Hindawi Publishing Corporation
Discrete Dynamics in Nature and Society
Volume 2016, Article ID 1267954, 10 pages
http://dx.doi.org/10.1155/2016/1267954

Research Article

Global Exponential Stability of Periodic Solution for Neutral-Type Complex-Valued Neural Networks

Song Guo and Bo Du

Department of Mathematics, Huaiyin Normal University, Huaian, Jiangsu 223300, China

Correspondence should be addressed to Bo Du; dubo7307@163.com

Received 24 April 2016; Accepted 11 August 2016

Academic Editor: Rigoberto Medina

This paper deals with a class of neutral-type complex-valued neural networks with delays. By means of Mawhin's continuation theorem, some criteria on existence of periodic solutions are established for the neutral-type complex-valued neural networks. By constructing an appropriate Lyapunov-Krasovskii functional, some sufficient conditions are derived for the global exponential stability of periodic solutions to the neutral-type complex-valued neural networks. Finally, numerical examples are given to show the effectiveness and merits of the present results.

1. Introduction

As it is well known, in a large amount of applications, complex signals often occur and the complex-valued neural networks (CVNNs) are preferable. Therefore, there have been increasing research interests in the dynamical behaviors of complex-valued recurrent neural networks; see [1–8] and references therein. Recently, Gong et al. [9] considered the complex-valued recurrent neural networks with time-varying delays by using the matrix measure method and the Halanay inequality as follows:

$$\dot{u}(t) = -Cu(t) + Af(u(t)) + Bf(u(t-\tau(t))), \quad u \in \mathbb{C}^n. \tag{1}$$

Zhang and Yu [10] studied a class of complex-valued Cohen-Grossberg neural networks with time delays

$$\begin{aligned} z'(t) &= -az(t)\left[h(z(t)) - Af(z(t)) - Bg(z(t-\tau)) - u\right], \end{aligned} \tag{2}$$

and some stability results were obtained. In [11], Song et al. considered the global exponential stability of complex-valued neural networks with both time-varying delays and impulsive effects as follows:

$$\begin{aligned} \dot{z}(t) &= -Cz(t) + Af(z(t)) + Bf(z(t-\tau(t))) + J_k, \quad t \neq t_k, \\ z(t) &= D_k h_k(z(t^-)) + E_k S_k(z(t^- - \tau(t^-))) + \bar{J}_k, \quad t = t_k. \end{aligned} \tag{3}$$

In [12], Wang and Huang obtained the stability criteria for complex-valued bidirectional associative memory (BAM) with time delay by using the Lyapunov function method and mathematical analysis technique. Pan et al. [13] obtained the global exponential stability of a class of CVNNs with time-varying delays by applying conjugate system of CVNNs, fixed point theorem, contraction mapping principle, and a delay differential inequality. In [3], applying delta differential operator, a complex-valued neural network on time scales was discussed. In [11], a class of CVNNs with probabilistic time-varying delays is considered, and several delay-distribution-dependent sufficient conditions to guarantee the global asymptotic and exponential stability were obtained by constructing proper Lyapunov-Krasovskii functional and employing inequality technique.

On the other hand, neutral-type CNNs can be used for describing these complicated dynamic properties of neural cells. Generally, it can be described as

$$\begin{aligned}(x_i)'(t) &= -a_i(t)x_i(t) + \sum_{j=1}^{m} b_{ij}(t) f_j\left(x_j\left(t-\tau_{ij}(t)\right)\right) \\ &\quad + \sum_{j=1}^{n} d_{ij}(t) g_j\left(x_j'\left(t-\delta_{ij}(t)\right)\right) + I_i(t), \\ x_i(t) &= \phi_i(t),\ t\in[-r,0],\ i=1,2,\ldots,n.\end{aligned} \tag{4}$$

There exist numerous attempts to show the properties of system (4). When $g_j(x_j'(t-\delta_{ij}(t))) = \alpha x_j'(t-\delta_{ij}(t))$, Park et al. [14–18] considered the stability problems of (4). Under the assumptions $n = m$ and $f_j(u) = g_j(u)$, $\forall u \in \mathbb{R}$, Gui et al. [19] studied the periodic oscillation of (4). Xu et al. [20] and Qin and Cao [21] considered the delay-dependent exponential stability and the delay-dependent robust stability of (4), respectively. Recently, Wang and Zhu [22] investigated the following generalized neutral-type neural networks with delays:

$$\begin{aligned}&(A_i x_i)'(t) \\ &= -a_i(t)x_i(t) \\ &\quad + \sum_{j=1}^{n}\left[b_{ij}(t) f_j\left(x_j(t)\right) + d_{ij}(t) g_j\left(x_j\left(t-\tau_{ij}(t)\right)\right)\right] \\ &\quad + I_i(t), \\ &\qquad x_i(t) = \phi_i(t),\ t\in[-\tau,0],\ i=1,2,\ldots,n,\end{aligned} \tag{5}$$

where A_i is a difference operator defined by

$$(A_i x_i)(t) = x_i(t) - \sum_{j=1}^{n} c_{ij}(t) x_i\left(t-\delta_{ij}(t)\right), \tag{6}$$

and they obtained some simple sufficient conditions for guaranteeing the existence, global asymptotic stability, and exponential stability of the unique almost periodic solution for (5) by using fixed point theorem and Lyapunov functional method.

To our knowledge, no studies have been reported on the properties of neutral-type CVNNs with time-varying delays until now. This motivated us to carry out a study in this paper. In this study, we discuss the existence and exponential stability of periodic solution for the following neutral-type CVNNs with time-varying delays:

$$\begin{aligned}\frac{dA_p z_p(t)}{dt} &= -d_p(t) z_p(t) + \sum_{q=1}^{n} a_{pq}(t) f_q\left(z_q(t)\right) \\ &\quad + \sum_{q=1}^{n} b_{pq}(t) g_q\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &\quad + H_p(t),\end{aligned} \tag{7}$$

where

$$\begin{aligned}A_p z_p(t) &= A_p x_p(t) + iA_p y_p(t), \\ A_p x_p(t) &= x_p(t) - c_p x_p(t-\tau), \\ A_p y_p(t) &= y_p(t) - c_p y_p(t-\tau),\end{aligned} \tag{8}$$

$p = 1,2,\ldots,n$, $\tau, c_p \in \mathbb{R}$ with $|c_p| \neq 1$, $d_p(t) \geq 0$ is the self-feedback connection weight, and $a_{pq}(t)$, $b_{pq}(t)$ are complex-valued connection weight matrices without and with time delays, respectively. $f_q(z_q)$, $g_q(z_q) : \mathbb{C} \to \mathbb{C}$ is the activation function of the neutrons. $H_p(t) \in \mathbb{C}$ is the external input vector. $\gamma_{pq}(t) \geq 0$ corresponds to the transmission delays.

System (7) can be written in vector form as follows:

$$\begin{aligned}\frac{dAz(t)}{dt} &= -D(t)z(t) + A(t) f(z(t)) \\ &\quad + B(t) g\left(z\left(t-\gamma(t)\right)\right) + H(t),\end{aligned} \tag{9}$$

where $Az(t) = (A_1 z_1(t), A_2 z_2(t), \ldots, A_n z_n(t))^\top \in \mathbb{C}^n$ is the state vector, $D(t) = \operatorname{diag}(d_1(t), d_2(t), \ldots, d_n(t))$ is the self-feedback connection weight matrix, and $A(t) = (a_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$ and $B(t) = (b_{pq})_{n\times n} \in \mathbb{C}^{n\times n}$ are complex-valued connection weight matrices without and with time delays, respectively. $f(z) = (f_1(z_1), f_2(z_2), \ldots, f_n(z_n))^\top : \mathbb{C}^n \to \mathbb{C}^n$ is the activation function of the neutrons. $H(t) = (H_1(t), H_2(t), \ldots, H_n(t))^\top \in \mathbb{C}^n$ is the external input vector.

