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*Research Article*

# Radical Structures of Fuzzy Polynomial Ideals in a Ring

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We investigate the radical structure of a fuzzy polynomial ideal induced by a fuzzy ideal of a ring and study its properties. Given a fuzzy ideal $\beta$ of $R$ and a homomorphism $f : R \to R'$, we show that if $f_x$ is the induced homomorphism of $f$, that is, $f_x(\sum_{i=0}^{n} a_i x^i) = \sum_{i=0}^{n} f(a_i)x^i$, then $f_x^{-1}[(\sqrt{\beta})_x] = (\sqrt{f^{-1}(\beta)})_x$.

## 1. Introduction

Zadeh [1] introduced the notion of a fuzzy subset $A$ of a set $X$ as a function from $X$ into $[0, 1]$. Rosenfeld [2] applied this concept to the theory of groupoids and groups. Liu [3] introduced and studied the notion of the fuzzy ideals of a ring. Following Liu, Mukherjee and Sen [4] defined and examined the fuzzy prime ideals of a ring. The concept of fuzzy ideals was applied to several algebras: $BN$-algebras [5], $BL$-algebras [6], semirings [7], and semigroups [8]. Ersoy et al. [9] applied the concept of intuitionistic fuzzy soft sets to rings, and Shah et al. [10] discussed intuitionistic fuzzy normal subrings over a nonassociative ring. Prajapati [11] investigated residual of ideals of an $L$-ring. Dheena and Mohanraj [12] obtained a condition for a fuzzy small right ideal to be fuzzy small prime right ideal.

The present authors [13] introduced the notion of a fuzzy polynomial ideal $\alpha_x$ of a polynomial ring $R[x]$ induced by a fuzzy ideal $\alpha$ of a ring $R$ and obtained an isomorphism theorem of a ring of fuzzy cosets of $\alpha_x$. It was shown that a fuzzy ideal $\alpha$ of a ring is fuzzy prime if and only if $\alpha_x$ is a fuzzy prime ideal of $R[x]$. Moreover, we showed that if $\alpha_x$ is a fuzzy maximal ideal of $R[x]$, then $\alpha$ is a fuzzy maximal ideal of $R$.

In this paper we investigate the radical structure of a fuzzy polynomial ideal induced by a fuzzy ideal of a ring and study their properties.

## 2. Preliminaries

In this section, we review some definitions which will be used in the later section. Throughout this paper unless stated otherwise all rings are commutative rings with identity.

*Definition 1* (see [3]). A *fuzzy ideal* of a ring $R$ is a function $\alpha : R \to [0, 1]$ satisfying the following axioms:

(1) $\alpha(x + y) \geq \min\{\alpha(x), \alpha(y)\}$.

(2) $\alpha(xy) \geq \max\{\alpha(x), \alpha(y)\}$.

(3) $\alpha(-x) = \alpha(x)$

for any $x, y \in R$.

*Definition 2* (see [2]). Let $f : R \to S$ be a homomorphism of rings and let $\beta$ be a fuzzy subset of $S$. We define a fuzzy subset $f^{-1}\beta$ of $R$ by $f^{-1}\beta(x) \coloneqq \beta(f(x))$ for all $x \in R$.

*Definition 3* (see [2]). Let $f : R \to S$ be a homomorphism of rings and let $\alpha$ be a fuzzy subset of $R$. We define a fuzzy subset $f(\alpha)$ of $S$ by

$$f(\alpha)(y) \coloneqq \begin{cases} \sup\{\alpha(t) \mid t \in R, f(t) = y\} & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{if } f^{-1}(y) = \emptyset. \end{cases} \quad (1)$$

*Definition 4* (see [2]). Let $R$ and $S$ be any sets and let $f : R \to S$ be a function. A fuzzy subset $\alpha$ of $R$ is called an $f$-*invariant* if $f(x) = f(y)$ implies $\alpha(x) = \alpha(y)$, where $x, y \in R$.

Zadeh [1] defined the following notions. The union of two fuzzy subsets $\alpha$ and $\beta$ of a set $S$, denoted by $\alpha \cup \beta$, is a fuzzy subset of $S$ defined by

$$(\alpha \cup \beta)(x) := \max\{\alpha(x), \beta(x)\} \tag{2}$$

for all $x \in S$.

