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Research Article

The Role of Initial Credit Distribution Scheme in Managing Network Mobility and Maximizing Reserve Capacity: Considering Traveler's Cognitive Illusion

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The role of initial credit distribution scheme (ICDS) in managing network mobility has long been overlooked in previous studies of tradable credit scheme (TCS), which may make their results disputable in the reality, as the travelers possessing leftover credits can get some subsidy from the credit market and offset part of travel cost. In this paper, the disequilibrium phenomenon of previous user equilibrium (UE) solution is shown when traveler's cognitive illusion (CI) is considered. Then, a new UE condition with TCS is defined with the ICDS and CI explicitly considered. To comprehensively reveal the impacts of ICDS on UE solution, four different types of ICDS are introduced and analyzed in a unified variational inequality (VI) modeling framework. The uniqueness of the UE link flow and market equilibrium (ME) credit price is also investigated. Furthermore, the mathematical program with equilibrium constraint (MPEC) for the optimal ICDS design problem is established, with the optimization objective being maximizing network reserve capacity. A modified relaxation algorithm is adopted to solve the MPEC. The numerical example shows that a properly designed ICDS can not only improve the network reserve capacity, but also decrease the travel cost of all the travelers in the network simultaneously.

1. Introduction

Traffic congestion has become one of the most severe social problems in almost all large cities worldwide. For years, the most commonly used solution for mitigating congestion is congestion pricing as it can perfectly eliminate the traffic delay in theory. However, although congestion price is perfect in theory, it has not yet been well accepted in practice because of the intractable issues of social equity and taxation neutrality. Given the general political resistance to congestion pricing, some transportation researchers and planners have turned to a sophisticated quantity-based instrument, that is, tradable credit scheme (TCS), to mitigate traffic congestion by directly regulating the travel demand. TCS can be traced back to Dales [1] for the purpose of managing water quality in a cost-effective manner. Then, Verhoef et al. [2] explored the potential application of TCS in transportation realm. They proposed a "tradable road-pricing smart card" to regulate road transportation externalities and discussed many promising applications of TCS in transportation on both demand and supply side. A similar idea was addressed by Viegas [3] who suggested allocating "mobility credits" to all taxpayers monthly and providing the flexibility to pay either transit fare or road toll for private cars, as well as permitting the free trading of credits. Other TCS in transportation areas are investigated in [4–6] and so on, with different credits endowed with different legitimated rights. Apart from these conceptual studies, Akamatsu [7] made some mathematical analyses on TCS. He proposed a tradable bottleneck permits scheme (TBPS) under which the queuing delay can be eliminated. In his paper, he established an equilibrium model under TBPS and discussed the theoretical relationships between TBPS and congestion pricing. However, such a TBPS has some problematic issues, such as the complicated permit trading market and the controversial market selling scheme. To develop an alternative simple but forceful TCS, Yang and Wang [8] proposed a new tradable credit distribution and charging scheme where credits were universal for all links

but link specific in the amount of credit charging, and then they quantitatively analyzed how to manage network mobility with TCS in a general network equilibrium context. Along this thread, numerous efforts have been devoted to expanding Yang and Wang's landmark work on TCS. Wang et al. [9] expanded the work by considering heterogeneous users with different value of time (VOT) and formulated an equivalent variational inequalities (VI) model. Nie [10] investigated the effects of credit transaction costs in both auction market and negotiated market. Shirmohammadi et al. [11] explored the deviation in managing network mobility between congestion pricing and TCS under uncertain conditions. He et al. [12] analyzed TCS on the network with Cournot-Nash (CN) players and Wardrop-equilibrium (WE) players. Bao et al. [13] extended the studies of managing network mobility with TCS by considering travelers' loss aversion behaviors and the transaction costs. Recently, Han and Cheng [14] investigated the TCS design problem for maximizing network reserve capacity in the SUE context. Besides, there are also many investigations on managing bottleneck congestion with TCS, such as [15–17]. For recent reviews on tradable credits scheme, see [18, 19].

It is worth mentioning that the UE/SUE conditions with TCS in previous studies are all defined according to the generalized path travel cost, which includes the path travel time and path credit charging (in time unit). Clearly, this generalizedpath travel cost does not involve the initial credit distribution scheme (ICDS), neither does it embody explicitly the subsidy/expense of selling/buying credit. Under a given TCS, the alternative paths for each traveler are multiple; some paths (called expensive paths) charge more credits than the traveler's initial distributed credits while some paths (called cheap paths) charge less credits than his initial distributed credits. If the traveler chooses the cheap paths, he will have some leftover credits to earn money by selling them, whereas if he chooses the expensive paths he has to buy extra credits. Naturally, such thinking logic may show that the windfall income can offset part of the generalized path travel cost for the travelers choosing cheap paths while extra credits must be bought for the travelers choosing expensive paths. Under this thinking logic, the travelers will figure out a different generalized path travel cost (e.g., the generalized path travel costs of cheap path are taken as "path travel time plus path credit charging minus windfall income"); thus, the final decision of path choice will be totally different. In fact, this deductive reasoning is a kind of thinking logic with cognitive illusion (CI), which often comes to an apparently true but actually wrong result. The CI exists widely in our daily life, which often leads people to make incorrect decision or judgment; for example, (1) someone believes he will earn ¥10 if he spends ¥100 to buy a watch and sells it for ¥110 and then he spends ¥120 to buy it back and sells it for ¥130 at last, as shown in an online test (access date: October 28, 2015; website of the online test (in Chinese): http://zhidao.baidu.com/link?url=J2HBN4fI2mXUG20VODABzD3i9_d-EdT4bH75kFErEip0CpvYr4sopv7fjG-PtM4v889gO6JH5B-qpHDZJdDTlINQlE2PdzbI0_wW-HhbTaa); (2) in the Chinese idiom story "Zhao San Mu Si," the monkeys believe that the total acorns got from their master become more when the acorn distribution scheme changes from three in the morning and four in the evening to four in the morning and three in the evening. Why does the phenomenon of CI exist in our decision/judgment-making processes? Some psychologists have found in their researches that people usually make the complex decisions relying on the mental shortcuts or heuristics. Although reliance on heuristics facilitates good judgment most of the time, however, in some circumstances, it causes people to draw systematically inaccurate inferences; that is, these heuristics can sometime create CI of judgment [20, 21]. There are various types of CI, such as CI of thinking, CI of judgment, and CI of memory [22], and, furthermore, the CI could have a great influence on people's rationality [23, 24]. The defining features of CI are as follows: (1) the CI leads to a perception, judgment, or memory that reliably deviates from "reality"; (2) the CI deviates from the normative standard in a systematic fashion (i.e., in a predictable direction) rather than just randomly; (3) the CI appears involuntarily without specific instructions or deliberate will; (4) the CI is hard if not impossible to avoid; (5) the CI is not typical error, misunderstanding, or faulty memory, and it often appears as rather distinct from the normal course of information processing [22].

This paper attempts to introduce the CI into travelers' path choice behavior; specifically, it is assumed that the CI appears in the travelers' thinking logic when they evaluate the generalized path travel cost. With this assumption, the generalized path travel costs in travelers' mind usually deviate from the real ones in a predictable direction, and thus the shortest paths in travelers' mind are generally not the real ones either. As no one can realize the occurrence of CI in his mind, he will definitely choose the "shortest paths" according to the paradoxical generalized path travel costs. Obviously, the realized network flow pattern is not likely to be predicted correctly by utilizing the previous UE models with TCS. To approach this misestimating question, a new UE condition with TCS is defined with consideration of CI in this paper. Specifically, since the ICDS is explicitly considered in the proposed UE condition with TCS, it can greatly influence the UE network flow and market equilibrium (ME) credit price. To comprehensively analyze the impacts of ICDS on the network mobility, several different types of ICDS are introduced and analyzed in a unified modeling framework.

The paper is organized as follows. Section 2 analyzes the disequilibrium phenomenon of the previous UE model with TCS in seemingly more reasonable thinking logic. In Section 3, a new UE condition with TCS considering the traveler's CI is defined, in which the ICDS is explicitly considered, and, meanwhile, four different types of ICDS are introduced. Section 3 provides an equivalent variational inequality (VI) model and analyzes the impacts of ICDS and transaction cost on a unified VI modeling framework. Furthermore, the sufficient conditions for the uniqueness of link flow pattern and the credit price are also discussed in this section. In Section 4, the model of mathematical program with equilibrium constraint (MPEC) for the optimal ICDS design is established, of which the optimization objective is maximizing network reserve capacity. The relaxation algorithm is adopted to solve the MPEC problem. Section 5

presents a numerical example, and Section 6 provides some conclusion remarks.

