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Research Article

Global Attractivity in a Discrete Mutualism Model with Infinite Deviating Arguments

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A set of sufficient conditions is obtained for the global attractivity of the following two-species discrete mutualism model with infinite deviating arguments: $x_1(k+1) = x_1(k)\exp\{r_1[(K_1+\alpha_1\sum_{s=0}^{+\infty}J_2(s)x_2(k-s))/(1+\sum_{s=0}^{+\infty}J_2(s)x_2(k-s))-x_1(k)]\}$ and $x_2(k+1) = x_2(k)\exp\{r_2[(K_2+\alpha_2\sum_{s=0}^{+\infty}J_1(s)x_1(k-s))/(1+\sum_{s=0}^{+\infty}J_1(s)x_1(k-s))-x_2(k)]\}$, where r_i, K_i, α_i, $i = 1, 2$, are all positive constants, $\sum_{j=1}^{+\infty}J_i(n) = 1$, and $\alpha_i > K_i$. Our results generalize the main result of Yang et al. (2014).

1. Introduction

The aim of this paper is to investigate the stability property of the following two-species discrete mutualism model with infinite deviating arguments:

$$\begin{aligned} x_1(k+1) &= x_1(k) \cdot \exp\left\{r_1\left[\frac{K_1+\alpha_1\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)}{1+\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)} - x_1(k)\right]\right\}, \\ x_2(k+1) &= x_2(k) \cdot \exp\left\{r_2\left[\frac{K_2+\alpha_2\sum_{s=0}^{+\infty}J_1(s)x_1(k-s)}{1+\sum_{s=0}^{+\infty}J_1(s)x_1(k-s)} - x_2(k)\right]\right\}, \end{aligned} \tag{1}$$

together with the initial conditions

$$\begin{aligned} &x_i(s) = \phi_i(s) \geq 0, \\ &x_i(0) > 0, \quad s = \cdots, -k, -k+1, \ldots, -2, -1, \quad i = 1, 2. \end{aligned} \tag{2}$$

Li and Xu [1] studied the following two-species integrodifferential model of mutualism:

$$\begin{aligned} N_1'(t) &= r_1(t)N_1(t) \cdot \left[\frac{K_1(t)+\alpha_1(t)\int_0^{\infty}J_2(s)N_2(t-s)\,ds}{1+\int_0^{\infty}J_2(s)N_2(t-s)\,ds} - N_1(t-\sigma_1(t))\right], \\ N_2'(t) &= r_2(t)N_2(t) \cdot \left[\frac{K_2(t)+\alpha_2(t)\int_0^{\infty}J_1(s)N_1(t-s)\,ds}{1+\int_0^{\infty}J_1(s)N_1(t-s)\,ds} - N_2(t-\sigma_2(t))\right]. \end{aligned} \tag{3}$$

Under the assumption $K_i(t), \alpha_i(t)$, $i = 1, 2$, are all positive periodic functions and $\alpha_i > K_i$, $i = 1, 2$, by applying the coincidence degree theory, they showed that system (3) admits at least one positive ω-periodic solution. Chen and You [2] argued that a general nonautonomous nonperiodic system is more appropriate, and for the general nonautonomous case, by using the differential inequality theory, they showed that the system is permanent. It brings to our attention that both [1, 2] did not consider the stability property of the system, and in [3], under the assumption r_i, K_i, α_i, $i = 1, 2$, are all positive constants, $\sigma_i(t) \equiv 0$, we investigated the stability property of the system, and we showed that the system admits a unique globally attractive positive equilibrium. At the end of the paper, we pointed out "whether some parallel result could be established for the discrete type mutualism system is still unknown, we leave this for future investigation."