The intersection of $\alpha$ and $\beta$, symbolized by $\alpha \cap \beta$, is a fuzzy subset of $S$, defined by

$$(\alpha \cap \beta)(x) := \min\{\alpha(x), \beta(x)\} \tag{3}$$

for all $x \in S$.

**Theorem 5** (see [13]). *Let $\alpha : R \to [0,1]$ be a fuzzy ideal of a ring $R$ and let $f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$ be a polynomial in $R[x]$. Define a fuzzy set $\alpha_x : R[x] \to [0,1]$ by $\alpha_x(f(x)) := \min_i\{\alpha(a_i) \mid a_i$'s are coefficients of $f(x)\}$. Then $\alpha_x$ is a fuzzy ideal of $R[x]$.*

The fuzzy ideal $\alpha_x$ discussed in Theorem 5 is called the *fuzzy polynomial ideal* [13] of $R[x]$ induced by a fuzzy ideal $\alpha$.

**Theorem 6** (see [13]). *Let $\alpha : R \to [0,1]$ be a fuzzy ideal of a ring $R$. Then $\alpha$ is a fuzzy prime ideal of $R$ if and only if $\alpha_x$ is a fuzzy prime ideal of $R[x]$.*

*Notation 1.* Let $\alpha : R \to [0,1]$ be a fuzzy subset of a set $R$. We denote a level set $\alpha_*$ by $\alpha_* := \{a \in R \mid \alpha(a) = \alpha(0)\}$, and we know that $\alpha(0) \geqq \alpha(x)$ for all $x \in R$. The set of all polynomials $f(x) = \sum_{i=0}^{m} a_ix^i \in R[x]$ whose $\alpha$'s values $\alpha(a_i)$ are equal to $\alpha(0)$ for all $i = 0, 1, \ldots, n$, is denoted by $\alpha_*[x]$.

**Theorem 7** (see [13]). *If $\alpha$ and $\beta$ are fuzzy ideals of a ring $R$, then*

(i) $(\alpha \cap \beta)_x = \alpha_x \cap \beta_x$,

(ii) $\alpha_x \cup \beta_x \subseteq (\alpha \cup \beta)_x$.

Let $f : R \to R'$ be a homomorphism of rings. A map $f_x : R[x] \to R'[x]$ defined by $f_x(a_0 + a_1x + \cdots + a_nx^n) := f(a_0) + f(a_1)x + \cdots + f(a_n)x^n$ is obviously a ring homomorphism, and we call it an *induced homomorphism* [13] by $f$.

**Theorem 8** (see [13]). *Let $f : R \to R'$ be an epimorphism of rings and let $f_x$ be an induced homomorphism of $f$. If $\alpha$ is an $f$-invariant fuzzy ideal of $R$, then $(f(\alpha))_x = f_x(\alpha_x)$.*

## 3. Fuzzy Polynomial Ideals

In this section, we study some relations between the radical of the fuzzy polynomial induced by a fuzzy ideal and the radical of a fuzzy ideal of a ring.

A fuzzy ideal $\alpha : R \to [0,1]$ of a ring $R$ is called a *fuzzy prime ideal* [14] of $R$ if $\alpha_*$ is a prime ideal of $R$. A fuzzy set $\sqrt{\alpha} : R \to [0,1]$, defined as $\sqrt{\alpha}(a) := \bigvee\{\alpha(a^n) \mid n > 0\}$, is called a *fuzzy nil radical* [15] of $\alpha$.

