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Research Article

Dynamic Analysis of General Integrated Pest Management Model with Double Impulsive Control

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A general predator-prey model with disease in the prey and double impulsive control is proposed and investigated for the purpose of integrated pest management. By using the Floquet theory, the comparison theorem of impulsive differential equations, and the persistence theory of dynamical systems, we obtain that if threshold value $R_0 < 1$, then the susceptible pest eradication periodic solution is globally asymptotically stable and if $R_0 > 1$, then the model is permanent. The numerical examples not only illustrate the theoretical results, but also show that when the model is permanent, then it may possess a unique globally attractive T-periodic solution.

1. Introduction

Integrated pest management (IPM) is a long term management tactic that uses a combination of chemical, biological strategies to reduce pests to tolerable level or below the threshold, with less cost to the farmers and minimal effect on the environment (see [1, 2]). Such techniques include mechanical methods (erecting pest barriers or using pest traps) and biological methods (breeding natural predators of the pest or using biological insecticides). Some successful biological control examples contain the use of the predatory arthropod *Orius sauteri* against the pest *Thrips palmi* Karny to protect eggplant crops in greenhouses (see [3]) and the use of the predatory mites *Phytoseiulus persimilis* and *Neoseiulus californicus* to regulate the red spider mite *Tetranychus urticae* Koch in field-grown strawberries (see [4]).

The discontinuity of human activities and the abrupt variation in the amount of the pest population, which occurs immediately after successful control measures (such as spraying pesticides, releasing natural enemies of the pest, and freeing infective pest individuals), may be described mathematically through making use of impulsive differential equations (see [5–16]).

Many scholars have been devoted to the analysis of impulsive differential equation models describing IPM strategies and some rich results have been obtained (see [6–15, 17]). They assumed that the disease incidence rate should be distinguished; as far as disease transmission is concerned, nonlinear, bilinear, and standard incidence rates have often been used in establishing ecoepidemic models, which depends on different infective disease and environment. Georgescu and Zhang (see [10]) investigated a predator-pest model with incidence rate given by $g(I)S$, Pang and Chen (see [12]) discussed an SI model with bilinear incidence rate βSI, Wang et al. (see [13]) analyzed an SI model with incidence rate given by $f(S)I$, and so forth. Main results of these theses have focused on conditions of pest eradication and permanence of the system. According to the authors' knowledge, at present stage, there are few studies of general incidence rate. So one of the goals of this paper is to generalize the incidence rate.

The functional response between pests and natural enemies plays an important role in assessing dynamical behavior of the system. People use natural enemy, as in some sense like a pesticide, to control pest via augmentation or releasing natural enemy once the quantity of pest has reached or exceeded the economic threshold (see [9, 10, 14, 15]). Shi et al. (see [14]) analyzed a predator-pest model with disease in the pest and functional response given by Holling-II type and the time-dependent impulsive strategy including release of infective pest individuals and those natural predators at different point in time; the threshold on pest eradication was obtained. However, little of paper has been devoted to analysis of models which combine release of infective pest individuals and those natural predators. The approach to biological control which we adopted is to release both infective pest individuals and natural predators periodically with constant amount at different point in time; what is more, the functional response is also a more general form. Motivated by the valuable contributions of Georgescu and Zhang [10], Wang et al. [13], and Shi et al. [14], in this paper general IPM model will be considered as follows:

$$
\begin{aligned}
S'(t) &= S(t)\, f(S(t), I(t)) - g(S(t), I(t)) \\
&\quad - h(S(t), y(t)), \\
I'(t) &= g(S(t), I(t)) - \omega I(t), \\
y'(t) &= \delta h(S(t), y(t)) - dy(t), \\
&\qquad\qquad t_n \neq (n+l-1)T,\ t_n \neq nT, \\
I(t_n^+) &= I(t_n) + p_1, \quad t_n = (n+l-1)T,\ 0 < l < 1, \\
y(t_n^+) &= y(t_n) + p_2, \quad t_n = nT.
\end{aligned}
\tag{1}
$$

The model is based on the following assumptions:

(H_1) The pest is divided into the susceptible and the infective, and the infective cannot produce offsprings as a result of the disease, but the infective still consume crop. The incidence rate of the infective is given by function $g(S, I)$. The growth rate of the susceptible is assumed to function $Sf(S, I)$.

(H_2) Parameter $p_1 > 0$ represents the amount of infective pest released periodically at time $t_n = (n + l - 1)T$, where $n \in Z_+ = \{1, 2, \ldots\}$ and $0 < l < 1$. Parameter $p_2 > 0$ represents the amount of natural enemy released periodically at time $t_n = nT$, $n \in Z_+$.

(H_3) The natural enemy only hunts the susceptible and the functional response is given by function $h(S, y)$; $\delta > 0$ is the conversion rate.

(H_4) Positive constants ω and d are the death rates of the infective pest and the natural enemy, respectively.

In model (1), $S(t)$ and $I(t)$ denote the density of the susceptible pest and the infective pest (prey) population, respectively. $y(t)$ is the density of natural enemy (predator) population. For model (1), in this paper we will investigate global stability of the susceptible pest eradication periodic solution and the permanence of model (1). In Section 2, the positivity and boundedness of solutions are presented. In Section 3, by using the Floquet theory for impulsive differential equations, the theorem on the global asymptotic stability of the susceptible pest eradication periodic solution is established. In Section 4, by using the persistence theory of dynamical systems, the theorem on the permanence of model (1) is established. In Section 5, we will give the numerical simulations to illustrate the main results obtained in this paper. Finally, in last section a brief discussion and some possible future researches are proposed.

2. Preliminaries

Denote $R_+ = [0, +\infty)$ and $R_+^n = \{(x_1, x_2, \ldots, x_n) : x_i \in R_+, i = 1, 2, \ldots, n\}$. For model (1), we introduce the following assumptions.

(A_1) Function $f(x, y)$ is continuous on R_+^2 and is nonincreasing for x and y, respectively. Consider $\sup_{x \in R_+}(xf(x, 0) + \gamma x) < \infty$ with $\gamma = \min\{\omega, d\}$.

