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Research Article

Asymptotic Stability Analysis of Binary Heterogeneous Traffic Based on Car-Following Model

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We study the asymptotic stability of Chandler Model for a heterogeneous traffic by using numerical simulations. A simple binary platoon is considered which consists of two types of vehicles. Platoon stabilities under various kinds of combinations of parameters are investigated. It is found that the stability of the binary platoon cannot be determined by the mean values of individual vehicle's parameters. Some combinations of parameters that benefit to the platoon stability are found. Several interesting properties of binary platoon's stability are summarized. The analytic stability criterion of heterogeneous traffic reported in the historical literature is studied. The result indicates the analytic criterion is not rigorous, which is apt to overestimate the stability of heterogeneous platoon.

1. Introduction

The stability of traffic flow is a very interesting problem in the research of traffic flow theory. It depicts how perturbation evolves with time and along the platoon. The pioneer work related to the stability analysis of traffic flow was started by Chandler et al. [1] and Herman et al. [2] in the late 1950s. In their work, the following simple linear car-following model which is also called Chandler Model was examined:

$$\ddot{x}_n(t+\tau) = \lambda\left(\dot{x}_{n-1}(t) - \dot{x}_n(t)\right), \quad n = 2, 3, \ldots, N, \tag{1}$$

where $x_n(t)$ denotes the position of nth vehicle at time t, τ is the response delay of the following driver, and λ is the sensitivity to stimulus. Based on Chandler Model, two types of stability were studied: local stability and asymptotic stability. Local stability is concerned with the time variations of the response of the following vehicle to a change in the motion of its direct leading vehicle. Asymptotic stability, unlikely, focuses on the manner in which perturbation on the leading vehicle is propagated down a line of traffic. The stability was examined analytically by Fourier analysis in Chandler et al.'s work [1] and Laplace transforms in Herman et al.'s study [2]. Perhaps the most important result of the stability analysis was that Chandler Model is asymptotically stable when the condition $\lambda\tau < 1/2$ holds.

In the following five decades, Chandler Model was further developed into some more general forms [3–6] by substituting a nonlinear sensitivity for the original constant one. Meanwhile, many other kinds of car-following models were also proposed, such as Newell Model [7], Optimal Velocity Model [8], Full Velocity Model [9], Intelligent Driver Model [10], and elaborate models [11, 12]. A lot of efforts were devoted into the analytic stability studies of these car-following models [13–17]. More recently, Treiber and Kesting introduced a new classification between convective and absolute stability [18] and described a mathematical framework for linear stability analyses of all sorts of microscopic models [19]. Most of the abovementioned works are concerned with the traffic consisting of identical drivers. In other words, all driver-vehicles are governed by uniform car-following model with uniform parameters in stability analyses. However, such an ideal homogeneous traffic does not exist in the real world. Field observations reveal that there are considerable heterogeneities between the driving behaviors of individual drivers as well as the performances of different types of vehicles [20–22]. In view of this fact, Ossen et al. investigated the asymptotic stability of Chandler Model in heterogeneous traffic via numerical simulation. They found that it cannot be simply stated that a heterogeneous platoon becomes asymptotically instable when the mean values of the model parameters fall outside the stable region for homogeneous platoons [21].

Compared with homogeneous traffic, only a few of attempts [23–25] were undertaken to examine the analytic stability criterion of heterogeneous platoon in last two decades, due to the complexity of this problem. Zhang and Jarrett [23] extended the traditional stability analysis method to consider different parameter values for different drivers in the platoon, while they only gave the sufficient condition for the asymptotic stability. Holland proposed a generalized method to analyze the stability of platoon, which was also extended to nonidentical drivers [24]. However, the explicit stability criterion for heterogeneous traffic was not provided. Though some works were carried out for analyzing the asymptotic stability of heterogeneous traffic, there are still several issues needed to be deeply studied as follows:

(a) Is that possible to describe the stability of heterogeneous traffic with the mean of the model parameters?

(b) How does the heterogeneity influence the stability of the platoon?

(c) Does the sequence of individual vehicles influence the stability of platoon?

In order to answer the above questions, this paper starts effects from a simple scenario, namely, a binary heterogeneous traffic which only consists of two types of vehicles (or drivers) under the car-following rule of Chandler Model. A simulation based method is used to investigate the asymptotic stability of the binary platoon with various combinations of car-following model parameters. The rest of the paper is organized as follows. In next section, the simulation based methodology of our work is introduced. Then, several simulation experiments are carried out and the properties of the asymptotic stability of a binary platoon are presented in Section 3. In Section 4, some possible explanations are presented for the results obtained from simulations. In Section 5, some discussions are given based on the simulation of Holland's criterion. Finally, conclusions are summarized in Section 6.

2. Methodology

Numerical simulations were performed to investigate the asymptotic stability of platoon. The platoon consists of two types of vehicles (or drivers), which are defined as type A and type B. Both of them were governed by Chandler Model. There are numerous possible combinations for type A vehicles and type B vehicles to constitute such a platoon. It hardly makes any sense to say whether a platoon is stable or not, if type A vehicles and type B vehicles distribute randomly in the platoon. In view of this fact, we mainly focus on the stability of binary platoon with periodic structures in our work, such as "ABAB..." and "AABBAABB...." Without loss of generality, we start the studies from the case "ABABAB...," within which the proportions of both type A vehicles and type B vehicles

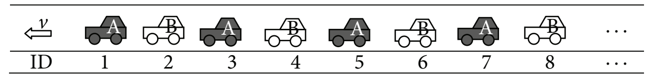


Figure 1: Schematic illustration of platoon "ABAB...."

are equal to 50%. The parameters of type A and type B are denoted as λ_A, τ_A and λ_B, τ_B, respectively.

