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Research Article

Dynamical Behaviors of Stochastic Delayed One-Predator and Two-Competing-Prey Systems with Holling Type IV and Crowley-Martin Type Functional Responses

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This paper is devoted to stochastic delayed one-predator and two-competing-prey systems with two kinds of different functional responses. By establishing appropriate Lyapunov functions, the globally positive solution and stochastic boundedness are investigated. In some case, the stochastic permanence and extinction are also obtained. Moreover, sufficient conditions of the global asymptotic stability of the system are established. Finally, some numerical examples are provided to explain our conclusions.

1. Introduction

It is well known that predator-prey system, cooperative system, and competitive system are three kinds of important ecological systems. The dynamic relationship among species is a significant theme whether in ecology or in mathematical ecology because of its importance and universal existence with many concerned biological systems (see [1]). A lot of systems about predator-prey behaviors have been proposed (see [2–4]).

A main objective for ecologists is to find the relationships among species. And the consumption rate of each predator on prey is an important component of the relationships between predator and prey, that is, predator's functional response. In order to describe different situations when predators search or compete for food, many significant functional responses have been proposed, such as L-V and Holling II–IV types (see [5–7]). A suitable functional response is not only related to the density of prey, but also to the predator. A statistics from 19 predator-prey systems indicates that Crowley-Martin type, Beddington-DeAngelis type, and Hassell-Varley type predator-dependent functions can provide a better description in some case. In [8], the following predator-prey system bas been studied:

$$\begin{aligned} \frac{dx}{dt} &= x\left(A - Bx - \frac{Cy}{A_1 + B_1 x + C_1 y + B_1 C_1 xy}\right), \\ \frac{dy}{dt} &= y\left(-D - Ey + \frac{Fx}{A_1 + B_1 x + C_1 y + B_1 C_1 xy}\right), \end{aligned} \tag{1}$$

where x is the density of prey and y is the density of predator at time t.

However, interaction of multiple species often occurs in nature and their relationships are much more complex than that of the two species (see [9, 10]). Therefore, it is more realistic to study the multiple species predator-prey systems. Motivated by above, we consider the following systems:

$$\begin{aligned} dy_1 = y_1 \Bigg(a_1 - b_1 y_1 - \frac{c_2 y_2}{1 + y_2} \\ - \frac{c_3 z}{1 + m y_1 + n z + k y_1 z} \Bigg) dt, \end{aligned}$$

$$dy_2 = y_2\left(a_2 - b_2y_2 - \frac{c_1y_1}{1+y_1} - \frac{c_4z}{1+m_1y_2^2}\right)dt,$$
$$dz = y\left(a_3 - b_3z + \frac{d_1y_1}{1+my_1+nz+ky_1z} + \frac{d_2y_2}{1+m_1y_2^2}\right)dt, \quad (2)$$

where y_1, y_2 and z denote two competed prey and predator densities, respectively. And the parameters a_1, a_2, and a_3 are the intrinsic growth of three species; b_1, b_2, and b_3 are the intraspecific competition rate of three species, respectively. c_1 and c_2 are the interspecific competition rates of two competed species, c_3 and c_4 are the predators' capturing rates, and d_1 and d_2 are the rates of conversion of nutrients into the production of predator. And all the parameters in system (2) are constants. It is very necessary to point out that $c_3z/(1+my_1+nz+ky_1z)$ is a special functional response; when $m=n=k=0$ it becomes a linear mass-action function response (or Holling type I functional response), when $n=k=0$ it becomes a Holling type II functional response, when $k=0$ it becomes a modified Holling type II functional response, and when $mn=k$ it becomes a Crowley-Martin functional response.

In the real world, population dynamics are often affected by white noise from the environment, which relate to climate, geographical distribution, geological features, human disaster, human intervention, and other environmental factors. Therefore, the flow of biological energy is a process of fluctuation. The oscillation of population biomass is directly related to the birth and death rate of random perturbation. Up to now, there have been many works considering the effect of random perturbation (see [11–13]). In this paper, we assume that white noise affects the intrinsic birth rate, capture rate of predator, and conversion rate of the predator population. On the other hand, the development trend of the real biological system is not only related to the status of the system, but also depends on the history of the system more or less, which is called time delays. Moreover, time delay widely exists in biological systems; for example, in the predator-prey system, the process for the conversion of prey to predators is not immediately translated into the predator population but after a certain period of time to digest the transformation. So a more realistic predator-prey model should consider the effects of time delays (see [14, 15]). As a matter of fact, delay differential systems have much more complicated dynamical behaviors than the differential equations without delays. Therefore, the following stochastic delay systems are considered:

$$dy_1 = y_1\left[a_1 - b_1y_1 - \frac{c_2y_2(t-\tau_2)}{1+y_2(t-\tau_2)} - \frac{c_3z}{1+my_1+nz+ky_1z}\right]dt + \sigma_1y_1dB_1(t) + \frac{\sigma_2y_1z}{1+my_1+nz+ky_1z}dB_2(t),$$
$$dy_2 = y_2\left[a_2 - b_2y_2 - \frac{c_1y_1(t-\tau_1)}{1+y_1(t-\tau_1)} - \frac{c_4z}{1+m_1y_2^2}\right]dt + \sigma_3y_2dB_3(t),$$
$$dz = z\left[a_3 - b_3z + \frac{d_1y_1(t-\tau_3)}{1+my_1(t-\tau_3)+nz(t-\tau_3)+ky_1(t-\tau_3)z(t-\tau_3)} + \frac{d_2y_2(t-\tau_3)}{1+m_1y_2^2(t-\tau_3)}\right]dt + \sigma_4zdB_4(t) + \frac{\sigma_5y_1z}{1+my_1+nz+ky_1z}dB_5(t). \quad (3)$$

σ_i^2 are the intensities of the noises, $i=1,2,3,4,5$. $B_i(t)$ are standard Brownian motions which are defined on a complete probability space (Ω, F, P), $i=1,2,3,4,5$. Let $\tau=\max\{\tau_1,\tau_2,\tau_3\}$ and $C=C([-\tau,0],R_+^3)$ be the set of continuous functions from $[-\tau,0]$ to R_+^3 with initial condition $\gamma \in C([-\tau,0],R_+^3)$ and the norm $\|\gamma\| = \sup_{-\tau\le\theta\le0}\xi(\theta) < +\infty$.

Any biological system, whether it is population, biological communities, ecosystems, or the biosphere, its dynamic behavior is one of the main objects of the study, such as the resistance of ecosystem, persistence, recoverability, variability, and consistency. Therefore, we use mathematical theory and methods to study the dynamics of biological populations, which can not only protect the ecological balance but also can improve the ecological environment for human survival. According to what we know, few current literatures are found to discuss stochastic delayed predator-prey systems with Holling type IV and Crowley-Martin type functional responses at the same time. In this paper, it is the first time to obtain the condition of global asymptotic stability of system (3).

This paper is carried out as follows. In Section 2 and in Section 3, global positive solution and stochastically ultimate boundedness of system (3) are investigated. In Sections 4 and 5, we study the stochastic permanence and extinction, respectively. In Section 6, we obtain the fact that system (3) is globally asymptotically stable. In the end, in Section 7, some numerical examples are provided to explain our findings.

2. Existence of Global Positive Solution

Theorem 1. *For arbitrary initial data $\gamma \in C([-\tau,0],R_+^3)$, system (3) has a unique positive solution on $\tau \ge 0$ which will remain in R_+^3 with probability 1.*

Proof. Define a C^3-function $V_1 \to R_+$ by

$$V_1(y_1, y_2, z) = y_1 - 1 - \ln y_1 + y_2 - 1 - \ln y_2 + z - 1 - \ln z. \quad (4)$$

Clearly, $V_1(y_1, y_2, z)$ is nonnegative when $y_1 > 0$, $y_2 > 0$, $z > 0$. Now we continue to define

$$V_2(y_1, y_2, z) = V_1(y_1, y_2, z) + c_1 \int_{t-\tau_1}^{t} y_1(s)\,ds + c_2 \int_{t-\tau_2}^{t} y_2(s)\,ds, \quad (5)$$

where c_1, c_2 are the same in system (3). By the same method in [13], we can complete the proof and omit it here. □

3. Stochastically Ultimate Boundedness

Stochastically ultimate boundedness of system (3) is studied in this part. Firstly, we present a useful lemma in the following.

Lemma 2. *For any initial data $\gamma \in C([-\tau, 0], R_+^3)$, $(y_1(t), y_2(t), z(t))$ is a positive solution of system (3); there exist three positive constants $K_1(p)$, $K_2(p)$, and $K_3(p)$, $p > 1$, which satisfy*

$$\lim_{t\to\infty} \mathbf{E}|y_1(t)|^p = K_1(p),$$
$$\lim_{t\to\infty} \mathbf{E}|y_2(t)|^p = K_2(p), \quad (6)$$
$$\lim_{t\to\infty} \mathbf{E}|z(t)|^p = K_3(p).$$

Proof. We define $V_1(y_1) = y_1^p$, $V_2(y_2) = y_2^p$, and $V_3(z) = z^p$. By Itô's formula, we have

$$\begin{aligned} dV_1 &= py_1^{p-1}dy_1 + \frac{1}{2}p(p-1)y_1^{p-2}(dy_1)^2 \\ &= py_1^p dy_1 \left[a_1 - b_1 y_1 - \frac{c_2 y_2(t-\tau_2)}{1 + y_2(t-\tau_2)} - \frac{c_3 z}{1 + my_1 + nz + ky_1 z} + \frac{1}{2}(p-1) \cdot \left(\sigma_1^2 + \frac{\sigma_2^2 z^2}{2(1 + my_1 + nz + ky_1 z)^2}\right)\right] dt \\ &\quad + \sigma_1 py_1^p dB_1(t) + \frac{\sigma_2 py_1^p z}{(1 + my_1 + nz + ky_1 z)} dB_2(t) \\ &\leqslant py_1^p \left[a_1 - b_1 y_1 + \frac{1}{2}(p-1) + \frac{(p-1)\sigma_2^2}{2n_2}\right] \\ &\quad + \sigma_1 py_1^p dB_1(t) + \frac{\sigma_2 py_1^p z}{(1 + my_1 + nz + ky_1 z)} dB_2(t). \end{aligned} \quad (7)$$