Previously, corresponding to system (3), Li and Yang [4] and Li [5] proposed the following two-species discrete model of mutualism with infinite deviating arguments:

$$\begin{aligned} x_1(k+1) &= x_1(k)\exp\Bigg\{r_1(k) \\ &\cdot \left[\frac{K_1(k) + \alpha_1(k)\sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s)} \right. \\ &\left. - x_1(k - \delta_1(k))\right]\Bigg\}, \\ x_2(k+1) &= x_2(k)\exp\Bigg\{r_2(k) \\ &\cdot \left[\frac{K_2(k) + \alpha_2(k)\sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s)}{1 + \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s)} \right. \\ &\left. - x_2(k - \delta_2(k))\right]\Bigg\}, \end{aligned} \tag{4}$$

where $x_i(k)$, $i = 1, 2$, is the density of mutualism species i at the kth generation and $\{r_i(k)\}, \{K_i(k)\}, \{\alpha_i(k)\}, \{J_i(k)\}$, and $\{\delta_i(k)\}$, $i = 1, 2$, are bounded nonnegative sequences such that

$$\begin{aligned} &0 < r_i^l \le r_i^u, \\ &0 < a_i^l \le a_i^u, \\ &0 < K_i^l \le K_i^u, \\ &0 < \delta_i^l \le \delta_i^u, \\ &\sum_{j=1}^{+\infty} J_i(n) = 1, \\ &\alpha_i > K_i. \end{aligned} \tag{5}$$

They showed that, under the above assumption, system (4) is permanent. Again, none of the papers [4, 5] considered the stability property of the system. To make an intensive study on this direction, in [6], we investigated the dynamic behaviors of the following autonomous mutualism system:

$$\begin{aligned} &x_1(k+1) \\ &\quad = x_1(k)\exp\left\{r_1\left[\frac{K_1 + \alpha_1 x_2(k)}{1 + x_2(k)} - x_1(k)\right]\right\}, \\ &x_2(k+1) \\ &\quad = x_2(k)\exp\left\{r_2\left[\frac{K_2 + \alpha_2 x_1(k)}{1 + x_1(k)} - x_2(k)\right]\right\}, \end{aligned} \tag{6}$$

where $x_i(k)$ $(i = 1, 2)$ are the population density of the ith species at k-generation. We showed that if

(H_1) r_i, K_i, α_i $(i = 1, 2)$ are all positive constants and $\alpha_i > K_i$ $(i = 1, 2)$;

(H_2) $r_i\alpha_i \le 1$, $(i = 1, 2)$

hold, system (6) admits a unique positive equilibrium (x_1^*, x_2^*), which is globally asymptotically stable. Our result shows that the dynamic behavior of the discrete type mutualism model is more complicated, and one could not expect to establish parallel result as that of continuous ones. Also, at the end of the paper, we pointed out "it seems interesting to incorporate the time delay to the system (6) and investigate the dynamic behaviors of the system, we leave this for future study." However, to this day, we still did not study the correspondence topic on this area. For more background of system (3), (4), and (6) one could refer to [1–24] and the references cited therein. We mention here that, with $\sigma_i(k) \ne 0$, $i = 1, 2$, and all the coefficients being time-dependent, system (4) is a nonautonomous pure-delay system, and it is not an easy thing to investigate the stability property of the system. This motivated us to discuss the simple one, that is, the autonomous simple non-pure-delay system (1).

Concerned with the stability property of system (1)-(2), we have the following result.

Theorem 1. *Assume that (H_1) and (H_2) hold, and then system (1)-(2) admits a unique positive equilibrium (x_1^*, x_2^*), which is globally attractive.*

Remark 2. Obviously, Theorem 1 generalizes the main results of Yang et al. [6] to the infinite deviating arguments case. Theorem 1 can also be seen as the parallel result of the continuous one in [3]. Thus, we push on the study of the mutualism model.

2. Existence and Uniqueness of Positive Equilibrium

This section focuses on the existence and uniqueness of positive equilibrium of system (1). More precisely, we will prove the following result.