**Theorem 9** (see [15]). *If $\alpha : R \to [0,1]$ is a fuzzy ideal of $R$, then the fuzzy set $\sqrt{\alpha}$ is a fuzzy ideal of $R$.*

**Lemma 10** (see [15]). *If $\alpha$ and $\beta$ are fuzzy ideals of $R$, then $\sqrt{\alpha \cap \beta} = \sqrt{\alpha} \cap \sqrt{\beta}$.*

**Lemma 11.** *If $\alpha$ and $\beta$ are fuzzy ideals of $R$, then $\sqrt{\alpha \cup \beta} = \sqrt{\alpha} \cup \sqrt{\beta}$.*

*Proof.* If $a$ is an element of $R$, then

$$\begin{aligned} \sqrt{\alpha \cup \beta}(a) &= \bigvee\{(\alpha \cup \beta)(a^n) \mid n > 0\} \\ &= \bigvee\{\max\{\alpha(a^n), \beta(a^n)\} \mid n > 0\} \\ &= \max\left\{\bigvee\{\alpha(a^n) \mid n > 0\}, \bigvee\{\beta(a^n) \mid n > 0\}\right\} \\ &= \max\left\{\sqrt{\alpha}(a), \sqrt{\beta}(a)\right\} = \left(\sqrt{\alpha} \cup \sqrt{\beta}\right)(a). \end{aligned} \tag{4}$$

This proves that $\sqrt{\alpha \cup \beta} = \sqrt{\alpha} \cup \sqrt{\beta}$. □

Since $\alpha_x$ is a fuzzy ideal of a polynomial ring $R[x]$ by Theorem 5, the fuzzy set $\sqrt{\alpha_x} : R[x] \to [0,1]$ is the fuzzy nil radical of $\alpha_x$. The following theorem gives that the two fuzzy nil radicals have the same value.

**Theorem 12.** *If $\alpha : R \to [0,1]$ is a fuzzy ideal of $R$, then*

$$(\sqrt{\alpha_x})_x = (\sqrt{\alpha})_x. \tag{5}$$

*Proof.* Let $f(x) := \sum_{i=0}^{m} \in R[x]$ be any element of $R[x]$. Then, by Theorem 5, we have $\alpha_x(a_j^n) = \alpha_x(a_j^n + 0x + 0x^2 + \cdots + 0x^m) = \min\{\alpha(a_j^n), \alpha(0), \ldots, \alpha(0)\} = \alpha(a_j^n)$. Since $\sqrt{\alpha_x}$ is a fuzzy ideal of $R[x]$, we obtain

$$\begin{aligned} (\sqrt{\alpha_x})_x(f(x)) &= \min_{i=0}^{m}\{\sqrt{\alpha_x}(a_i)\} \\ &= \min_{i=0}^{m}\left\{\bigvee\{\alpha_x(a_i^n) \mid n > 0\}\right\} \\ &= \min_{i=0}^{m}\left\{\bigvee\{\alpha(a_i^n) \mid n > 0\}\right\} \\ &= \min_{i=0}^{m}\{\sqrt{\alpha}(a_i)\} = (\sqrt{\alpha})_x(f(x)). \end{aligned} \tag{6}$$

This proves that $(\sqrt{\alpha})_x = (\sqrt{\alpha_x})_x$. □

**Theorem 13.** *If $\alpha$ and $\beta$ are fuzzy ideals of $R$, then*

$$\left(\sqrt{\alpha \cap \beta}\right)_x = (\sqrt{\alpha})_x \cap \left(\sqrt{\beta}\right)_x. \tag{7}$$

*Proof.* If $\alpha$ and $\beta$ are fuzzy ideals of $R$, then $\alpha_x$ and $\beta_x$ are fuzzy ideals of $R[x]$ by Theorem 5. It follows from Theorems 12 and 7(i) and Lemma 10 that

$$\begin{aligned}\left(\sqrt{\alpha\cap\beta}\right)_x &= \left(\sqrt{(\alpha\cap\beta)_x}\right)_x \quad \text{[Theorem 12]}\\ &= \left(\sqrt{\alpha_x\cap\beta_x}\right)_x \quad \text{[Theorem 7 (i)]}\\ &= \left(\sqrt{\alpha_x}\cap\sqrt{\beta_x}\right)_x \quad \text{[Lemma 10]}\\ &= \left(\sqrt{\alpha_x}\right)_x\cap\left(\sqrt{\beta_x}\right)_x \quad \text{[Theorem 7 (i)]}\\ &= \left(\sqrt{\alpha}\right)_x\cap\left(\sqrt{\beta}\right)_x \quad \text{[Theorem 12]}\end{aligned} \tag{8}$$

proving the theorem. □

**Theorem 14.** *If $\alpha$ and $\beta$ are fuzzy ideals of $R$, then*

$$\left(\sqrt{\alpha}\right)_x\cup\left(\sqrt{\beta}\right)_x\subseteq\left(\sqrt{\alpha\cup\beta}\right)_x. \tag{9}$$