Also, we can obtain that

$$\begin{aligned} dV_2 &= py_2^{p-1}dy_2 + \frac{1}{2}p(p-1)y_2^{p-2}(dy_2)^2 = py_2^p \left[a_2 - b_2 y_2 - \frac{c_1 y_1(t-\tau_1)}{1 + y_1(t-\tau_1)} - \frac{c_4 z}{1 + m_1 y_2^2}\right] + \sigma_3 py_2^p dB_3(t) \\ &\leqslant py_2^p \left[a_2 - b_2 y_2 + \frac{1}{2}(p-1)\sigma_3^2\right] dt + py_2^p \sigma_3 dB_3(t), \end{aligned} \quad (8)$$

$$\begin{aligned} dV_3 &= pz^{p-1}dz + \frac{1}{2}p(p-1)z^{p-2}(dy)^2 = pz^p \left[a_3 - b_3 z + \frac{d_1 y_1(t-\tau_3)}{1 + my_1(t-\tau_3) + nz(t-\tau_3) + ky_1(t-\tau_3)z(t-\tau_3)} \right. \\ &\quad \left. + \frac{d_2 y_2(t-\tau_3)}{1 + m_1 y_2^2(t-\tau_3)} + \frac{1}{2}(p-1)\left(\sigma_4^2 + \frac{\sigma_5^2 y_1^2}{2(1 + my_1 + nz + ky_1 z)^2}\right)\right] dt \\ &\quad + \sigma_4 py^p dB_4(t) + \frac{\sigma_5 x_1 y^p}{1 + mx_1 + nz + kx_1 z} dB_5(t) \\ &\leqslant pz^p \left[a_3 + \frac{d_1}{m} + \frac{d_2^2}{2} + \frac{1}{2m_1} - b_3 z + \frac{1}{2}(p-1)\left(\sigma_4^2 + \frac{\sigma_5^2}{m^2}\right)\right] dt + \sigma_4 pz^p dB_4(t) \\ &\quad + \frac{\sigma_5 y_1 z^p}{1 + my_1 + nz + ky_1 z} dB_5(t). \end{aligned} \quad (9)$$

By taking expectation in both sides of inequality (9), we have that

$$\begin{aligned} \frac{d\mathbf{E}[y_1^p(t)]}{dt} &\leqslant p\left[a_1 + \frac{(p-1)\sigma_1^2}{2} + \frac{(p-1)\sigma_2^2}{2n^2}\right] \cdot \mathbf{E}[y_1^p(t)] - b_1 p\mathbf{E}[y_1^{p+1}(t)] \leqslant p\left[a_1 + \frac{(p-1)\sigma_1^2}{2} + \frac{(p-1)\sigma_2^2}{2n^2}\right]\mathbf{E}[y_1^p(t)] \\ &\quad - b_1 p\left[\mathbf{E}(y_1^p(t))\right]^{(p+1)/p} \leqslant p\mathbf{E}[y_1^p(t)]\left\{a_1 + \frac{(p-1)\sigma_1^2}{2} + \frac{(p-1)\sigma_2^2}{2n^2} - b_1\left[\mathbf{E}(y_1^p(t))\right]^{1/p}\right\}. \end{aligned} \quad (10)$$

By the same way, we can obtain that

$$\begin{aligned}\frac{d\mathbf{E}\left[y_2^p(t)\right]}{dt} &\leqslant p\mathbf{E}\left[y_2^p(t)\right]\left\{a_2+\frac{(p-1)\sigma_3^2}{2}\right.\\&\left.-b_2\left[\mathbf{E}\left(y_2^p(t)\right)\right]^{1/p}\right\},\end{aligned} \quad (11)$$

$$\begin{aligned}\frac{d\mathbf{E}\left[z^p(t)\right]}{dt} &\leqslant p\mathbf{E}\left[z^p(t)\right]\left\{a_3+\frac{d_1}{m}+\frac{d_2^2}{2}+\frac{1}{2m_1}\right.\\&\left.+\frac{1}{2}(p-1)\left(\sigma_4^2+\frac{\sigma_5^2}{m^2}\right)-b_3\left[\mathbf{E}\left(z^p(t)\right)\right]^{1/p}\right\}.\end{aligned} \quad (12)$$

Therefore, we have

$$\begin{aligned}\lim_{n\to\infty}\mathbf{E}\left|y_1(t)\right|^p &= K_1(p),\\ \lim_{n\to\infty}\mathbf{E}\left|y_2(t)\right|^p &= K_2(p),\\ \lim_{n\to\infty}\mathbf{E}\left|z(t)\right|^p &= K_3(p),\end{aligned} \quad (13)$$

where

$$\begin{aligned}K_1(p) &= \left(\frac{a_1+(p-1)\sigma_1^2/2+(p-1)\sigma_2^2/2n^2}{b_1}\right)^p,\\ K_2(p) &= \left(\frac{a_2+(p-1)\sigma_3^2/2}{b_2}\right)^p,\\ K_3(p) &= \left(\frac{a_3+d_1/m+d_2^2/2+1/2m_1+(1/2)(p-1)\left(\sigma_4^2+\sigma_5^2/m^2\right)}{b_3}\right)^p.\end{aligned} \quad (14)$$

This completes the proof. □

Theorem 3. *For arbitrary initial data $\gamma \in C([-\tau,0],R_+^3)$, by the definition of stochastic boundedness, one has the fact that system (3) is stochastically ultimately bounded.*

Proof. From Lemma 2, we can see that $\mathbf{E}[|y_1(t), y_2(t), z(t)|^p] \leqslant K(p)$. For any $\epsilon \in (0,1)$, let $G = [K(p)/\epsilon]^{1/p}$, $p > 1$; then using Chebyshev inequality, we can obtain that

$$\begin{aligned}&P\left\{\left|y_1(t), y_2(t), z(t)\right| > G\right\}\\&\quad < \frac{\mathbf{E}\left[\left|y_1(t), y_2(t), z(t)\right|^p\right]}{G^p} \leqslant \epsilon,\end{aligned} \quad (15)$$

where $K(p) = 3^{p/2}(K_1(p)+K_2(p)+K_3(p))$.

This completes the proof of Theorem 3. □

4. Stochastic Permanence

Theorem 4. *If*

$$\begin{aligned}&\frac{1}{2}\max\left\{\sigma_1^2+\frac{\sigma_2^2+\sigma_5^2}{n^2},\sigma_3^2,\sigma_4^2+\frac{\sigma_2^2+\sigma_5^2}{m^2}\right\}\\&\quad < \min\left\{a_1-c_2,a_2-c_1,a_3-\frac{c_3}{m}\right\}\end{aligned} \quad (16)$$

holds, by the definition of stochastic permanence, we say that system (3) is stochastically permanent.

Proof. Define $V(y_1,y_2,z) = y_1+y_2+z$ for $(y_1(t), y_2(t), z(t)) \in R_+^3$; then

$$\begin{aligned}dV(y_1,y_2,z) &= y_1\left[a_1-b_1y_1-\frac{c_2y_2(t-\tau_2)}{1+y_2(t-\tau_2)}\right.\\&\left.-\frac{c_3z}{1+my_1+nz+ky_1z}\right]dt+y_2\left[a_2-b_2y_2\right.\\&\left.-\frac{c_1y_1(t-\tau_1)}{1+y_1(t-\tau_1)}-\frac{c_4z}{1+m_1y_2^2}\right]dt+z\left[a_3-b_3z\right.\\&+\frac{d_1y_1(t-\tau_3)}{1+my_1(t-\tau_3)+nz(t-\tau_3)+ky_1(t-\tau_3)z(t-\tau_3)}\\&\left.+\frac{d_2y_2(t-\tau_3)}{1+m_1y_2^2(t-\tau_3)}\right]dt+\sigma_1y_1dB_1(t)+\sigma_3y_2dB_3(t)\\&+\sigma_4zdB_4(t)+\frac{y_1z}{1+my_1+nz+ky_1z}(\sigma_2dB_2(t)\\&+\sigma_5dB_5(t)).\end{aligned} \quad (17)$$

Let $W(y_1,y_2,z) = 1/V(y_1,y_2,z)$; using Itô's formula, we can obtain

$$\begin{aligned}dW &= -\frac{1}{V^2}dV+\frac{1}{V^3}dV = -W^2\left\{y_1\left[a_1-b_1y_1\right.\right.\\&\left.-\frac{c_2y_2(t-\tau_2)}{1+y_2(t-\tau_2)}-\frac{c_3z}{1+my_1+nz+ky_1z}\right]+y_2\left[a_2\right.\end{aligned}$$

$$- b_2 y_2 - \frac{c_1 y_1 (t-\tau_1)}{1 + y_1 (t-\tau_1)} - \frac{c_4 z}{1 + m_1 y_2^2} \Bigg] + z \Bigg[a_3 - b_3 z + \frac{d_1 y_1 (t-\tau_3)}{1 + m y_1 (t-\tau_3) + n z (t-\tau_3) + k x_1 (t-\tau_3) z (t-\tau_3)} + \frac{d_2 y_2 (t-\tau_3)}{1 + m_1 y_2^2 (t-\tau_3)} \Bigg] \Bigg\} dt + W^3 \Bigg[\sigma_1^2 y_1^2 + \sigma_3^2 y_2^2 + \sigma_4^2 z^2 + \frac{\sigma_2^2 y_1^2 z^2}{(1 + m y_1 + n z + k y_1 z)^2} + \frac{\sigma_5^2 y_1^2 z^2}{(1 + m y_1 + n z + k y_1 z)^2} \Bigg] dt - W^2 \Bigg[\sigma_1 y_1 dB_1 (t) + \sigma_3 y_2 dB_3 (t) + \sigma_4 z dB_4 (t) + \frac{y_1 z}{1 + m y_1 + n z + k y_1 z} (\sigma_2 dB_2 (t) + \sigma_5 dB_5 (t)) \Bigg] = LW dt - W^2 \Bigg[\sigma_1 y_1 dB_1 (t) + \sigma_3 y_2 dB_3 (t) + \sigma_4 z dB_4 (t) + \frac{y_1 z}{1 + m y_1 + n z + k y_1 z} (\sigma_2 dB_2 (t) + \sigma_5 dB_5 (t)) \Bigg], \tag{18}$$

where

$$LW = -W^2 \Bigg\{ y_1 \Bigg[a_1 - b_1 y_1 - \frac{c_2 y_2 (t-\tau_2)}{1 + y_2 (t-\tau_2)} - \frac{c_3 z}{1 + m y_1 + n z + k y_1 z} \Bigg] + y_2 \Bigg[a_2 - b_2 y_2 - \frac{c_1 y_1 (t-\tau_1)}{1 + y_1 (t-\tau_1)} - \frac{c_4 z}{1 + m_1 y_2^2} \Bigg] + z \Bigg[a_3 - b_3 z + \frac{d_1 y_1 (t-\tau_3)}{1 + m y_1 (t-\tau_3) + n z (t-\tau_3) + k x_1 (t-\tau_3) z (t-\tau_3)} + \frac{d_2 y_2 (t-\tau_3)}{1 + m_1 y_2^2 (t-\tau_3)} \Bigg] \Bigg\} + W^3 \Bigg[\sigma_1^2 y_1^2 + \sigma_3^2 y_2^2 + \sigma_4^2 z^2 + \frac{(\sigma_2^2 + \sigma_5^2) y_1^2 z^2}{(1 + m y_1 + n z + k y_1 z)^2} \Bigg]. \tag{19}$$

If (16) holds, we can find a positive constant μ which satisfies the following condition:

$$\frac{1+\mu}{2} \max \left\{ \sigma_1^2 + \frac{\sigma_2^2 + \sigma_5^2}{n^2}, \sigma_3^2, \sigma_4^2 + \frac{\sigma_2^2 + \sigma_5^2}{m^2} \right\} < \min \left\{ a_1 - c_2, a_2 - c_1, a_3 - \frac{c_3}{m} \right\}. \tag{20}$$