Throughout the paper, we give some notations:

$$\begin{aligned}a_{pq}^- &= \min_{t\in[0,T]}\left\{\left|a_{pq}\right|\right\}, \\ a_{pq}^+ &= \max_{t\in[0,T]}\left\{\left|a_{pq}\right|\right\}, \\ b_{pq}^- &= \min_{t\in[0,T]}\left\{\left|b_{pq}\right|\right\}, \\ b_{pq}^+ &= \max_{t\in[0,T]}\left\{\left|b_{pq}\right|\right\}, \\ d_{pq}^- &= \min_{t\in[0,T]}\left\{\left|d_{pq}\right|\right\}, \\ d_{pq}^+ &= \max_{t\in[0,T]}\left\{\left|d_{pq}\right|\right\}, \\ \overline{f} &= \frac{1}{T}\int_0^T f(t)\,dt.\end{aligned} \tag{10}$$

Remark 1. The neural network model (7) shows the neutral character by the A_p operator, which is different from the corresponding results of other papers; see, for example, [23–27].

Remark 2. When $c_p = 0$, $p = 1,2,\ldots,n$, system (7) is changed into non-neutral-type CVNNs which have been extensively studied; see, for example, [2, 5, 9, 12, 13].

Remark 3. In general, when $|c_p| \geq 1$ in (7), the operator A_p has no inverse operator. Hence, it is very difficult for obtaining existence and stability results to (7).

We also make the following assumptions:

(H_1) $d_p(t), a_{pq}(t), b_{pq}(t), d_{pq}(t)$, and $H_p(t)$ are T-periodic continuous functions.

(H_2) There exist nonnegative constants $M_{q,R}, M_{q,I}, N_{q,R}$, and $N_{q,R}$ such that

$$\begin{aligned} &\left|f_q^R(z)\right| \le M_{q,R}, \\ &\left|f_q^I(z)\right| \le M_{q,I}, \\ &\left|g_q^R(z)\right| \le N_{q,R}, \\ &\left|g_q^I(z)\right| \le N_{q,I}, \\ &\qquad \forall z \in \mathbb{C}^n,\ q = 1, 2, \ldots, n. \end{aligned} \tag{11}$$

The distinctive contributions of this paper are outlined as follows: (1) the neutral-type neural network model (7) shows the neutral character by the A_p operator, which is different from other papers. Hence, when the neutral delay term is studied as a neutral operator A_p, novel analysis technique is developed since the conventional analysis tool no longer applies. (2) We use a novel method for studying the stability of periodic solutions, which is different from traditional methods. (3) A unified framework is established to handle complex-valued neural networks, neutral terms, and time-varying delays.

By separating the state, the connection weight, the activation function, and the external input into its real and imaginary part, then system (7) can be rewritten as follows for $p = 1, 2, \ldots, n$:

$$\begin{aligned} \frac{dA_p x_p(t)}{dt} = &-d_p(t)\, x_p(t) + \sum_{q=1}^{n} a_{pq}^R(t)\, f_q^R\left(z_q(t)\right) \\ &- \sum_{q=1}^{n} a_{pq}^I(t)\, f_q^I\left(z_q(t)\right) \\ &+ \sum_{q=1}^{n} b_{pq}^R(t)\, g_q^R\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &- \sum_{q=1}^{n} b_{pq}^I(t)\, g_q^I\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &+ H_p^R(t) = \Xi_{p,x}(t), \\ \frac{dA_p y_p(t)}{dt} = &-d_p(t)\, y_p(t) + \sum_{q=1}^{n} a_{pq}^R(t)\, f_q^I\left(z_q(t)\right) \\ &+ \sum_{q=1}^{n} a_{pq}^I(t)\, f_q^R\left(z_q(t)\right) \\ &+ \sum_{q=1}^{n} b_{pq}^R(t)\, g_q^I\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &+ \sum_{q=1}^{n} b_{pq}^I(t)\, g_q^R\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &+ H_p^I(t) = \Xi_{n+p,y}(t), \end{aligned} \tag{12}$$

where $x_p(t) = \mathrm{Re}(z_p(t))$, $y_p(t) = \mathrm{Im}(z_p(t))$, $a_{pq}^R(t) = \mathrm{Re}(a_{pq}(t))$, $a_{pq}^I(t) = \mathrm{Im}(a_{pq}(t))$, $b_{pq}^R(t) = \mathrm{Re}(b_{pq}(t))$, $b_{pq}^I(t) = \mathrm{Im}(b_{pq}(t))$, $H_p^R(t) = \mathrm{Re}(H_p(t))$, $H_p^I(t) = \mathrm{Im}(H_p(t))$, $f_q^R(z_q(t)) = \mathrm{Re}(f_q(z_q(t)))$, $f_q^I(z_q(t)) = \mathrm{Im}(f_q(z_q(t)))$, $g_q^R(z_q(t-\gamma_{pq}(t))) = \mathrm{Re}(g_q(z_q(t-\gamma_{pq}(t))))$, $g_q^I(z_q(t-\gamma_{pq}(t))) = \mathrm{Im}(g_q(z_q(t-\gamma_{pq}(t))))$.

2. Preliminaries

In this section, we state some useful definitions and lemmas.

Lemma 4 (see [28, 29]). *Define operator A on C_T:*

$$\begin{aligned} &A : C_T \longrightarrow C_T, \\ &[Ax](t) = x(t) - cx(t-\tau), \quad \forall t \in \mathbb{R}, \end{aligned} \tag{13}$$

where $C_T = \{x : x \in C(\mathbb{R}, \mathbb{R}),\ x(t+T) \equiv x(t)\}$ and c is a constant. When $|c| \neq 1$, then A has a unique continuous bounded inverse A^{-1} satisfying

$$\begin{aligned} &\left[A^{-1} f\right](t) \\ &\quad = \begin{cases} \displaystyle\sum_{j\ge 0} c^j f(t-j\tau), & \text{if } |c| < 1,\ \forall f \in C_T, \\ \displaystyle -\sum_{j\ge 1} c^{-j} f(t+j\tau), & \text{if } |c| > 1,\ \forall f \in C_T, \end{cases} \end{aligned} \tag{14}$$

and A^{-1} has the following inequality properties:

(a) $\|A^{-1}\| \le 1/|1-|c||$;

(b) $\int_0^T |[A^{-1}f](t)|dt \le (1/|1-|c||)\int_0^T |f(t)|dt$, $\forall f \in C_T$;

(c) $\int_0^T |[A^{-1}f](t)|^2 dt \le (1/|1-|c||)\int_0^T |f(t)|^2 dt$, $\forall f \in C_T$.