*Proof.* Since $\alpha_x$ and $\beta_x$ are fuzzy ideals of $R[x]$ by Theorem 5, we obtain

$$\begin{aligned}\left(\sqrt{\alpha}\right)_x\cup\left(\sqrt{\beta}\right)_x &= \left(\sqrt{\alpha_x}\right)_x\cup\left(\sqrt{\beta_x}\right)_x \quad \text{[Theorem 12]}\\ &\subseteq \left(\sqrt{\alpha_x}\cup\sqrt{\beta_x}\right)_x \quad \text{[Theorem 7 (ii)]}\\ &= \left(\sqrt{\alpha_x\cup\beta_x}\right)_x \quad \text{[Lemma 11]}\\ &\subseteq \left(\sqrt{(\alpha\cup\beta)_x}\right)_x \quad \text{[Theorem 7 (ii)]}\\ &= \left(\sqrt{\alpha\cup\beta}\right)_x \quad \text{[Theorem 12]}.\end{aligned} \tag{10}$$

This proves that $(\sqrt{\alpha})_x\cup(\sqrt{\beta})_x\subseteq(\sqrt{\alpha\cup\beta})_x$. □

**Theorem 15.** *Let $\beta$ be a fuzzy ideal of $R$ and let $f : R \to R'$ be a homomorphism of rings. If $f_x$ is the induced homomorphism of $f$, that is, $f_x(\sum_{i=0}^{n} a_i x^i) = \sum_{i=0}^{n} f(a_i)x^i$, then*

$$f_x^{-1}\left[\left(\sqrt{\beta}\right)_x\right]=\left(\sqrt{f^{-1}(\beta)}\right)_x. \tag{11}$$

*Proof.* Given a polynomial $g(x) = b_0+b_1x+\cdots+b_mx^m \in R[x]$, we have

$$\begin{aligned}&\left(\sqrt{f^{-1}(\beta)}\right)_x(g(x)) = \min\left\{\sqrt{f^{-1}\beta}(b_0),\right.\\ &\left.\sqrt{f^{-1}\beta}(b_1),\ldots,\sqrt{f^{-1}\beta}(b_m)\right\}\\ &= \min\left\{\bigvee\left\{f^{-1}\beta(b_0{}^n)\mid n>0\right\},\right.\\ &\bigvee\left\{f^{-1}\beta(b_1{}^n)\mid n>0\right\},\ldots,\\ &\left.\bigvee\left\{f^{-1}\beta(b_m{}^n)\mid n>0\right\}\right\}\\ &= \min\left\{\bigvee\left\{\beta\left(f(b_0)^n\right)\mid n>0\right\},\right.\\ &\bigvee\left\{\beta\left(f(b_1)^n\right)\mid n>0\right\},\ldots,\\ &\left.\bigvee\left\{\beta\left(f(b_m)^n\right)\mid n>0\right\}\right\} = \min\left\{\sqrt{\beta}(f(b_0)),\right.\\ &\left.\sqrt{\beta}(f(b_1)),\ldots,\sqrt{\beta}(f(b_m))\right\} = \left(\sqrt{\beta}\right)_x\\ &\cdot(f_x(g(x))) = f_x{}^{-1}\left[\left(\sqrt{\beta}\right)_x\right](g(x)).\end{aligned} \tag{12}$$

This proves that $f_x{}^{-1}[(\sqrt{\beta})_x] = (\sqrt{f^{-1}(\beta)})_x$. □

**Proposition 16** (see [15]). *Let $f : R \to R'$ be a ring epimorphism from $R$ onto $R'$, and let $\alpha : R \to [0,1]$ be a fuzzy ideal of $R$. If $\alpha$ is constant on* Ker $f$, *then $f(\sqrt{\alpha}) = \sqrt{f(\alpha)}$.*

**Theorem 17.** *Let $f : R \to R'$ be a homomorphism of rings and let $f_x$ be the induced homomorphism of $f$. If a fuzzy ideal $\alpha$ of $R$ is constant on* Ker $f$, *then the fuzzy polynomial ideal $\alpha_x$ is constant on* Ker $f_x$.