2. Disequilibrium Phenomenon of Previous UE Considering Traveler's CI

Consider a general transportation network $\mathbf{G} = (\mathbf{N}, \mathbf{A})$, with a set $\mathbf{N}$ of nodes and a set $\mathbf{A}$ of directed links. It is assumed that the network is strongly connected; that is, at least one path joins each OD pair. Let the set of OD pairs be denoted by $\mathbf{W}$ and let the traffic demand for OD pair $w \in \mathbf{W}$ be denoted by q_w; let $\mathbf{R}_w$ denote the set of all paths between OD pair $w \in \mathbf{W}$; let f_r^w denote the flow on path $r \in \mathbf{R}_w$; let c_r^w denote the path travel cost on the path $r \in \mathbf{R}_w$; and let v_a and t_a denote the traffic flow and travel time on link $a \in \mathbf{A}$. The set $\mathbf{\Omega}$ of feasible network flow patterns $(\mathbf{f}, \mathbf{v})$ for the transportation network can be described as

$$\mathbf{\Omega} = \left\{ (\mathbf{f}, \mathbf{v}) \mid \sum_{r \in \mathbf{R}_w} f_r^w = q_w,\ v_a = \sum_{w \in \mathbf{W}} \sum_{r \in \mathbf{R}_w} f_r^w \delta_{a,r}^w,\ f_r^w \geq 0,\ \forall r \in \mathbf{R}_w,\ w \in \mathbf{W},\ \forall a \in \mathbf{A} \right\}, \tag{1}$$

where $\mathbf{f} = (f_r^w, r \in \mathbf{R}_w, w \in \mathbf{W})$, $\mathbf{v} = (v_a, a \in \mathbf{A})$, and $\delta_{a,r}^w = 1$ if route r uses link a and 0 otherwise.

Assume that the government issues a certain amount of credits, denoted by K, and distributes them to all eligible travelers according to a certain ICDS, denoted by $\mathbf{\Phi}$, such as uniform credit distribution and OD specific distribution. Meanwhile, let $\mathbf{k} = \{k_a, a \in \mathbf{A}\}$ denote the credit charging scheme, where k_a is the credit charge for using link $a \in \mathbf{A}$. In what follows, we will use $(K, \mathbf{k}, \mathbf{\Phi})$ to represent a TCS (only the feasible and active tradable credit scheme is considered in this paper; i.e., under any given credit scheme, there always exists a feasible UE solution and credit price is positive). According to Yang and Wang [8], the network flow pattern reaches user equilibrium (UE) under a given TCS $(K, \mathbf{k}, \mathbf{\Phi})$ when the following equations are satisfied:

$$\begin{aligned}
&\left(\sum_{a \in \mathbf{A}} \left(t_a(v_a) + p\kappa_a \right) \delta_{a,r}^w - \mu_w \right) f_r^w = 0, \quad \forall r \in \mathbf{R}_w,\ w \in \mathbf{W}, \\
&\sum_{a \in \mathbf{A}} \left(t_a(v_a) + p\kappa_a \right) \delta_{a,r}^w - \mu_w \geq 0, \quad f_r^w \geq 0, \quad \forall r \in \mathbf{R}_w,\ w \in \mathbf{W}, \\
&\left(K - \sum_a \kappa_a v_a \right) p = 0, \quad K - \sum_a \kappa_a v_a \geq 0,\ p \geq 0,
\end{aligned} \tag{2}$$

where p is the market equilibrium (ME) credit price in time unit and μ_w is the equilibrium path cost between OD pair w. Note that in this paper the credit price p is in time unit instead of money unit for the simplicity of the mathematical expressions; thus, the generalized path travel cost is also in time unit. Obviously, one can also rewrite the generalized path cost in money unit by simply multiplying the value of time.

It can be seen clearly that the above UE conditions do not involve the ICDS; that is, no matter how the ICDS $\mathbf{\Phi}$ changes, the resulting UE network flow pattern will never change as long as the total credit amount K and the credit charge $\mathbf{k}$ are invariant. However, is this result always true, especially when the travelers evaluate the generalized path cost in a thinking logic with CI? In order to answer this question, a simple numerical example is presented as follows.

Example 1. This example shows the disequilibrium phenomenon of traditional UE solution under a given TCS (K, $\mathbf{k}$, $\mathbf{\Phi}$) in seemingly more reasonable thinking logic. The toy network with two parallel links connecting a single OD pair is taken as the example. The input data are as follows: $q_{\mathrm{OD}} = 6$, $K = 8$, $k_1 = 1$, $k_2 = 2$, and $\Phi = 4/3$, and the link travel time functions are $t_1(x_1) = 1 + 3x_1$ and $t_2(x_2) = 3 + x_2$.

When the network reaches the UE state defined as (2), the equilibrium network flow pattern under the given credit scheme is $x_1^* = 4$, $x_2^* = 2$, and the equilibrium credit price $p^* = 8$. However, is this UE always a steady one? It is noteworthy that all the travelers have equal initial credit distribution amount $\Phi = 4/3$, and the travelers who choose route 1 will have leftover credits ($\Phi > k_1$) while the travelers who choose route 2 will have to buy extra credit ($\Phi < k_2$); in other words, the credit trading will happen. Therefore, under the thinking logic mentioned in the previous section, the generalized travel cost of route 1 evaluated in traveler's mind is 18.33, while the generalized path cost of route 2 is 21. Obviously, some travelers on route 2 will switch to route 1 until a new UE is reached; thus, the previous UE is not a steady one when the CI appears in the traveler's thinking logic.

3. Network Equilibrium with TCS and Equivalent Variational Inequality Model

3.1. UE Condition with TCS Considering Traveler's CI. As mentioned above, since the CI appears in the traveler's thinking logic, the generalized path travel cost in traveler's mind will deviate from the real one in a predictable direction. Specifically, the traveler will take the subsidy of selling credits to offset part of the generalized path travel cost if he chooses the cheap paths (charging less credits than his initial credits), or, conversely, the traveler will take all the expense into the generalized path travel cost if he chooses the expensive paths (charging more credits than his initial credits).

Along this thinking logic with CI, the generalized path cost c_r^w in traveler's mind under a given feasible TCS (K, $\mathbf{k}$, $\mathbf{\Phi}$) can be expressed as follows, in which the ICDS is explicitly considered:

$$c_r^w = \sum_{a \in \mathbf{A}} \left(t_a + p\kappa_a \right) \delta_{a,r}^w - p \left[\Phi - \sum_{a \in \mathbf{A}} \kappa_a \delta_{a,r}^w \right]_+, \quad r \in \mathbf{R}_w,\ w \in \mathbf{W}, \tag{3}$$

where $[a]_+ = a$ if $a \geq 0$ and 0 otherwise. Equation (3) indicates that the traveler will have to buy extra credits when he chooses the route with credit charging amount more than his initial credit amount, while he can sell the leftover credits to get some subsidy when he chooses the route with less credit charging amount. Therefore, a new UE condition with TCS considering the traveler's CI can be defined as follows.

Definition 2 (UE condition considering CI). A stable condition is reached only when no traveler can improve his generalized path cost evaluated in his own thinking logic by unilaterally changing routes, if the CI appears in traveler's path choice decision-making process. Hereafter, the generalized path travel cost in this paper means that evaluated in traveler's own thinking logic with CI if no specific instruction is made.

In such a route choice process, travelers will minimize their reckoned generalized path cost; therefore, the corresponding UE&ME conditions under a given TCS (K, $\mathbf{k}$, $\mathbf{\Phi}$) can be written as

$$\left(\sum_{a\in\mathbf{A}}(t_a + p\kappa_a)\delta_{a,r}^w - p\left[\Phi - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r}^w\right]_+ - \mu_w\right) f_r^w = 0, \quad \forall r \in \mathbf{R}_w,\ w \in \mathbf{W}, \tag{4a}$$

$$\sum_{a\in\mathbf{A}}(t_a + p\kappa_a)\delta_{a,r}^w - p\left[\Phi - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r}^w\right]_+ - \mu_w \geq 0, \quad f_r^w \geq 0, \quad \forall r \in \mathbf{R}_w,\ w \in \mathbf{W}, \tag{4b}$$

$$\left(K - \sum_a \kappa_a v_a\right) p = 0, \tag{4c}$$

$$K - \sum_a \kappa_a v_a \geq 0, \quad p \geq 0, \tag{4d}$$

where $(\mathbf{f}, \mathbf{v}) \in \mathbf{\Omega}$. The above equilibrium conditions (4a), (4b), (4c), and (4d) state that, at traffic equilibrium in a road network, all used routes between the same OD pair $w \in \mathbf{W}$ have equal and minimum generalized travel cost μ_w. Besides, the ICDS is explicitly considered in the UE conditions; thus, different ICDS will generally result in a different UE network flow pattern. The difference between the proposed UE condition and the previous UE condition (without consideration of the benefit from credit trading) is as shown in Figure 1.