Lemma 5 (see [30]). *Suppose that X and Y are two Banach spaces, and $L : D(L) \subset X \to Y$ is a Fredholm operator with index zero. Furthermore, $\Omega \subset X$ is an open bounded set and $N : \overline{\Omega} \to Y$ is L-compact on $\overline{\Omega}$. If all the following conditions hold,*

(1) $Lx \neq \lambda Nx$, $\forall x \in \partial\Omega \cap D(L)$, $\forall \lambda \in (0, 1)$,

(2) $Nx \notin \mathrm{Im}\, L$, $\forall x \in \partial\Omega \cap \mathrm{Ker}\, L$,

(3) $\deg\{JQN,\ \Omega \cap \mathrm{Ker}\, L, 0\} \neq 0$,

where $J : \mathrm{Im}\, Q \to \mathrm{Ker}\, L$ is an isomorphism, then equation $Lx = Nx$ has a solution on $\overline{\Omega} \cap D(L)$.

Definition 6. The periodic solution of (7) $z_p^* = (z_1^*, z_2^*, \ldots, z_n^*)^\top$ is globally exponentially stable if there exist constants $\alpha > 0$ and $\beta \ge 1$ such that

$$\left|z_p - z_p^*\right| \le \beta e^{-\alpha t}\left|z_p(0) - z_p^*(0)\right|. \tag{15}$$

3. Existence of Periodic Solution

In order to investigate the periodicity of system (7) by means of Mawhin's continuation theorem, we need to introduce some function spaces. Let

$$\begin{aligned} C_T &= \{\phi : \mathbb{R} \\ &\quad \longrightarrow \mathbb{R} \mid \phi(t) \text{ is continuous and } T\text{-periodic}\}, \\ C_T^1 &= \{\phi \in C_T \mid \phi' \in C_T\}, \\ I_N &= \{1, 2, \ldots, n\}, \\ U &= \{u(t) = (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n)^\top \mid x_p, y_p \\ &\quad \in C_T,\ p \in I_N\}, \end{aligned} \tag{16}$$

with the norm

$$\begin{aligned} \|u\|_U &= \left\|(x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n)^\top\right\|_U \\ &= \sum_{p=1}^{n} \sup_{t\in[0,T]} \left|x_p(t)\right| + \sum_{p=1}^{n} \sup_{t\in[0,T]} \left|y_p(t)\right|. \end{aligned} \tag{17}$$

Theorem 7. *Assume that conditions (H_1) and (H_2) hold. Then system (7) has at least one T-periodic solution, if $T[d_p(t)]^+/|1-c_p| > 1$.*

Proof. From (12), let

$$\begin{aligned} L &: D(L) \subset U \longrightarrow U, \\ Lu &= (Au)'(t), \\ N &: U \longrightarrow U, \\ Nu &= \begin{pmatrix} \Xi_{p,x}(t) \\ \Xi_{n+p,y}(t) \end{pmatrix}, \\ & \qquad u \in U. \end{aligned} \tag{18}$$

By Hale's terminology [31], a solution of system (7) is $u = (x_1, x_2, \ldots, x_n, y_1, y_2, \ldots, y_n)^\top \in C(\mathbb{R}, \mathbb{R}^{2n})$ such that $Au \in C^1(\mathbb{R}, \mathbb{R}^{2n})$ and the equalities in (7) are satisfied on $\mathbb{R}$. Nevertheless, it is easy to see that $(Au)'(t) = Au'(t)$. So a T-periodic solution of the system (7) must be from $C^1(\mathbb{R}, \mathbb{R}^{2n})$. According to Lemma 5, we can easily obtain that

$$\begin{aligned} \operatorname{Im} L &= \left\{u \in U \mid \int_0^T Au(s)\,ds = \mathbf{0}\right\}, \\ \operatorname{Ker} L &= \left\{u \in U \mid u \equiv c \in \mathbb{R}^{2n}\right\}. \end{aligned} \tag{19}$$

Obviously, $\operatorname{Im} L$ is a closed set in C_T and $\dim \operatorname{Ker} L = \operatorname{con}\dim \operatorname{Im} L = 2n$. So L is a Fredholm operator with index zero. Define continuous projectors P, Q:

$$\begin{aligned} P &: C_T \longrightarrow \operatorname{Ker} L, \\ (Pu)(t) &= \frac{1}{T}\int_0^T u(s)\,ds, \\ Q &: C_T \longrightarrow \frac{C_T}{\operatorname{Im} L}, \\ Qv &= \frac{1}{T}\int_0^T v(s)\,ds. \end{aligned} \tag{20}$$

Let

$$L_P = L|_{D(L)\cap \operatorname{Ker} P} : D(L) \cap \operatorname{Ker} P \longrightarrow \operatorname{Im} L, \tag{21}$$

and then

$$L_P^{-1} = K_P : \operatorname{Im} L \longrightarrow D(L) \cap \operatorname{Ker} P. \tag{22}$$

Since $\operatorname{Im} L \subset C_T$ and $D(L) \cap \operatorname{Ker} P \subset C_T^1$, K_P is an embedding operator. Hence K_P is a complete operator in $\operatorname{Im} L$. By the definitions of Q and N, it is known that $QN(\overline{\Omega})$ is bounded on $\overline{\Omega}$. Hence nonlinear operator N is L-compact on $\overline{\Omega}$. We complete the proof by three steps.

Step 1. Let $\Omega_1 = \{u \in D(L) \subset C_T^1 : Lu = \lambda Nu,\ \lambda \in (0,1)\}$. We show that Ω_1 is a bounded set. If $u \in \Omega_1$, then $Lu = \lambda Nu$; that is, for $p = 1, 2, \ldots, n$,

$$\begin{aligned} \frac{dA_p x_p(t)}{dt} &= \lambda \Bigg[-d_p(t)\, x_p(t) \\ &\quad + \sum_{q=1}^{n} a_{pq}^R(t)\, f_q^R\left(z_q(t)\right) - \sum_{q=1}^{n} a_{pq}^I(t)\, f_q^I\left(z_q(t)\right) \\ &\quad + \sum_{q=1}^{n} b_{pq}^R(t)\, g_q^R\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &\quad - \sum_{q=1}^{n} b_{pq}^I(t)\, g_q^I\left(z_q\left(t-\gamma_{pq}(t)\right)\right) + H_p^R(t) \Bigg], \end{aligned} \tag{23}$$

$$\begin{aligned} \frac{dA_p y_p(t)}{dt} &= \lambda \Bigg[-d_p(t)\, y_p(t) + \sum_{q=1}^{n} a_{pq}^R(t)\, f_q^I\left(z_q(t)\right) \\ &\quad + \sum_{q=1}^{n} a_{pq}^I(t)\, f_q^R\left(z_q(t)\right) \\ &\quad + \sum_{q=1}^{n} b_{pq}^R(t)\, g_q^I\left(z_q\left(t-\gamma_{pq}(t)\right)\right) \\ &\quad + \sum_{q=1}^{n} b_{pq}^I(t)\, g_q^R\left(z_q\left(t-\gamma_{pq}(t)\right)\right) + H_p^I(t) \Bigg]. \end{aligned} \tag{24}$$

There exists $t_i \in [0,T]$ such that $A_p x_p(t_i) = [A_p x_p(t)]^+$. Hence $(A_p x_p)'(t_i) = 0$. In view of (23), this implies that

$$\begin{aligned} d_p(t_i)\, x_p(t_i) &= \sum_{q=1}^{n} a_{pq}^R(t_i)\, f_q^R\left(z_q(t_i)\right) \\ &\quad - \sum_{q=1}^{n} a_{pq}^I(t_i)\, f_q^I\left(z_q(t_i)\right) \end{aligned}$$

$$+\sum_{q=1}^{n} b_{pq}^{R}(t_i)\, g_q^{R}\left(z_q\left(t_i-\gamma_{pq}(t_i)\right)\right)$$
$$-\sum_{q=1}^{n} b_{pq}^{I}(t_i)\, g_q^{I}\left(z_q\left(t_i-\gamma_{pq}(t_i)\right)\right)$$
$$+H_p^{R}(t_i). \tag{25}$$