The platoon used in the simulation consists of 20 type A vehicles and 20 type B vehicles, as shown in Figure 1. We give the first vehicle in the platoon the ID number "1" and the nth vehicle number "n." Then all the vehicles with odd-numbered ID are type A vehicles, and the even-numbered vehicles are type B vehicles. At the beginning of the simulation, all vehicles move at the same velocity v_0; the headway of each vehicle obeys the implicit velocity-headway function of Chandler Model as follows [24]:

$$x_{n-1}(0) - x_n(0) = \begin{cases} \dfrac{v_0}{\lambda_A} + b_{\text{jam}}, & n = 3, 5, 7, \ldots \\ \dfrac{v_0}{\lambda_B} + b_{\text{jam}}, & n = 2, 4, 6, \ldots, \end{cases} \tag{2}$$

where b_{jam} denotes the headway at jam state. Then we add small perturbation on the first vehicle and see how it propagates along the platoon.

According to Holland's study [24], the platoon is asymptotically stable only when the amplitude of perturbation keeps reducing along with its propagation. Define f_n as the amplitude of velocity for the nth vehicle in the platoon:

$$f_n = v_n^{\max} - v_n^{\min}, \quad n = 1, 2, \ldots, N, \tag{3}$$

where $v_n^{\max}$ and $v_n^{\min}$ are the maximal velocity and minimal velocity of the nth vehicle under the perturbation, respectively. As the parameters of type A vehicle and type B vehicle are different, they may have different amplification effects during the propagation of the perturbation. However, for the same type of vehicle, the reactions to the perturbation are the same. Then, for the asymptotically stable platoon "ABAB...," we have $f_n > f_{n+2i}$, $(i = 1, 2, \ldots)$. Considering that instability may take long distance to manifest itself, we use the condition $f_3 - f_{39} > 0$ and $f_4 - f_{40} > 0$ to define the asymptotic stability of the platoon.

3. Simulation and Result

Half century ago, Chandler et al. [1] and Herman et al. [2] gave out the famous asymptotic stability criterion of Chandler Model; that is, $\lambda\tau < 1/2$. Let $\lambda_{AB} = (\lambda_A + \lambda_B)/2$ and $\tau_{AB} = (\tau_A + \tau_B)/2$; can we describe the stability of binary platoon only with the mean of model parameters λ_{AB} and τ_{AB}? There are three cases needed to be noticed.

Case 1 (neither type A vehicle nor type B vehicle obeys the asymptotic stability criterion $(\lambda_A\tau_A \geq 1/2$ & $\lambda_B\tau_B \geq 1/2)$). From the physical point of view, the binary platoon is not asymptotically stable in this case. If we draw λ-τ plot as Figure 2(a), it is easy to see that the point $(\tau_{AB}, \lambda_{AB})$ locates