(A_2) Function $g(x, y)$ is continuously differentiable for x and y on R_+^2, $g(0, y) = 0$, and $\partial g(x, y)/\partial x \geq 0$, $\partial g(x, y)/\partial y \geq 0$ for all $(x, y) \in R_+^2$.

(A_3) Function $h(x, y)$ is continuously differentiable for x and y on R_+^2, $h(0, y) = 0$, and $\partial h(x, y)/\partial x \geq 0$, $\partial h(x, y)/\partial y \geq 0$ for all $(x, y) \in R_+^2$.

The solution of model (1), denoted by $x(t) = (S(t), I(t), y(t)) : R_+ \to R_+^3$, is piecewise continuous on $((n-1)T, (n+l-1)T]$ and $((n+l-1)T, nT]$, $n \in Z_+$, $x((n+l-1)T^+) = \lim_{t \to (n+l-1)T^+} x(t)$, and $x(nT^+) = \lim_{t \to nT^+} x(t)$ exist. The global existence and uniqueness of solution for model (1) with any initial value $x(0^+) \geq 0$ are guaranteed by the smoothness of the right-hand functions of model (1) (see [18]). Firstly, the following results are obtained easily.

Lemma 1. *Assume that $x(t)$ is the solution of model (1) with $x(0^+) \geq 0$, and then $x(t) \geq 0$ for all $t \geq 0$. Furthermore, if $x(0^+) > 0$, then $x(t) > 0$ for all $t > 0$.*

Lemma 2. *Let α be a positive constant. Then system*

$$
\begin{aligned}
u'(t) &= -\alpha u(t), \\
&\qquad t_n \neq (n+l-1)T,\ 0 \leq l \leq 1,\ n \in Z_+, \\
u(t_n^+) &= u(t_n) + u_0,\ t_n = (n+l-1)T,
\end{aligned}
\tag{2}
$$

has a positive periodic solution

$$
u^*(t) = \begin{cases} u^*(0^+)\exp\{-\alpha[t-(n-1)T]\}, & (n-1)T < t \leq (n+l-1)T, \\ \left(u^*(0^+)\exp(-\alpha lT) + u_0\right)\exp\{-\alpha[t-(n+l-1)T]\}, & (n+l-1)T < t \leq nT, \end{cases}
\tag{3}
$$

where $u^*(0^+) = u_0/(1-\exp(-\alpha T))$. *Furthermore, any solution* $u(t)$ *of system (2) with initial value* $u(0^+)$ *can be expressed as*

$$u(t) = \left(u(0^+) - \frac{u_0}{1-\exp(-\alpha T)}\right)\exp(-\alpha t) + u^*(t), \qquad (n-1)T < t \le nT, \tag{4}$$

and satisfies $u(t) \to u^*(t)$ *as* $t \to +\infty$.

If susceptible pest $S(t)$ is absent, then model (1) reduces to

$$\begin{aligned} I'(t) &= -\omega I(t), \\ y'(t) &= -dy(t), \\ &\qquad t_n \ne (n+l-1)T,\ t_n \ne nT, \\ I(t_n^+) &= I(t_n) + p_1, \quad t_n = (n+l-1)T,\ 0<l<1, \\ y(t_n^+) &= y(t_n) + p_2, \quad t_n = nT. \end{aligned} \tag{5}$$

By Lemma 2, positive periodic solution $(I^*(t), y^*(t))$ of system (5) is

$$\begin{aligned} I^*(t) &= \begin{cases} I^*(0^+)\exp\{-\omega[t-(n-1)T]\}, & (n-1)T < t \le (n+l-1)T, \\ (I^*(0^+)\exp(-\omega lT) + p_1)\exp\{-\omega[t-(n+l-1)T]\}, & (n+l-1)T < t \le nT, \end{cases} \\ y^*(t) &= y^*(0^+)\exp\{-d[t-(n-1)T]\}, \quad (n-1)T < t \le nT, \end{aligned} \tag{6}$$

where $I^*(0^+) = p_1/(1-\exp(-\omega T))$ and $y^*(0^+) = p_2/(1-\exp(-dT))$. Furthermore, any solution $(I(t), y(t))$ of system (5) with initial values $I(0^+)$ and $y(0^+)$ can be expressed as

$$\begin{aligned} I(t) &= (I(0^+) - I^*(0^+))\exp(-\omega t) + I^*(t), \\ &\qquad (n-1)T < t \le nT, \\ y(t) &= (y(0^+) - y^*(0^+))\exp(-dt) + y^*(t), \\ &\qquad (n-1)T < t \le nT. \end{aligned} \tag{7}$$

It is easy to get the following conclusion.

Lemma 3. *System (5) has positive periodic solutions* $z^*(t) = (I^*(t), y^*(t))$. *For any solution* $z(t) = (I(t), y(t))$ *of system (5) with initial value* $z(0^+) = (I(0^+), y(0^+))$, *one has* $z(t) \to z^*(t)$ *as* $t \to \infty$.

On the ultimate boundedness of solutions for model (1), we have the following conclusion.

Lemma 4. *There exists a constant* $M > 0$ *such that, for any solution* $x(t) = (S(t), I(t), y(t))$ *of model (1) with initial value* $x(0^+) = (S(0^+), I(0^+), y(0^+)) \in R_+^3$, *one has* $S(t) \le M$, $I(t) \le M$, *and* $y(t) \le M$ *with* t *large enough.*

Proof. Define

$$V(t) = V(x(t)) = S(t) + I(t) + \frac{1}{\delta}y(t). \tag{8}$$

By calculating the derivative of $V(t)$ with respect to model (1), when $t_n \ne (n+l-1)T$ and $t_n \ne nT$,

$$\begin{aligned} D^+V(t) &= S(t)f(S(t), I(t)) - \omega I(t) - \frac{d}{\delta}y(t) \\ &\le S(t)f(S(t), 0) - \omega I(t) - \frac{d}{\delta}y(t) \\ &\le \widetilde{M} - \gamma V(t), \end{aligned} \tag{9}$$

where $\gamma = \min\{\omega, d\}$ and $\widetilde{M} = \sup_{S\ge 0}\{Sf(S,0) + \gamma S\} < \infty$. When $t_n = (n+l-1)T$, $V((n+l-1)T^+) \le V((n+l-1)T) + p_1$, and when $t_n = nT$, $V(nT^+) \le V(nT) + p_2$. By Lemma 2.2 given in [18], it is obvious that

$$\limsup_{t\to\infty} V(t) \le \frac{\widetilde{M}}{\gamma} + \frac{(p_1+p_2)\exp(\gamma T)}{\exp(\gamma T) - 1}. \tag{10}$$

From this, there exists a constant $M > 0$ such that $S(t) \le M$, $I(t) \le M$, and $y(t) \le M$ for t large enough. This completes the proof. □

In the following, we introduce some necessary definitions and lemma on the persistence of dynamical systems, which will be used for the discussion of permanence of model (1). For more details, see [19, 20].