By (25) and assumption (H_1), for $p=1,2,\ldots,n$, we have

$$\begin{aligned}\left|x_p(t_i)\right| &= \left|\sum_{q=1}^{n}\frac{a_{pq}^{R}(t_i)\, f_q^{R}\left(z_q(t_i)\right)}{d_p(t_i)}\right.\\ &\quad-\sum_{q=1}^{n}\frac{a_{pq}^{I}(t_i)\, f_q^{I}\left(z_q(t_i)\right)}{d_p(t_i)}\\ &\quad+\sum_{q=1}^{n}\frac{b_{pq}^{R}(t_i)\, g_q^{R}\left(z_q\left(t-\gamma_{pq}(t_i)\right)\right)}{d_p(t_i)}\\ &\quad\left.-\sum_{q=1}^{n}\frac{b_{pq}^{I}(t_i)\, g_q^{I}\left(z_q\left(t_i-\gamma_{pq}(t_i)\right)\right)}{d_p(t)}+\frac{H_p^{R}(t_i)}{d_p(t_i)}\right|\\ &\le \sum_{q=1}^{n}\left[\frac{a_{pq}^{R}(t_i)}{d_p(t_i)}\right]^{+} M_{q,R}+\sum_{q=1}^{n}\left[\frac{a_{pq}^{I}(t_i)}{d_p(t_i)}\right]^{+} M_{q,I}\\ &\quad+\sum_{q=1}^{n}\left[\frac{b_{pq}^{R}(t_i)}{d_p(t_i)}\right]^{+} N_{q,R}+\sum_{q=1}^{n}\left[\frac{b_{pq}^{I}(t)}{d_p(t_i)}\right]^{+} N_{q,I}\\ &\quad+\left[\frac{H_p^{R}(t_i)}{d_p(t_i)}\right]^{+}.\end{aligned} \tag{26}$$

Then by (26) there exists a positive constant h_p such that

$$\left|x_p(t_i)\right| \le h_p, \quad p=1,2,\ldots,n, \tag{27}$$

where h_p is the pth component of vector h. Clearly, h_p, $p=1,2,\ldots,n$, are independent of λ. From (27) and Lemma 4 we have

$$\begin{aligned}\left|x_p(t)\right| &\le h_p+\int_0^T\left|x_p'(s)\right| ds\\ &= h_p+\int_0^T\left|A_p^{-1}A_p x_p'(s)\right| ds\\ &\le h_p+\frac{1}{\left|1-c_p\right|}\int_0^T\left|A_p x_p'(s)\right| ds.\end{aligned} \tag{28}$$

Thus

$$\left[x_p(t)\right]^{+} \le h_p+\frac{1}{\left|1-c_p\right|}\int_0^T\left|A_p x_p'(s)\right| ds. \tag{29}$$

Moreover, it follows from (23) that

$$\begin{aligned}\left|A_p x_p'(t)\right| &= \left|\left[A_p x_p\right]'(t)\right|\\ &\le \left[d_p(t)\right]^{+}\left|x_p(t)\right|+\sum_{q=1}^{n}\left[a_{pq}^{R}(t)\right]^{+} M_{q,R}\\ &\quad+\sum_{q=1}^{n}\left[a_{pq}^{I}(t)\right]^{+} M_{q,I}\\ &\quad+\sum_{q=1}^{n}\left[b_{pq}^{R}(t)\right]^{+} N_{q,R}+\sum_{q=1}^{n}\left[b_{pq}^{I}(t)\right]^{+} N_{q,I}\\ &\quad+\left[H_p^{R}(t)\right]^{+}=\left[d_p(t)\right]^{+}\left[x_p(t)\right]^{+}+\kappa,\end{aligned} \tag{30}$$

where

$$\begin{aligned}\kappa &= \sum_{q=1}^{n}\left[a_{pq}^{R}(t)\right]^{+} M_{q,R}+\sum_{q=1}^{n}\left[a_{pq}^{I}(t)\right]^{+} M_{q,I}\\ &\quad+\sum_{q=1}^{n}\left[b_{pq}^{R}(t)\right]^{+} N_{q,R}+\sum_{q=1}^{n}\left[b_{pq}^{I}(t)\right]^{+} N_{q,I}\\ &\quad+\left[H_p^{R}(t)\right]^{+}.\end{aligned} \tag{31}$$

From (29) and (30), we get

$$\left[x_p(t)\right]^{+} \le h_p+\frac{T\left[d_p(t)\right]^{+}}{\left|1-c_p\right|}\left[x_p(t)\right]^{+}+\frac{T\kappa}{\left|1-c_p\right|}, \tag{32}$$

in which, together with $T[d_p(t)]^{+}/|1-c_p|>1$, there exists a constant M_p such that

$$\left[x_p(t)\right]^{+} \le M_p, \quad p=1,2,\ldots,n. \tag{33}$$

Similar to the above proof, there exists a constant N_p such that

$$\left[x_p(t)\right]^{+} \le N_p, \quad p=1,2,\ldots,n. \tag{34}$$

It follows from (33) and (34) that

$$\|u\|_U < \sum_{p=1}^{n}\left(M_p+N_p\right)+1 := M_0. \tag{35}$$

Step 2. Let $\Omega_2=\{u\in \operatorname{Ker} L : QNu=\mathbf{0}\}$, and we shall prove that if $u\in\partial\Omega_2\subset \operatorname{Ker} L$, then

$$Lu\ne\lambda Nu \quad \lambda\in(0,1). \tag{36}$$

Consider the following algebraic equation:

$$\begin{aligned}&-\overline{d}_p x_p+\sum_{q=1}^{n}\overline{a}_{pq}^{R} f_q^{R}\left(z_q\right)-\sum_{q=1}^{n}\overline{a}_{pq}^{I} f_q^{I}\left(z_q\right)\\ &\quad+\sum_{q=1}^{n}\overline{b}_{pq}^{R} g_q^{R}\left(z_q\right)-\sum_{q=1}^{n}\overline{b}_{pq}^{I} g_q^{I}\left(z_q\right)+\overline{H}_p^{R}=0,\end{aligned}$$

$$-\overline{d}_p y_p + \sum_{q=1}^{n} \overline{a}_{pq}^R f_q^I(z_q) + \sum_{q=1}^{n} \overline{a}_{pq}^I f_q^R(z_q) + \sum_{q=1}^{n} \overline{b}_{pq}^R g_q^I(z_q) + \sum_{q=1}^{n} \overline{b}_{pq}^I g_q^R(z_q) + \overline{H}_p^I = 0. \tag{37}$$

If system (37) has a solution $u^* = (x_1^*, \ldots, x_n^*, y_1^*, \ldots, y_n^*)^\top$, then similar to the above argument, we have $\|u^*\|_U < M_0$. Thus, (36) holds.