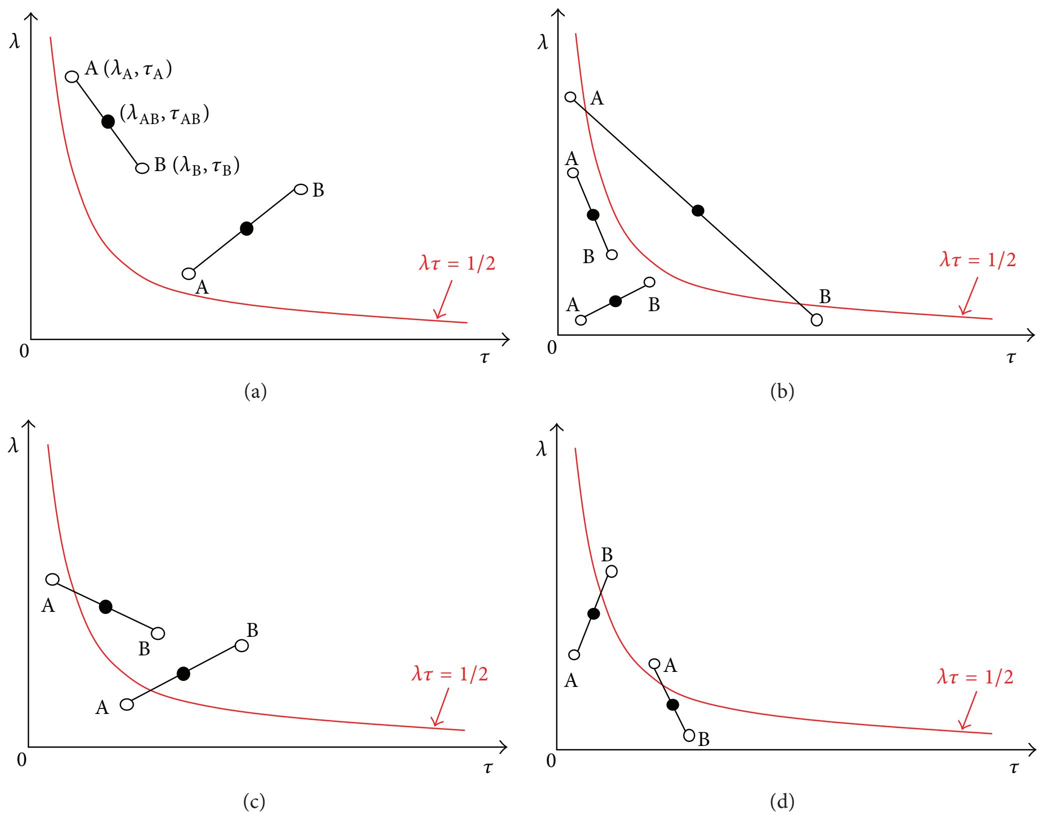


FIGURE 2: Model parameters in λ-τ plot. (a) Case 1, (b) Case 2, (c) Case 3 with $\lambda_{AB}\tau_{AB} \geq 1/2$, and (d) Case 3 with $\lambda_{AB}\tau_{AB} < 1/2$ (color online).

at the midpoint of the straight line connecting point (τ_A, λ_A) and point (τ_B, λ_B). As the critical curve $\lambda\tau = 1/2$ is convex, the point $(\tau_{AB}, \lambda_{AB})$ always falls above the critical curve; namely, $\lambda_{AB}\tau_{AB} \geq 1/2$.

Case 2 (both type A vehicle and type B vehicle obey the asymptotic stability criterion ($\lambda_A\tau_A < 1/2$ & $\lambda_B\tau_B < 1/2$)). Obviously, the binary platoon is stable in this case. However, it is uncertain whether $\lambda_{AB}\tau_{AB} < 1/2$ or not as shown in Figure 2(b).

Case 3 (either type A vehicle or type B vehicle obeys the asymptotic stability criterion ($\lambda_A\tau_A < 1/2$ or $\lambda_B\tau_B < 1/2$)). In this case, the stability of binary platoon is not clear, and it is also uncertain whether $\lambda_{AB}\tau_{AB} < 1/2$ or not as shown in Figures 2(c) and 2(d).

From Figure 2, we can see that there are two types of line AB. The one with positive slope corresponds to the case of $(\lambda_A - \lambda_B)(\tau_A - \tau_B) \geq 0$; and the one with negative slope corresponds to the case of $(\lambda_A - \lambda_B)(\tau_A - \tau_B) < 0$. In the following section, we will show that these two cases have different effects on the stability of binary platoon.

3.1. Distribution of Binary Platoon Stability in λ_{AB}-τ_{AB} Plot. Table 1 indicates that we cannot know whether a binary platoon is stable or not only with its mean values of model parameters. In order to take deep insight into the stability performance of the binary platoon, numerical simulations were undertaken. We give model parameters the same ranges for both type A vehicle and type B vehicle, $\lambda_A \mid \lambda_B \in (0, 3]$ and $\tau_A \mid \tau_B \in (0, 3]$. Then, $\lambda_{AB} \in (0, 3]$ and $\tau_{AB} \in (0, 3]$. All the four parameters are enumerated by step of 0.1 within the above given ranges. The initial velocity of the platoon is 20 m/s. A small perturbation was added 5 seconds after beginning, which reduced the velocity of the first vehicle by 1 m/s, lasting for 2 seconds, and then removed. Similarly, λ_{AB}-τ_{AB} plot (Figure 3) was used to present the stability performance of the binary platoon. For a certain point (τ_{AB}, λ_{AB}) in Figure 3, we enumerated all the possible combinations of parameters τ_A, λ_A, τ_B, and λ_B to see whether the binary platoon is stable. For every combination, the platoon stability was determined according to the methodology introduced in Section 2. The results are presented in Figure 3.

There are three categories of results in Figure 3, namely, absolute stability, absolute instability, and uncertainty, which are denoted by red dots, blue crosses, and black circles. The absolute stability (instability) means that the binary platoon is asymptotically stable (unstable) under all the combinations of τ_A, λ_A, τ_B, and λ_B, while the uncertainty means that some combinations of τ_A, λ_A, τ_B, and λ_B are stable whereas other combinations are unstable. There are two black solid curves that divide λ_{AB}-τ_{AB} plot into three regions in Figure 3 (marked as I, II, and III). The first curve is formulated by the function $\lambda_{AB}\tau_{AB} = 1/8$, which ensures that any point $(\tau_{AB}, \lambda_{AB})$ beneath the curve belongs to Case 2 in Table 1.

Table 1: Three cases of binary platoon.

	Case 1	Case 2	Case 3	
Type A vehicle	$\lambda_A\tau_A \geq 1/2$	$\lambda_A\tau_A < 1/2$	$\lambda_A\tau_A \geq 1/2$	$\lambda_A\tau_A < 1/2$
Type B vehicle	$\lambda_B\tau_B \geq 1/2$	$\lambda_B\tau_B < 1/2$	$\lambda_B\tau_B < 1/2$	$\lambda_B\tau_B \geq 1/2$
Mean of parameters	$\lambda_{AB}\tau_{AB} \geq 1/2$		$\lambda_{AB}\tau_{AB} \geq 1/2$ or $\lambda_{AB}\tau_{AB} < 1/2$	
Binary platoon stability	Unstable	Stable	Uncertain	