*Proof.* Let $\alpha(a) = k_0$ for all $a \in \operatorname{Ker} f$ and let $g(x) = b_0+b_1x+\cdots+b_mx^m$ be any element of $\operatorname{Ker} f_x$. Then $0 = f_x(g(x)) = f(b_0)+f(b_1)x+\cdots+f(b_m)x^m$. It follows that $f(b_i) = 0$ for all $i = 0,1,\ldots,m$. Hence $b_i \in \operatorname{Ker} f$ for all $i = 0,1,\ldots,m$; that is, $\alpha(b_i) = k_0$ for all $i = 0,1,\ldots,m$. This shows that $\alpha_x(g(x)) = \min\{\alpha(b_0),\alpha(b_1),\ldots,\alpha(b_m)\} = k_0$, proving the theorem. □

**Corollary 18.** *Let $f : R \to R'$ be an epimorphism of rings and let $f_x$ be the induced homomorphism of $f$. If an $f$-invariant fuzzy ideal $\alpha$ of $R$ is constant on* Ker $f$, *then*

$$f_x\left(\sqrt{\alpha_x}\right)=\sqrt{(f(\alpha))_x}. \tag{13}$$

*Proof.* It follows from Proposition 16 and Theorem 8 that $f_x(\sqrt{\alpha_x}) = \sqrt{f_x(\alpha_x)} = \sqrt{(f(\alpha))_x}$. □

**Theorem 19.** *Let $\alpha$ be a fuzzy ideal of $R$ and let $\alpha_x$ be its fuzzy polynomial ideal of $R[x]$. If $\beta$ is a fuzzy prime ideal of $R[x]$ such that $\alpha_x \subseteq \beta$, then there exists a fuzzy prime ideal $\alpha_0$ of $R$ such that $(\alpha_0)_* = \beta_* \cap R$ and $\alpha \subseteq \alpha_0$.*

*Proof.* Since $\beta$ is a fuzzy prime ideal of $R[x]$, $\beta_*$ is a prime ideal of $R[x]$. If we define $\gamma := \beta_* \cap R$, then it is easy to show that $\gamma$ is a prime ideal of $R$. Define a fuzzy subset $\alpha_0 : R \to [0,1]$ by

$$\alpha_0(a) = \begin{cases} \beta(0) & \text{if } a \in \gamma, \\ \beta(a) & \text{if } a \notin \gamma. \end{cases} \quad (14)$$

Then, by routine calculations, we show that $\alpha_0$ is a fuzzy ideal of $R$ satisfying $(\alpha_0)_* = \gamma$. We claim that $\alpha \subseteqq \alpha_0$. Given $a \in R$, if $a \in \gamma$, then $\alpha(a) = \alpha_x(a) \leq \beta(a) \leq \beta(0) = \alpha_0(a)$. If $a \notin \gamma$, then $\alpha(a) = \alpha_x(a) \leq \beta(a) = \alpha_0(a)$. Since $\gamma$ is a prime ideal of $R$, $(\alpha_0)_*$ is a prime ideal of $R$. This shows that $\alpha_0$ is a fuzzy prime ideal of $R$, proving the theorem. □

*Definition 20* (see [15]). Let $\alpha : R \to [0,1]$ be a fuzzy ideal of $R$. The fuzzy ideal $r(\alpha)$ defined by

$$r(\alpha) := \bigcap \{\beta \mid \alpha \subseteqq \beta,\ \beta: \text{a fuzzy prime ideal of } R\} \quad (15)$$

is called the *prime fuzzy radical* of $\alpha$.