Step 3. Let $\Omega = \{u \in U : \|u\|_U < M_0\}$, and then $\Omega_1 \cup \Omega_1 \subset \Omega$. We will show that condition (3) in Lemma 5 holds. Take the homotopy

$$H(u,\mu) = \begin{pmatrix} -\overline{d}_p x_p + (1-\mu)\sum_{q=1}^{n}\left[\overline{a}_{pq}^R f_q^R(z_q) - \overline{a}_{pq}^I f_q^I(z_q) + \overline{b}_{pq}^R g_q^R(z_q) - \overline{b}_{pq}^I g_q^I(z_q)\right] + \overline{H}_p^R \\ -\overline{d}_p y_p + (1-\mu)\sum_{q=1}^{n}\left[\overline{a}_{pq}^R f_q^R(z_q) - \overline{a}_{pq}^I f_q^I(z_q) + \overline{b}_{pq}^R g_q^R(z_q) - \overline{b}_{pq}^I g_q^I(z_q)\right] + \overline{H}_p^I \end{pmatrix}_{2n\times 1}, \tag{38}$$

where $\mu \in [0,1]$. We will show that if $u \in \partial\Omega \cap \operatorname{Ker} L$, and u is a constant vector in $\mathbb{R}^{2n}$ with $\|u\|_U = M_0$, then $H(u,\mu) \neq 0$. Otherwise, if $u \in \mathbb{R}^{2n}$ with $\|u\|_U = M_0$ satisfying $H(u,\mu) = 0$,

$$\begin{aligned} &-\overline{d}_p x_p + (1-\mu)\sum_{q=1}^{n}\left[\overline{a}_{pq}^R f_q^R(z_q) - \overline{a}_{pq}^I f_q^I(z_q) + \overline{b}_{pq}^R g_q^R(z_q) - \overline{b}_{pq}^I g_q^I(z_q)\right] + \overline{H}_p^R = 0, \\ &-\overline{d}_p y_p + (1-\mu)\sum_{q=1}^{n}\left[\overline{a}_{pq}^R f_q^R(z_q) - \overline{a}_{pq}^I f_q^I(z_q) + \overline{b}_{pq}^R g_q^R(z_q) - \overline{b}_{pq}^I g_q^I(z_q)\right] + \overline{H}_p^I = 0, \end{aligned} \tag{39}$$

which is a contradiction to $\|u\|_U < M_0$. And then by the degree theory,

$$\begin{aligned} \deg\{QN, \Omega \cap \operatorname{Ker} L, 0\} &= \deg\{H(\cdot,0), \Omega \cap \operatorname{Ker} L, 0\} \\ &= \deg\{H(\cdot,1), \Omega \cap \operatorname{Ker} L, 0\} \\ &= \deg\{-I, \Omega \cap \operatorname{Ker} L, 0\} \neq 0. \end{aligned} \tag{40}$$

Applying Lemma 5, we reach the conclusion. □

Remark 8. Since the condition $|c_p| \neq 1$, $p = 1,2,\ldots,n$ ensures that operator A_p has inverse operator which is important for obtaining the existence results of periodic solution for (7). In critical case, Serra [32] studied a class of neutral differential equations

$$x'(t) \pm x'(t-\tau) = f(t, x(t)), \quad x\text{: } 2\pi\text{-periodic} \tag{41}$$

and obtained some existence results of periodic solutions. In a very recent paper, Junca and Lombard [33] investigated the following neutral differential equation:

$$\begin{aligned} &y'(t) \pm y'(t-1) + f(y(t)) + g(y(t-1)) = s(t), \quad t > 0, \\ &y(t) = y_0(t) \in H^1((-1,0),\mathbb{R}), \quad -1 < t < 0. \end{aligned} \tag{42}$$

Based on the energy method, the authors obtained new results about asymptotic stability of constant and periodic solutions. Schmitt in [34] has shown how to reduce the case $|a| > 1$ to $|a| < 1$. However, when $|c_p| = 1$, $p = 1,2,\ldots,n$ (in this case, difference A_p has no inverse operator), there are no existence results of periodic solutions for system (7). We hope that the authors are interested in doing further research on this issue.

Remark 9. In this paper, we use Mawhin's continuous theorem and some mathematical analysis technique for obtaining existence results. We can also use other methods (e.g., some fixed points theorem) to discuss the existence of T-periodic solution for system (7).

4. Global Exponential Stability of Periodic Solution

In this section, we establish some results for the uniqueness and exponential stability of the T-periodic solution of system (7).

Theorem 10. *Under conditions of Theorem 7, assume further that*

(H_3) *there exist $L_q > 0$ and $\widetilde{L}_q > 0$ such that for any $u, v \in \mathbb{C}$, one has*

$$\begin{aligned} &\left|f_q(u) - f_q(v)\right| \le L_q |u - v|, \\ &\left|g_q(u) - g_q(v)\right| \le \widetilde{L}_q |u - v|, \\ &\qquad q = 1,2,\ldots,n; \end{aligned} \tag{43}$$

(H_4) $[d_p(t)]^- - \sum_{q=1}^{n}[a_{pq}(t)]^+ L_q - \sum_{q=1}^{n}\max_{t\in[0,T]}(1/(1-\gamma'_{pq}(\nu_{pq}(t))))\widetilde{L}_q[b_{pq}(t)]^+ > 0$, $p = 1,2,\ldots,n$,

where $\gamma'_{pq}(t) < 1$ and $\nu_{pq}(t)$ is an inverse function of $t - \gamma_{pq}(t)$. Then system (7) has unique T-periodic solution which is globally exponentially stable.

Proof. Using conditions of Theorem 7 and condition (H_3), we know that system (7) has unique T-periodic solution $z^*(t) = (z_1^*(t), z_2^*(t), \ldots, z_n^*(t))^\top$. Suppose that $z(t) =$

$(z_1(t), z_2(t), \ldots, z_n(t))^\top$ is an arbitrary solution of (7). Then by (7), for $p = 1, 2, \ldots, n,\ t > 0$ we have

$$\begin{aligned}\frac{d\left[A_p z_p(t) - A_p z_p^*(t)\right]}{dt} &= -d_p(t)\left[z_p(t) - z_p^*(t)\right] \\ &+ \sum_{q=1}^{n} a_{pq}(t)\left[f_q\left(z_q(t)\right) - f_q\left(z_q^*(t)\right)\right] + \sum_{q=1}^{n} b_{pq}(t) \\ &\cdot \left[g_q\left(z_q\left(t-\gamma_{pq}(t)\right)\right) - g_q\left(z_q^*\left(t-\gamma_{pq}(t)\right)\right)\right].\end{aligned} \tag{44}$$

In view of condition (H_3), we have

$$\begin{aligned}\frac{d^+\left|A_p z_p(t) - A_p z_p^*(t)\right|}{dt} &= -\left[d_p(t)\right]^- \left|z_p(t)\right. \\ &\left. - z_p^*(t)\right| + \sum_{q=1}^{n}\left[a_{pq}(t)\right]^+ L_q \left|z_q(t) - z_q^*(t)\right| \\ &+ \sum_{q=1}^{n}\left[b_{pq}(t)\right]^+ \\ &\cdot \widetilde{L}_q \left|z_q\left(t-\gamma_{pq}(t)\right) - z_q^*\left(t-\gamma_{pq}(t)\right)\right|,\end{aligned} \tag{45}$$

where $p = 1, 2, \ldots, n,\ t > 0$, and d^+/dt denotes the upper right derivative. For $t \geq 0$, define a Lyapunov functional $V(t)$ by

$$\begin{aligned}V(t) = \sum_{p=1}^{n} \Bigg\{ &\left|A_p z_p(t) - A_p z_p^*(t)\right| \\ &+ \sum_{q=1}^{n} \int_{t-\gamma_{pq}(t)}^{t} \frac{\widetilde{L}_q\left[b_{pq}(t)\right]^+}{1-\gamma'_{pq}\left(\nu_{pq}(s)\right)} \left|z_p(s) - z_p^*(s)\right| ds \Bigg\},\end{aligned} \tag{46}$$