Let X be a metric space with metric d and let $f : X \to X$ be a continuous map. For any $x \in X$, we represent $f^n(x) = f(f^{n-1}(x))$ for any integer $n > 1$ and $f^1(x) = f(x)$. f is said to be compact in X if, for any bounded set $H \subset X$, set $f(H) = \{f(x) : x \in H\}$ is precompact in X. f is said to be point dissipative if there is a bounded set $B_0 \subset X$ such that, for any $x \in X$,

$$\lim_{n\to\infty} d(f^n(x), B_0) = 0. \tag{11}$$

For any $x_0 \in X$, the positive semiorbit through x_0 is defined by $\gamma^+(x_0) = \{f^n(x_0) = x_n, n = 1, 2, \ldots\}$, the negative semiorbit through x_0 is defined as a sequence $\gamma^-(x_0) = \{x_k\}$ satisfying $f(x_{k-1}) = x_k$ for integers $k \leq 0$, the omega limit set of $\gamma^+(x_0)$ is defined by $\omega(x_0) = \{y \in X$: there is a sequence $n_k \to \infty$ such that $\lim_{k\to\infty} x_{n_k} = y\}$, and the alpha limit set of $\gamma^-(x_0)$ is defined by $\alpha(x_0) = \{y \in X$: there is a sequence $n_k \to -\infty$ such that $\lim_{k\to\infty} x_{n_k} = y\}$.

A nonempty set $B \subset X$ is said to be invariant if $f(B) \subseteq B$. A nonempty invariant set M of X is called isolated in X if it is the maximal invariant set in a neighborhood of itself. For a nonempty set M of X, set $W^s(M) := \{x \in X : \lim_{n\to\infty} d(f^n(x), M)\}$ is called the stable set of M.

Let M_1 and M_2 be two isolated invariant sets and set M_1 is said to be chained to set M_2, usually expressed as $M_1 \to M_2$, if there exists a full orbit though some $x \notin M_1 \cup M_2$ such that $\omega(x) \subset M_2$ and $\alpha(x) \subset M_1$. A finite sequence $\mathscr{M} = \{M_1, \ldots, M_n\}$ of isolated invariant sets is called a chain if $M_1 \to M_2 \to \cdots \to M_n$, and if $M_n = M_1$ the chain is called a cycle.

Let X_0 be a nonempty open set of X. We denote

$$\begin{aligned} \partial X_0 &:= X \setminus X_0, \\ M_\partial &:= \{x \in \partial X_0 : f^n(x) \in \partial X_0, \forall n \geq 0\}. \end{aligned} \tag{12}$$

Lemma 5 (see [19, 20]). *Let $f : X \to X$ be a continuous map. Assume that the following conditions hold:*

(C_1) *Map f is compact and point dissipative and $f(X_0) \subseteq X_0$.*

(C_2) *There exists a finite sequence $\mathscr{M} = \{M_1, \ldots, M_n\}$ of compact and isolated invariant sets in ∂X_0 such that*

(1) *$M_i \cap M_j = \varnothing$ for any $i, j = 1, 2, \ldots, n$ and $i \neq j$;*

(2) *$\Omega(M_\partial) := \cup_{x\in M_\partial} \omega(x) \subset \cup_{i=1}^n M_i$;*

(3) *no subset of $\mathscr{M}$ forms a cycle in ∂X_0;*

(4) *$W^s(M_i) \cap X_0 = \varnothing$ for each $1 \leq i \leq n$.*

Then map f is uniformly persistent with respect to $(X_0, \partial X_0)$; that is, there exists a constant $\eta > 0$ such that $\liminf_{n\to\infty} d(f^n(x), \partial X_0) \geq \eta$ for all $x \in X_0$.

3. Global Stability of Susceptible Pest Eradication Periodic Solution

From Lemma 3, we know that model (1) has a susceptible pest eradication periodic solution $(0, I^*(t), y^*(t))$. On the global asymptotic stability of this periodic solution, we have the following theorem.

Theorem 6. *Assume that*

$$\int_0^T \left\{ f(0, I^*(t)) - \frac{\partial g}{\partial S}(0, I^*(t)) - \frac{\partial h}{\partial S}(0, y^*(t)) \right\} dt < 0. \tag{13}$$

Then periodic solution $(0, I^*(t), y^*(t))$ of model (1) is globally asymptotically stable.

Proof. To investigate the local stability of susceptible pest eradication periodic solution $(0, I^*(t), y^*(t))$, let $S(t) = u_1(t)$, $I(t) = u_2(t) + I^*(t)$, $y(t) = u_3(t) + y^*(t)$. We have

$$\begin{aligned} \Delta u_2(t_n) &= 0, \quad t_n = (n + l - 1)T, \ 0 < l < 1, \\ \Delta u_3(t_n) &= 0, \quad t_n = nT. \end{aligned} \tag{14}$$

The corresponding linearized system is

$$\begin{aligned} u_1'(t) &= \left[f(0, I^*(t)) - \frac{\partial g}{\partial S}(0, I^*(t)) - \frac{\partial h}{\partial S}(0, y^*(t)) \right] \cdot u_1(t), \\ u_2'(t) &= \frac{\partial g}{\partial S}(0, I^*(t)) u_1(t) - \omega u_2(t), \\ u_3'(t) &= \delta \frac{\partial h}{\partial S}(0, y^*(t)) u_1(t) - d u_3(t). \end{aligned} \tag{15}$$