Theorem 3. *Under the assumption of Theorem 1, system (1)-(2) admits a unique positive equilibrium.*

Proof. The positive equilibrium of system (1) satisfies

$$\frac{K_1+\alpha_1 x_2}{1+x_2}-x_1=0,\qquad \frac{K_2+\alpha_2 x_1}{1+x_1}-x_2=0, \tag{7}$$

which is equivalent to

$$A_1x_1^2+A_2x_1+A_3=0,\qquad B_1x_2^2+B_2x_2+B_3=0, \tag{8}$$

where

$$\begin{aligned} A_1&=\alpha_2+1,\\ A_2&=-\alpha_1\alpha_2-K_1+K_2+1,\\ A_3&=-K_2\alpha_1-K_1,\\ B_1&=\alpha_1+1,\\ B_2&=-\alpha_1\alpha_2+K_1-K_2+1,\\ B_3&=-K_1\alpha_2-K_2. \end{aligned} \tag{9}$$

Now let us consider the function

$$F_1(x_1)=A_1x_1^2+A_2x_1+A_3, \tag{10}$$

and since $A_1>0$, $A_3<0$, it follows that $F_1(-\infty)=+\infty$, $F_1(0)=0$, $F_1(+\infty)=+\infty$, and, hence, from the continuity of F_1, there exist two points x_1^{**} and x_1^*, $x_1^{**}<0<x_1^*$, such that $F_1(x_1^{**})=F_1(x_1^*)=0$, and since $F_1(x_1)=0$ has at most two solutions, it means that $F_1(x_1)=0$ admits unique positive solution x_1^*. Similarly, from $B_1>0$, $B_3<0$, one could prove $F_2(x_2)=B_1x_2^2+B_2x_2+B_3=0$ admits unique positive solution x_2^*. By simple computation, system (7) admits a unique positive solution $E_+(x_1^*,x_2^*)$, where

$$x_1^*=\frac{-A_2+\sqrt{A_2^2-4A_1A_3}}{2A_1},\qquad x_2^*=\frac{-B_2+\sqrt{B_2^2-4B_1B_3}}{2B_1}. \tag{11}$$

This ends the proof of Theorem 3. □

3. Proof of Theorem 1

Now we state several lemmas which will be useful in the proof of Theorem 1.

Lemma 4 (see [25]). *Let $f(u)=u\exp(\alpha-\beta u)$, where α and β are positive constants, and then $f(u)$ is nondecreasing for $u\in(0,1/\beta]$.*

Lemma 5 (see [25]). *Assume that sequence $\{u(k)\}$ satisfies*

$$u(k+1)=u(k)\exp(\alpha-\beta u(k)),\quad k=1,2,\ldots, \tag{12}$$

where α and β are positive constants and $u(0)>0$. Then

(i) *If $\alpha<2$, then $\lim_{k\to+\infty}u(k)=\alpha/\beta$.*

(ii) *If $\alpha\le 1$, then $u(k)\le 1/\beta$, $k=2,3,\ldots$.*

Lemma 6 (see [26]). *Suppose that functions $f,g : Z_+\times[0,\infty)\to[0,\infty)$ satisfy $f(k,x)\le g(k,x)(f(k,x)\ge g(k,x))$ for $k\in Z_+$ and $x\in[0,\infty)$ and $g(k,x)$ is nondecreasing with respect to x. If $\{x(k)\}$ and $\{u(k)\}$ are the nonnegative solutions of the following difference equations:*

$$x(k+1)=f(k,x(k)),\qquad u(k+1)=g(k,u(k)), \tag{13}$$

respectively, and $x(0)\le u(0)(x(0)\ge u(0))$, then

$$x(k)\le u(k)\,(x(k)\ge u(k)),\quad \forall k\ge 0. \tag{14}$$

Lemma 7 (see [27]). *Let $x : Z\to R$ be nonnegative bounded sequences, and let $H : N\to R$ be nonnegative sequences such that $\sum_{n=0}^{\infty}H(n)=1$. Then*

$$\begin{aligned}\liminf_{n\to+\infty}x(n)&\le\liminf_{n\to+\infty}\sum_{s=-\infty}^{n}H(n-s)x(s)\\ &\le\limsup_{n\to+\infty}\sum_{s=-\infty}^{n}H(n-s)x(s)\\ &\le\limsup_{n\to+\infty}x(n).\end{aligned} \tag{15}$$