Using Itô's formula, we can obtain

$$d (1 + W)^{\mu} = \Bigg[\mu (1 + W)^{\mu - 1} LW + \frac{\mu (\mu - 1)}{2} W^4 (1 + W)^{\mu - 2} \Bigg(\sigma_1^2 y_1^2 + \sigma_3^2 y_2^2 + \sigma_4^2 z^2 + \frac{(\sigma_2^2 + \sigma_5^2) y_1^2 z^2}{(1 + m y_1 + n z + k y_1 z)^2} \Bigg) \Bigg] dt + \mu W^2 (1 + W)^{\mu - 1} \Bigg(\sigma_1 y_1 dB_1 (t) + \sigma_3 y_2 dB_3 (t) + \sigma_4 z dB_4 (t) + \frac{y_1 z}{1 + m y_1 + n z + k y_1 z} (\sigma_2 dB_2 (t) + \sigma_5 dB_5 (t)) \Bigg). \tag{21}$$

Hence,

$$L (1 + W)^{\mu} = \mu (1 + W)^{\mu - 1} LW + \frac{\mu (\mu - 1)}{2} W^4 (1 + W)^{\mu - 2} \Bigg(\sigma_1^2 y_1^2 + \sigma_3^2 y_2^2 + \sigma_4^2 z^2 + \frac{(\sigma_2^2 + \sigma_5^2) y_1^2 z^2}{(1 + m x_1 + n z + k x_1 z)^2} \Bigg). \tag{22}$$

Under the condition of (20), we can choose another positive constant k making it satisfy the following condition:

$$\frac{\mu (1+\mu)}{2} \max \left\{ \sigma_1^2 + \frac{\sigma_2^2 + \sigma_5^2}{n^2}, \sigma_3^2, \sigma_4^2 + \frac{\sigma_2^2 + \sigma_5^2}{m^2} \right\} < \min \left\{ a_1 - c_2, a_2 - c_1, a_3 - \frac{c_3}{m} \right\} - k. \tag{23}$$

Using Itô's formula, we can obtain

$$d \left[e^{kt} (1 + W)^{\mu} \right] = e^{kt} d (1 + W)^{\mu} + k e^{kt} (1 + W)^{\mu} dt. \tag{24}$$

Hence,

$$L e^{kt} (1 + W)^{\mu} = e^{kt} LW + k e^{kt} (1 + W)^{\mu} + \frac{\mu (\mu - 1)}{2} \cdot W^4 (1 + W)^{\mu - 2} \Bigg(\sigma_1^2 y_1^2 + \sigma_3^2 y_2^2 + \sigma_4^2 z^2 + \frac{(\sigma_2^2 + \sigma_5^2) y_1^2 z^2}{(1 + m y_1 + n z + k y_1 z)^2} \Bigg) \leqslant k e^{kt} (1 + W)^{\mu - 2}$$

$$\cdot\left\{\left(k+\mu\max\left\{b_1,\left(b_2+\frac{c_4}{2m_1}\right),\left(\frac{c_4}{2}+b_3\right)\right\}\right)\right.$$
$$+\left(2k-\mu\min\left\{(a_1-c_2),(a_2-c_1),\left(a_3-\frac{c_3}{m}\right)\right\}\right.$$
$$+\mu\max\left\{\sigma_1^2+\frac{\sigma_2^2+\sigma_5^2}{n^2},\sigma_3^2,\sigma_4^2+\frac{\sigma_2^2+\sigma_5^2}{m^2}\right\}+\mu$$
$$\left.\cdot\max\left\{b_1,\left(b_2+\frac{c_4}{2m_1}\right),\left(\frac{c_4}{2}+b_3\right)\right\}\right)W+\left(k-\mu\right.$$
$$\cdot\min\left\{(a_1-c_2),(a_2-c_1),\left(a_3-\frac{c_3}{m}\right)\right\}$$
$$+\frac{\mu(1+\mu)}{2}$$
$$\left.\left.\cdot\max\left\{\sigma_1^2+\frac{\sigma_2^2+\sigma_5^2}{n^2},\sigma_3^2,\sigma_4^2+\frac{\sigma_2^2+\sigma_5^2}{m^2}\right\}\right)W^2\right\}. \quad (25)$$

Obviously, we can find a positive constant K which satisfies

$$Le^{kt}(1+W)^{\mu}\leqslant Ke^{kt}. \quad (26)$$

Then

$$de^{kt}(1+W)^{\mu}\leqslant Ke^{kt}-e^{kt}\mu W^2(1+W)^{\mu-1}$$
$$\cdot\left(\sigma_1y_1dB_1(t)+\sigma_3y_2dB_3(t)+\sigma_4zdB_4(t)\right.$$
$$\left.+\frac{y_1z}{1+my_1+nz+ky_1z}\left(\sigma_2dB_2(t)+\sigma_5dB_5(t)\right)\right). \quad (27)$$

We can integrate above inequality and then take expectation

$$\begin{aligned}\mathbf{E}\left[e^{kt}(1+W)^{\mu}\right]&\leqslant(1+W(0))^{\mu}+\frac{K}{k}e^{kt}\\&=(1+W(0))^{\mu}+M_1e^{kt},\end{aligned} \quad (28)$$

where $M_1=K/k$, so

$$\begin{aligned}\lim_{t\to+\infty}\sup\mathbf{E}\left[W(t)^{\mu}\right]&\leqslant\lim_{t\to+\infty}\sup\mathbf{E}\left[(1+W(t))^{\mu}\right]\\&\leqslant M_1.\end{aligned} \quad (29)$$

Since $(y_1(t)+y_2(t)+z(t))^{\mu}\leqslant 3^{\mu}(y_1(t)+y_2(t)+z(t))^{\mu/2}=3^{\mu}|y_1(t),y_2(t),z(t)|$, where $(y_1(t),y_2(t),z(t))\in R_+^3$, therefore

$$\begin{aligned}&\lim_{t\to+\infty}\sup\mathbf{E}\left[\frac{1}{|y_1(t),y_2(t),z(t)|^{\mu}}\right]\\&\leqslant 3^{\mu}\lim_{t\to+\infty}\sup\mathbf{E}\left[W(t)^{\mu}\right]\leqslant 3^{\mu}M_1.\end{aligned} \quad (30)$$

From Theorem 3, we have

$$\lim_{t\to+\infty}\sup\mathbf{E}\left[|y_1(t),y_2(t),z(t)|^{p}\right]\leqslant K(p), \quad (31)$$

for any $\varepsilon>0$; let $\chi=(K(p)/\varepsilon)^{1/p}$, using Chebyshev's inequality, we can obtain the conclusion. □

5. Extinction

The extinction of system (3) will be investigated in this part.

Definition 5. For arbitrary initial data $\gamma\in C([-\tau,0],R_+^3)$, if

$$\begin{aligned}&\lim_{t\to+\infty}\sup\frac{\ln y_1(t)}{t}<0,\\&\lim_{t\to+\infty}\sup\frac{\ln y_2(t)}{t}<0,\\&\lim_{t\to+\infty}\sup\frac{\ln z(t)}{t}<0\end{aligned} \quad (32)$$

hold, then system (3) is extinct.

Theorem 6. *For arbitrary initial data $\gamma\in C([-\tau,0],R_+^3)$, assume that*

$$\begin{aligned}&a_1-\frac{\sigma_1^2}{2}<0,\\&a_2-\frac{\sigma_3^2}{2}<0,\\&a_3+\frac{d_1}{m}+\frac{d_2^2}{2}+\frac{1}{2m_1}-\frac{\sigma_4^2}{2}<0\end{aligned} \quad (33)$$

holds; by the definition of extinction, system (3) is said to be extinct.

Proof. We establish three Lyapunov functions as follows:

$$\begin{aligned}V_1(y_1(t))&=\ln y_1(t),\\V_2(y_2(t))&=\ln y_2(t),\\V_3(z(t))&=\ln z(t).\end{aligned} \quad (34)$$

Making use of the generalized Itô's formula results in

$$dV_1(y_1)=\frac{dy_1}{y_1}-\frac{(dy_1)^2}{2y_1^2}=\left(a_1-b_1y_1-\frac{c_2y_2(t-\tau_2)}{1+y_2(t-\tau_2)}\right.$$
$$-\frac{c_3z}{1+my_1+nz+ky_1z}$$
$$\left.-\frac{1}{2}\left(\sigma_1^2+\frac{\sigma_2^2z^2}{(1+my_1+nz+ky_1z)^2}\right)\right)dt$$
$$+\sigma_1dB_1(t)+\frac{\sigma_2z}{1+my_1+nz+ky_1z}dB_2(t),$$

$$dV_2(y_2) = \frac{dy_2}{y_2} - \frac{(dy_2)^2}{2y_2^2} = \left(a_2 - b_2 y_2 - \frac{c_1 y_1(t-\tau_1)}{1+y_1(t-\tau_1)} - \frac{c_4 z}{1+m_1 y_2^2} - \frac{\sigma_3^2}{2}\right)dt + \sigma_3 dB_3(t),$$

$$dV_3(z) = \frac{dz}{z} - \frac{(dz)^2}{2z^2} = \left(a_3 - b_3 z + \frac{d_2 y_2(t-\tau_3)}{1+m_1 y_2^2(t-\tau_3)} + \frac{d_1 y_1(t-\tau_3)}{1+my_1(t-\tau_3)+nz(t-\tau_3)+ky_1(t-\tau_3)z(t-\tau_3)} - \frac{1}{2}\left(\sigma_4^2 + \frac{\sigma_5^2 y_1^2}{(1+my_1+nz+ky_1z)^2}\right)\right)dt + \sigma_4 dB_4(t) + \frac{\sigma_5 y_1}{1+my_1+nz+ky_1z}dB_5(t). \quad (35)$$