Let $\Phi(t)$ be the fundamental matrix of system (15); then

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} f(0, I^*(t)) - \frac{\partial g}{\partial S}(0, I^*(t)) - \frac{\partial h}{\partial S}(0, y^*(t)) & 0 & 0 \\ \frac{\partial g}{\partial S}(0, I^*(t)) & -\omega & 0 \\ \delta \frac{\partial h}{\partial S}(0, y^*(t)) & 0 & -d \end{pmatrix} \Phi(t), \tag{16}$$

with $\Phi(0) = I$, a 3×3 identity matrix. By calculating, we get

$$\Phi(t) = \begin{pmatrix} \exp \int_0^t [\rho_0(s)] \, ds & 0 & 0 \\ a_{21} & \exp \int_0^t [-\omega] \, ds & 0 \\ a_{31} & 0 & \exp \int_0^t [-d] \, ds \end{pmatrix}, \tag{17}$$

where

$$\rho_0(s) = f(0, I^*(s)) - \frac{\partial g}{\partial S}(0, I^*(s)) - \frac{\partial h}{\partial S}(0, y^*(s)),$$
$$a_{21} = \int_0^t \frac{\partial g}{\partial S}(0, I^*(s)) \exp\left\{\omega(s-t) + \int_0^s \rho_0(\tau)\, d\tau\right\} ds,$$
$$a_{31} = \int_0^t \delta \frac{\partial h}{\partial S}(0, y^*(s)) \exp\left\{d(s-t) + \int_0^s \rho_0(\tau)\, d\tau\right\} ds. \quad (18)$$

Obviously, the eigenvalues of matrix $M = \Phi(T)$ are

$$\lambda_2 = \exp\left\{\int_0^T -\omega\, ds\right\} < 1,$$
$$\lambda_3 = \exp\left\{\int_0^T -d\, ds\right\} < 1,$$
$$\lambda_1 = \exp\left\{\int_0^T \left[f(0, I^*(t)) - \frac{\partial g}{\partial S}(0, I^*(t)) - \frac{\partial h}{\partial S}(0, y^*(t))\right] dt\right\}. \quad (19)$$

If (13) holds, then $\lambda_1 < 1$. By the Floquet theory (see [21]), $(0, I^*(t), y^*(t))$ is locally asymptotically stable.

From (13), we can choose a small enough constant $\epsilon_1 > 0$ such that

$$\rho = \exp\left\{\int_0^T \left[f(0, I^*(t) - \epsilon_1) - \min_{0\le\xi_1\le\epsilon_1} \frac{\partial g(\xi_1, I^*(t) - \epsilon_1)}{\partial S} - \min_{0\le\xi_2\le\epsilon_1} \frac{\partial h(\xi_2, y^*(t) - \epsilon_1)}{\partial S}\right] dt\right\} = \exp\left\{\int_0^T \rho_0(t, \epsilon_1)\, dt\right\} < 1. \quad (20)$$

From the second equation of model (1) and assumption (A_2), we obtain

$$I'(t) \ge -\omega I(t), \quad t_n \ne (n+l-1)T. \quad (21)$$

Consider the following impulsive differential equations:

$$\begin{aligned} v_2'(t) &= -\omega v_2(t), \quad t_n \ne (n+l-1)T, \\ v_2(t_n^+) &= v_2(t_n) + p_1, \quad t_n = (n+l-1)T, \end{aligned} \quad (22)$$

and, by the comparison theorem of impulsive differential equations (see [18]) and Lemma 3, it follows that $I(t) \ge v_2(t)$ and $v_2(t) \to I^*(t)$ as $t \to \infty$. Therefore,

$$I(t) \ge v_2(t) \ge I^*(t) - \epsilon_1 \quad (23)$$

for t large enough. Similarly,

$$y(t) \ge y^*(t) - \epsilon_1 \quad (24)$$

for t large enough.

From assumptions (A_2) and (A_3), there exist $0 < \xi_1 < S$ and $0 < \xi_2 < S$ such that $g(S, I) = (\partial g(\xi_1, I)/\partial S)S$ and $h(S, I) = (\partial h(\xi_2, I)/\partial S)S$. From (23), (24), and the first equation of model (1), there exists a $T_0 > 0$ such that

$$\begin{aligned} S'(t) &= S\left\{f(S, I) - \frac{g(S, I)}{S} - \frac{h(S, y)}{S}\right\} \\ &\le S\left\{f(0, I^*(t) - \epsilon_1) - \frac{\partial g(\xi_1, I^*(t) - \epsilon_1)}{\partial S} - \frac{\partial h(\xi_2, y^*(t) - \epsilon_1)}{\partial S}\right\} \le S\left\{f(0, I^*(t) - \epsilon_1) \right. \\ &\quad \left. - \min_{0\le\xi_1\le\epsilon_1} \frac{\partial g(\xi_1, I^*(t) - \epsilon_1)}{\partial S} - \min_{0\le\xi_2\le\epsilon_1} \frac{\partial h(\xi_2, y^*(t) - \epsilon_1)}{\partial S}\right\}, \end{aligned} \quad (25)$$

for all $t \ge T_0$. For any $t > T_0$ we can choose an integer $n \ge 0$ such that $t = T_0 + nT + \tilde{t}$, where $\tilde{t} \in [0, T)$. Integrating the above inequality from T_0 to t, we obtain

$$\begin{aligned} S(t) &\le S(T_0) \exp\left\{\int_{T_0}^t \rho_0(s, \epsilon_1)\, ds\right\} = S(T_0) \cdot \exp\left\{\int_{T_0}^{T_0+nT} \rho_0(s, \epsilon_1)\, ds\right\} \\ &\quad \cdot \exp\left\{\int_{T_0+nT}^{T_0+nT+\tilde{t}} \rho_0(s, \epsilon_1)\, ds\right\} \le S(T_0) e^{M_0 T} \rho^n \longrightarrow 0 \quad \text{as } n \longrightarrow \infty, \end{aligned} \quad (26)$$

where $M_0 = \sup_{t\geq 0}\rho_0(t,\epsilon_1) > 0$, which implies $S(t) \to 0$ as $t \to \infty$.