Lemma 8. *Let $g_i(x)=(K_i+\alpha_i x)/(1+x)$, $i=1,2$, assume that $\alpha_i>K_i$, and then $g_i(x)$ are the strictly increasing function of x.*

Proof. Since

$$g_i'(x)=-\frac{(K_i-\alpha_i)}{(1+x)^2}>0,\quad i=1,2, \tag{16}$$

the conclusion of Lemma 8 immediately follows. □

Now we are in the position to prove the main result of this paper.

Proof of Theorem 1. Let $(x_1(k),x_2(k))$ be arbitrary solution of system (1) with initial condition (2). Denote

$$U_i=\limsup_{k\to+\infty}x_i(k),\qquad V_i=\liminf_{k\to+\infty}x_i(k),\qquad i=1,2. \tag{17}$$

We claim that $U_1=V_1=x_1^*$ and $U_2=V_2=x_2^*$.

From the first equation of system (1), we obtain

$$\begin{aligned} x_1(k+1) &= x_1(k) \\ &\cdot \exp\left\{r_1\left[\frac{K_1+\alpha_1\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)}{1+\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)} - x_1(k)\right]\right\} \le x_1(k) \\ &\cdot \exp\left\{r_1\left[\frac{\alpha_1+\alpha_1\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)}{1+\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)} - x_1(k)\right]\right\} \le x_1(k)\exp\{r_1\alpha_1 - r_1x_1(k)\}, \\ &\quad k = 0,1,2,\ldots, \end{aligned} \tag{18}$$

considering the auxiliary equation as follows:

$$u(k+1) = u(k)\exp\{r_1\alpha_1 - r_1u(k)\}, \quad k = 0,1,2,\ldots. \tag{19}$$

Because of $0 < r_1\alpha_1 \le 1$, according to (ii) of Lemma 5, we can obtain $u(k) \le 1/r_1$ for all $k \ge 2$, where $u(k)$ is arbitrary positive solution of (18) with initial value $u(0) > 0$. From Lemma 4, $f(u) = u\exp(r_1\alpha_1 - r_1u)$ is nondecreasing for $u \in (0, 1/r_1]$. According to Lemma 6 we can obtain $x_1(k) \le u(k)$ for all $k \ge 2$, where $u(k)$ is the solution of (19) with the initial value $u(2) = x_1(2)$. According to (i) of Lemma 5, we can obtain

$$U_1 = \limsup_{k\to+\infty} x_1(k) \le \lim_{k\to+\infty} u(k) = \alpha_1. \tag{20}$$

From (20) and Lemma 7 we have

$$\begin{aligned} &\limsup_{k\to+\infty}\sum_{s=0}^{+\infty}J_1(s)x_1(k-s) \\ &= \limsup_{n\to+\infty}\sum_{s=-\infty}^{k}J_1(k-s)x_1(s) \le \limsup_{k\to+\infty}x_1(k) \\ &\le \alpha_1. \end{aligned} \tag{21}$$

From the second equation of system (1), we obtain

$$x_2(k+1) \le x_2(k)\exp\{r_2\alpha_2 - r_2x_2(k)\}, \quad k = 0,1,2,\ldots. \tag{22}$$

Similar to the above analysis, we have

$$U_2 = \limsup_{k\to+\infty} x_2(k) \le \alpha_2. \tag{23}$$

From (23) and Lemma 7 we have

$$\begin{aligned} &\limsup_{k\to+\infty}\sum_{s=0}^{+\infty}J_2(s)x_2(k-s) \\ &= \limsup_{n\to+\infty}\sum_{s=-\infty}^{k}J_2(k-s)x_2(s) = \limsup_{k\to+\infty}x_2(k) \\ &\le \alpha_2. \end{aligned} \tag{24}$$