**Theorem 21.** *Let $\alpha$ be a fuzzy ideal of $R$ and let $\alpha_x$ be its fuzzy polynomial ideal of $R[x]$. Then*

$$r(\alpha_x) \subseteq (r(\alpha))_x. \quad (16)$$

*Proof.* By Theorem 6, $\beta_i$ is a fuzzy prime ideal of $R$ with $\alpha \subseteq \beta_i$ if and only if $(\beta_i)_x$ is a fuzzy prime ideal of $R[x]$ with $\alpha_x \subseteq (\beta_i)_x$. It follows from Theorem 7(i) that

$$\begin{aligned} (r(\alpha))_x &= \Big(\bigcap \{\beta_i \mid \alpha \\ &\subseteq \beta_i,\ \beta_i \text{ is a fuzzy prime ideal of } R\}\Big)_x \\ &= \Big(\bigcap \{(\beta_i)_x \mid \alpha \\ &\subseteq \beta_i,\ \beta_i \text{ is a fuzzy prime ideal of } R\}\Big) = \bigcap \{(\beta_i)_x \mid \\ &\alpha_x \subseteq (\beta_i)_x,\ (\beta_i)_x \text{ is a fuzzy prime ideal of } R[x]\} \\ &\supseteq \bigcap \{\gamma_i \mid \alpha_x \subseteq \gamma_i, \\ &\gamma_i \text{ is a fuzzy prime ideal of } R[x]\} = r(\alpha_x), \end{aligned} \quad (17)$$

proving the theorem. □

*Notation 2.* Let $\alpha$ be a fuzzy ideal of $R$ and let $\alpha_x$ be its fuzzy polynomial ideal of $R[x]$. We denote $\mathrm{FPI}(\alpha)$ by

$$\mathrm{FPI}(\alpha) := \{\beta \mid \alpha \subseteq \beta,\ \beta \text{ is a fuzzy prime ideal of } R\} \quad (18)$$

and $\mathrm{FPI}(\alpha_x)$ by

$$\begin{aligned} &\mathrm{FPI}(\alpha_x) \\ &\quad := \{\gamma \mid \alpha_x \subseteq \gamma,\ \gamma \text{ is a fuzzy prime ideal of } R[x]\}. \end{aligned} \quad (19)$$

**Theorem 22.** *Let $\alpha$ be a fuzzy ideal of $R$ and let $\alpha_x$ be its fuzzy polynomial ideal of $R[x]$. Then a map $\phi : \mathrm{FPI}(\alpha) \to \mathrm{FPI}(\alpha_x)$ defined by $\phi(\beta) := \beta_x$ is one-one.*

*Proof.* If $\beta, \gamma \in \mathrm{FPI}(\alpha)$ such that $\phi(\beta) = \phi(\gamma)$, then $\beta_x = \gamma_x$. It follows that $\beta_x(a) = \gamma_x(a)$ for all $a \in R$, and hence $\beta(a) = \gamma(a)$ for all $a \in R$, proving that $\beta = \gamma$. Hence $\phi$ is one-one. □

**Corollary 23.** *Let $\alpha$ be a fuzzy ideal of $R$ and let $\alpha_x$ be its fuzzy polynomial ideal of $R[x]$. If the map $\phi$ defined in Theorem 22 is an onto map, then*

$$(r(\alpha))_x = r(\alpha_x). \quad (20)$$

*Proof.* If $\beta$ is any element of $\mathrm{FPI}(\alpha_x)$, then there exists $\gamma \in \mathrm{FPI}(\alpha)$ such that $\gamma_x = \phi(\gamma) = \beta$ with $\alpha \subseteq \gamma$. Thus $(r(\alpha))_x = r(\alpha_x)$. This shows that the reverse inclusion in Theorem 21 holds. □

*Example 24.* Let $\mathbf{Z}$ be set of all integers. Let

$$\alpha(x) := \begin{cases} 1 & \text{if } x \in 2\mathbf{Z}, \\ 0 & \text{if } x \notin 2\mathbf{Z}. \end{cases} \quad (21)$$

Then $\alpha$ is a fuzzy prime ideal of $\mathbf{Z}$, since $\alpha_* = 2\mathbf{Z}$ is a prime ideal of $\mathbf{Z}$, and its induced polynomial ideal $\alpha_x$ is

$$\alpha_x(f(x)) := \begin{cases} 1 & \text{if } f(x) \in 2\mathbf{Z}[x], \\ 0 & \text{if } f(x) \notin 2\mathbf{Z}[x]. \end{cases} \quad (22)$$

By Theorem 6, the fuzzy polynomial ideal $\alpha_x$ induced by $\alpha$ is a fuzzy prime ideal of $\mathbf{Z}[x]$. Hence $(r(\alpha))_x = \alpha_x = r(\alpha_x)$.