Hence, we can integrate them from 0 to t:

$$\begin{aligned}\ln y_1(t) &= \ln y_1(0) + \int_0^t \left(a_1 - b_1 y_1(s) - \frac{c_2 y_2(s-\tau_2)}{1+y_2(s-\tau_2)} - \frac{c_3 z(s)}{1+my_1(s)+nz(s)+ky_1(s)z(s)} - \frac{1}{2}\left(\sigma_1^2 + \frac{\sigma_2^2 z^2(s)}{(1+my_1(s)+nz(s)+ky_1(s)z(s))^2}\right)\right)ds + \int_0^t \sigma_1 dB_1(s) + M_1(t),\\
\ln y_2(t) &= \ln y_2(0) + \int_0^t \left(a_2 - b_2 y_2(s) - \frac{c_1 y_1(s-\tau_1)}{1+y_1(s-\tau_1)} - \frac{c_4 z(s)}{1+m_1 y_2^2(s)} - \frac{\sigma_3^2}{2}\right)ds + \int_0^t \sigma_3 dB_3(s),\\
\ln z(t) &= \ln z(0) + \int_0^t \left(a_3 - b_3 z(s) + \frac{d_2 y_2(s-\tau_3)}{1+m_1 y_2^2(s-\tau_3)} + \frac{d_1 y_1(s-\tau_3)}{1+my_1(s-\tau_3)+nz(s-\tau_3)+ky_1(s-\tau_3)z(s-\tau_3)} - \frac{1}{2}\left(\sigma_4^2 + \frac{\sigma_5^2 y_1^2(s)}{(1+my_1(s)+nz(s)+ky_1(s)z(s))^2}\right)\right)ds + \int_0^t \sigma_4 dB_4(s) + M_2(t),\end{aligned} \quad (36)$$

where $M_1(t) = \int_0^t (\sigma_2 z(s)/(1+my_1(s)+nz(s)+ky_1(s)z(s)))dB_2(s)$ and $M_2(t) = \int_0^t (\sigma_5 y_1(s)/(1+my_1(s)+nz(s)+ky_1(s)z(t)))dB_5(s)$ are martingales. And

$$\begin{aligned}\langle M_1(t), M_1(t)\rangle &= \int_0^t \left(\frac{\sigma_2 y(s)}{1+my_1(s)+ny(s)+ky_1(s)z(s)}\right)^2 ds,\\
\langle M_2(t), M_2(t)\rangle &= \int_0^t \left(\frac{\sigma_5 y_1(s)}{1+my_1(s)+nz(s)+ky_1(s)z(s)}\right)^2 ds\end{aligned} \quad (37)$$

are the quadratic variations, respectively. Using the exponential martingale inequality, for arbitrary positive constants N, η, and v, we can obtain

$$P\left\{\sup_{0\leqslant t\leqslant N}\left[M(t) - \frac{1}{2}\langle M(t), M(t)\rangle\right] > \beta\right\} = e^{-\eta v}. \quad (38)$$

Choosing $N = n$, $\eta = 1$, and $v = 2\ln n$, we have that

$$\begin{aligned}P\left\{\sup_{0\leqslant t\leqslant n}\left[M_1(t) - \frac{1}{2}\langle M_1(t), M_1(t)\rangle\right] > 2\ln n\right\} &\leqslant \frac{1}{n^2},\\
P\left\{\sup_{0\leqslant t\leqslant n}\left[M_2(t) - \frac{1}{2}\langle M_2(t), M_2(t)\rangle\right] > 2\ln n\right\} &\leqslant \frac{1}{n^2}.\end{aligned} \quad (39)$$

Applying the Borel-Cantelli lemma, there exists a random integer $n_0 = n_0(\omega)$ for almost all $\omega \in \Omega$ such that

$$\begin{aligned}\sup_{0\leqslant t\leqslant n}\left[M_1(t) - \frac{1}{2}\langle M_1(t), M_1(t)\rangle\right] &\leqslant 2\ln n,\\
\sup_{0\leqslant t\leqslant n}\left[M_2(t) - \frac{1}{2}\langle M_2(t), M_2(t)\rangle\right] &\leqslant 2\ln n,\end{aligned} \quad (40)$$

for $n \geqslant n_0$.

Thus

$$\begin{aligned}M_1(t) &\leqslant 2\ln n + \frac{1}{2}\int_0^t \left(\frac{\sigma_2 z(s)}{1+my_1(s)+nz(s)+ky_1(s)z(s)}\right)^2 ds,\\
M_2(t) &\leqslant 2\ln n + \frac{1}{2}\int_0^t \left(\frac{\sigma_5 y_1(s)}{1+my_1(s)+nz(s)+ky_1(s)z(s)}\right)^2 ds,\end{aligned} \quad (41)$$

for all $0 \leqslant t \leqslant n,\ n \geqslant n_0$ a.s. We substitute it into (36)

$$\begin{aligned} \ln y_1(t) - \ln y_1(0) &\leqslant \left(a_1 - \frac{1}{2}\sigma_1^2\right)t + \int_0^t \sigma_1 dB_1(s) \\ &\quad + 2\ln n, \\ \ln y_2(t) - \ln y_2(0) &\leqslant \left(a_2 - \frac{1}{2}\sigma_3^2\right)t + \int_0^t \sigma_3 dB_3(s), \\ \ln z(t) - \ln z(0) &\leqslant \left(a_3 + \frac{d_1}{m} + \frac{d_2^2}{2} + \frac{1}{2m_1} - \frac{\sigma_4^2}{2}\right)t \\ &\quad + \int_0^t \sigma_4 dB_4(s) + 2\ln n. \end{aligned} \tag{42}$$

We divide t on both sides of (42) and let $t \to +\infty$:

$$\begin{aligned} \lim_{t\to+\infty}\sup \frac{\ln y_1(t)}{t} &\leqslant a_1 - \frac{\sigma_1^2}{2} < 0, \\ \lim_{t\to+\infty}\sup \frac{\ln y_2(t)}{t} &\leqslant a_2 - \frac{\sigma_3^2}{2} < 0, \\ \lim_{t\to+\infty}\sup \frac{\ln z(t)}{t} &\leqslant a_3 + \frac{d_1}{m} + \frac{d_2^2}{2} + \frac{1}{2m_1} - \frac{\sigma_4^2}{2} < 0. \end{aligned} \tag{43}$$

The desired assertion is derived. □

6. Global Asymptotic Stability

Definition 7. Let $(y_1(t), y_2(t), z(t))$ and $(y_1^*(t), y_2^*(t), z^*(t))$ be two arbitrary solutions of system (3) with initial data $\gamma, \gamma^* \in ([-\tau, 0], R_+^3)$, respectively. If

$$\begin{aligned} P\Big\{\lim_{t\to+\infty} \mathbf{E}\Big[\big|(y_1(t), y_2(t), z(t)) \\ - (y_1^*(t), y_2^*(t), z^*(t))\big|\Big] = 0\Big\} = 1, \end{aligned} \tag{44}$$

we say that system (3) is globally asymptotically stable in expectation.

Lemma 8. *For any initial data $\gamma \in C([-\tau, 0], R_+^3)$, $(y_1(t), y_2(t), z(t))$ is a solution of system (3); then almost every sample track of $(y_1(t), y_2(t), z(t))$ is uniformly continuous for $t \geqslant -\tau$.*

Proof. Considering system (3), we have that

$$\begin{aligned} y_1(t) = y_1(0) + \int_0^t f_1(s)\,ds + \int_0^t g_1(s)\,dB_1(s) \\ + \int_0^t g_2(s)\,dB_2(s), \end{aligned} \tag{45}$$

where

$$\begin{aligned} f_1(s) &= y_1(s)\Bigg(a_1 - b_1 y_1(s) - \frac{c_2 y_2(s-\tau_2)}{1 + y_2(s-\tau_2)} \\ &\quad - \frac{c_3 z(s)}{1 + m y_1(s) + n z(s) + k y_1(s) z(s)}\Bigg), \\ g_1(s) &= \sigma_1 y_1(s), \\ g_2(s) &= \frac{\sigma_2 y_1(s) z(s)}{1 + m y_1(s) + n z(s) + k y_1(s) z(s)}. \end{aligned} \tag{46}$$

Letting $\vartheta > 2$, we obtain that

$$\begin{aligned} \mathbf{E}\left[|f_1(t)|^{\vartheta}\right] &= \mathbf{E}\left[\left|y_1\left(a_1 - b_1 y_1 - \frac{c_2 y_2(t-\tau_2)}{1 + y_2(t-\tau_2)}\right.\right.\right. \\ &\quad \left.\left.\left. - \frac{c_3 z}{1 + m y_1 + n z + k y_1 z}\right)\right|^{\vartheta}\right] \leqslant \frac{1}{2}\mathbf{E}\left[|y_1|^{2\vartheta}\right] + \frac{1}{2} \\ &\quad \cdot 4^{2\vartheta-1}\left[|a_1|^{2\vartheta} + |b_1|^{2\vartheta}\mathbf{E}\left[|y_1|^{2\vartheta}\right] + |c_2|^{2\vartheta}\right| \\ &\quad \mathbf{E}\left[|y_2(t-\tau_2)|^{2\vartheta}\right] + |c_3|^{2\vartheta}\mathbf{E}\left[|z|^{2\vartheta}\right]\Big] \leqslant \frac{1}{2}K_1(2\vartheta) \\ &\quad + \frac{1}{2}4^{2\vartheta-1}\left[|a_1|^{2\vartheta} + |b_1|^{2\vartheta}K_1(2\vartheta) + |c_2|^{2\vartheta}K_2(2\vartheta)\right. \\ &\quad \left. + |c_3|^{2\vartheta}K_3(2\vartheta)\right] \triangleq L_1(\vartheta), \\ \mathbf{E}\left[|g_1(s)|^{\vartheta}\right] &= \mathbf{E}\left[|\sigma_1|^{\vartheta}|y_1|^{\vartheta}\right] = |\sigma_1|^{\vartheta}\mathbf{E}\left[|y_1|^{\vartheta}\right] \\ &\leqslant \sigma_1^p K_1(\vartheta) \triangleq L_2(\vartheta), \\ \mathbf{E}\left[|g_2(s)|^{\vartheta}\right] &= \mathbf{E}\left[\left|\frac{\sigma_2 y_1 z}{1 + m y_1 + n z + k y_1 z}\right|^{\vartheta}\right] \\ &\leqslant \mathbf{E}\left[\left|\frac{\sigma_2 y_1}{n}\right|^{\vartheta}\right] \leqslant \left(\frac{\sigma_2}{n}\right)^{\vartheta} K_1(\vartheta) \triangleq L_3(\vartheta). \end{aligned} \tag{47}$$

For another, using the moment inequality of stochastic integral (see [16]), for $0 \leqslant t_1 < t_2 < +\infty$ and $\vartheta > 2$, we have that

$$\begin{aligned} &\mathbf{E}\left[\left|\int_{t_1}^{t_2} g_1(s)\,dB_1(s)\right|^{\vartheta}\right] \\ &\quad \leqslant \left[\frac{\vartheta(\vartheta-1)}{2}\right]^{\vartheta/2}(t_2 - t_1)^{(\vartheta-2)/2}\int_{t_1}^{t_2}\mathbf{E}\left[|g_1(s)|^{\vartheta}\right]ds \\ &\quad \leqslant \left[\frac{\vartheta(\vartheta-1)}{2}\right]^{\vartheta/2}(t_2 - t_1)^{\vartheta/2} L_2(\vartheta), \end{aligned} \tag{48}$$

$$\mathbf{E}\left[\left|\int_{t_1}^{t_2} g_2(s)\,dB_2(s)\right|^{\vartheta}\right] \leqslant \left[\frac{\vartheta(\vartheta-1)}{2}\right]^{\vartheta/2}(t_2-t_1)^{(\vartheta-2)/2}\int_{t_1}^{t_2}\mathbf{E}\left[|g_2(s)|^{\vartheta}\right]ds \leqslant \left[\frac{\vartheta(\vartheta-1)}{2}\right]^{\vartheta/2}(t_2-t_1)^{\vartheta/2}L_3(\vartheta). \quad (49)$$