3.2. Four Types of ICDS and Variational Inequality Model. The most commonly used ICDS are uniform credit distribution scheme and OD specific distribution scheme. Under the former, every traveler has distributed equal initial credit amount, while under the latter each traveler between the same OD pair has equal initial credit amount. In addition to these two simple ICDS, there are also many other different ICDS. In order to comprehensively consider the roles of various types of ICDS in managing network mobility, another two different schemes are also considered in this section, that is, group specific distribution scheme and route specific distribution scheme. It is assumed that the population can be divided into several groups according to certain feasible criteria (it is noteworthy that the division criteria do not involve the value of travel time, and thus the travelers in different groups have the same value of time) (e.g., age group) and that the travel demand for each specific group is also known a priori; hence, in the group specific distribution scheme, the travelers in the same group will have equal initial credit amount. In the route specific distribution scheme, the travelers on the same route will have equal initial credit amount. For convenience, we use ICDS-1, ICDS-2, ICDS-3, and ICDS-4 to represent the four types of ICDS, respectively. Although ICDS-4 is very difficult, if not impossible, to be implemented in the practical road network, it can still be taken as a theoretical benchmark.

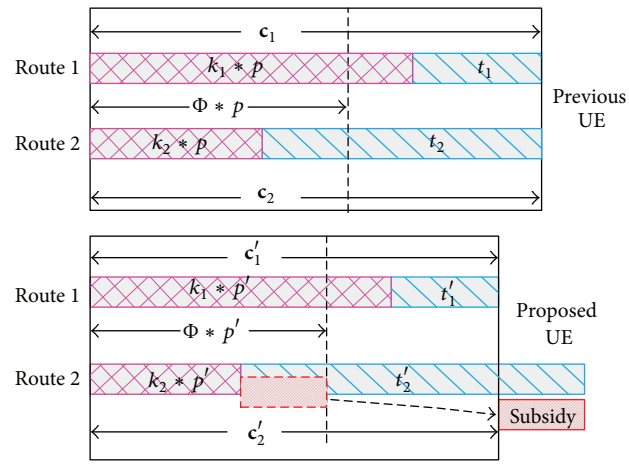


Figure 1: The diagram of the two types of UE conditions.

With a little abuse of notations, we denote ICDS-1 as $\overline{\Phi}$ $(= K/\sum_w q_w)$, ICDS-2 as $\mathbf{\Phi}_w$, $w \in \mathbf{W}$, ICDS-3 as $\mathbf{\Phi}_i^w$, $i \in \mathbf{I}_w$, $w \in \mathbf{W}$, and ICDS-4 as $\mathbf{\Phi}_r^w$, $w \in \mathbf{W}$, $r \in \mathbf{R}_w$, where $\mathbf{I}_w$ represent the groups of travelers between OD pair w. According to the definitions of these four types of ICDS, the following equations can be obtained:

$$\begin{aligned}
\overline{\Phi}\sum_{w\in\mathbf{W}} q_w &= K \quad \text{(ICDS-1)},\\
\sum_{w\in\mathbf{W}} \Phi_w q_w &= K \quad \text{(ICDS-2)},\\
\sum_{w\in\mathbf{W}}\sum_{i\in I_w} \Phi_i^w q_i^w &= K \quad \text{(ICDS-3)},\\
\sum_{w\in\mathbf{W}}\sum_{r\in\mathbf{R}_w} \Phi_r^w f_r^w &= K \quad \text{(ICDS-4)},
\end{aligned} \tag{5}$$

where q_i^w, $i \in \mathbf{I}_w$, $w \in \mathbf{W}$, denotes the travel demands of group i between OD pair w, and thus $\sum_{i\in\mathbf{I}_w} q_i^w = q_w$. The above equations show that in any ICDS the total initial credit amount distributed to the travelers is always equal to the credit amount issued by the government.

Besides, the generalized path cost defined in (3) can still apply to any ICDS, and the detailed expression under a particular ICDS can be obtained by replacing Φ with the corresponding ICDS notation, for example, $\mathbf{\Phi}_w$. Likewise, the UE conditions defined in (4a), (4b), (4c), and (4d) can also describe the corresponding network equilibrium condition by the same treatment. Therefore, the four types of ICDS can be theoretically analyzed in a unified modeling framework.

If we denote $\mathbf{c} = (c_r^w,\ r \in \mathbf{R}_w,\ w \in \mathbf{W})$ as the function vector of generalized path cost in the network under a given credit scheme $(K, \mathbf{k}, \mathbf{\Phi})$, UE conditions (4a), (4b), (4c), and (4d) can be formulated as the following variational inequality (VI) problem.

Find $(\mathbf{f}^*, \mathbf{v}^*) \in \mathbf{\Omega}$, $p^* \in \mathbf{R}^+$ such that

$$\text{(VI model)}\ \ \mathbf{c}(\mathbf{f}^*, p^*)(\mathbf{f} - \mathbf{f}^*) + \left(\sum_{a\in\mathbf{A}} \kappa_a v_a^* - K\right)(p - p^*) \geq 0, \quad \forall \mathbf{f} \in \mathbf{\Omega},\ p \in \mathbf{R}^+, \tag{6}$$

where $\mathbf{R}^+$ denotes the nonnegative orthant of $\mathbf{R}$.

It is worth mentioning that VI model (6) is equivalent to the UE conditions under any ICDS mentioned above except ICDS-4. Actually, it is very difficult, if not impossible, to find an equivalent VI model for the UE conditions under ICDS-4 as there is an additional linear constraint with respect to the path flow in this case.

Proposition 3. *Given a credit scheme $(K, \mathbf{k}, \mathbf{\Phi})$, the VI problem (6) is equivalent to the UE conditions under the credit scheme if ICDS-1, ICDS-2, or ICDS-3 is adopted.*

Proof. Note that the set $\mathbf{\Omega}$ of feasible network flow patterns becomes $\mathbf{\Omega}'$ under ICDS-3 as follows:

$$\mathbf{\Omega}' = \left\{(\mathbf{f}, \mathbf{v}) \mid \sum_r f_r^{w,i} = q_i^w,\ v_a = \sum_w \sum_i \sum_r f_r^{w,i} \delta_{a,r}^w,\ \forall w \in \mathbf{W},\ i \in I_w,\ r \in \mathbf{R}_w,\ a \in \mathbf{A}\right\}, \tag{7}$$

where $\mathbf{f} = (f_r^{w,i},\ r \in \mathbf{R}_w,\ i \in I_w,\ w \in \mathbf{W})$ and $\mathbf{c} = (c_r^{w,i},\ r \in \mathbf{R}_w,\ i \in I_w,\ w \in \mathbf{W})$. Obviously, the VI model in ICDS-3 contains that in ICDS-1 or ICDS-2 as the special case from the angle of modeling form; hence, only the equivalence between VI model (6) and UE conditions under ICDS-3 is proved in the following, and the proofs for the other two cases, ICDS-1 and ICDS-2, are the same as the ICDS-3 case.