Next, we show that $I(t) \to I^*(t)$ and $y(t) \to y^*(t)$ as $t \to \infty$. Choose constant $\epsilon_2 > 0$ small enough such that $\eta = \sup_{0\leq I\leq M}[g(\epsilon_2, I)/I] - \omega < 0$. From $\lim_{t\to\infty}S(t) = 0$, it follows that $S(t) \leq \epsilon_2$ for t large enough. From assumption (A_3) and the second equation in model (1),

$$-\omega I(t) \leq I'(t) \leq \eta I(t), \quad t_n \neq (n+l-1)T. \tag{27}$$

Consequently, $v_2(t) \leq I(t) \leq w_2(t)$ and $v_2(t) \to I^*(t)$, $w_2(t) \to w_2^*(t)$ as $t \to \infty$, where $v_2(t)$ is the solution of (22) and $w_2(t)$ is the solution of the following system:

$$\begin{gathered} w_2'(t) = \eta w_2(t), \quad t_n \neq (n+l-1)T, \\ w_2(t_n^+) = w_2(t_n) + p_1, \quad t_n = (n+l-1)T, \\ w^*(t) = \begin{cases} \dfrac{p_1}{1-\exp(\eta T)}\exp\{\eta[t-(n-1)T]\}, & (n-1)T < t \leq (n+l-1)T, \\ \left[\dfrac{p_1\exp(\eta l T)}{1-\exp(\eta T)} + p_1\right]\exp\{\eta[t-(n+l-1)T]\}, & (n+l-1)T < t \leq nT. \end{cases} \end{gathered} \tag{28}$$

Since $\lim_{\epsilon_2\to 0}\eta = -\omega$, then

$$\lim_{\epsilon_2\to 0} w_2^*(t) = I^*(t). \tag{29}$$

Therefore, for any $\epsilon_3 > 0$ small enough, we have $I^*(t) - \epsilon_3 \leq I(t) \leq I^*(t) + \epsilon_3$ for t large enough, which implies $I(t) \to I^*(t)$ as $t \to \infty$.

Lastly, a similar argument as from (27) to (29), we also can obtain that $y(t) \to y^*(t)$ as $t \to \infty$. This completes the proof. □

Corollary 7. *When the right-hand functions in model (1) are*

$$\begin{aligned} f(S,I) &= r\left\{1 - \frac{S+\theta I}{K}\right\}, \\ g(S,I) &= \beta S I^q, \\ h(S,y) &= aSy, \end{aligned} \tag{30}$$

then condition (13) is equivalent to the following form:

$$rT < \frac{r\theta p_1}{K\omega} + \frac{\beta p_1^q}{q\omega}\frac{1-\exp(-q\omega T)}{[1-\exp(-\omega T)]^q} + \frac{ap_2}{d}, \tag{31}$$

where r, θ, K, β, q, and a are positive constants.

Remark 8. In (31), if $p_1 = 0$, then $p_2 > drT/a$, which means that if only natural enemies are released periodically, then the release amount must be larger than drT/a to ensure the eradication of the pest. If $p_2 = 0$, then the release amount must satisfy the inequality

$$rT < \frac{r\theta p_1}{K\omega} + \frac{\beta p_1^q}{q\omega}\frac{1-\exp(-q\omega T)}{[1-\exp(-\omega T)]^q} \tag{32}$$

to ensure the eradication of the pest.

4. Permanence of the Model

Theorem 9. *Assuming that*

$$\int_0^T\left\{f(0,I^*(t)) - \frac{\partial g}{\partial S}(0,I^*(t)) - \frac{\partial h}{\partial S}(0,y^*(t))\right\}dt > 0, \tag{33}$$

then model (1) is permanent.

Proof. Since the impulsive effects in model (1) are periodic, model (1) can be regarded as periodic model with period T. Therefore, we can use the persistence theory of dynamical systems to discuss the permanence of model (1). Define

$$\begin{aligned} X &= \{(S,I,y) : S \geq 0, I \geq 0, y \geq 0\}, \\ X_0 &= \{(S,I,y) \in X : S > 0, I \geq 0, y \geq 0\}. \end{aligned} \tag{34}$$

Thus

$$\partial X_0 = X \setminus X_0 = \{(S,I,y) \in X : S = 0\}. \tag{35}$$

From Lemma 1, we claim that X and X_0 are positively invariant with respect to model (1). ∂X_0 is a relatively closed set in X.

Let $P : X \to X$ be a Poincaré map associated with model (1); that is,

$$P(S_0, I_0, y_0) = u(T, S_0, I_0, y_0), \quad (S_0, I_0, y_0) \in X, \tag{36}$$

where $u(t, S_0, I_0, y_0)$ is the unique solution of model (1) with initial value $u(0^+, S_0, I_0, y_0) = (S_0, I_0, y_0)$. By Lemma 4, Poincaré map P is compact and point dissipative on X. Therefore, condition (C_1) of Lemma 5 holds.

Let

$$M_\partial = \{(S_0, I_0, y_0) \in \partial X_0 : P^n(S_0, I_0, y_0) \in \partial X_0, n = 1, 2, \ldots\}, \tag{37}$$

where $P^n = P(P^{n-1})$, $n > 1$, and $P^1 = P$.

Firstly, we will testify

$$M_\partial = \partial X_0. \quad (38)$$

Clearly, $M_\partial \subseteq \partial X_0$. For any $(0, I_0, y_0) \in \partial X_0$, by $S_0 = 0$, the solution $(S(t), I(t), y(t))$ of model (1) with initial value $(S(0^+), I(0^+), y(0^+)) = (0, I_0, y_0)$ satisfies $S(t) = 0, I(t) \geq 0$, and $y(t) \geq 0$ for all $t \geq 0$. Therefore, for any integer $n > 0$, we obtain

$$P^n(0, I_0, y_0) \in \partial X_0. \quad (39)$$

This implies $(0, I_0, y_0) \in M_\partial$. Therefore, (38) holds.