For $\varepsilon > 0$ enough small, without loss of generality, we may assume that $\varepsilon < (1/2)\min\{K_1, K_2\}$, and it follows from (20)–(24) that there is an integer $k_1 > 2$ such that, for all $k > k_1$,

$$\begin{aligned} x_1(k) &< \alpha_1 + \varepsilon \overset{\text{def}}{=} M_1^{x_1}, \\ x_2(k) &< \alpha_2 + \varepsilon \overset{\text{def}}{=} M_1^{x_2}, \end{aligned} \tag{25}$$

$$\begin{aligned} \sum_{s=0}^{+\infty}J_1(s)x_1(k-s) &< \alpha_1 + \varepsilon \overset{\text{def}}{=} M_1^{x_1}, \\ \sum_{s=0}^{+\infty}J_2(s)x_2(k-s) &< \alpha_2 + \varepsilon \overset{\text{def}}{=} M_1^{x_2}. \end{aligned} \tag{26}$$

For $k \ge k_1$, according to the first equation of system (1) we can obtain

$$\begin{aligned} x_1(k+1) &= x_1(k) \\ &\cdot \exp\left\{r_1\left[\frac{K_1+\alpha_1\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)}{1+\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)} - x_1(k)\right]\right\} \ge x_1(k) \\ &\cdot \exp\left\{r_1\left[\frac{K_1+K_1\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)}{1+\sum_{s=0}^{+\infty}J_2(s)x_2(k-s)} - x_1(k)\right]\right\} \ge x_1(k)\exp\{r_1K_1 - r_1x_1(k)\}, \end{aligned} \tag{27}$$

considering the auxiliary equation as follows:

$$u(k+1) = u(k)\exp\{r_1K_1 - r_1u(k)\}. \tag{28}$$

According to (ii) of Lemma 5, we can obtain $u(k) \le 1/r_1$ for all $k \ge k_2$, where $u(k)$ is arbitrary positive solution of (28) with initial value $u(k_2) > 0$. From Lemma 4, $f(u) = u\exp(r_1K_1 - r_1u)$ is nondecreasing for $u \in (0, 1/r_1]$. According to Lemma 6 we can obtain $x_1(k) \ge u(k)$ for all $k \ge 2$, where $u(k)$ is the solution of (28) with the initial value $u(k_2) = x_1(k_2)$. According to (i) of Lemma 5, we have

$$V_1 = \liminf_{k\to+\infty} x_1(k) \ge \lim_{k\to+\infty} u(k) = K_1. \tag{29}$$

From (29) and Lemma 7 we can obtain

$$\liminf_{k\to+\infty}\sum_{s=0}^{+\infty}J_1(s)x_1(k-s) \ge \liminf_{k\to+\infty}x_1(k) \ge K_1. \tag{30}$$

From the second equation of system (1), we obtain

$$x_2(k+1) \ge x_2(k)\exp\{r_2K_2 - r_2x_2(k)\}. \tag{31}$$

Similar to the analysis of (27)–(30), we have

$$\begin{aligned} V_2 &= \liminf_{k\to+\infty} x_2(k) \ge K_2, \\ &\liminf_{k\to+\infty}\sum_{s=0}^{+\infty}J_2(s)x_2(k-s) \ge K_2. \end{aligned} \tag{32}$$

Then, for the above $\varepsilon > 0$, there is an integer $k_2 > k_1$ such that, for all $k > k_2$,

$$\begin{aligned} x_1(k) &> K_1 - \varepsilon \overset{\text{def}}{=} m_1^{x_1}, \\ x_2(k) &> K_2 - \varepsilon \overset{\text{def}}{=} m_1^{x_2}; \\ \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s) &> K_1 - \varepsilon \overset{\text{def}}{=} m_1^{x_1}, \\ \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s) &> K_2 - \varepsilon \overset{\text{def}}{=} m_1^{x_2}. \end{aligned} \tag{33}$$