Thus for $t_2 - t_1 < 1$ and $1/\vartheta + 1/q = 1$, we can obtain

$$\begin{aligned}\mathbf{E}\left|y_1(t_2)-y_1(t_1)\right|^{\vartheta} &= \mathbf{E}\left[\left|\int_{t_1}^{t_2} f_1(s)\,ds + \int_{t_1}^{t_2} g_1(s)\,dB_1(s) + \int_{t_1}^{t_2} g_2(s)\,dB_2(s)\right|^{\vartheta}\right] \\ &\leqslant 3^{\vartheta-1}\left\{\mathbf{E}\left[\left|\int_{t_1}^{t_2} f_1(s)\,ds\right|\right]^{\vartheta} + \mathbf{E}\left[\left|\int_{t_1}^{t_2} g_1(s)\,dB_1(s)\right|\right]^{\vartheta} + \mathbf{E}\left[\left|\int_{t_1}^{t_2} g_1(s)\,dB_1(s)\right|\right]^{\vartheta}\right\} \leqslant 3^{\vartheta-1}\left\{1 + \left[\frac{\vartheta(\vartheta-1)}{2}\right]^{\vartheta/2}\right\}L_4(\vartheta)(t_2-t_1)^{\vartheta/2},\end{aligned} \quad (50)$$

where $L_4(\vartheta) = \max\{L_1(\vartheta), L_2(\vartheta) + L_3(\vartheta)\}$. From Lemma 2.3 of [13], we have that almost every sample path of $y_1(t)$ is uniformly continuous. In the same method, the uniform continuity of $y_2(t)$ and $z(t)$ can be obtained. □

Lemma 9 (see [17]). *$g(t) \geqslant 0$ is a uniformly continuous and integrable function defined on $[0, +\infty)$; then $\lim_{t\to\infty} g(t) = 0$.*

Theorem 10. *If*

$$\begin{aligned}A &:= -b_1 + c_1 + 2d_1 + \frac{2\sigma_5^2}{m} + \frac{(mn+k)c_3}{n^2} + \frac{(mn+k)\sigma_2^2}{n^3} < 0, \\ B &:= -b_2 + c_2 + \frac{3d_2}{2} + \frac{2c_4m_1(a_2+\sigma_3^2)(a_3+d_1/m+d_2^2/2+1/2m_1+\sigma_4^2+\sigma_5^2/m^2)}{b_2b_3} < 0, \\ C &:= -b_3 + 2c_3 + 2c_4 + \frac{2\sigma_2^2}{n} + \frac{(mn+k)d_1}{m^2} + \frac{(mn+k)\sigma_5^2}{m^3} < 0,\end{aligned} \quad (51)$$

then system (3) is globally asymptotically stable.

Proof. $(y_1(t), y_2(t), z(t))$ and $(y_1^*(t), y_2^*(t), z^*(t))$ are two arbitrary solutions of system (3) with initial data $\gamma, \gamma^* \in ([-\tau, 0], R_+^3)$, respectively. Define

$$\begin{aligned}V(t) &= \left|\ln y_1 - \ln y_1^*\right| + \left|\ln y_2 - \ln y_2^*\right| + \left|\ln z - \ln z^*\right| + c_1\int_{t-\tau_1}^{t}\left|y_1(s)-y_1^*(s)\right|ds \\ &+ c_2\int_{t-\tau_2}^{t}\left|y_2(s)-y_2^*(s)\right|ds + 2d_1\int_{t-\tau_3}^{t}\left|y_1(s)-y_1^*(s)\right|ds \\ &+ \frac{3d_2}{2}\int_{t-\tau_3}^{t}\left|y_2(s)-y_2^*(s)\right|ds + \frac{(mn+k)d_1}{m^2}\int_{t-\tau_3}^{t}\left|z(s)-z^*(s)\right|ds.\end{aligned} \quad (52)$$

Calculating the right differential $d^+V(t)$, by Itô's formula

$$\begin{aligned}d^+V &= \operatorname{sgn}\left(y_1 - y_1^*\right)\Bigg\{\Bigg[\Bigg(a_1 - b_1y_1 - \frac{c_2y_2(t-\tau_2)}{1+y_2(t-\tau_2)} - \frac{c_3z}{1+my_1+nz+ky_1z} - \frac{\sigma_2^2z^2}{2(1+my_1+nz+ky_1z)^2}\Bigg)dt \\ &+ \sigma_1 dB_1(t) + \frac{\sigma_2 z}{1+my_1+nz+ky_1z}dB_2(t)\Bigg] - \Bigg[\Bigg(a_1 - b_1y_1^* - \frac{c_2y_2^*(t-\tau_2)}{1+y_2^*(t-\tau_2)} - \frac{c_3z^*}{1+my_1^*+nz^*+ky_1^*z^*} \\ &- \frac{\sigma_2^2(z^*)^2}{2(1+my_1^*+nz^*+ky_1^*z^*)^2}\Bigg)dt + \sigma_1 dB_1(t) + \frac{\sigma_2z^*}{1+my_1^*+nz^*+ky_1^*z^*}dB_2(t)\Bigg]\Bigg\} + \operatorname{sgn}\left(y_2 - y_2^*\right) \\ &\cdot\Bigg\{\Bigg[\Bigg(a_2 - b_2y_2 - \frac{c_1y_1(t-\tau_1)}{1+y_1(t-\tau_1)} - \frac{c_4y}{1+m_1y_2^2}\Bigg)dt + \sigma_3 dB_3(t)\Bigg] - \Bigg[\Bigg(a_2 - b_2y_2^* - \frac{c_1y_1^*(t-\tau_1)}{1+y_1^*(t-\tau_1)} \\ &- \frac{c_4z^*}{1+m_1(y_2^*)^2}\Bigg)dt + \sigma_3 dB_3(t)\Bigg]\Bigg\} + \operatorname{sgn}\left(z - z^*\right) \\ &\cdot\Bigg\{\Bigg[\Bigg(a_3 - b_3z + \frac{d_2y_2(t-\tau_3)}{1+m_1y_2^2(t-\tau_3)}\end{aligned}$$

$$
\begin{aligned}
&+\frac{d_1 y_1(t-\tau_3)}{1+my_1(t-\tau_3)+nz(t-\tau_3)+ky_1(t-\tau_3)z(t-\tau_3)} \\
&-\left.\frac{\sigma_5^2 y_1^2}{2\left(1+my_1+nz+ky_1z\right)^2}\right)dt+\sigma_4 dB_4(t) \\
&+\left.\frac{\sigma_5 y_1}{1+my_1+nz+ky_1z}dB_5(t)\right]-\left[\left(a_3-b_3z^*\right.\right. \\
&+\frac{d_2 y_2^*(t-\tau_3)}{1+m_1 y_2^{*2}(t-\tau_3)} \\
&+\frac{d_1 y_1^*(t-\tau_3)}{1+my_1^*(t-\tau_3)+nz^*(t-\tau_3)+ky_1^*(t-\tau_3)z^*(t-\tau_3)} \\
&-\left.\frac{\sigma_5^2 y_1^{*2}}{2\left(1+my_1^*+nz^*+ky_1^*z^*\right)^2}\right)dt+\sigma_4 dB_4(t) \\
&+\left.\left.\frac{\sigma_5 y_1^*}{1+my_1^*+nz^*+ky_1^*z^*}dB_5(t)\right]\right\}+c_1\left[\left|y_1-y_1^*\right|\right. \\
&-\left.\left|y_1(t-\tau_1)-y_1^*(t-\tau_1)\right|\right]+c_2\left[\left|y_2-y_2^*\right|-\left|y_2(t-\tau_2)\right.\right. \\
&-\left.\left.y_2^*(t-\tau_2)\right|\right]+2d_1\left[\left|y_1-y_1^*\right|-\left|y_1(t-\tau_3)-y_1^*(t\right.\right. \\
&-\left.\left.\tau_3)\right|\right]+\frac{3d_2}{2}\left[\left|y_2-y_2^*\right|-\left|y_2(t-\tau_3)-y_2^*(t-\tau_3)\right|\right] \\
&+\frac{(mn+k)d_1}{m^2}\left[\left|z-z^*\right|-\left|z(t-\tau_3)-z^*(t-\tau_3)\right|\right].
\end{aligned}
\tag{53}
$$

We can take integration from 0 to t and then take expectation

$$
\begin{aligned}
&\mathbf{E}[V(t)]-\mathbf{E}[V(0)] \\
&=\mathbf{E}\left\{\int_0^t\left[\operatorname{sgn}\left(y_1(s)-y_1^*(s)\right)\left(-b_1\left(y_1(s)-y_1^*(s)\right)\right.\right.\right. \\
&\quad-c_2\left(\frac{y_2(s-\tau_2)}{1+y_2(s-\tau_2)}-\frac{y_2^*(s-\tau_2)}{1+y_2^*(s-\tau_2)}\right) \\
&\quad-c_3\left(\frac{z(s)}{1+my_1(s)+nz(s)+ky_1(s)z(s)}\right. \\
&\quad-\left.\frac{z^*(s)}{1+my_1^*(s)+nz^*(s)+ky_1^*(s)z^*(s)}\right) \\
&\quad-\frac{\sigma_2^2}{2}\left(\frac{z^2(s)}{\left(1+my_1(s)+nz(s)+ky_1(s)z(s)\right)^2}\right. \\
&\quad-\left.\left.\frac{\left(z^*(s)\right)^2}{\left(1+my_1^*(s)+nz^*(s)+ky_1^*(s)z^*(s)\right)^2}\right)\right) \\
&\quad+\operatorname{sgn}\left(y_2(s)-y_2^*(s)\right)\left(-b_2\left(y_2(s)-y_2^*(s)\right)\right. \\
&\quad-c_1\left(\frac{y_1(s-\tau_1)}{1+y_1(s-\tau_1)}-\frac{y_1^*(s-\tau_1)}{1+y_1^*(s-\tau_1)}\right) \\
&\quad-\left.c_4\left(\frac{z(s)}{1+m_1y_2^2(s)}-\frac{z^*(s)}{1+m_1\left(y_2^*(s)\right)^2}\right)\right) \\
&\quad+\operatorname{sgn}\left(z(s)-z^*(s)\right)\left(-b_3\left(z(s)-z^*(s)\right)\right. \\
&\quad+d_1\left(\frac{y_1(s-\tau_3)}{1+my_1(s-\tau_3)+nz(s-\tau_3)+ky_1(s-\tau_3)z(s-\tau_3)}\right. \\
&\quad-\left.\frac{y_1^*(s-\tau_3)}{1+my_1^*(s-\tau_3)+nz^*(s-\tau_3)+ky_1^*(s-\tau_3)z^*(s-\tau_3)}\right) \\
&\quad+d_2\left(\frac{y_2(s-\tau_3)}{1+m_1y_2^2(s-\tau_3)}-\frac{y_2^*(s-\tau_3)}{1+m_1y_2^{*2}(s-\tau_3)}\right) \\
&\quad-\frac{\sigma_5^2}{2}\left(\frac{y_1^2(s)}{\left(1+my_1(s)+nz(s)+ky_1(s)z(s)\right)^2}\right. \\
&\quad-\left.\left.\frac{\left(y_1^*(s)\right)^2}{\left(1+my_1^*(s)+nz^*(s)+ky_1^*(s)z^*(s)\right)^2}\right)\right) \\
&\quad+c_1\left[\left|y_1(s)-y_1^*(s)\right|-\left|y_1(s-\tau_1)-y_1^*(s-\tau_1)\right|\right] \\
&\quad+c_2\left[\left|y_2(s)-y_2^*(s)\right|-\left|y_2(s-\tau_2)-y_2^*(s-\tau_2)\right|\right] \\
&\quad+2d_1\left[\left|y_1(s)-y_1^*(s)\right|-\left|y_1(s-\tau_3)-y_1^*(s-\tau_3)\right|\right] \\
&\quad+\frac{3d_2}{2}\left[\left|y_2(s)-y_2^*(s)\right|-\left|y_2(s-\tau_3)-y_2^*(s-\tau_3)\right|\right] \\
&\quad+\frac{(mn+k)d_1}{m^2} \\
&\quad\left.\left.\cdot\left[\left|z(s)-z^*(s)\right|-\left|z(s-\tau_3)-z^*(s-\tau_3)\right|\right]\right]ds\right\}.
\end{aligned}
\tag{54}
$$