Model (1) can be simplified as model (5) in ∂X_0. By Lemma 3, model (1) has globally attractive periodic solution $(0, I^*(t), y^*(t))$ in ∂X_0. This shows that map P has a global attractor $M_1 = \{(0, I^*(0), y^*(0))\}$ in ∂X_0. It is clear that, in $\partial X_0, \{M_1\}$ is isolated, invariant, and does not form a cycle. Therefore, conditions (1)–(3) of (C_2) in Lemma 5 hold.

Secondly, let $x_0 = (S_0, I_0, y_0) \in X_0$. By the continuity of solutions with respect to the initial value, for any $\varepsilon > 0$, there is a $\delta_1 > 0$; when $\|x_0 - M_1\| \leq \delta_1$, we have

$$\|u(t, x_0) - u(t, M_1)\| < \varepsilon, \quad \forall t \in [0, T]. \quad (40)$$

Now, we claim that

$$\limsup_{n\to\infty} d(P^n(x_0), M_1) \geq \delta_1. \quad (41)$$

Suppose the conclusion is not true; then we have

$$\limsup_{n\to\infty} d(P^n(x_0), M_1) < \delta_1 \quad (42)$$

for some $x_0 \in X_0$. For the sake of simplicity, one may assume that

$$d(P^n(x_0), M_1) < \delta_1 \quad \forall n \geq 0. \quad (43)$$

Also, from (40) we obtain

$$\|u(t, P^n(x_0)) - u(t, M_1)\| < \varepsilon \quad \forall n \geq 0,\ t \in [0, T]. \quad (44)$$

Then, for any $t \geq 0$, let $t = nT + \tilde{t}$, where $\tilde{t} \in [0, T)$ and $n = [t/T]$ is the greatest integer less than or equal to t/T; we can get

$$\begin{aligned} &\|u(t, x_0) - u(t, M_1)\| \\ &\quad = \|u(\tilde{t}, P^n(x_0)) - u(\tilde{t}, M_1)\| < \varepsilon. \end{aligned} \quad (45)$$

Since $u(t, x_0) = (S(t), I(t), y(t))$ and $u(t, M_1) = (0, I^*(t), y^*(t))$, (45) signifies that

$$\begin{gathered} 0 < S(t) \leq \varepsilon, \\ I(t) \leq I^*(t) + \varepsilon, \\ y(t) \leq y^*(t) + \varepsilon \end{gathered} \quad (46)$$

for all $t \geq 0$. By condition (33), we can choose constant $\varepsilon > 0$ such that

$$\varrho = \exp\left\{\int_0^T \left[f(\varepsilon, I^*(t) + \varepsilon) - \max_{0\leq\theta_1\leq\varepsilon} \frac{\partial g(\varepsilon, I^*(t) + \varepsilon)}{\partial S} - \max_{0\leq\theta_1\leq\varepsilon} \frac{\partial h(\varepsilon, y^*(t) + \varepsilon)}{\partial S}\right] dt\right\} = \exp\left\{\int_0^T \varrho_0(t, \varepsilon)\, dt\right\} > 1. \quad (47)$$

Further, from assumptions (A_1) and (A_2) and inequalities (46), we have

$$\begin{aligned} \dot{S}(t) &= S\left\{f(S, I) - \frac{g(S, I)}{S} - \frac{h(S, y)}{S}\right\} \\ &\geq S\left\{f(\varepsilon, I^*(t) + \varepsilon) - \frac{\partial g(\theta_1, I^*(t) + \varepsilon)}{\partial S}\right. \\ &\quad \left. - \frac{\partial h(\theta_2, y^*(t) + \varepsilon)}{\partial S}\right\} \geq S\left\{f(\varepsilon, I^*(t) + \epsilon)\right. \\ &\quad - \max_{0\leq\theta_1\leq\epsilon} \frac{\partial g(\theta_1, I^*(t) + \varepsilon)}{\partial S} \\ &\quad \left. - \max_{0\leq\theta_2\leq\epsilon} \frac{\partial h(\theta_2, y^*(t) + \varepsilon)}{\partial S}\right\}, \end{aligned} \quad (48)$$

where $0 < \theta_1, \theta_2 < S$. For any $t \geq 0$, choose an integer $k \geq 0$ such that $t = kT + \hat{t}$, where $\hat{t} \in [0, T)$. Integrating (48) from 0 to t and noticing (47), then

$$\begin{aligned} &S(t) \\ &\quad \geq S(0^+) \exp\left(\int_0^t \varrho_0(s, \varepsilon)\, ds\right) \\ &\quad = S(0^+) \exp\left(k\int_0^T \varrho_0(s, \varepsilon)\, ds + \int_{kT}^{kT+\hat{t}} \varrho_0(s, \varepsilon)\, ds\right) \\ &\quad \geq \frac{S(0^+)}{\exp(TN_0)} \varrho^k, \end{aligned} \quad (49)$$

where $N_0 = \sup_{t\geq 0}\varrho_0(t, \epsilon) > 0$. Thus, $\lim_{t\to\infty} S(t) = \infty$, which is a contradiction with $0 \leq S(t) \leq \varepsilon$ for all $t \geq 0$. Hence, claim (41) holds. This shows $W^s(M_1) \cap X_0 = \emptyset$. Therefore, condition (4) of (C_2) holds. Consequently, by Lemma 5, P is uniformly persistent with respect to $(X_0, \partial X_0)$.