Note that $(\mathbf{f}^*, p^*)$ is a solution of VI problem (6) if and only if it is a solution of the following linear problem:

$$\min_{\mathbf{f}\in\mathbf{\Omega}',\, p\in\mathbf{R}^+} \sum_{w\in\mathbf{W}} \sum_{i\in I_w} \sum_{r\in\mathbf{R}_w} c_r^{w,i}(\mathbf{f}^*, p^*) f_r^{w,i} + \left(\sum_{a\in\mathbf{A}} \kappa_a v_a^* - K\right) p. \tag{8}$$

The Lagrangian function for the linear problem (8) is

$$L = \sum_{w\in\mathbf{W}} \sum_{i\in I_w} \sum_{r\in\mathbf{R}_w} c_r^{w,i}(\mathbf{f}^*, p^*) f_r^{w,i} + \left(\sum_{a\in\mathbf{A}} \kappa_a v_a^* - K\right) p - \sum_{w\in\mathbf{W}} \sum_{i\in I_w} \mu_i^w \left(\sum_{r\in\mathbf{R}_w} f_r^{w,i} - q_i^w\right). \tag{9}$$

The first-order conditions (KKT conditions) for this problem are

$$\begin{aligned}
&\left(c_r^{w,i}(\mathbf{f}^*, p^*) - \mu_i^w\right) f_r^{w,i*} = 0, \quad \forall w \in \mathbf{W},\ i \in I_w,\ r \in \mathbf{R}_w,\\
&c_r^{w,i}(\mathbf{f}^*, p^*) - \mu_i^w \geq 0, \quad \forall w \in \mathbf{W},\ i \in I_w,\ r \in \mathbf{R}_w,\\
&p^*\left(\sum_a \kappa_a v_a^* - K\right) = 0, \quad \sum_a \kappa_a v_a^* - K \leq 0,\ p^* \geq 0,
\end{aligned} \tag{10}$$

where μ_i^w, $i \in I_w$, $w \in \mathbf{W}$, is the Lagrange multiplier associated with the flow conservation constraint for the group i between OD pair w. Obviously, the above KKT conditions are just an alternative expression of the UE conditions (4a), (4b), (4c), and (4d), and hence any solution of the VI model (6) is identical with the UE flow pattern under ICDS-3. This completes the proof. □

Proposition 4. *The VI problem (6) admits at least one solution, if any one of ICDS-1, ICDS-2, and ICDS-3 is adopted.*

Proof. Likewise, only the case of ICDS-3 is proved, and the proofs for the other two cases are still the same. Note that the feasible set $\mathbf{\Omega}'$ and $\mathbf{R}^+$ are nonempty and convex. Given the TCS $(K, \mathbf{k}, \mathbf{\Phi})$, the generalized path cost functions $c_r^{w,i} = \sum_{a\in\mathbf{A}}(t_a + p\kappa_a)\delta_{a,r}^w - p[\Phi_i^w - \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r}^w]_+$, $r \in \mathbf{R}_w$, $i \in \mathbf{I}_w$, $w \in \mathbf{W}$, and $\sum_{a\in\mathbf{A}} \kappa_a v_a - K$ are continuous with respect to path flow vector $\mathbf{f}$ and credit price p. According to Facchinei and Pang [25], the VI problem (6) admits at least one solution $(\mathbf{f}^*, p^*)$. This completes the proof. □

Proposition 5. *Given a TCS $(K, \mathbf{k}, \mathbf{\Phi})$ with ICDS-4, the UE network flow pattern must exist if the following condition is satisfied: $\Phi_r^w = \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r}^w$, $\forall w \in \mathbf{W}$, $r \in \mathbf{R}_w$; otherwise, the existence of such a UE network flow pattern depends on the specific values of ICDS-4.*

Proof. Firstly, let $(\mathbf{f}^*, \mathbf{v}^*) \in \mathbf{\Omega}$ denote the UE network flow pattern under ICDS-4 and p^* denote the equilibrium credit price. If $\Phi_r^w = \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r}^w$, then the generalized path cost becomes $c_r^w = \sum_{a\in\mathbf{A}}(t_a + p\kappa_a)\delta_{a,r}^w$, $r \in \mathbf{R}_w$, $w \in \mathbf{W}$, which is exactly the case studied in Yang and Wang [8], and the solution must exist in this case.

If $\Phi_r^w \neq \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r}^w$, then we have

$$\begin{aligned}
\sum_{r\in\mathbf{R}_w} f_r^{w*} &= q_w, \quad \forall w \in \mathbf{W},\\
\sum_{w\in\mathbf{W}} \sum_{r\in\mathbf{R}_w} \Phi_r^w f_r^{w*} &= K,\\
\left(c_r^w\left(\mathbf{f}^*, p^*\right) - \mu_w\right) f_r^{w*} &= 0, \quad \forall w \in \mathbf{W},\ r \in \mathbf{R}_w,\\
\sum_a \kappa_a v_a^* &= K,\\
p^* &> 0,\\
f_r^{w*} &\geq 0,\\
c_r^w\left(\mathbf{f}^*, p^*\right) - \mu_w &\geq 0,\\
&\forall w \in \mathbf{W},\ r \in \mathbf{R}_w.
\end{aligned} \tag{11}$$

It can be seen that the dimension of the variables $(\mathbf{f}^*, p^*, \boldsymbol{\mu}^*)$ is less than the number of the equalities in the above equation system in this case, and thus the existence of UE solution depends on the specific values of ICDS-4. This completes the proof. □

3.3. Uniqueness of the UE Link Flow Pattern and Credit Price. In Yang and Wang [8], the uniqueness of UE link flow pattern is readily verified, and a mild and sufficient condition for the uniqueness of equilibrium credit price is given. However, due to the nonadditivity of the generalized path cost, the VI problem (6) fails to coincide with any optimization problem, and the uniqueness of aggregate link flow is not straightforwardly clear. Here, we firstly construct a sufficient condition for the uniqueness of link flow pattern solution $\mathbf{v}^*$, and then we present the conditions for the uniqueness of the equilibrium credit price p^*. Hereafter, only the cases of ICDS-1, ICDS-2, and ICDS-3 are considered in what follows, as ICDS-4 case is exactly the same as in Yang and Wang [8] if the UE solution exists.

Proposition 6. *The link flow solution $\mathbf{v}^*$ of the VI problem (6) is unique if the following conditions are satisfied: (1) the link travel time functions are strictly increasing, in the sense that $(\mathbf{t}(\mathbf{v}^1) - \mathbf{t}(\mathbf{v}^2))(\mathbf{v}^1 - \mathbf{v}^2) > 0$ for any $\mathbf{v}^1, \mathbf{v}^2 \in \boldsymbol{\Omega}'^1$ $(\mathbf{v}^1 \neq \mathbf{v}^2)$; (2) the total sold credit amount is nonincreasing with respect to the credit price, in the sense that $(p_1 - p_2)\sum_{w\in\mathbf{W}}\sum_{i\in I_w}\sum_{r\in\mathbf{R}_w}[\Phi_i^w - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r}^w]_+(f_r^{1w,i} - f_r^{2w,i}) \leq 0$ for any $p_1, p_2 \in \mathbf{R}^+$ $(p_1 \neq p_2)$ under ICDS-3.*

Proof. Firstly, note that if the path flow solution of VI model (6) is unique, or if the path flow solution is not unique, but its induced link flow solutions are the same, the link flow solution is always unique. So we only need to consider the case when more than one path flow solution exists and the induced link flow solutions are different, and two of them are denoted by $(\mathbf{f}^1, \mathbf{v}^1), (\mathbf{f}^2, \mathbf{v}^2) \in \boldsymbol{\Omega}'$ $(\mathbf{f}^1 \neq \mathbf{f}^2, \mathbf{v}^1 \neq \mathbf{v}^2)$, $p_1, p_2 \in \mathbf{R}^+$ $(p_1 \neq p_2)$. Note that $\mathbf{c}(\mathbf{f}^*, p^*)(\mathbf{f} - \mathbf{f}^*) \geq 0$, $\forall \mathbf{f} \in \boldsymbol{\Omega}$, always holds (the second term in the left of VI model (6) is always equal to zero if $p^* \neq 0$; thus, the inequality holds; if $p^* = 0$, the tradable credit scheme is inactive; thus the second term can be omitted and the inequality also holds); then we have $\mathbf{c}(\mathbf{f}^1, p_1)(\mathbf{f}^2 - \mathbf{f}^1) \geq 0$ and $\mathbf{c}(\mathbf{f}^2, p_2)(\mathbf{f}^1 - \mathbf{f}^2) \geq 0$. Combine the two inequalities; we can obtain $(\mathbf{c}(\mathbf{f}^1, p_1) - \mathbf{c}(\mathbf{f}^2, p_2))(\mathbf{f}^2 - \mathbf{f}^1) \geq 0$ and then unfold the equation

$$\begin{aligned}
&\left(\mathbf{c}\left(\mathbf{f}^1, p_1\right) - \mathbf{c}\left(\mathbf{f}^2, p_2\right)\right)\left(\mathbf{f}^2 - \mathbf{f}^1\right)\\
&\quad = \sum_a \left(t_a^1 - t_a^2\right)\left(v_a^2 - v_a^1\right) + \left(p_1 - p_2\right)\sum_a \kappa_a\left(v_a^2 - v_a^1\right)\\
&\quad + \left(p_2 - p_1\right)\\
&\quad \cdot \sum_w \sum_i \sum_r \left[\Phi_i^w - \sum_a \kappa_a \delta_{a,r}^w\right]_+ \left(f_r^{2w,i} - f_r^{1w,i}\right).
\end{aligned} \tag{12}$$

Note that the first term in the right hand side of the above equation is less than zero, the second term is equal to zero (note that the total credit amounts of two solutions are the same and equal to K), and the third term is less than or equal to zero; that is, the right hand side is less than zero. Clearly, it is in contradiction to the left hand side; thus the link flow solution of the VI problem (6) must be unique. This completes the proof. □

Remark 7. It can be obtained easily that the second condition of Proposition 6 will become $(p_1 - p_2)\sum_{w\in\mathbf{W}}\sum_{r\in\mathbf{R}_w}[\overline{\Phi} - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r}^w]_+(f_r^{1w} - f_r^{2w}) \leq 0$ under ICDS-1 and will become $(p_1 - p_2)\sum_{w\in\mathbf{W}}\sum_{r\in\mathbf{R}_w}[\Phi_w - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r}^w]_+(f_r^{1w} - f_r^{2w}) \leq 0$ under ICDS-2. Besides, although under any ICDS the first condition in Proposition 6 is weak and can be satisfied in most cases, the second one cannot be ensured. Therefore, the uniqueness of the link flow pattern of the VI problem (6) cannot always be ensured.