and by (45) we have

$$\begin{aligned}\frac{d^+V(t)}{dt} \leq \sum_{p=1}^{n} \Bigg\{ &-\left[d_p(t)\right]^- \left|z_p(t) - z_p^*(t)\right| \\ &+ \sum_{q=1}^{n}\left[a_{pq}(t)\right]^+ L_q \left|z_q(t) - z_q^*(t)\right| + \sum_{q=1}^{n}\left[b_{pq}(t)\right]^+ \\ &\cdot \widetilde{L}_q \left|z_q\left(t-\gamma_{pq}(t)\right) - z_q^*\left(t-\gamma_{pq}(t)\right)\right| \\ &+ \sum_{q=1}^{n} \frac{\widetilde{L}_q\left[b_{pq}(t)\right]^+}{1-\gamma'_{pq}\left(\nu_{pq}(t)\right)} \left|z_p(t) - z_p^*(t)\right| \\ &- \sum_{q=1}^{n} \frac{\widetilde{L}_q\left[b_{pq}(t)\right]^+}{1-\gamma'_{pq}(t)} \left|z_p(t) - z_p^*(t)\right| \left(1-\gamma'_{pq}(t)\right) \Bigg\} \\ \leq -\sum_{q=1}^{n} \Bigg(&\left[d_p(t)\right]^- - \left[a_{pq}(t)\right]^+ L_q \\ &- \max_{t\in[0,T]} \frac{1}{1-\gamma'_{pq}\left(\nu_{pq}(t)\right)} \widetilde{L}_q\left[b_{pq}(t)\right]^+ \Bigg) \left|z_p(t)\right. \\ &\left. - z_p^*(t)\right|.\end{aligned} \tag{47}$$

According to condition (H_4) there exists a real number $\alpha > 0$ such that

$$\begin{aligned}&\left[d_p(t)\right]^- - \sum_{q=1}^{n}\left[a_{pq}(t)\right]^+ L_q \\ &\quad - \sum_{q=1}^{n} \max_{t\in[0,T]} \frac{1}{1-\gamma'_{pq}\left(\nu_{pq}(t)\right)} \widetilde{L}_q\left[b_{pq}(t)\right]^+ > \alpha, \\ &\qquad p = 1, 2, \ldots, n,\end{aligned} \tag{48}$$

and it follows that

$$\frac{d^+V(t)}{dt} \leq -\alpha V(t), \quad t > 0. \tag{49}$$

Then by (49) we have

$$V(t) \leq e^{-\alpha t} V(0), \quad t > 0. \tag{50}$$

Thus

$$\sum_{p=1}^{n} \left|z_p(s) - z_p^*(s)\right| \leq e^{-\alpha t}\left|z_p(0) - z_p^*(0)\right|, \quad t > 0. \tag{51}$$

Then system (7) has unique T-periodic solution $z_q^*(t)$ which is globally exponentially stable. □

Remark 11. It is well known that Lyapunov method has been widely used for studying stability problems. In this paper we construct a novel Lyapunov functional for studying the stability of periodic solutions, which is different from traditional Lyapunov functional method. And the proposed analysis method is also easy to extend to the case of other type neural networks. In the future, we will further study the synchronization problem and/or the Markovian jumping problem of complex-valued neural networks.

Remark 12. In studying the stability problems of time-delay systems, various methods have been developed. The important methods are based on the LKF methods [35], combined with other techniques such as the free-weighting matrix approach [36], the descriptor system approach [37], state estimation [38], and the triple-integral terms approach [39]. The results by the above methods are comparatively conservative. The important feature of the above LKF method is that the resulting conditions can check systems with interval delays, which may be unstable for small delays or delay-free cases. For more details, see, for example, [40–42].

5. Example

In order to verify the feasibility of our results, consider the following neutral-type CVNNs:

$$\begin{aligned}\frac{dAz(t)}{dt} &= -D(t)z(t) + A(t)f(z(t)) \\ &\quad + B(t)g\left(z\left(t-\gamma(t)\right)\right) + H(t),\end{aligned} \tag{52}$$

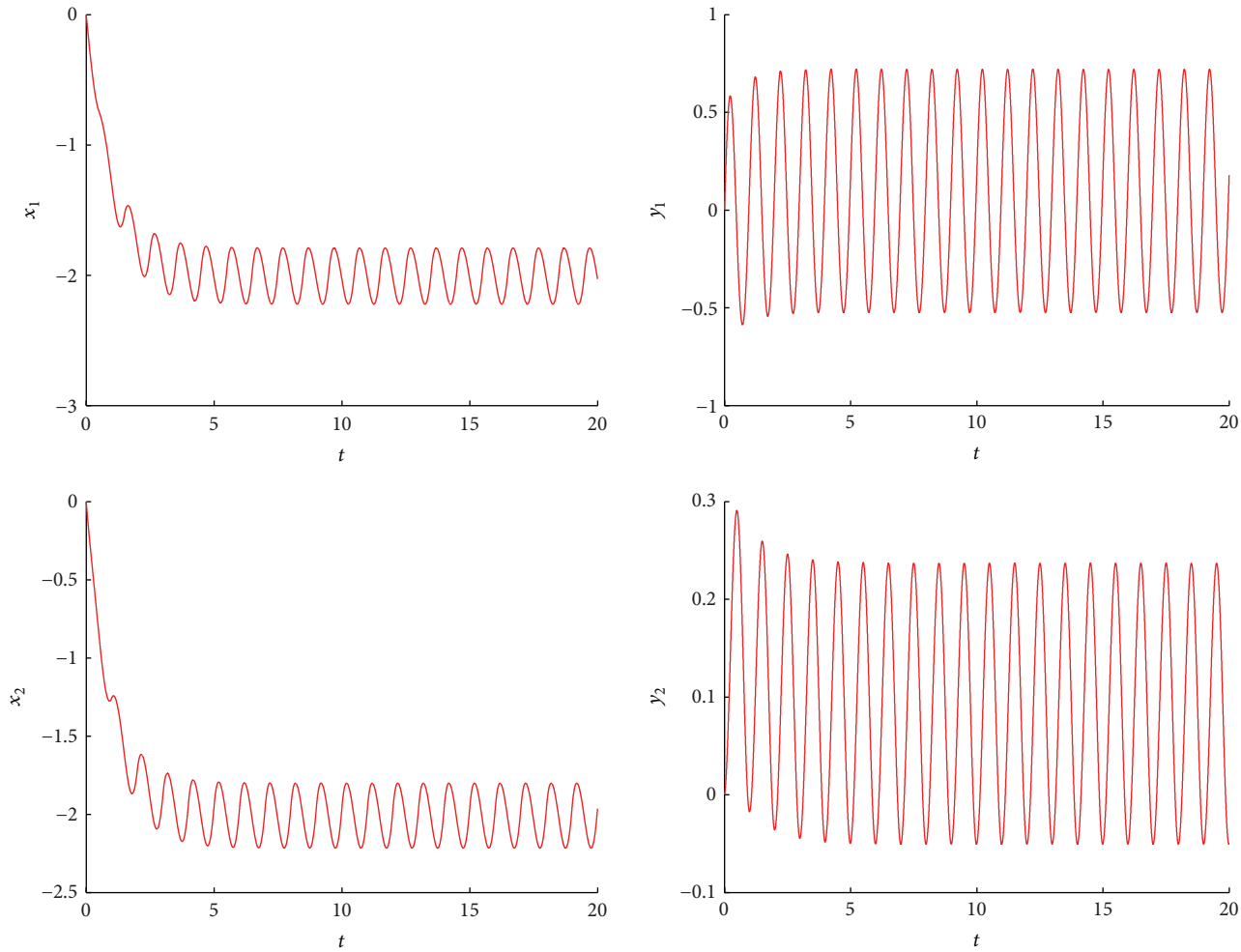

Figure 1: Transient states of the real and imaginary parts of system (52).