Hence,

$$
\begin{aligned}
\frac{d\mathbf{E}[V(t)]}{dt} &\leqslant \left[-b_1+c_1+2d_1+\frac{2\sigma_5^2}{m}+\frac{(mn+k)c_3}{n^2}\right. \\
&\quad+\left.\frac{(mn+k)\sigma_2^2}{n^3}\right]\mathbf{E}\left[\left|y_1-y_1^*\right|\right]+\left[-b_2+c_2+\frac{3d_2}{2}\right. \\
&\quad+c_4m_1\left(\left[\mathbf{E}|z|^3\right]^{1/3}\left[\mathbf{E}\left|y_2\right|^3\right]^{1/3}\right. \\
&\quad+\left.\left.\left[\mathbf{E}|z|^3\right]^{1/3}\left[\mathbf{E}\left|y_2^*\right|^3\right]^{1/3}\right)\left[\mathbf{E}\left|y_2-y_2^*\right|\right]\right]+\left[-b_3\right.
\end{aligned}
$$

$$+ 2c_3 + 2c_4 + \frac{2\sigma_2^2}{n} + \frac{(mn+k)\,d_1}{m^2} + \frac{(mn+k)\,\sigma_5^2}{m^3}\Bigg] \cdot \mathbf{E}\left[\left|z - z^*\right|\right]. \tag{55}$$

By Lemma 2, we have

$$\begin{aligned}\left[\mathbf{E}\left|y_1(t)\right|^3\right]^{1/3} &\leqslant \frac{a_1 + \sigma_1^2 + \sigma_2^2/n^2}{b_1},\\ \left[\mathbf{E}\left|y_2(t)\right|^3\right]^{1/3} &\leqslant \frac{a_2 + \sigma_3^2}{b_2},\\ \left[\mathbf{E}\left|z(t)\right|^3\right]^{1/3} &\leqslant \frac{a_3 + d_1/m + d_2^2/2 + 1/2m_1 + \sigma_4^2 + \sigma_5^2/m^2}{b_3}.\end{aligned} \tag{56}$$

Thus

$$\begin{aligned}\frac{d\mathbf{E}[V(t)]}{dt} &\leqslant \left(-b_1 + c_1 + 2d_1 + \frac{2\sigma_5^2}{m} + \frac{(mn+k)\,c_3}{n^2} + \frac{(mn+k)\,\sigma_2^2}{n^3}\right)\mathbf{E}\left[\left|y_1(t) - y_1^*(t)\right|\right]\\ &+ \left(-b_2 + c_2 + \frac{3d_2}{2} + \frac{2c_4m_1\left(a_2+\sigma_3^2\right)\left(a_3+\sigma_4^2+d_1/m+d_2^2/2+1/2m_1+\sigma_5^2/m^2\right)}{b_2b_3}\right)\mathbf{E}\left[\left|y_2(t) - y_2^*(t)\right|\right]\\ &+ \left(-b_3 + 2c_3 + 2c_4 + \frac{2\sigma_2^2}{n} + \frac{(mn+k)\,d}{m^2} + \frac{(mn+k)\,\sigma_5^2}{m^3}\right)\mathbf{E}\left[\left|z(t) - z^*(t)\right|\right]\\ &= A\mathbf{E}\left[\left|y_1(t) - y_1^*(t)\right|\right] + B\mathbf{E}\left[\left|y_2(t) - y_2^*(t)\right|\right] + C\mathbf{E}\left[\left|z(t) - z^*(t)\right|\right].\end{aligned} \tag{57}$$

Integrating both sides of (57) yields that

$$\begin{aligned}V(t) - V(0) \leqslant A\int_0^t \mathbf{E}\left[\left|y_1(s) - y_1^*(s)\right|\right]ds &+ B\int_0^t \mathbf{E}\left[\left|y_2(s) - y_2^*(s)\right|\right]ds\\ &+ C\int_0^t \mathbf{E}\left[\left|z(s) - z^*(s)\right|\right]ds.\end{aligned} \tag{58}$$

Therefore

$$\begin{aligned}V(t) - \Bigg(A\int_0^t \mathbf{E}\left[\left|y_1(s) - y_1^*(s)\right|\right]ds &+ B\int_0^t \mathbf{E}\left[\left|y_2(s) - y_2^*(s)\right|\right]ds\\ &+ C\int_0^t \mathbf{E}\left[\left|z(s) - z^*(s)\right|\right]ds\Bigg) \leqslant V(0) < +\infty.\end{aligned} \tag{59}$$

Then we have

$$\begin{aligned}\mathbf{E}&\left[\left|\left(y_1(t), y_2(t), z(t)\right) - \left(y_1^*(t), y_2^*(t), z^*(t)\right)\right|\right]\\ &\leqslant \mathbf{E}\left[\left|y_1(t) - y_1^*(t)\right| + \left|y_2(t) - y_2^*(t)\right| + \left|z(t) - z^*(t)\right|\right] \in L^1[0,+\infty).\end{aligned} \tag{60}$$

Thus it follows from Lemma 9 that

$$\begin{aligned}\lim_{t\to\infty}\left|y_1(t) - y_1^*(t)\right| &= 0,\\ \lim_{t\to\infty}\left|y_2(t) - y_2^*(t)\right| &= 0,\\ \lim_{t\to\infty}\left|z(t) - z^*(t)\right| &= 0.\end{aligned} \tag{61}$$

□

7. Numerical Simulation

We give some illustrative examples to evidence our results in this part. Using the Milstein method mentioned in ([18]), we get the discretization equations of system (3):

$$\begin{aligned}y_{1,i+1} = y_{1,i} + y_{1,i}&\left(a_1 - b_1y_{1,i} - \frac{c_2y_{2,(i-s_2)}}{1+y_{2,(i-s_2)}} - \frac{c_3z_i}{1+my_{1,i}+nz_i+ky_{1,i}z_i}\right)\Delta t + \sigma_1y_{1,i}\sqrt{\Delta t}\eta_{1,i}\\ &+ \frac{\sigma_2y_{1,i}z_i}{1+my_{1,i}+nz_i+ky_{1,i}z_i}\sqrt{\Delta t}\eta_{2,i} + \frac{\sigma_1^2}{2}y_{1,i}\left(\eta_{1,i}^2 - 1\right)\Delta t\\ &+ \frac{\sigma_2^2y_{1,i}z_i^2\left(1+nz_i\right)}{2\left(1+my_{1,i}+nz_i+ky_{1,i}z_i\right)^3}\left(\eta_{2,i}^2 - 1\right) \cdot \Delta t,\end{aligned}$$

$$y_{2,i+1} = y_{2,i} + y_{2,i}\left(a_2 - b_2 y_{2,i} - \frac{c_1 y_{1,(i-s_1)}}{1 + y_{1,(i-s_1)}} - \frac{c_4 z_i}{1 + m_1 y_{2,i}^2}\right)\Delta t + \sigma_3 y_{2,i}\sqrt{\Delta t}\eta_{3,i} + \frac{\sigma_3^2}{2} y_{2,i}\left(\eta_{3,i}^2 - 1\right)\Delta t,$$

$$z_{i+1} = z_i + z_i\left(a_3 - b_3 z_i + \frac{d_1 y_{1,(i-s_3)}}{1 + m y_{1,(i-s_3)} + n z_{i-s_3} + k y_{1,(i-s_3)} z_{i-s_3}} + \frac{d_2 y_{2,(i-s_3)}}{1 + m_1 y_{2,(i-s_3)}^2}\right)\Delta t + \sigma_4 z_i \sqrt{\Delta t}\eta_{4,i} + \frac{\sigma_5 y_{1,i} z_i}{1 + m y_{1,i} + n z_i + k y_{1,i} z_i}\sqrt{\Delta t}\eta_{5,i} + \frac{\sigma_4^2}{2} z_i\left(\eta_{4,i}^2 - 1\right)\Delta t + \frac{\sigma_5^2 y_{1,i}^2 z_i\left(1 + m_1 y_{1,i}\right)}{2\left(1 + m y_{1,i} + n z_i + k y_{1,i} z_i\right)^3}\left(\eta_{5,i}^2 - 1\right) \cdot \Delta t, \quad (62)$$

where $\eta_{1,i}$, $\eta_{2,i}$, $\eta_{3,i}$, $\eta_{4,i}$, and $\eta_{5,i}$ are independent Gaussian random variables which follow $N(0,1)$. Let step size $\Delta t = 0.01$ and steps = 20000. We choose

$$a_1 = 0.4; \quad b_1 = 0.5; \quad c_2 = 0.2; \quad c_3 = 0.2; \quad m = 1; \quad n = 0.8; \quad k = 0.1; \quad a_2 = 0.5; \quad b_2 = 0.55; \quad c_1 = 0.1; \quad c_4 = 0.1; \quad m_1 = 1; \quad a_3 = 0.6; \quad b_3 = 0.7; \quad d_1 = 0.02; \quad d_2 = 0.01;$$

$$\tau_1 = 0.1; \quad \tau_2 = 0.2; \quad \tau_3 = 0.3. \quad (63)$$

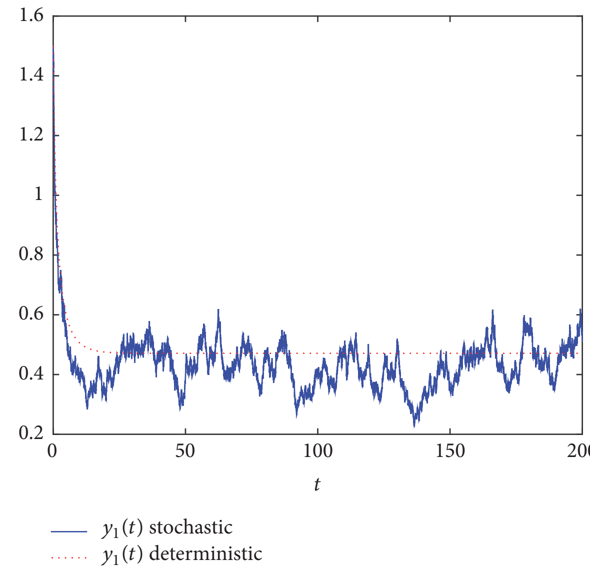

FIGURE 1: Solution of model (62) for $\gamma_1(\theta) = (1.5, 1.3, 1)$. Blue: $\sigma_i = 0.1$ $(i = 1, 2, 3, 4, 5)$. Red: $\sigma_i = 0$ $(i = 1, 2, 3, 4, 5)$.