We now move to examine the credit price. The uniqueness of the credit price p^*, at network equilibrium, is described in the following proposition.

Proposition 8. *Given a TCS $(K, \mathbf{k}, \boldsymbol{\Phi})$ with any ICDS, the equilibrium credit price p^* is unique if the following two conditions are satisfied: (1) $\mathbf{v}^*$ is unique; (2) there exists at least one OD pair whose used equilibrium route set always contains two (or more) routes with different credit charges.*

Proof. Suppose two routes $r_1, r_2 \in \mathbf{R}_w$ with different path credit charges are always equilibrium routes between an OD pair $w \in \mathbf{W}$ for any equilibrium path flow patterns and credit prices. That is,

$$\begin{aligned}
\sum_{a\in\mathbf{A}} \left(t_a + p\kappa_a\right)\delta_{a,r_1}^w - p\left[\Phi - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r_1}^w\right]_+ &= \mu_w,\\
\sum_{a\in\mathbf{A}} \left(t_a + p\kappa_a\right)\delta_{a,r_2}^w - p\left[\Phi - \sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r_2}^w\right]_+ &= \mu_w.
\end{aligned} \tag{13}$$

Since the two routes are charged with different credits, $\sum_a \kappa_a\delta_{a,r_1}^w \neq \sum_a \kappa_a\delta_{a,r_2}^w$, the equilibrium credit price p^* can be expressed as

$$p^* = \frac{\sum_a t_a \delta_{a,r_2}^w - \sum_a t_a \delta_{a,r_1}^w}{\sum_a \kappa_a \delta_{a,r_1}^w - \sum_a \kappa_a \delta_{a,r_2}^w + \left[\Phi - \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r_2}^w\right]_+ - \left[\Phi - \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r_1}^w\right]_+}. \tag{14}$$

From the given assumptions, the link flow pattern $\mathbf{v}^*$ is unique; we can conclude that the equilibrium credit price p^* is unique if the denominator in the above equation is not equal to zero.

Suppose that $\sum_a \kappa_a \delta_{a,r_1}^w > \sum_a \kappa_a \delta_{a,r_2}^w$; there exist three cases: $\Phi > \sum_a \kappa_a \delta_{a,r_1}^w > \sum_a \kappa_a \delta_{a,r_2}^w$, $\sum_a \kappa_a \delta_{a,r_1}^w > \Phi > \sum_a \kappa_a \delta_{a,r_2}^w$ and $\sum_a \kappa_a \delta_{a,r_1}^w > \sum_a \kappa_a \delta_{a,r_2}^w > \Phi$, and the resultant denominators are, respectively, $2(\sum_a \kappa_a \delta_{a,r_1}^w - \sum_a \kappa_a \delta_{a,r_2}^w) > 0$, $\Phi + \sum_a \kappa_a \delta_{a,r_1}^w - 2\sum_a \kappa_a \delta_{a,r_2}^w > 0$, and $\sum_a \kappa_a \delta_{a,r_1}^w - \sum_a \kappa_a \delta_{a,r_2}^w > 0$. Similarly, it can be proved that the denominator is not equal to zero for $\sum_a \kappa_a \delta_{a,r_1}^w < \sum_a \kappa_a \delta_{a,r_2}^w$. This completes the proof. □

Besides, if the equations defined in (4a) are added up for all paths ($\mathbf{R}_w$) and OD pairs ($\mathbf{W}$), then a more general expression can be obtained for the equilibrium credit price:

$$p^* = \frac{\sum_w \mu_w q_w - \sum_a t_a^* v_a^*}{K - \sum_w \sum_r \left[\Phi - \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r}^w\right]_+ f_r^{w*}}. \tag{15}$$

The above equation is always valid since the total sold credit amount is necessarily less than the total credit amount issued by the government in the network; that is, $K - \sum_w \sum_r [\Phi - \sum_{a\in\mathbf{A}} \kappa_a \delta_{a,r}^w]_+ f_r^{w*} > 0$ always holds.

4. Optimal ICDS Design for Maximizing Reserve Capacity

4.1. MPEC for Optimal ICDS Design. From the above sections, we can see that different ICDS will lead to different UE solution, and hence it is meaningful and valuable to find such an ICDS that the resultant UE solution makes a certain performance index of transportation network achieve the optimal point. In this section, the network reserve capacity, of which the concept is the same as that in the literature [26], is taken as the network performance index. Thus, the model of mathematical program with equilibrium constraint (MPEC) for the optimal ICDS design is established in this section, of which the optimization objective is maximizing network reserve capacity, and the equilibrium constraint is the UE assignment problem under a certain TCS. The unified modeling framework of MPEC for the three ICDS cases is as follows:

$$\max_{\sigma,\mathbf{\Phi}} \quad \sigma \tag{16a}$$

$$\text{s.t.} \quad v_a(\sigma\mathbf{q}, \mathbf{\Phi}) \le C_a, \quad \forall a \in \mathbf{A} \tag{16b}$$

$$\mathbf{\Phi} \cdot \sigma\mathbf{q} = \sigma K \tag{16c}$$

$$\sigma \ge 0, \ \mathbf{\Phi} \ge 0, \ \forall a \in \mathbf{A}. \tag{16d}$$

Note that constraint (16c) indicates that the total credit amount issued by the government will increase by the same scaling factor as the total OD demands, and constraint (16c) can be replaced by the corresponding equation in (5) when a particular ICDS is adopted. Meanwhile, $v_a(\sigma\mathbf{q}, \mathbf{\Phi})$ can be obtained by solving the following VI model under a certain ICDS $\mathbf{\Phi}$ with a certain OD demand level $\sigma\mathbf{q}$.

Find $(\mathbf{f}^*, \mathbf{v}^*) \in \mathbf{\Omega}$, $p^* \in \mathbf{R}^+$ such that

$$\mathbf{c}(\mathbf{f}^*(\sigma\mathbf{q}, \mathbf{\Phi}), p^*)(\mathbf{f} - \mathbf{f}^*(\sigma\mathbf{q}, \mathbf{\Phi})) + \left(\sum_{a\in\mathbf{A}} \kappa_a v_a^*(\sigma\mathbf{q}, \mathbf{\Phi}) - \sigma K\right)(p - p^*) \ge 0, \quad \forall \mathbf{f} \in \mathbf{\Omega}_{(\sigma\mathbf{q},\mathbf{\Phi})}, \ p \in \mathbf{R}^+, \tag{16e}$$

where $\mathbf{\Omega}_{(\sigma\mathbf{q},\mathbf{\Phi})} = \{(\mathbf{f}, \mathbf{v}) \mid \sum_{r\in\mathbf{R}_w} f_r^w = \sigma q_w, v_a = \sum_{w\in\mathbf{W}} \sum_{r\in\mathbf{R}_w} f_r^w \delta_{a,r}^w, f_r^w \ge 0, \forall r \in \mathbf{R}_w, w \in \mathbf{W}, \forall a \in \mathbf{A}\}$.