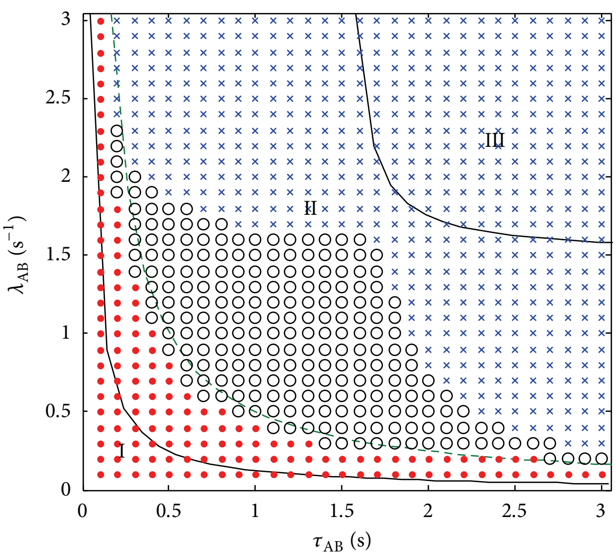


Figure 3: Stability results of binary platoon in λ_{AB}-τ_{AB} plot (color online).

The second curve is expressed functionally as $(\lambda_{AB}-1.5)(\tau_{AB}-1.5) = 1/8$, which ensures that all points $(\tau_{AB}, \lambda_{AB})$ above the curve belong to Case 1 in Table 1. The green dash curve denotes the critical condition according to the stability criterion of Chandler Model; that is, $\lambda_{AB}\tau_{AB} = 1/2$.

It is clear to see that all points in region I are absolute stable, while points in region III are absolute unstable, which are consistent with the conclusion drawn from the analysis for Cases 1 and 2 in Table 1. In region II, most points beneath the green dash curve are absolute stable, while there are still some points marked as "uncertainty" beneath the green dash curve. That is to say, in some cases the binary platoon is not stable even though the mean values of model parameters satisfy the stability criterion of Chandler Model $\lambda_{AB}\tau_{AB} < 1/2$. Furthermore, a broad range of points above the green dash curve are marked as "uncertainty," which indicates that the binary platoon can still be asymptotically stable when the mean values of model parameters do not satisfy the stability criterion of Chandler Model. This is consistent with the result found by Ossen et al. [21].

According to the analysis about the three cases in Table 1, the points marked as "uncertainty" beneath the green dash curve can have many different combinations of parameters τ_A, λ_A, τ_B, and λ_B. Among them, some combinations belong to Case 2 as shown in Figure 2(b), while others belong to Case 3 as illustrated in Figure 2(d). For those points marked as "uncertainty" above the green dash curve, the combinations of parameters can come from all the three cases as shown in Figures 2(a), 2(b), and 2(c).

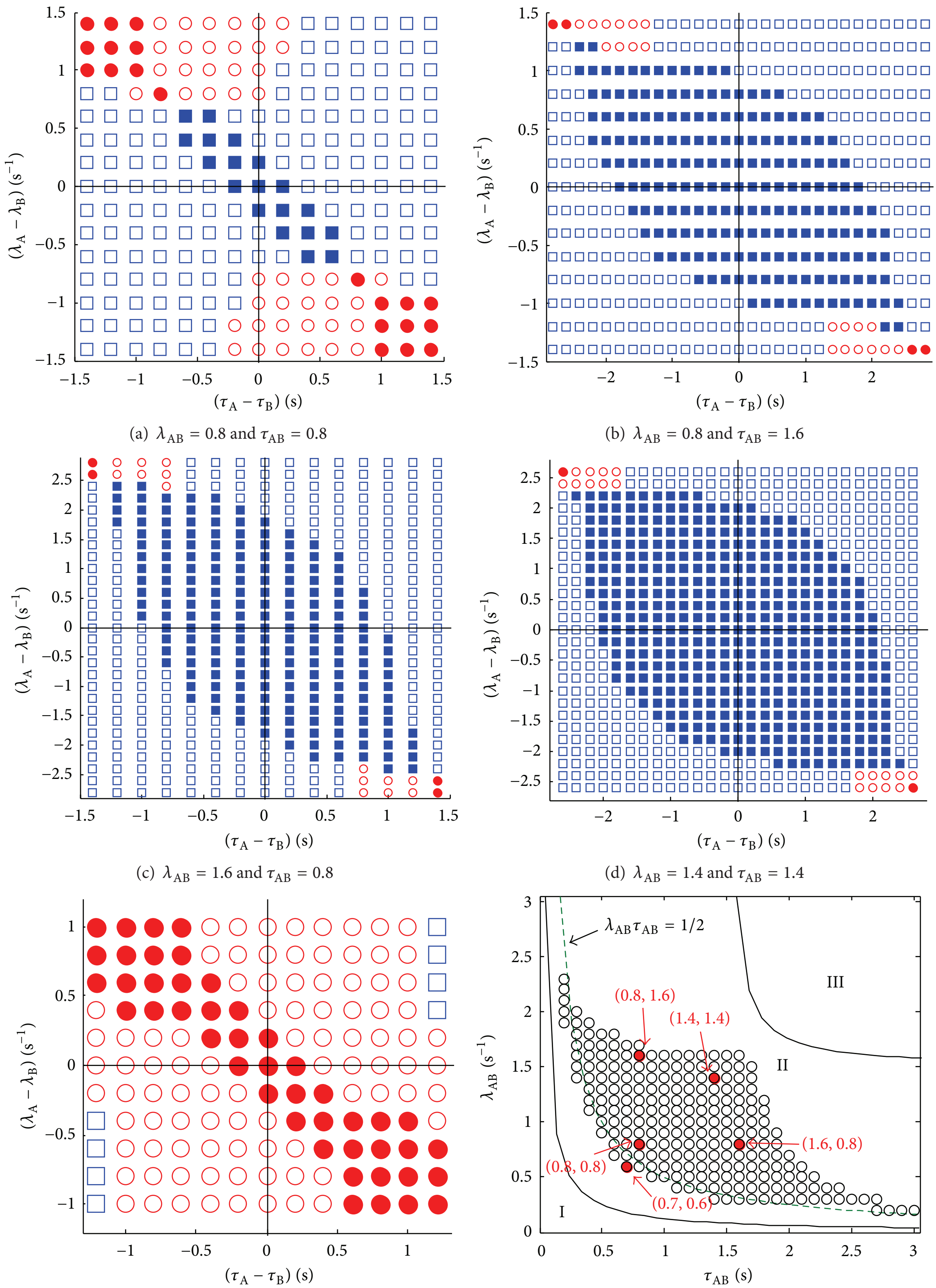
3.2. Stabilities of Various Combinations of τ_A, λ_A, τ_B, and λ_B. In order to take deep insight into the "uncertainty" cases in Figure 3, we plot the stability analysis results of all the combinations for some typical "uncertainty" cases, as shown in Figure 4. The x-axis and y-axis in Figure 4 represent $(\tau_A - \tau_B)$ and $(\lambda_A - \lambda_B)$, respectively. We selected four points marked as "uncertainty" in Figure 3, with the coordinates of (0.8, 0.8), (1.6, 0.8), (0.8, 1.6), (1.4, 1.4), and (0.7, 0.6). For each point, stabilities of all the possible parameter-combinations are presented in Figures 4(a)–4(e). There are four different types of markers and each of them represents a specific case. The red dots denote those points that belong to Case 2 in Table 1, and the binary platoon is asymptotic stable. The blue solid squares denote points that belong to Case 1 in Table 1, and the binary platoon is instable. The remaining red circles and blue hollow squares all belong to Case 3 in Table 1, while the former represents stability and the latter instability. Figure 4(f) shows the coordinates of the five chosen points in λ_{AB}-τ_{AB} plot.