## Competing Interests

The authors declare that they have no competing interests.

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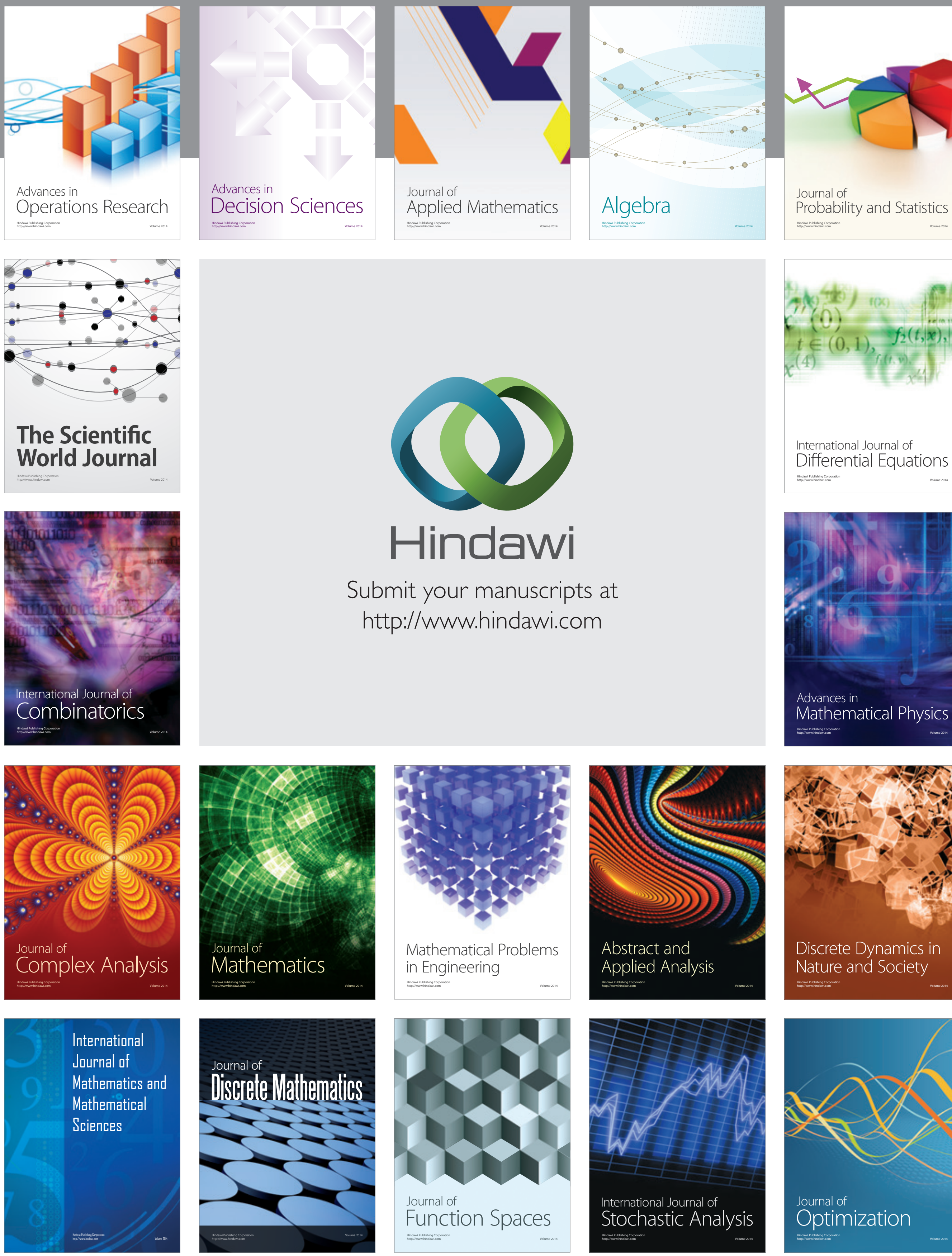
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