hartley, C. F. Lorenzo, and H. K. Qammer, "Chaos in a fractional order Chua's system," *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, vol. 42, no. 8, pp. 485–490, 1995.

[10] P. Arena, R. Caponetto, L. Fortuna, and D. Porto, "Bifurcation and chaos in noninteger order cellular neural networks," *International Journal of Bifurcation and Chaos*, vol. 8, no. 7, pp. 1527–1539, 1998.

[11] X. Gao and J. Yu, "Chaos in the fractional order periodically forced complex Duffing's oscillators," *Chaos, Solitons and Fractals*, vol. 24, no. 4, pp. 1097–1104, 2005.

[12] M. P. Aghababa and H. P. Aghababa, "The rich dynamics of fractional-order gyros applying a fractional controller," *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering*, vol. 227, no. 7, pp. 588–601, 2013.

[13] M. P. Aghababa, "Chaos in a fractional-order micro-electro-mechanical resonator and its suppression," *Chinese Physics B*, vol. 21, no. 10, Article ID 100505, 9 pages, 2012.

[14] I. Grigorenko and E. Grigorenko, "Chaotic dynamics of the fractional Lorenz system," *Physical Review Letters*, vol. 91, no. 3, Article ID 034101, 2003.

[15] I. Petráš, "Chaos in the fractional-order Volta's system: modeling and simulation," *Nonlinear Dynamics*, vol. 57, no. 1-2, pp. 157–170, 2009.

[16] H. Y. Jia, Z. Q. Chen, and G. Y. Qi, "Chaotic characteristics analysis and circuit implementation for a fractional-order system," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 61, no. 3, pp. 845–853, 2014.

[17] Z. M. Odibat, "Adaptive feedback control and synchronization of non-identical chaotic fractional order systems," *Nonlinear Dynamics*, vol. 60, no. 4, pp. 479–487, 2010.

[18] S. Bhalekar and V. Daftardar-Gejji, "Synchronization of different fractional order chaotic systems using active control," *Communications in Nonlinear Science and Numerical Simulation*, vol. 15, no. 11, pp. 3536–3546, 2010.

[19] R. Zhang and S. Yang, "Adaptive synchronization of fractional-order chaotic systems via a single driving variable," *Nonlinear Dynamics*, vol. 66, no. 4, pp. 831–837, 2011.

[20] D.-Y. Chen, Y.-X. Liu, X.-Y. Ma, and R.-F. Zhang, "Control of a class of fractional-order chaotic systems via sliding mode," *Nonlinear Dynamics*, vol. 67, no. 1, pp. 893–901, 2012.

[21] Y. Li, Y. Chen, and I. Podlubny, "Stability of fractional-order nonlinear dynamic systems: lyapunov direct method and generalized Mittag-Leffler stability," *Computers and Mathematics with Applications*, vol. 59, no. 5, pp. 1810–1821, 2010.

[22] J.-B. Hu, G.-P. Lu, S.-B. Zhang, and L.-D. Zhao, "Lyapunov stability theorem about fractional system without and with delay," *Communications in Nonlinear Science and Numerical Simulation*, vol. 20, no. 3, pp. 905–913, 2015.

[23] N. Aguila-Camacho, M. A. Duarte-Mermoud, and J. A. Gallegos, "Lyapunov functions for fractional order systems," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 9, pp. 2951–2957, 2014.

[24] C. Li and G. Peng, "Chaos in Chen's system with a fractional order," *Chaos, Solitons and Fractals*, vol. 22, no. 2, pp. 443–450, 2004.

[25] P. Zhou and K. Huang, "A new 4-D non-equilibrium fractional-order chaotic system and its circuit implementation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 19, no. 6, pp. 2005–2011, 2014.