There are several interesting properties that can be found from Figure 4 as follows:

(i) All the four types of markers are symmetrical about the origin in the plot. This property indicates that the stability of the binary platoon is identical for both the pattern "ABAB. . ." and the pattern "BABA. . . ."

(ii) The closer the mean parameters $(\tau_{AB}, \lambda_{AB})$ to the homogeneous traffic stable criterion curve, the more the chances for binary platoon to be asymptotically stable. Among all five points in Figure 4(f), the point (0.7, 0.6) is the closest to the stable curve, and the corresponding binary platoon is stable under most of the combinations of parameters τ_A, λ_A, τ_B, and λ_B, as shown in Figure 4(e). However, for the point (1.4, 1.4) that locates farthest from the stable curve in Figure 4(f), only a small proportion of its combinations can guarantee the binary platoon stability, as illustrated in Figure 4(d).

(iii) There are four quadrants in the coordinates system of Figures 4(a)–4(e). It is found that the parameters in the upper left quadrant and lower right quadrant get more chances to have the binary platoon stable. Coordinates in these two quadrants satisfy the inequation $(\lambda_A - \lambda_B)(\tau_A - \tau_B) < 0$, which correspond to line AB with negative slope in Figure 2. Moreover, the farther the coordinates from the origin, the more the chances for the binary platoon to be asymptotically stable.



(a) $\lambda_{AB} = 0.8$ and $\tau_{AB} = 0.8$

(b) $\lambda_{AB} = 0.8$ and $\tau_{AB} = 1.6$

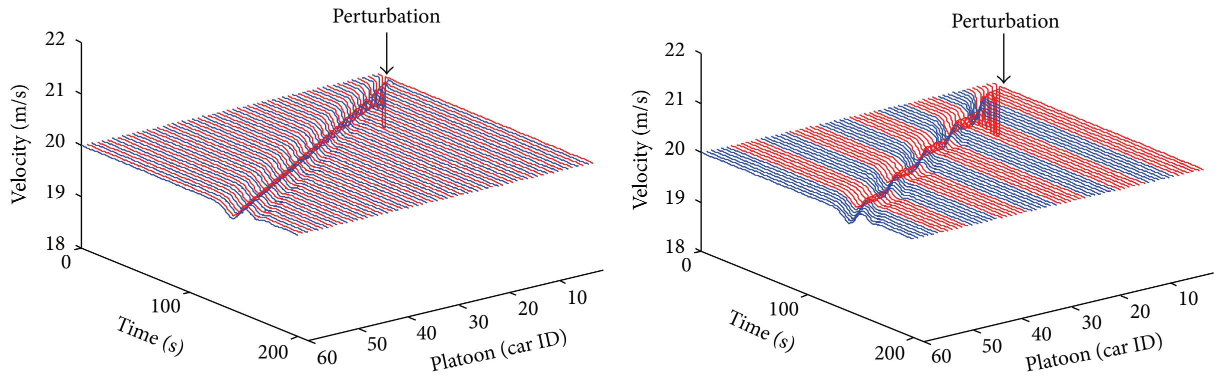
(c) $\lambda_{AB} = 1.6$ and $\tau_{AB} = 0.8$

(d) $\lambda_{AB} = 1.4$ and $\tau_{AB} = 1.4$

(e) $\lambda_{AB} = 0.6$ and $\tau_{AB} = 0.7$

(f)

FIGURE 4: Stabilities under various combinations of parameters (color online).