where

$$\begin{aligned} A_p z_p(t) &= A_p x_p(t) + i A_p y_p(t), \\ A_p x_p(t) &= x_p(t) - \frac{1}{2} x_p\left(t - \frac{\pi}{2}\right), \\ A_p y_p(t) &= y_p(t) - \frac{1}{2} y_p\left(t - \frac{\pi}{2}\right), \\ & \qquad p = 1, 2, \\ D(t) &= \begin{pmatrix} 2.1 - 1.6 \sin \pi t & 0 \\ 0 & 2.1 - 1.6 \cos \pi t \end{pmatrix}, \\ A(t) &= \begin{pmatrix} \frac{1}{25} - \frac{i}{23} \sin 2\pi t & \frac{1}{25} + \frac{i}{23} \cos 2\pi t \\ \frac{1}{25} + \frac{i}{23} \cos 2\pi t & \frac{1}{25} - \frac{i}{23} \sin 2\pi t \end{pmatrix}, \\ B(t) &= \begin{pmatrix} \frac{1}{21} - \frac{i}{21} \sin 2\pi t & \frac{1}{21} - \frac{i}{23} \cos 2\pi t \\ \frac{1}{21} - \frac{i}{23} \cos 2\pi t & \frac{1}{21} - \frac{i}{23} \sin 2\pi t \end{pmatrix}, \\ H(t) &= \begin{pmatrix} -2 + 4 \sin 2\pi t \\ -2 + \cos 2\pi t \end{pmatrix}, \\ \gamma(t) &= \frac{1}{10} \sin t, \\ f_q(z_q) &= g_q(z_q) \\ &= \frac{2}{3 + \exp(x_q + y_q)} \\ &\quad + i \frac{1 - \exp(-3x_q - y_q)}{2 + \exp(-x_q + y_q)}, \quad q = 1, 2. \end{aligned} \tag{53}$$

Obviously, the conditions in Theorem 10 are all satisfied. It follows from Theorem 10 that the periodic solution of system (52) is globally exponentially stable.

The numerical simulations of (52) are shown in Figures 1 and 2. Figure 1 shows the state trajectories of the real part

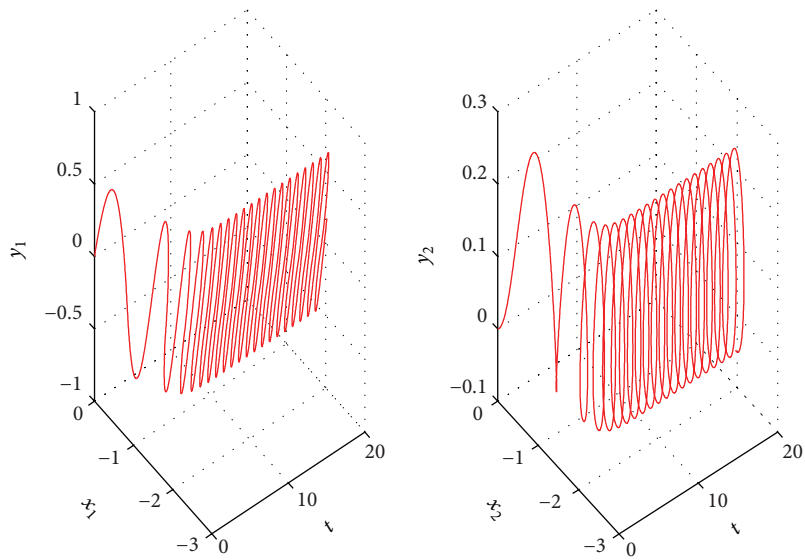

Figure 2: State trajectories of the real and imaginary parts of system (52).

and the imaginary part of (52). Figure 2 shows the amplitude curves of neuro states of (52). From the simulation results, it can be seen that the periodic solution of (52) is unique and stable.

Remark 13. We greatly want to provide the circuit diagram of system (7). However, system (52) contains neutral-type operators and time-varying delays influence is more complicated. So far, we can not obtain the circuit diagram of (52). We hope that some authors research this subject in the future.

6. Conclusions

In this paper, we have investigated stability problems of periodic solutions for a class of neutral-type complex-valued neural networks with time-varying delays. By utilizing novel Lyapunov-Krasovskii functionals, the sufficient conditions are derived to guarantee global exponential stability for the involved systems. A simulation example has been provided to show the usefulness of the proposed global exponential stability conditions.

We mention here that some finer approaches to deal with time delays would be the delay-slope-dependent method [43] and the delay-fraction approach [44], which could be the further work to reduce the possible conservatism in the dynamical analysis. And another future research topics would be the extension of the present results to more general cases, for example, the case that there exists impulsive influence, and the case that the neural network of neutral-type is a difference system. The results will appear in the near future.

Competing Interests

The authors declare to have no competing interests in this paper.

Acknowledgments

The authors acknowledge the funding of NNSF (no. 11571136) of China.

References

[1] A. Hirose, *Complex-Valued Neural Networks: Theories and Applications*, World Scientific Publishing, River Edge, NJ, USA, 2003.

[2] J. Hu and J. Wang, "Global stability of complex-valued recurrent neural networks with time-delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 23, no. 6, pp. 853–865, 2012.

[3] M. Bohner, V. S. H. Rao, and S. Sanyal, "Global stability of complex-valued neural networks on time scales," *Differential Equations and Dynamical Systems*, vol. 19, no. 1-2, pp. 3–11, 2011.

[4] R. Ceylan, M. Ceylan, Y. Özbay, and S. Kara, "Fuzzy clustering complex-valued neural network to diagnose cirrhosis disease," *Expert Systems with Applications*, vol. 38, no. 8, pp. 9744–9751, 2011.

[5] W. Zhou and J. M. Zurada, "Discrete-time recurrent neural networks with complex-valued linear threshold neurons," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 56, no. 8, pp. 669–673, 2009.

[6] R. Rakkiyappan, G. Velmurugan, and X. Li, "Complete stability analysis of complex-valued neural networks with time delays and impulses," *Neural Processing Letters*, vol. 41, no. 3, pp. 435–468, 2015.

[7] N. Özdemir, B. B. İskender, and N. Y. Özgür, "Complex valued neural network with Möbius activation function," *Communications in Nonlinear Science and Numerical Simulation*, vol. 16, no. 12, pp. 4698–4703, 2011.

[8] V. S. H. Rao and G. R. Murthy, "Global dynamics of a class of complex valued neural networks," *International Journal of Neural Systems*, vol. 18, no. 2, pp. 165–171, 2008.

[9] W. Gong, J. Liang, and J. Cao, "Matrix measure method for global exponential stability of complex-valued recurrent neural

networks with time-varying delays," *Neural Networks*, vol. 70, pp. 81–89, 2015.