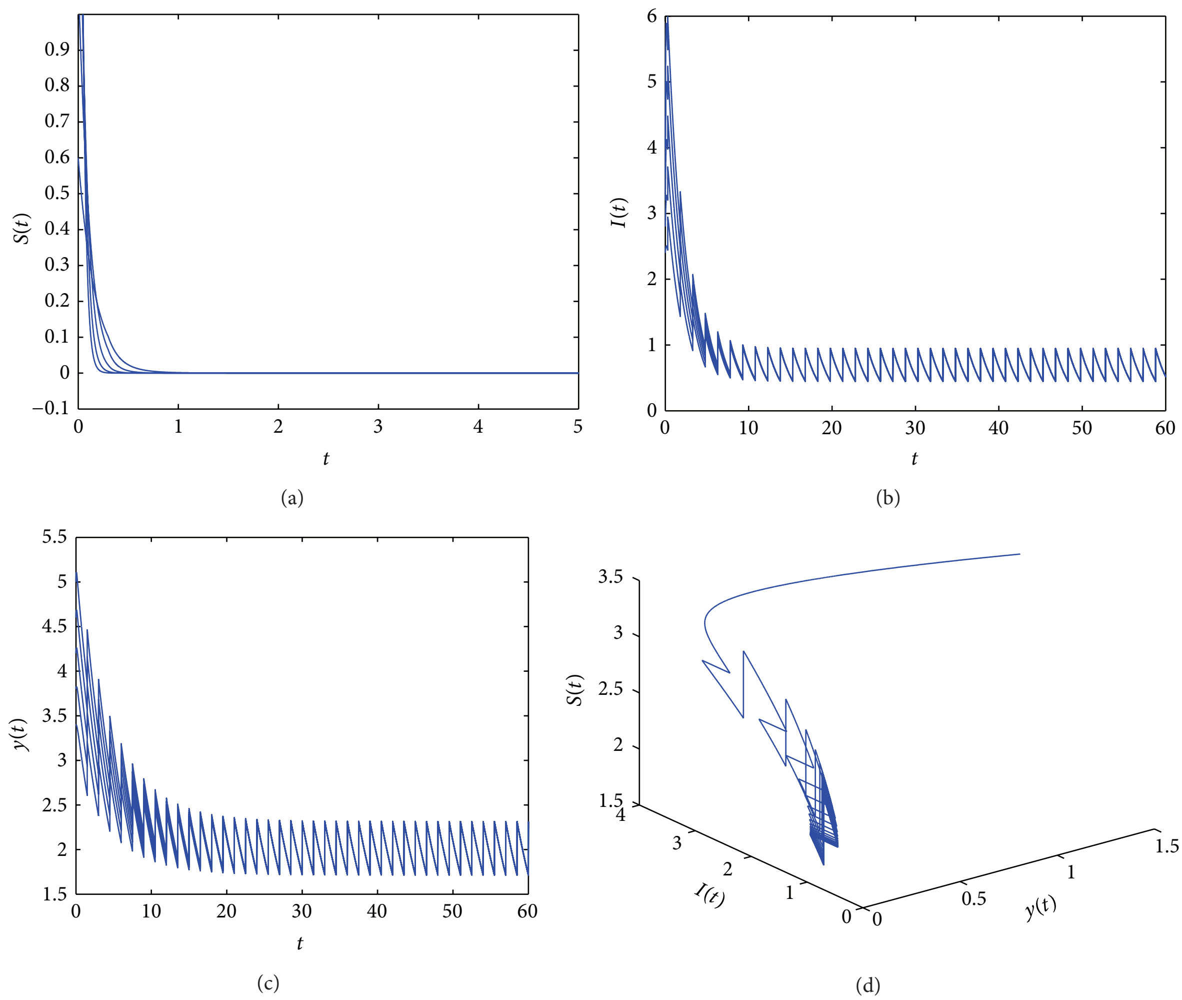

FIGURE 1: Time sires of $S(t)$ (a), $I(t)$ (b), $y(t)$ (c), and phase portrait (d) in model (1) with the initial values $S(0^+) = 0.1 + 0.5k$, $I(0^+) = 2 + 0.4k$, and $y(0^+) = 3 + 0.4k$, $k = 1, 2, 3, 4, 5$.

Lastly, since model (1) is periodic, we obtain that model (1) is uniformly persistent. From Lemma 4, model (1) also is permanent. This completed the proof. □

Corollary 10. *When functions $f(S, I)$, $g(S, I)$, and $h(S, y)$ are given in (30), then condition (33) is equivalent to the following condition:*

$$rT > \frac{r\theta p_1}{K\omega} + \frac{\beta p_1^q}{q\omega}\frac{1 - \exp(-q\omega T)}{[1 - \exp(-\omega T)]^q} + \frac{ap_2}{d}. \tag{50}$$

Remark 11. In (50), if $p_1 = 0$, then we have $p_2 < drT/a$, which means that if only natural enemies are released periodically and the amount is less than drT/a, then the system is permanent and the pest will not be eradicated. If $p_2 = 0$, then the release amount satisfies

$$rT > \frac{r\theta p_1}{K\omega} + \frac{\beta p_1^q}{q\omega}\frac{1 - \exp(-q\omega T)}{[1 - \exp(-\omega T)]^q} \tag{51}$$

to ensure the system is permanent and the pest will not be eradicated.

Remark 12. Applying Theorem 1 given in [16], it is clear that when condition (33) holds, model (1) at least has one positive T-periodic solution.

Remark 13. Taking the functions in model (1) as

$$\begin{aligned} f(S, I) &= r\left\{1 - \frac{S + I}{K}\right\}, \\ g(S, I) &= \beta SI, \\ h(S, y) &= \frac{aSy}{1 + \omega S}, \end{aligned} \tag{52}$$

the model has been discussed in [14]. Furthermore, noticing (30), we have reason to confirm that our study makes the model in [14] more general.

Corollary 14. *Define constant*

$$\begin{aligned} &R_0 \\ &= \exp\left\{\int_0^T \left(f(0, I^*(t)) - \frac{\partial g}{\partial S}\left(0, I^*(t) - \frac{\partial h}{\partial S}(0, y^*(t))\right)\right) dt\right\}. \end{aligned} \tag{53}$$