(a) Platoon "ABABAB..."

(b) Platoon "AAAAAABBBBBB..."

Figure 5: Propagation of perturbation along two types of binary platoon (color online).

Such properties indicate that, for the binary platoon of Case 3 (i.e., one type of vehicle satisfies the asymptotically stable criterion while the other type does not satisfy the asymptotically stable criterion), you would better not have both the sensitivity λ and response delay τ of one type vehicle bigger than those of the other type; otherwise the binary platoon may not be stable.

3.3. Binary Platoon with Other Periodic Structures. If we extend the pattern "ABAB..." into some other periodic structures, such as "AABBAABB..." or "AAABBBAAABBB..." and repeat the simulations by the identical methodology for pattern "ABAB...," we find the same properties of binary platoon stability. The only issue needed to notice is that the small perturbation should not be amplified into shockwaves during its propagation along the successive identical type of vehicle.

Figure 5 presents the simulation result of perturbation propagation along both the binary platoon "ABABAB..." and the platoon "AAAAAABBBBBB...." The red lines and blue lines represent type A vehicles and type B vehicles, respectively. The parameters of type A vehicle and type B vehicle are set as follows: $\lambda_{\mathrm{A}} = 1.0$, $\tau_{\mathrm{A}} = 0.3$, $\lambda_{\mathrm{B}} = 0.3$, and $\tau_{\mathrm{B}} = 1.7$. The combination belongs to Case 3 as illustrated in Figure 2(c). At the beginning, a small perturbation was added onto the first vehicle, which reduced its velocity by 1 m/s for two seconds. From the simulation results, we can see that the perturbation declines along its propagation in both platoons. The amplitude of velocity reduced to less than 0.5 m/s when the perturbation propagated to the 60th vehicle in both platoons.

4. Possible Explanations

The numerical simulations in Section 3 show a few interesting properties such that (i) the binary platoons "ABABAB..." and "BABABA..." have the same stable criterion; (ii) the binary platoons "ABABAB..." and "$(\mathrm{A}\ldots\mathrm{A})_m(\mathrm{B}\ldots\mathrm{B})_m\ldots$" have the same stable criterion. Here are some possible explanations.

The perturbation propagating along an equilibrium moving platoon is manifested as the amplitude of vehicle velocity. The vehicle in platoon can either amplify the perturbation or damp it. If we define f_0 as the initial perturbation and f_n as the amplitude of the nth velocity after the perturbation passing it, then the propagation of perturbation can be illustrated as follows:

$$f_0 \longrightarrow \mathrm{A} \longrightarrow f_1 \longrightarrow \mathrm{B} \longrightarrow f_2 \longrightarrow \mathrm{A} \longrightarrow f_3 \longrightarrow \mathrm{B} \longrightarrow f_4 \longrightarrow \cdots . \tag{4}$$

If the binary platoon "ABABAB..." is stable, then the amplitude f_1 must be small, no matter how the first vehicle amplifies the initial perturbation f_0. Let us ignore the initial perturbation and the first vehicle and just focus on the rest part of the original platoon:

$$f_1 \longrightarrow \mathrm{B} \longrightarrow f_2 \longrightarrow \mathrm{A} \longrightarrow f_3 \longrightarrow \mathrm{B} \longrightarrow f_4 \longrightarrow \mathrm{A} \longrightarrow f_5 \longrightarrow \cdots . \tag{5}$$

We find that the rest binary platoon "BABABA..." is also stable. That is to say, if platoon "ABABAB..." is stable, then the platoon "BABABA..." must be stable as well and vice versa. Therefore, the binary platoons "ABABAB..." and "BABABA..." have the same stable criterion.

Furthermore, if we regard the binary platoon "ABABAB..." as "(AB)(AB)(AB)...," then the propagation of the perturbation can be illustrated as

$$\begin{aligned} &f_0 \longrightarrow (\mathrm{A} \longrightarrow f_1 \longrightarrow \mathrm{B}) \longrightarrow \\ &f_2 \longrightarrow (\mathrm{A} \longrightarrow f_3 \longrightarrow \mathrm{B}) \longrightarrow \\ &f_4 \longrightarrow (\mathrm{A} \longrightarrow f_5 \longrightarrow \mathrm{B}) \longrightarrow \cdots . \end{aligned} \tag{6}$$

As the platoon is asymptotically stable, we have $f_0 > f_2 > f_4 > \cdots$. In other words, the combination "(AB)" has the effect of damping perturbations. Define $f_n^- = f_n - \varepsilon_n$, where $0 < \varepsilon_n < f_n$. Then, we have the following property:

$$\cdots \longrightarrow f_n \longrightarrow (\mathrm{AB}) \longrightarrow f_n^- \longrightarrow \cdots . \tag{7}$$

If we insert some combinations "(AB)" into the platoon, then we get

$$f_0 \longrightarrow \mathrm{A} \longrightarrow f_1 \longrightarrow (\mathrm{AB}) \longrightarrow f_1^- \longrightarrow \mathrm{B} \longrightarrow f_2^- \longrightarrow \cdots . \quad (8)$$

Considering $f_0 > f_2$, then we have $f_0 > f_2^-$. If we regard "(AABB)" as a combination, we find that the above pattern (8) can be rewritten as

$$f_0 \longrightarrow (\mathrm{AABB}) \longrightarrow f_2^- \longrightarrow (\mathrm{AABB}) \longrightarrow \cdots \quad (9)$$

which means the binary platoon "AABBAABB..." is stable. Similarly, it is easy to get that platoon "$(\mathrm{A}\ldots\mathrm{A})_m(\mathrm{B}\ldots\mathrm{B})_m \ldots$" is also stable by recursive method, as long as the initial small perturbation will not be amplified into the shockwave when it passes the first m successive type A vehicles.