[10] Z. Zhang and S. Yu, "Global asymptotic stability for a class of complex-valued Cohen-Grossberg neural networks with time delays," *Neurocomputing*, vol. 171, pp. 1158–1166, 2016.

[11] Q. Song, H. Yan, Z. Zhao, and Y. Liu, "Global exponential stability of complex-valued neural networks with both time-varying delays and impulsive effects," *Neural Networks*, vol. 79, pp. 108–116, 2016.

[12] Z. Wang and L. Huang, "Global stability analysis for delayed complex-valued BAM neural networks," *Neurocomputing*, vol. 173, pp. 2083–2089, 2016.

[13] J. Pan, X. Liu, and W. Xie, "Exponential stability of a class of complex-valued neural networks with time-varying delays," *Neurocomputing*, vol. 164, pp. 293–299, 2015.

[14] J. H. Park and O. M. Kwon, "Design of state estimator for neural networks of neutral-type," *Applied Mathematics and Computation*, vol. 202, no. 1, pp. 360–369, 2008.

[15] J. H. Park, O. M. Kwon, and S. M. Lee, "LMI optimization approach on stability for delayed neural networks of neutral-type," *Applied Mathematics and Computation*, vol. 196, no. 1, pp. 236–244, 2008.

[16] J. H. Park, O. M. Kwon, and S. M. Lee, "State estimation for neural networks of neutral-type with interval time-varying delays," *Applied Mathematics and Computation*, vol. 203, no. 1, pp. 217–223, 2008.

[17] J. H. Park and O. M. Kwon, "Global stability for neural networks of neutral-type with interval time-varying delays," *Chaos, Solitons & Fractals*, vol. 41, no. 3, pp. 1174–1181, 2009.

[18] J. H. Park, C. H. Park, O. M. Kwon, and S. M. Lee, "A new stability criterion for bidirectional associative memory neural networks of neutral-type," *Applied Mathematics and Computation*, vol. 199, no. 2, pp. 716–722, 2008.

[19] Z. Gui, W. Ge, and X.-S. Yang, "Periodic oscillation for a Hopfield neural networks with neutral delays," *Physics Letters A*, vol. 364, no. 3-4, pp. 267–273, 2007.

[20] S. Xu, J. Lam, D. W. Ho, and Y. Zou, "Delay-dependent exponential stability for a class of neural networks with time delays," *Journal of Computational and Applied Mathematics*, vol. 183, no. 1, pp. 16–28, 2005.

[21] J. Qin and J. Cao, "Delay-dependent robust stability of neutral-type neural networks with time delays," *Journal of Mathematical Control Science and Applications*, vol. 1, pp. 179–188, 2007.

[22] K. Wang and Y. Zhu, "Stability of almost periodic solution for a generalized neutral-type neural networks with delays," *Neurocomputing*, vol. 73, no. 16–18, pp. 3300–3307, 2010.

[23] S. Mohamad and K. Gopalsamy, "Exponential stability of continuous-time and discrete-time cellular neural networks with delays," *Applied Mathematics and Computation*, vol. 135, no. 1, pp. 17–38, 2003.

[24] L. Wang and Z. Xu, "Sufficient and necessary conditions for global exponential stability of discrete-time recurrent neural networks," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 53, no. 6, pp. 1373–1380, 2006.

[25] Z. Yuan, D. Hu, and L. Huang, "Stability and bifurcation analysis on a discrete-time neural network," *Journal of Computational and Applied Mathematics*, vol. 177, no. 1, pp. 89–100, 2005.

[26] L. Zou and Z. Zhou, "Periodic solutions for nonautonomous discrete-time neural networks," *Applied Mathematics Letters*, vol. 19, no. 2, pp. 174–185, 2006.

[27] J. Liang, J. Cao, and J. Lam, "Convergence of discrete-time recurrent neural networks with variable delay," *International Journal of Bifurcation and Chaos*, vol. 15, no. 2, pp. 581–595, 2005.

[28] M. R. Zhang, "Periodic solutions of linear and quasilinear neutral functional-differential equations," *Journal of Mathematical Analysis and Applications*, vol. 189, no. 2, pp. 378–392, 1995.

[29] S. Lu, W. Ge, and Z. Zheng, "Periodic solutions to neutral differential equation with deviating arguments," *Applied Mathematics and Computation*, vol. 152, no. 1, pp. 17–27, 2004.

[30] R. E. Gaines and J. L. Mawhin, *Coincidence Degree, and Nonlinear Differential Equations*, Lecture Notes in Mathematics, Springer, Berline, Germany, 1977.

[31] J. K. Hale, *Theory of Functional Differential Equations*, Springer, New York, NY, USA, 1977.

[32] E. Serra, "Periodic solutions for some nonlinear differential equations of neutral type," *Nonlinear Analysis: Theory, Methods & Applications*, vol. 17, no. 2, pp. 139–151, 1991.

[33] S. Junca and B. Lombard, "Stability of a critical nonlinear neutral delay differential equation," *Journal of Differential Equations*, vol. 256, no. 7, pp. 2368–2391, 2014.

[34] K. Schmitt, *Fixed Point and Coincidence Theorems with Applications to Nonlinear Differential and Integral Equations*, vol. 97 of *Seminaires de Mathematique Appliquée et Mécanique*, Université Catholique de Louvain, 1976.

[35] J. K. Hale and S. M. Verduyn Lunel, *Introduction to Functional Differential Equations*, Springer, New York, NY, USA, 1993.

[36] Y. He, Q.-G. Wang, C. Lin, and M. Wu, "Delay-range-dependent stability for systems with time-varying delay," *Automatica*, vol. 43, no. 2, pp. 371–376, 2007.

[37] E. Fridman and U. Shaked, "A descriptor system approach to H_∞ control of linear time-delay systems," *IEEE Transactions on Automatic Control*, vol. 47, no. 2, pp. 253–270, 2002.

[38] C. Lin, Q.-G. Wang, and T. H. Lee, "Stability and stabilization of a class of fuzzy time-delay descriptor systems," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 4, pp. 542–551, 2006.

[39] J. Sun, G. P. Liu, J. Chen, and D. Rees, "Improved delay-range-dependent stability criteria for linear systems with time-varying delays," *Automatica*, vol. 46, no. 2, pp. 466–470, 2010.

[40] R. Rakkiyappan, A. Chandrasekar, S. Lakshmanan, J. H. Park, and H. Y. Jung, "Effects of leakage time-varying delays in Markovian jump neural networks with impulse control," *Neurocomputing*, vol. 121, pp. 365–378, 2013.

[41] A. Chandrasekar, R. Rakkiyappan, F. A. Rihan, and S. Lakshmanan, "Exponential synchronization of Markovian jumping neural networks with partly unknown transition probabilities via stochastic sampled-data control," *Neurocomputing*, vol. 133, pp. 385–398, 2014.

[42] S. Lakshmanan and P. Balasubramaniam, "New results of robust stability analysis for neutral-type neural networks with time-varying delays and Markovian jumping parameters," *Canadian Journal of Physics*, vol. 89, no. 8, pp. 827–840, 2011.

[43] T. Li, W. X. Zheng, and C. Lin, "Delay-slope-dependent stability results of recurrent neural networks," *IEEE Transactions on Neural Networks*, vol. 22, no. 12, pp. 2138–2143, 2011.

[44] Z. Wang, Y. Wang, and Y. Liu, "Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time delays," *IEEE Transactions on Neural Networks*, vol. 21, no. 1, pp. 11–25, 2010.