We assume that the parameters are the same above in the following discussion. Let the initial data be

$$\gamma_1(\theta) = (1.5, 1.3, 1), \quad -\tau \leqslant \theta \leqslant 0, \quad (64)$$

where $\tau = \max\{\tau_1, \tau_2, \tau_3\}$. We choose $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0.1$; from Theorem 3 we know that system (62) is stochastically ultimately bounded (see Figures 1–3: blue curves). To understand the influence of white noise, we choose $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0$; we obtain deterministic system of system (62) and it is also ultimately bounded (see Figures 1–3: red curves).

For stochastic permanence, we also choose $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0.1$; by calculating we obtain that

$$\min\left\{a_1 - c_2, a_2 - c_1, a_3 - \frac{c_3}{m}\right\} - \frac{1}{2}\max\left\{\sigma_1^2 + \frac{\sigma_2^2 + \sigma_5^2}{n^2}, \sigma_3^2, \sigma_4^2 + \frac{\sigma_2^2 + \sigma_5^2}{m^2}\right\} = \frac{287}{1600}, \quad (65)$$

which satisfies the condition (16) (see Figure 4).

In the following we choose larger noises $\sigma_1 = 0.95$, $\sigma_2 = 0.1$, $\sigma_3 = 1.05$, $\sigma_4 = 1.5$, and $\sigma_5 = 0.1$, which satisfy the condition of Theorem 6; then system (62) will go to extinction (see Figure 5).

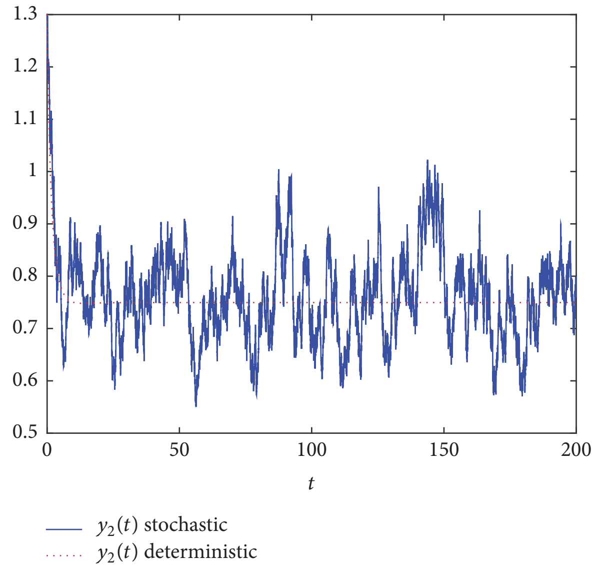

Figure 2: Solution of model (62) for $\gamma_1(\theta) = (1.5, 1.3, 1)$. Blue: $\sigma_i = 0.1$ $(i = 1, 2, 3, 4, 5)$. Red: $\sigma_i = 0$ $(i = 1, 2, 3, 4, 5)$.

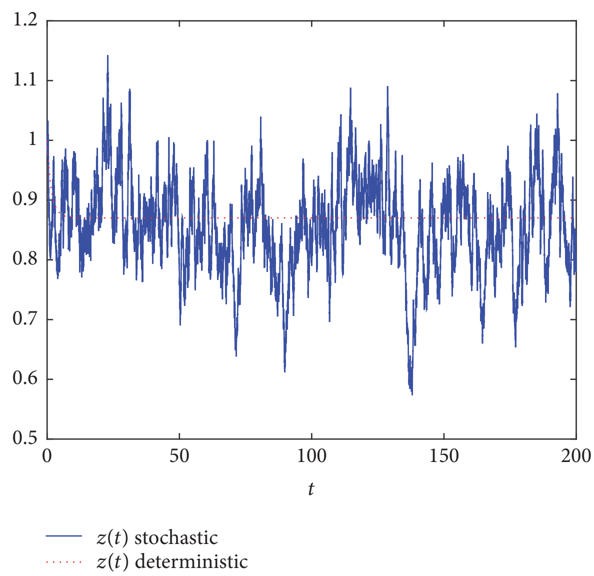

Figure 3: Solution of model (62) for $\gamma_1(\theta) = (1.5, 1.3, 1)$. Blue: $\sigma_i = 0.1$ $(i = 1, 2, 3, 4, 5)$. Red: $\sigma_i = 0$ $(i = 1, 2, 3, 4, 5)$.

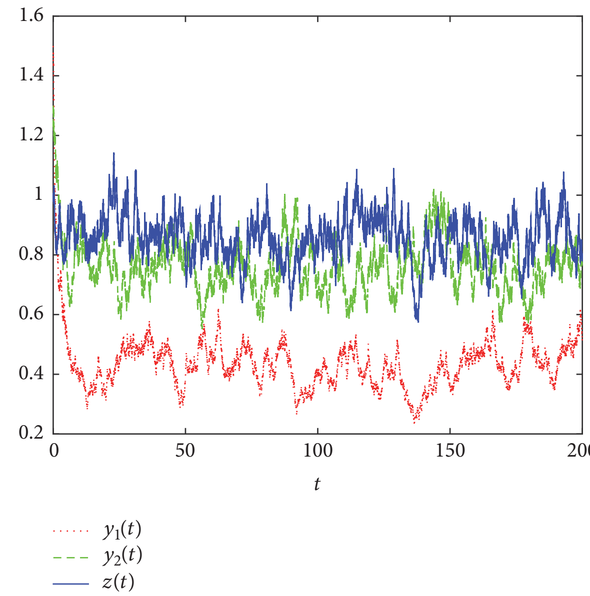

Figure 4: Solution of model (62) for $\gamma_1(\theta) = (1.5, 1.3, 1)$ and $\sigma_i = 0.1$ $(i = 1, 2, 3, 4, 5)$.

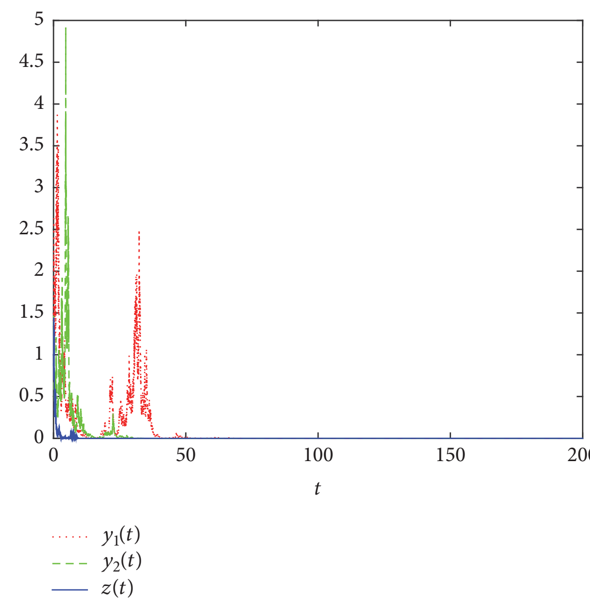

Figure 5: Solution of model (62) for $\gamma_1(\theta) = (1.5, 1.3, 1)$ and $\sigma_1 = 0.95$, $\sigma_2 = 0.1$, $\sigma_3 = 1.05$, $\sigma_4 = 1.5$, and $\sigma_5 = 0.1$.

In the end, we discuss the global asymptotic stability. Let

$$\gamma_1^* = (0.7, 0.8, 0.6), \quad -\tau \leqslant \theta \leqslant 0; \tag{66}$$

we also choose $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = \sigma_5 = 0.1$; by calculating we have that

$$\begin{aligned} A &:= -b_1 + c_1 + 2d_1 + \frac{2\sigma_5^2}{m} + \frac{(mn+k)\,c_3}{n^2} + \frac{(mn+k)\,\sigma_2^2}{n^3} \\ &= -\frac{527}{12800}, \\ B &:= -b_2 + c_2 + \frac{3d_2}{2} \\ &\quad + \frac{2c_4 m_1 \left(a_2 + \sigma_3^2\right)\left(a_3 + d_1/m + d_2^2/2 + 1/2m_1 + \sigma_4^2 + \sigma_5^2/m^2\right)}{b_2 b_3} \\ &= -\frac{174}{5279}, \\ C &:= -b_3 + 2c_3 + 2c_4 + \frac{2\sigma_2^2}{n} + \frac{(mn+k)\,d_1}{m^2} + \frac{(mn+k)\,\sigma_5^2}{m^3} \\ &= -\frac{6}{125}, \end{aligned} \tag{67}$$

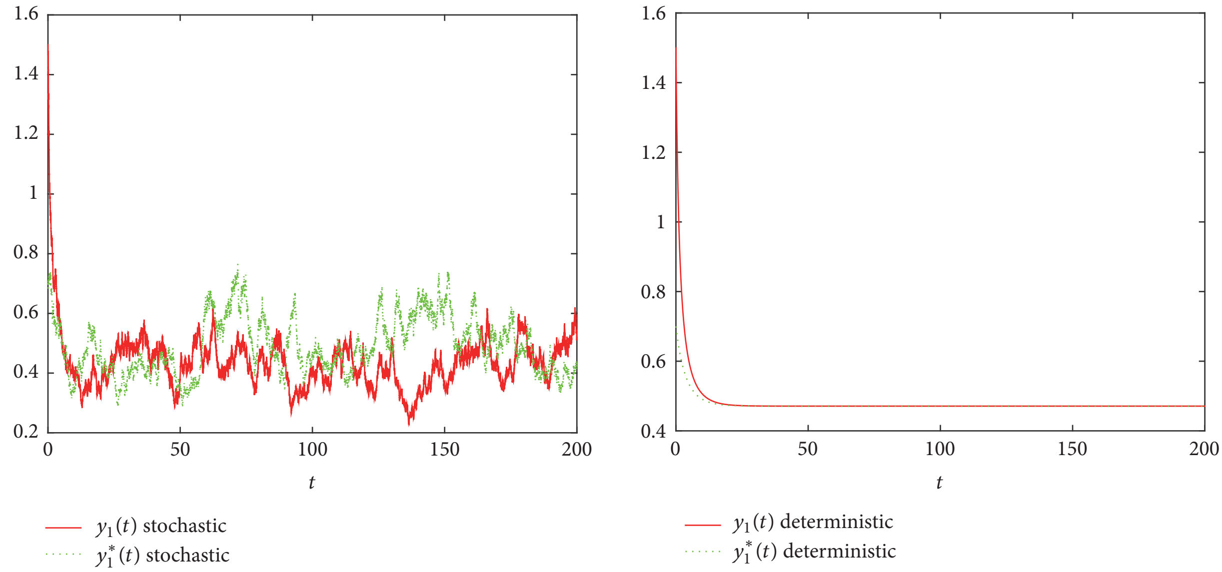

FIGURE 6: The path curve of $y_1(t)$ and $y_1^*(t)$. Red: solution of model (62) for $y_1(\theta) = (1.5, 1.3, 1)$. Green: solution of model (62) for $y_1^* = (0.7, 0.8, 0.6)$.