Moreover, the above properties can be extended into more general form as follows. If the binary platoon "ABAB..." is stable, then type A vehicles and type B vehicles in all kinds of sequences are also stable. In other words, the stability of binary platoon does not depend on the sequence of individual vehicles but on the parameters of both types of vehicles. To prove this, we need the aforementioned two lemmas.

Lemma 1. *If the binary platoon "ABAB..." is asymptotically stable, then the platoon "BABA..." is also asymptotically stable.*

Lemma 2. *If the binary platoon "ABAB..." is asymptotically stable, then the platoon "$(A\ldots A)_m(B\ldots B)_m \ldots$" is also asymptotically stable.*

It is easy to derive the third and fourth lemmas from the above two, as follows.

Lemma 3. *If the binary platoon "ABAB..." is asymptotically stable, then the platoon "$(B\ldots B)_m(A\ldots A)_m \ldots$" is also asymptotically stable.*

Lemma 4. *If the binary platoon "ABAB..." is asymptotically stable, then all the combinations "AB," "BA," "$(A\ldots A)_m(B\ldots B)_m$," and "$(B\ldots B)_m(A\ldots A)_m$" have the effects of damping perturbations.*

Suppose that there is a binary platoon with type A vehicles and type B vehicles arranged randomly. The proportions of both types of vehicles are equal to 50%. Without loss of generality, we give out a segment of such a platoon; for instance,

$$\ldots \mathrm{ABBBBBABAAABBABAAABAAB} \ldots \quad (10)$$

Step 1. Find out all the combinations "AB," "BA," "$(\mathrm{A}\ldots\mathrm{A})_m(\mathrm{B}\ldots\mathrm{B})_m$," and "$(\mathrm{B}\ldots\mathrm{B})_m(\mathrm{A}\ldots\mathrm{A})_m$" as mentioned in Lemma 4:

$$\longrightarrow f_n \longrightarrow (\mathrm{AB}) \longrightarrow \mathrm{BBB} \longrightarrow (\mathrm{BABA}) \longrightarrow (\mathrm{AABB}) \longrightarrow (\mathrm{AB}) \longrightarrow \mathrm{AAA} \longrightarrow (\mathrm{BA}) \longrightarrow (\mathrm{AB}) \longrightarrow \quad (11)$$

Step 2. Because all the selected combinations have the effects of damping perturbations, they can be all removed from the propagation path of the perturbation without changing the stability of the platoon. Then we get a simplified platoon:

$$\longrightarrow f_n \longrightarrow \mathrm{BBB} \longrightarrow \mathrm{AAA} \longrightarrow \quad (12)$$

Step 3. We know that the numbers of type A vehicles and type B vehicles are equal, and all the combinations we removed contain same numbers of type A vehicles and type B vehicles. Therefore, the remaining part of platoon also consists of same numbers of type A vehicles and type B vehicles. Obviously, if we continue the simplification for the remaining part of platoon as Steps 1 and 2, we find that the original platoon can be discomposed into such combinations completely. Therefore, we verify the conjecture: if the binary platoon "ABAB..." is stable, then type A vehicles and type B vehicles in all kinds of sequences are also stable.

5. Discussions

In Section 4, we prove that if the binary platoon "ABAB..." is asymptotically stable, then the platoons "$(\mathrm{A}\ldots\mathrm{A})_m(\mathrm{B}\ldots\mathrm{B})_m \ldots$" and "$(\mathrm{B}\ldots\mathrm{B})_m(\mathrm{A}\ldots\mathrm{A})_m \ldots$" are also stable. It is noteworthy that this statement has a prerequisite; namely, the perturbation should not be amplified into shockwave during its propagation at the beginning of the platoon. Suppose that the combination of type A vehicle and type B vehicle belongs to Case 3 in Table 1. Let us take $\lambda_{\mathrm{A}}\tau_{\mathrm{A}} \geq 1/2$ and $\lambda_{\mathrm{B}}\tau_{\mathrm{B}} < 1/2$, for example. Because the perturbation will be amplified by type A vehicle during its propagation, it may finally turn to be shockwave when it passes enough number of successive type A vehicles. Thus, platoon "$(\mathrm{A}\ldots\mathrm{A})_m(\mathrm{B}\ldots\mathrm{B})_m \ldots$" is not asymptotically stable when the number m is big enough to let the perturbation evolve into shockwave. However, the stability of platoon "$(\mathrm{B}\ldots\mathrm{B})_m(\mathrm{A}\ldots\mathrm{A})_m \ldots$" is consistent with that of platoon "ABAB...," according to the analysis in Section 4.

Similarly, for the binary platoon with type A vehicles and type B vehicles arranged randomly, the platoon is unstable as long as there are enough number of successive identical types of vehicles which can let the perturbation evolve into shockwave.