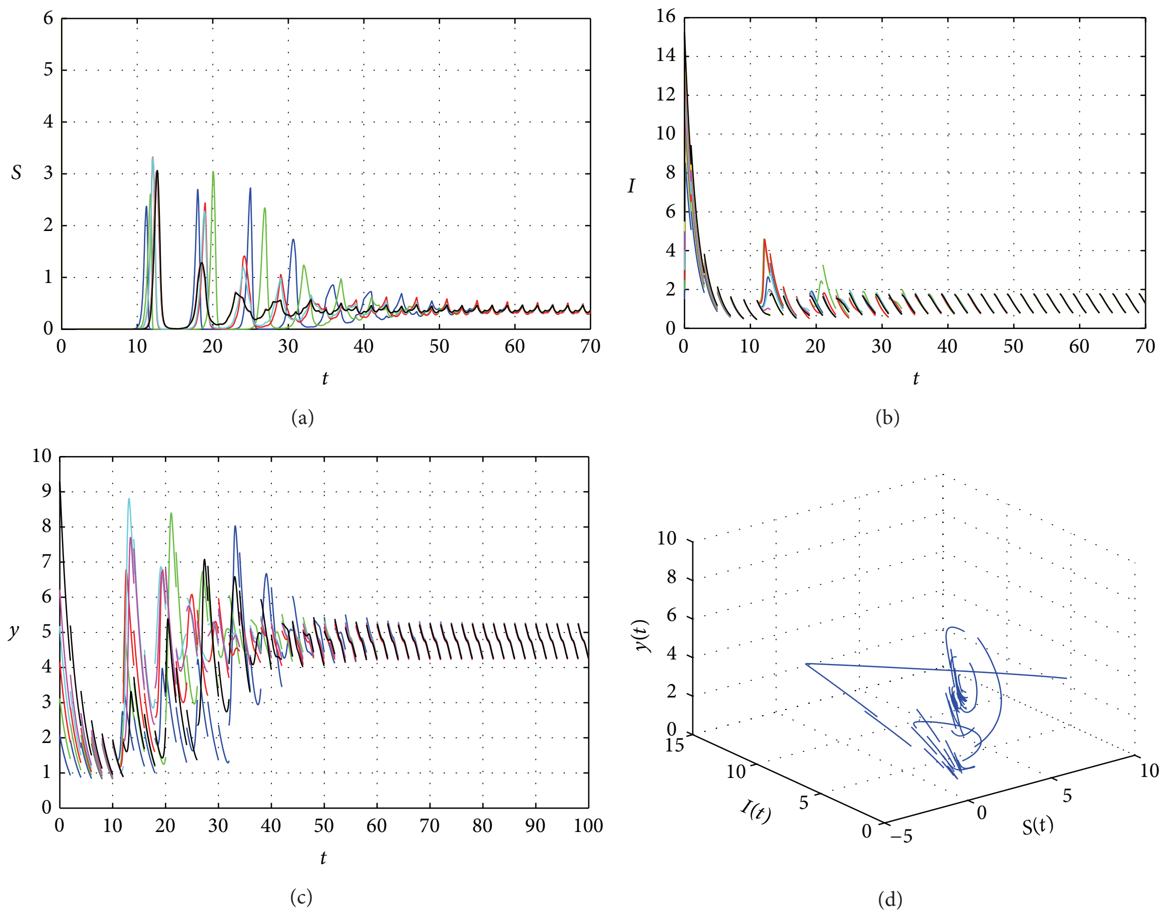

FIGURE 2: Time sires of $S(t)$ (a), $I(t)$ (b), $y(t)$ (c), and phase portrait (d) in model (1) with the initial values $S(0^+) = 0.1 + 0.5k$, $I(0^+) = 2 + 0.4k$, and $y(0^+) = 3 + 0.4k$, $k = 1, 2, 3, 4, 5$.

If $R_0 < 1$, then susceptible pest eradication periodic solution $(0, I^*(t), y^*(t))$ of model (1) is globally asymptotically stable, and if $R_0 > 1$, then model (1) is permanent.

5. Numerical Example

In this section, we give some examples and numerical simulations to confirm the above theoretical analysis. Let us consider model (1) with functions $f(S, I)$, $g(S, I)$, and $h(S, y)$ given in (30) and parameters as follows.

Example 1. Take $r = 1.5$, $K = 20$, $\theta = 0.5$, $\beta = 0.8$, $q = 2$, $\delta = 0.5$, $d = 0.2$, $\omega = 0.5$, $a = 0.7$, $p_1 = 0.5$, $p_2 = 0.6$, and $T = 1.5$. By computing, we have

$$rT = 2.25 < \frac{r\theta p_1}{K\omega} + \frac{\beta p_1^q}{q\omega} \frac{1 - \exp(-q\omega T)}{[1 - \exp(-\omega T)]^q} + \frac{ap_2}{d} \approx 5.31. \quad (54)$$

Therefore, inequality (31) holds; from Theorem 6, periodic solution $(0, I^*(t), y^*(t))$ of model (1) is globally asymptotically stable, which is illustrated in Figure 1.

Example 2. Take $r = 6$, $K = 13$, $\theta = 0.5$, $\beta = 0.3$, $q = 2$, $\delta = 0.8$, $d = 0.4$, $\omega = 0.6$, $a = 1$, $p_1 = 2$, $p_2 = 2$, and $T = 2$. By computing, $M \approx 63$, $m \approx 1.5$ in Lemma 4, and

$$rT = 12 > \frac{r\theta p_1}{K\omega} + \frac{\beta p_1^q}{q\omega} \frac{1 - \exp(-q\omega T)}{[1 - \exp(-\omega T)]^q} + \frac{ap_2}{d} \approx 9.57. \quad (55)$$

Therefore, inequality (50) holds; from Theorem 9, model (1) is permanent. Numerical simulation (see Figure 2) shows that there exists a unique positive T-periodic solution $(\widetilde{S}(t), \widetilde{I}(t), \widetilde{y}(t))$ of model (1) which is globally attractive.

From the above example, we can guess that only inequality (33) holds; then model (1) has a unique positive T-periodic solution which is globally attractive.

6. Discussion

In this paper, a general ecoepidemic model with impulsive control strategy is proposed and its dynamical behavior is analyzed for the purpose of integrated pest management. Meanwhile, the model which the researchers obtained in [14] was generalized. By using Floquet theorem and theory of persistence of dynamical systems, we show that if condition (13) holds, the susceptible pest eradication periodic solution $(0, I^*(t), y^*(t))$ is globally asymptotically stable (see Figure 1), which means that the pest has been eradicated; when condition (33) holds, model (1) is permanent (see Figure 2), which means the pest and their natural enemy can coexist in the area. Furthermore, from Remark 12, model (1) has at least one positive T-periodic solution.

Further study includes investigating whether or not a nontrivial periodic solution emerges when the threshold $R_0 = 1$ holds and analyzing dynamical behavior about model with delay.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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