It should be pointed out that the total credit amount K is also multiplied by the common OD demand multiplier σ in the VI model (16e), in addition to the OD demands q_w, $w \in \mathbf{W}$, in the feasible network pattern set $\mathbf{\Omega}_{(\sigma\mathbf{q},\mathbf{\Phi})}$. There are mainly three practical considerations for this modeling form: firstly, if the common demand multiplier σ is not added to the set $\mathbf{\Omega}_{(\sigma\mathbf{q},\mathbf{\Phi})}$, the undesirable problem that the total amount K of credits issued originally is not large enough to ensure all travelers completing their trips may happen just before the link capacity constraint (16b) is violated, with the common demand multiplier σ continuously increasing. Secondly, in order to make sure that the initial credit amount $\mathbf{\Phi}$ distributed to all the eligible travelers is invariant when the OD demands q_w, $w \in \mathbf{W}$, accommodated on the network are increasing, thus the total credit amount to be issued must be enlarged by the same scaling factor; and, furthermore, the modeling form can also ensure that, under the new credit scheme (σK, $\mathbf{k}$, $\mathbf{\Phi}$), there always exists at least one solution satisfying the feasible network pattern set $\mathbf{\Omega}_{(\sigma\mathbf{q},\mathbf{\Phi})}$ [14].

4.2. Solution Algorithm. Since the proposed MPEC is NP-hard and nonconvex, and, most importantly, the Mangasarian-Fromovitz Constraint Qualification (MFCQ) does not hold [27], thus solving the MPEC problem directly is usually difficult. Consequently, in this paper, we adopt a relaxation algorithm similar to Ban et al. [28] to solve the problem as the VI model (16e) is equivalent to the strict complementarity slackness constraints and nonnegativity conditions defined in (4a), (4b), (4c), and (4d). The main idea of the relaxation algorithm is to introduce auxiliary parameters $\theta^w > 0$, $w \in \mathbf{W}$, and $\theta^p > 0$ for the strict complementarity slackness constraints (4a) and (4c), which can be used to define the relaxed complementarity slackness conditions. In other words, at each iteration, the strict

complementarity slackness constraints (4a) and (4c) will be replaced by the following two equations:

$$\left(\sum_{a\in\mathbf{A}}(t_a+p\kappa_a)\delta_{a,r}^{w}-p\left[\Phi-\sum_{a\in\mathbf{A}}\kappa_a\delta_{a,r}^{w}\right]_{+}-\mu_w\right)f_r^w \le \theta^w,\quad \forall r\in\mathbf{R}_w,\ w\in\mathbf{W}, \tag{17a}$$

$$\left(K-\sum_a\kappa_a v_a\right)p\le\theta^p. \tag{17b}$$

Then, the relaxed MPEC problem can be solved repeatedly by constantly reducing the value of $\boldsymbol{\theta}=(\theta^w,\theta^p)>0$, by some predefined factor. It is clear that although this relaxed MPEC is still nonconvex, the MFCQ holds, which implies that existing NLP solution algorithms can be adopted to solve the relaxed MPEC. The solution procedure is as follows.

Step 1 (initialization). Choose an initial auxiliary parameter $\boldsymbol{\theta}^0>0$ for each complementarity slackness constraint. Set maximum iteration number M, error tolerance $\varepsilon>0$, update factor $0<\lambda<1$, and iteration number $n=0$.

Step 2 (major iteration). By setting $\boldsymbol{\theta}^n$ as the auxiliary parameter for the relaxed complementarity conditions in (17a) and (17b), solve the current relaxed MPEC

$$\begin{aligned}&\text{(Relaxed-MPEC)}\quad \max_{\sigma,\boldsymbol{\Phi},\boldsymbol{\mu},\mathbf{f},p}\quad \sigma\\ &\text{subject to:}\quad (16b),(16c),(16d),(17a),(17b),(4b),(4d).\end{aligned} \tag{18}$$

Step 3 (stop condition). If $\eta^{(n)}<\varepsilon$, then go to Step 4, where $\eta^{(n)}=\sqrt{\sum_w\sum_r[(c_r^{w(n)}-\mu_w^{(n)})f_r^{w(n)}]^2+[(K-\sum_a\kappa_a v_a^{(n)})p^{(n)}]^2}$; else, if $n\le M$, then set $\boldsymbol{\theta}^{n+1}=\lambda\boldsymbol{\theta}^n$, $n=n+1$, and go to Step 2; otherwise, go to Step 4.

Step 4 (report solution). The optimal solution $(\mathbf{f}^*,p^*)$ is achieved from the last run of Step 2.

The relaxation algorithm presented here is straightforward and easily implemented, and many existing solver software programs can be used to compute the nonlinear programming problem (NLP) in Step 2, such as the fmincon solver in MATLAB.

5. Numerical Example

Example 1. The six-node seven-link network, as shown in Figure 2, is taken as the example, the link travel time functions are standard BPR function $t_a(v_a)=t_a^0[1+0.15(v_a/C_a)^4]$, $a\in\mathbf{A}$, and the input data is as shown in Figure 2. The OD travel demands are, respectively, $q_1=60$, $q_2=50$, and the total credit amount $K=660$ is issued to all the travelers according to a certain ICDS. Assume that under ICDS-3 the travel demand q_1 can be divided into three groups, $q_1^1=q_2^1=q_3^1=20$, and q_2 can be divided into two groups, $q_1^2=q_2^2=25$.

For the experiments in this section, the initial parameters are set as $M=10$, $\boldsymbol{\theta}^0=(1,1,1,1,1)^T$, $\lambda=0.2$, and $\varepsilon=10^{-6}$. All the tests are computed based on a personal notebook computer with Intel® Core™ i5-3210M 2.50 GHz CPU, 4 GB RAM, and Windows 7 (Flagship Edition) operating system, and the MATLAB program is coded for the relaxation algorithm.

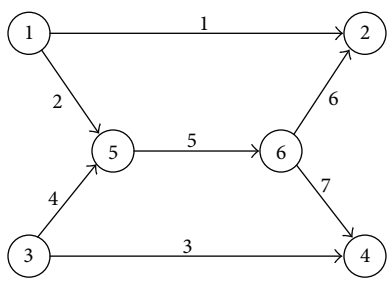
In order to show the impacts of ICDS on the network mobility, the market equilibrium (ME) credit price and UE path flow at different ICDS combination values (Φ_1,Φ_2) under ICDS-2 are compared in Figure 3. It can be seen from Figure 3 that the changing trend of UE solution with respect to the ICDS combination values (Φ_1,Φ_2) can be divided into three stages; specifically, the UE solution keeps invariant in the first and the third stage while it changes in the second stage. The reasons for this result are as follows: in the first stage, all paths between OD-1 are charging more credits than traveler's initial credit amount Φ_1 of this OD pair, and all paths between OD-2 are charging fewer credits than the initial credit amount Φ_2; therefore, no matter how (Φ_1,Φ_2) changes, the relative differences of paths between the same OD pair keep invariant; in the second stage, there always exists at least one OD pair between which some path credit charges are more than the initial credit amount while the others are fewer than that; thus the relative differences of paths between the same OD pair will change with respect to (Φ_1,Φ_2); in the third stage, the case is just the counterpart of the first stage. Besides, the total credit trade amount and total travel time (TTT) at different (Φ_1,Φ_2) under ICDS-2 are also compared with the previous TCS model (without consideration of ICDS in the UE model), as shown in Figure 4. It can be seen that, for the previous TCS model, the ICDS just influence the credit trade amount; however, for the model proposed in this paper, an appropriate ICDS combination (Φ_1,Φ_2) under ICDS-2 can not only reduce the total credit trade amount, but also reduce the total travel time of traffic network.

Figure 5 shows the convergence trend of relaxation algorithm under different types of ICDS for the optimal ICDS design problem, and the red dashed line represents the critical line of convergence (i.e., $\varepsilon=10^{-6}$). We can see that the convergence rate under ICDS-1 is much faster than that under ICDS-2 and ICDS-3, the reason for which is that the initial distributed credit amount Φ under ICDS-1 is a constant value while the other two are unknown variables.

In order to comprehensively illustrate the effect of ICDS on network reserve capacity, we compare the numerical results of ICDS-1, ICDS-2, and ICDS-3 with the result of previous TCS model (see Table 1).

Table 1: Comparison of the optimal solutions under different ICDS.