The second noteworthy point is that the initial velocity and spacing have no influence on the stability of Chandler Model. In this paper, the initial velocity is set as 20 m/s and the initial spacing is set according to the velocity-headway function of Chandler Model. If we use different initial velocity and spacing, we can obtain the same results.

Actually, there are some analytic studies about the stability of heterogeneous traffic flow. Zhang and Jarrett [23] derived the stability criterion analytically for heterogeneous platoon as $\lambda_i\tau_i < 1/2$, $i = 2, 3, \ldots, N$. Zhang's criterion corresponds to Case 2 in our paper. However, this condition is sufficient but not necessary. Holland also proposed a stability criterion for heterogeneous platoon [24], which states that the

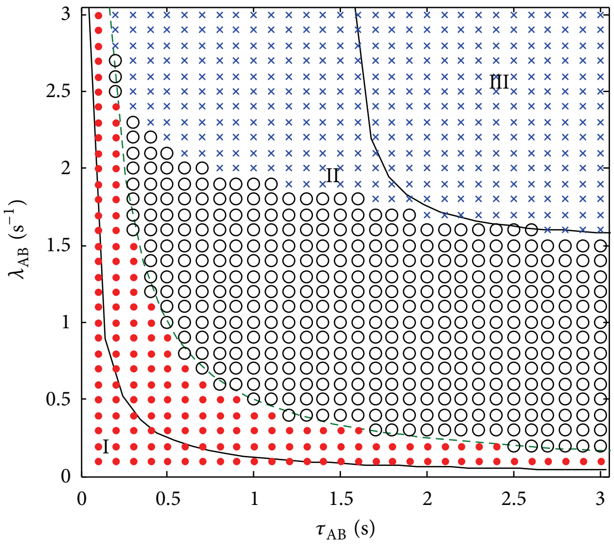


Figure 6: Stability results of binary platoon according to Holland's analytic stability criterion [24] (color online).

platoon is stable if the sum of the diffusion coefficients is positive:

$$\sum_{i=1}^{N}\left[\frac{1}{\lambda_i}\left(\frac{1}{2\lambda_i}-\tau_i\right)\right]>0. \tag{13}$$

Similar stability criterion for OVM model [8] was proposed by Mason and Woods [25]. In our study, for a binary platoon "ABAB. . .," the above condition is specified as

$$\frac{1}{2\lambda_{\mathrm{A}}^2}+\frac{1}{2\lambda_{\mathrm{B}}^2}-\frac{\tau_{\mathrm{A}}}{\lambda_{\mathrm{A}}}-\frac{\tau_{\mathrm{B}}}{\lambda_{\mathrm{B}}}>0. \tag{14}$$

We present the distributions of platoon stability in λ_{AB}-τ_{AB} plot as shown in Figure 6. All curves and markers are of the same meanings with Figure 3.

We find that region I and region III in Figure 6 are the same as those in Figure 3, which indicates that the analytic stability criterion is consistent with the simulation results for the cases of absolute stability and absolute instability. However, in region II, the analytic stability criterion shows a much wider area of "uncertainty" than the simulation results, especially for the lower right part of the plot. As the results in Figure 3 are obtained based on rigorous numerical simulations, the analytic stability criterion that Holland presented is insufficient; namely, it is likely to overestimate the stability of heterogeneous traffic in some cases.

6. Conclusions

In this paper, the asymptotic stability of Chandler Model for the binary heterogeneous traffic is studied based on numerical simulations. The binary traffic flow consists of two types of vehicles. Each type of vehicle accounts for 50% in the platoon and obeys the rule of Chandler Model with its own parameters. It is found that the stability of binary platoon cannot be determined by the mean values of model parameters. On the one hand, the binary platoon may not be stable even though the mean values of parameters satisfy the stability criterion of Chandler Model. On the other hand, we find a lot of cases, in which the binary platoons are stable whereas their mean parameters do not satisfy the stability criterion.

As far as the binary heterogeneous platoon is concerned, the stability of the platoon depends not on the orders of individual vehicles but on the parameters of these two types of vehicles. There are three cases regarding the combinations of two types of vehicles. First, if both types of vehicles satisfy the stability criterion, then the binary platoon is stable. Second, if neither type of vehicle satisfies the stability criterion, then the binary platoon is unstable. Third, if one type of vehicle satisfies the stability criterion while the other type does not satisfy the stability criterion, the stability of the binary platoon is uncertain. In the third case, it seems that the platoon is likely to be stable when the parameters satisfy the condition $(\lambda_{\mathrm{A}}-\lambda_{\mathrm{B}})(\tau_{\mathrm{A}}-\tau_{\mathrm{B}})<0$. Besides, the bigger the terms $|\lambda_{\mathrm{A}}-\lambda_{\mathrm{B}}|$ and $|\tau_{\mathrm{A}}-\tau_{\mathrm{B}}|$ are, the higher likelihood the platoon will have to be stable.

The analytic stability criterion for heterogeneous traffic reported in the literature is not rigorous, which is apt to overestimate the stability of heterogeneous traffic flow. In view of this fact, the numerical simulation is still a reliable approach for the analysis of traffic stability. We should mention that Chandler Model is a very simple model which may not be representative for all types of car-following models. The heterogeneous traffic we considered in this paper is a very simple binary platoon which only contains two types of vehicles with the same proportion. The stability studies on more general heterogeneous traffic and other car-following models will be our work in the next step.

Competing Interests

The authors declare that they have no competing interests.

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