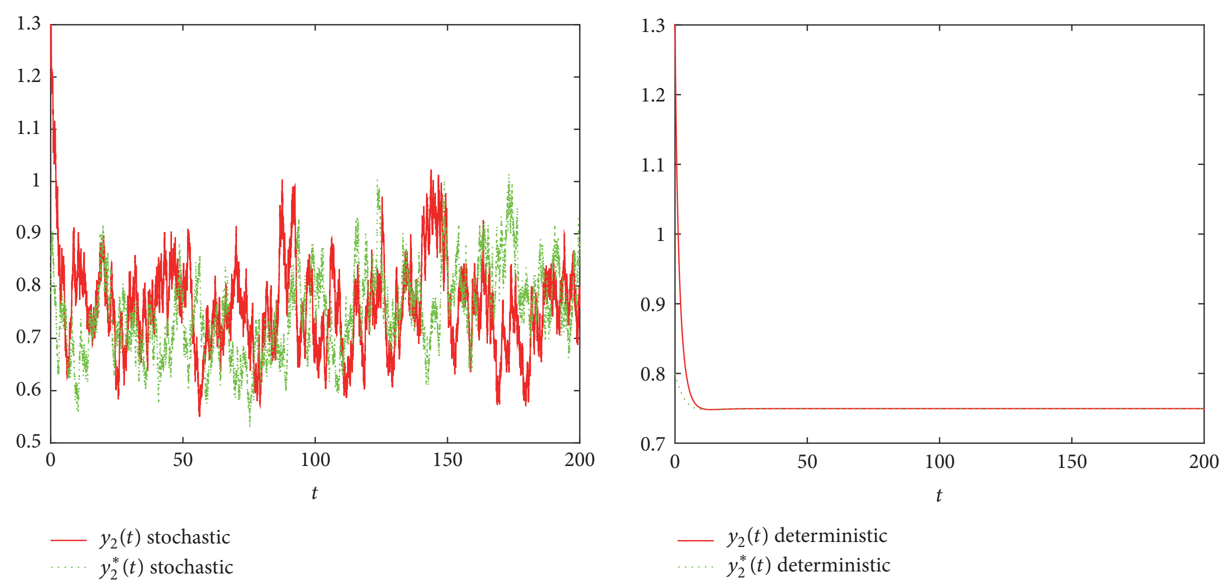

FIGURE 7: The path curve of $y_2(t)$ and $y_2^*(t)$. Red: solution of model (62) for $y_1(\theta) = (1.5, 1.3, 1)$. Green: solution of model (62) for $y_1^* = (0.7, 0.8, 0.6)$.

which satisfy the condition of Theorem 10; then system (62) is globally asymptotically stable. By the same way, we can obtain the fact that the deterministic system of (3) is also globally stable (see Figures 6–8).

8. Conclusions and Discussion

In this work, stochastic delayed one-predator and two-competing-prey systems with two different kinds of functional responses have been studied. Globally positive solution, stochastically ultimate boundedness, and the stochastic permanence and extinction for system (3) are investigated. Moreover, sufficient criteria for the global asymptotic stability of the system are established. In the end, some numerical examples are provided to explain our results. Through the study of the dynamic behavior of system (3), by comparing Theorems 4 and 6, we can obtain that if the environment noise is small, the stochastic system can maintain permanent while the system can be extinct under sufficiently large environmental noise (see Figures 4 and 5). Therefore, from Theorem 10, we can choose the appropriate parameters in a suitable environment noise intensity to make system (3) asymptotically stable.

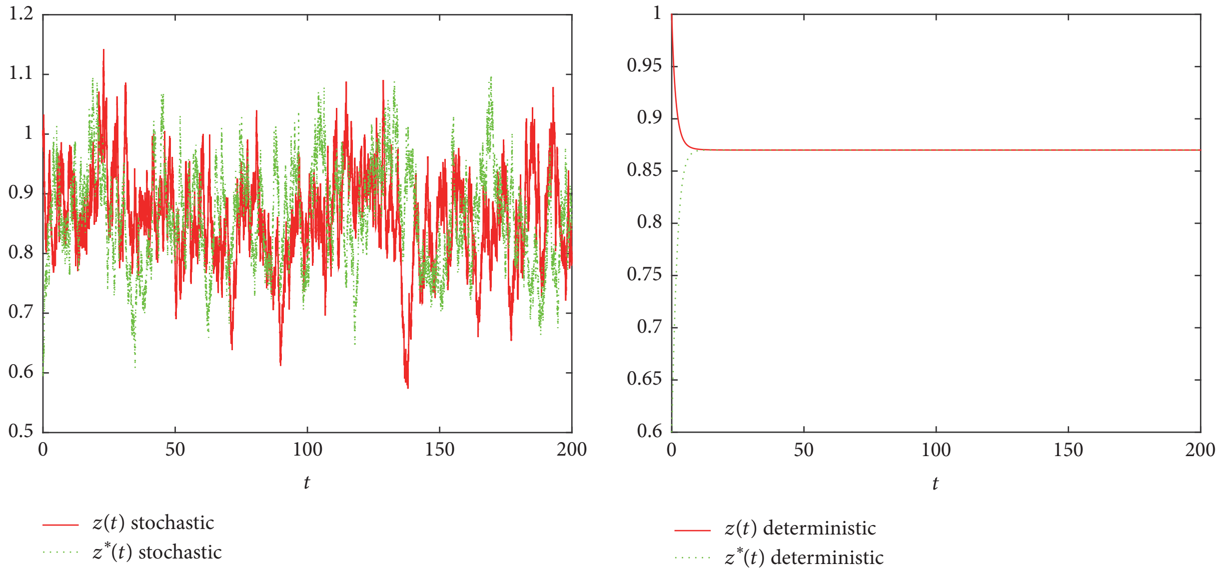

Figure 8: The path curve of $z(t)$ and $z^*(t)$. Red: solution of model (62) for $\gamma_1(\theta) = (1.5, 1.3, 1)$. Green: solution of model (62) for $\gamma_1^* = (0.7, 0.8, 0.6)$.

Recently, predator-prey models have been investigated extensively for their theoretical and practical significance. In the current literatures, most of this work is restricted to two-dimensional predator-prey system; few has been done on predator-prey system with interspecific competition in preys. In system (3), we study a three-dimensional hybrid system where the predator can capture two kinds of preys with different functional responses. In fact, the population models are often subjected to the influence of environmental noises inevitably. In most of predator-prey models, there is only a white noise which affects intrinsic rate of increase of predator or prey. In system (3), we consider the white noise not only has effect on the intrinsic rate of population growth but also on capture rate of predator and conversion rate of the predator population. Three time delays are introduced to make the model closer to the reality. It is interesting to point out that system (3) contains two kinds of different mathematical models; if we remove the predator, system (3) is reduced to a competitive model in [13]; if we remove any one of the two preys system (3) is reduced to a simple predator-prey system. Some meaningful questions deserve further investigation. One may investigate the stationary distribution of system (3). Moreover, it is worth considering the corresponding nonautonomous system of system (3). One may discuss dynamics behaviors contained predators with a mutual cooperation in other ecosystems.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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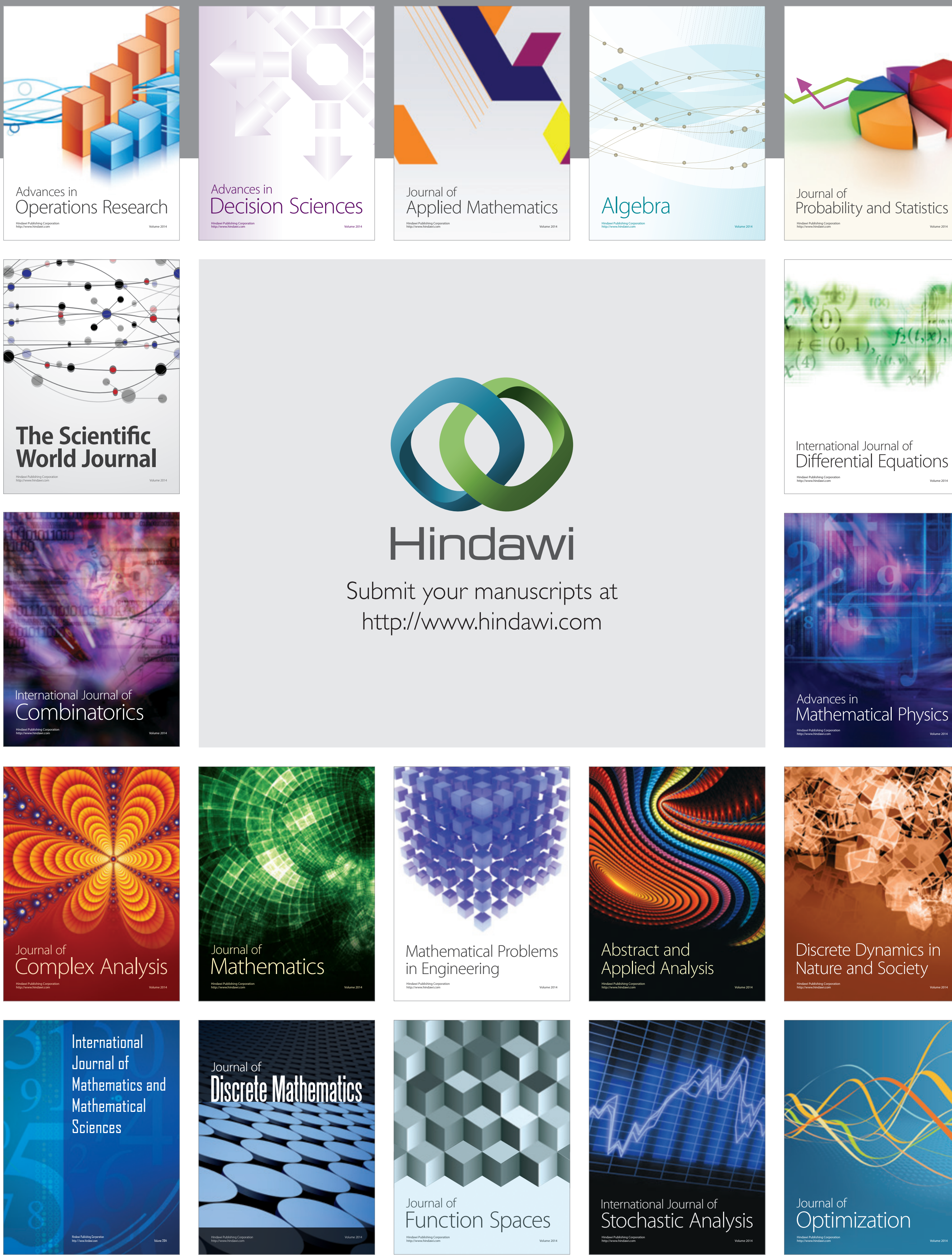

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