Link #		1	2	3	4	5	6	7	Max σ	p^*
Previous model	$\mathbf{\Phi}$	Any ICDS								
	v_a	19.44	14.87	8.47	20.13	35.00	14.87	20.13	***0.572***	0.41
	V/C	0.56	0.50	0.24	0.58	1.00	0.42	0.81		
ICDS-1	$\mathbf{\Phi}$	$\Phi = 6.00$								
	v_a	23.50	11.53	5.72	23.47	35.00	11.53	23.47	***0.584***	0.29
	V/C	0.67	0.38	0.16	0.67	1.00	0.33	0.94		
ICDS-2	$\mathbf{\Phi}^w$	$\Phi_1 = 5.57, \Phi_2 = 6.51$								
	v_a	25.36	10.00	4.46	25.00	35.00	10.00	25.00	***0.589***	0.29
	V/C	0.72	0.33	0.13	0.71	1.00	0.29	1.00		
ICDS-3	$\mathbf{\Phi}_i^w$	$\Phi_1^1 = 0.00, \Phi_2^1 = 6.37, \Phi_3^1 = 6.37, \Phi_1^2 = 8.21$, and $\Phi_2^2 = 8.00$								
	v_a	25.36	10.00	4.46	25.00	35.00	10.00	25.00	***0.589***	0.25
	V/C	0.72	0.33	0.13	0.71	1.00	0.29	1.00		



OD1: $1 \to 2$
Route 1: link 1; route 2: link 2-5-6

OD2: $3 \to 4$
Route 3: link 4-5-7; route 4: link 3

Link	t_a^0	C_a	k_a
1	10	35	9
2	3	30	2
3	12	35	8
4	4	35	1
5	5	35	1
6	3	35	2
7	4	25	1

Figure 2: The six-node seven-link network.

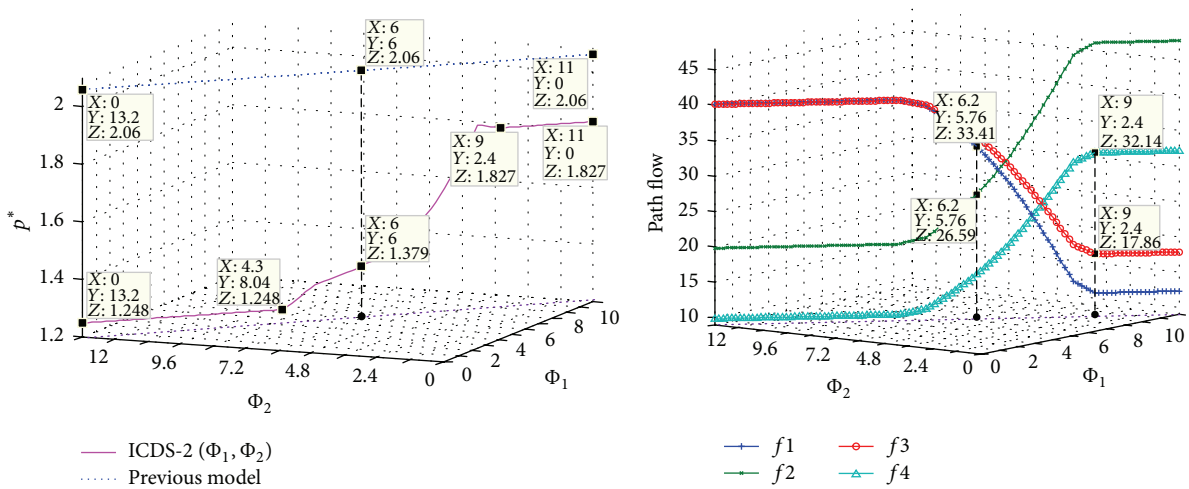
It can be seen from Table 1 that the maximum demand multiplier (i.e., network reserve capacity) for the previous TCS model is 0.572, no matter which type of ICDS is adopted. However, when the traveler's route choice behavior follows the UE condition proposed in this paper, the corresponding maximum demand multipliers under ICDS-1, ICDS-2, and ICDS-3 are all greater than that of the previous TCS model. Furthermore, the maximum demand multipliers under ICDS-2 and ICDS-3 are the same, and both are greater than that under ICDS-1. The reasons for these results are as follows: firstly, the network reserve capacity is determined by the external traffic management measures as well as the internal physical characteristics of the transportation network. Obviously, the ICDS is also a traffic management measure just like the credit charge scheme, which is yet overlooked in the previous TCS model; secondly, the initial credit amounts distributed to the travelers under ICDS-1 are a constant while they are unknown variables satisfying a linear constraint under ICDS-2 and ICDS-3, which limits the traffic regulation flexibility of the ICDS.

Besides, we are also pleasantly surprised to find that the ME credit prices under the optimal ICDS-1, ICDS-2, and ICDS-3 are very close to each other, which is a desirable and important feature in ICDS design problem. Since the volatility of credit price is usually undesirable in the realistic credit market, thus it is necessary and valuable to design an optimal ICDS with a steady (or almost steady) credit price.

Figure 6 compares the minimum generalized path costs between each OD pair (OD1: $1 \to 2$, OD2: $3 \to 4$) under three ICDS with those of the previous TCS model at the respective maximum demand multiplier σ. We can see that the generalized travel cost of each traveler under any ICDS is less than that of the previous TCS model, although the total OD travel demands in the network are larger under these three ICDS. Furthermore, the minimum generalized travel cost of each OD pair under ICDS-3 is the smallest among the three ICDS, which illustrates that the TCS with ICDS-3 may perform better in terms of making a pareto-improving transportation system.

6. Conclusions

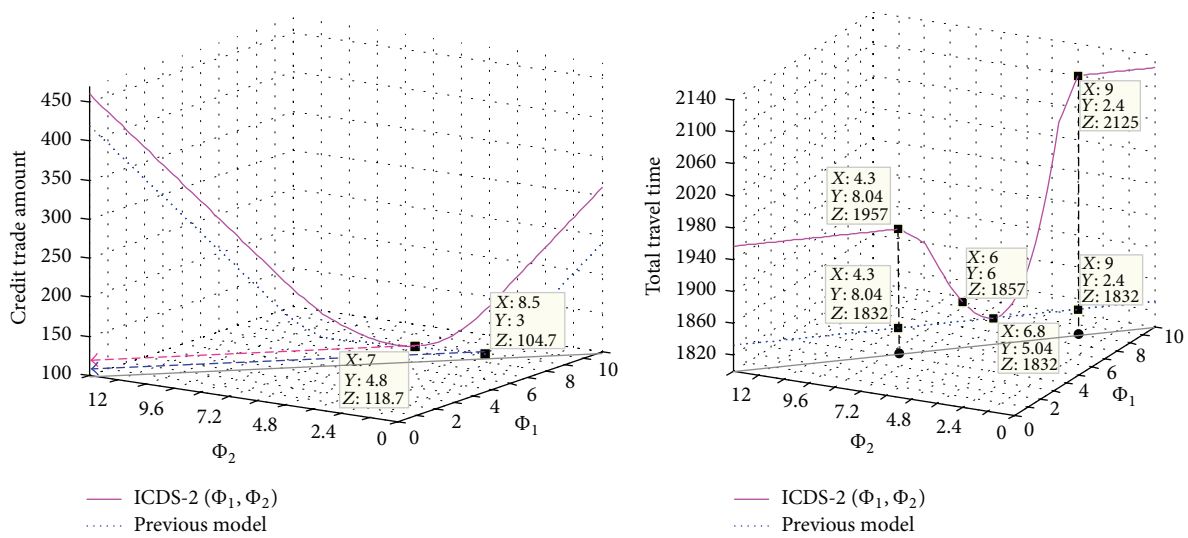
This paper mainly investigates the role of ICDS in managing network mobility and the optimal ICDS design problem. It is well known that the TCS is a sophisticated quantity-based traffic demand management instrument, which can effectively and efficiently mitigate the network congestion. However, in previous studies only the roles of total credit amount and the link-specific charge scheme are considered, whereas the role of ICDS has been overlooked for a long time.



(a) ME credit at different (Φ_1, Φ_2)

(b) UE path flow at different (Φ_1, Φ_2)

Figure 3: UE solution at different combination (Φ_1, Φ_2) under ICDS-2.



(a) Credit trade amount at different (Φ_1, Φ_2)

(b) TTT at different (Φ_1, Φ_2)

Figure 4: Credit trade amount and TTT at different (Φ_1, Φ_2) under ICDS-2.

This paper has attempted to make up this theory deficiency, and the main contributions of this paper are as follows.

Firstly, a new UE condition with TCS considering traveler's CI is defined, which explicitly considers the ICDS in the generalized path travel cost.

Secondly, four different types of ICDS are introduced and a unified variational inequality (VI) modeling framework is established for the UE condition under a given TCS with particular ICDS. Specifically, when ICDS-4 is adopted in the TCS, the UE solution must exist if the initial credit amount distributed to each traveler is equal to the total credit charge on the route he chooses, and the UE conditions in this case are exactly the same as that in [8]. Meanwhile, the uniqueness of UE solution under ICDS-1, ICDS-2, or ICDS-3 is discussed and the corresponding sufficient conditions are provided.

Last but not least, the model of mathematical program with equilibrium constraint (MPEC) is established for the optimal ICDS design problem, of which the optimization objective is maximizing network reserve capacity, and the equilibrium constraint is the UE assignment problem under a certain ICDS. The relaxation algorithm is used to solve the MPEC, and the numerical example shows that a properly designed ICDS can not only improve the network reserve

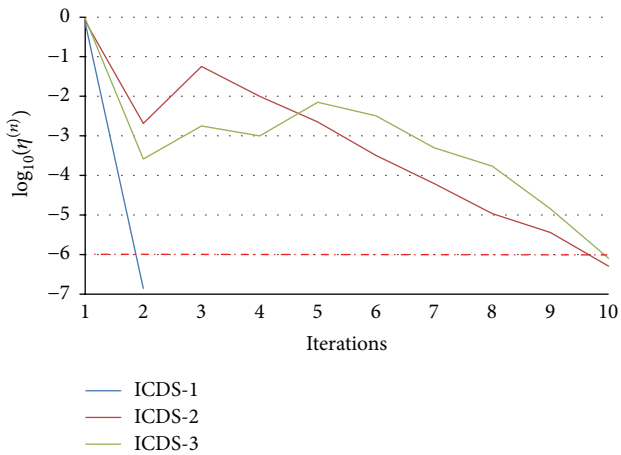


Figure 5: Convergence trend of relaxation algorithm under three ICDS.

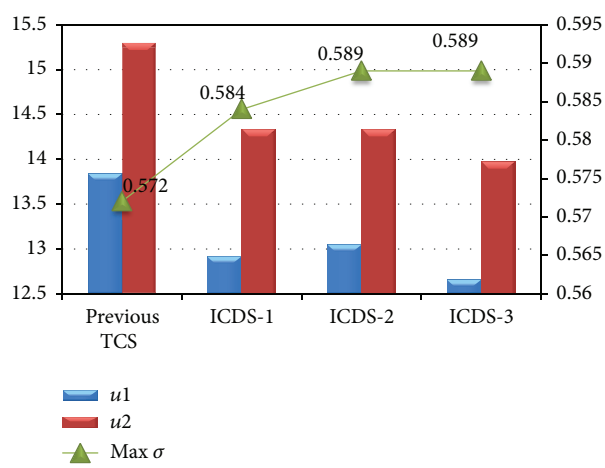


Figure 6: Comparison of minimum cost between each OD pair at maximum σ.

capacity, but also decrease the generalized travel cost of all the travelers in the network simultaneously.

Competing Interests

The authors declare that there are no competing interests regarding the publication of this paper.

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References

[1] J. H. Dales, *Pollution, Property, and Prices*, University of Toronto Press, Toronto, Canada, 1968.

[2] E. Verhoef, P. Nijkamp, and P. Rietveld, "Tradeable permits: their potential in the regulation of road transport externalities," *Environment and Planning B: Planning and Design*, vol. 24, no. 4, pp. 527–548, 1997.

[3] J. M. Viegas, "Making urban road pricing acceptable and effective: searching for quality and equity in urban mobility," *Transport Policy*, vol. 8, no. 4, pp. 289–294, 2001.

[4] H. C. Goddard, "Using tradeable permits to achieve sustainability in the world's large cities: policy design issues and efficiency conditions for controlling vehicle emissions, congestion and urban decentralization with an application to mexico city," *Environmental and Resource Economics*, vol. 10, no. 1, pp. 63–99, 1997.

[5] C. Raux, "The use of transferable permits in transport policy," *Transportation Research Part D: Transport and Environment*, vol. 9, no. 3, pp. 185–197, 2004.

[6] K. M. Kockelman and S. Kalmanje, "Credit-based congestion pricing: a policy proposal and the public's response," *Transportation Research Part A: Policy and Practice*, vol. 39, no. 7–9, pp. 671–690, 2005.

[7] T. Akamatsu, "Tradable Network Permits: A New Scheme for the Most Efficient Use of Network Capacity," 2007.

[8] H. Yang and X. L. Wang, "Managing network mobility with tradable credits," *Transportation Research Part B: Methodological*, vol. 45, no. 3, pp. 580–594, 2011.

[9] X. Wang, H. Yang, D. Zhu, and C. Li, "Tradable travel credits for congestion management with heterogeneous users," *Transportation Research Part E: Logistics and Transportation Review*, vol. 48, no. 2, pp. 426–437, 2012.

[10] Y. M. Nie, "Transaction costs and tradable mobility credits," *Transportation Research Part B: Methodological*, vol. 46, no. 1, pp. 189–203, 2012.

[11] N. Shirmohammadi, M. Zangui, Y. Yin, and Y. Nie, "Analysis and design of tradable credit schemes under uncertainty," *Transportation Research Record*, vol. 2333, pp. 27–36, 2013.

[12] F. He, Y. Yin, N. Shirmohammadi, and Y. M. Nie, "Tradable credit schemes on networks with mixed equilibrium behaviors," *Transportation Research Part B: Methodological*, vol. 57, pp. 47–65, 2013.

[13] Y. Bao, Z. Gao, M. Xu, and H. Yang, "Tradable credit scheme for mobility management considering travelers' loss aversion," *Transportation Research Part E: Logistics and Transportation Review*, vol. 68, pp. 138–154, 2014.

[14] F. Han and L. Cheng, "Stochastic user equilibrium with tradable credit schemes and optimal scheme design based on sensitivity analysis," in *Proceedings of the 94th Annual Meeting of the Transportation Research Board*, (15-0404), Washington, DC, USA, January 2015.

[15] Y. M. Nie and Y. F. Yin, "Managing rush hour travel choices with tradable credit scheme," *Transportation Research Part B: Methodological*, vol. 50, no. 4, pp. 1–19, 2013.

[16] L.-J. Tian, H. Yang, and H.-J. Huang, "Tradable credit schemes for managing bottleneck congestion and modal split with heterogeneous users," *Transportation Research Part E: Logistics and Transportation Review*, vol. 54, no. 8, pp. 1–13, 2013.

[17] F. Xiao, Z. Qian, and H. M. Zhang, "Managing bottleneck congestion with tradable credits," *Transportation Research Part B: Methodological*, vol. 56, pp. 1–14, 2013.

[18] W. B. Fan and X. G. Jiang, "Tradable mobility permits in roadway capacity allocation: review and appraisal," *Transport Policy*, vol. 30, no. 6, pp. 132–142, 2013.

[19] S. Grant-Muller and M. Xu, "The role of tradable credit schemes in road traffic congestion management," *Transport Reviews*, vol. 34, no. 2, pp. 128–149, 2014.

[20] A. Tversky and D. Kahneman, "Judgment under uncertainty: heuristics and biases. Biases in judgments reveal some heuristics of thinking under uncertainty," *Science*, vol. 185, no. 4157, pp. 1124–1131, 1974.

[21] C. Guthrie, J. J. Rachlinski, and A. J. Wistrich, "Judging by heuristic: cognitive illusions in judicial decision making," *Judicature*, vol. 86, no. 1, pp. 44–50, 2002.

[22] R. Pohl, *Cognitive Illusions: A Handbook on Fallacies and Biases in Thinking, Judgement and Memory*, Psychology Press, Hove, UK, 2004.

[23] G. Gigerenzer, "On cognitive illusions and rationality," *Poznan Studies in the Philosophy of the Sciences and the Humanities*, vol. 21, pp. 225–249, 1991.

[24] R. R. Scott, "Error and rationality in individual decisionmaking: an essay on the relationship between cognitive illusions and the management of choices," *Southern California Law Review*, vol. 59, pp. 329–362, 1985.

[25] F. Facchinei and J. S. Pang, *Finite-Dimensional Variational Inequality and Complementarity Problems*, Springer, New York, NY, USA, 2003.

[26] H. Yang and J. Y. T. Wang, "Travel time minimization versus reserve capacity maximization in the network design problem," *Transportation Research Record*, vol. 1, no. 1783, pp. 17–26, 2002.

[27] Z.-Q. Luo, J.-S. Pang, and D. Ralph, *Mathematical Programs with Equilibrium Constraints*, Cambridge University Press, Cambridge, UK, 1996.

[28] J. X. Ban, H. X. Liu, M. C. Ferris, and B. Ran, "A general MPCC model and its solution algorithm for continuous network design problem," *Mathematical and Computer Modelling*, vol. 43, no. 5-6, pp. 493–505, 2006.