Noting that, from Lemma 8, $g_i(x) = (K_i + \alpha_i x)/(1+x)$ $(\alpha_i > K_i)$ is a strictly increasing function, then, from the first and second equations of system (1) and (26), we have

$$\begin{aligned} x_1(k+1) &= x_1(k) \\ &\cdot \exp\left\{ r_1 \left[\frac{K_1 + \alpha_1 \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s)}{1 + \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s)} \right.\right. \\ &\left.\left. - x_1(k) \right] \right\} \le x_1(k) \exp\left\{ r_1 \left[\frac{K_1 + \alpha_1 M_1^{x_2}}{1 + M_1^{x_2}} \right.\right. \\ &\left.\left. - x_1(k) \right] \right\}, \\ x_2(k+1) &= x_2(k) \\ &\cdot \exp\left\{ r_2 \left[\frac{K_2 + \alpha_2 \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s)}{1 + \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s)} \right.\right. \\ &\left.\left. - x_2(k) \right] \right\} \le x_2(k) \exp\left\{ r_2 \left[\frac{K_2 + \alpha_2 M_1^{x_1}}{1 + M_1^{x_1}} \right.\right. \\ &\left.\left. - x_2(k) \right] \right\}. \end{aligned} \tag{34}$$

From (34), similarly to the analysis of (18)–(24), we can finally obtain

$$\begin{aligned} \limsup_{k\to+\infty} \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s) &\le \limsup_{k\to+\infty} x_1(k) \\ &\le \frac{K_1 + \alpha_1 M_1^{x_2}}{1 + M_1^{x_2}}, \\ \limsup_{k\to+\infty} \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s) &\le \limsup_{k\to+\infty} x_2(k) \\ &\le \frac{K_2 + \alpha_2 M_1^{x_1}}{1 + M_1^{x_1}}. \end{aligned} \tag{35}$$

For the above $\varepsilon > 0$, it follows from (35) that there exists an integer $k_3 > k_2$ such that, for all $k > k_3$,

$$\begin{aligned} x_1(k) &< \frac{K_1 + \alpha_1 M_1^{x_2}}{1 + M_1^{x_2}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{x_1}, \\ x_2(k) &< \frac{K_2 + \alpha_2 M_1^{x_1}}{1 + M_1^{x_1}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{x_2}; \\ \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s) &< \frac{K_1 + \alpha_1 M_1^{x_2}}{1 + M_1^{x_2}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{x_1}, \\ \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s) &< \frac{K_2 + \alpha_2 M_1^{x_1}}{1 + M_1^{x_1}} + \frac{\varepsilon}{2} \overset{\text{def}}{=} M_2^{x_2}. \end{aligned} \tag{36}$$

It then follows from (25), (26), and (36) that

$$M_2^{x_i} < M_1^{x_i}, \quad i = 1, 2. \tag{37}$$

For $k \ge k_3$, from the strictly increasing function $g_i(x) = (K_i + \alpha_i x)/(1+x)$, $\alpha_i > K_i$, $i = 1, 2$, and (33), we can obtain

$$\begin{aligned} x_1(k+1) &\ge x_1(k) \exp\left\{ r_1 \left[\frac{K_1 + \alpha_1 m_1^{x_2}}{1 + m_1^{x_2}} - x_1(k) \right] \right\}, \\ x_2(k+1) &\ge x_2(k) \exp\left\{ r_2 \left[\frac{K_2 + \alpha_2 m_1^{x_1}}{1 + m_1^{x_1}} - x_2(k) \right] \right\}. \end{aligned} \tag{38}$$

From (38), similar to the analysis of (27)–(32), we can obtain

$$\begin{aligned} V_1 = \liminf_{k\to+\infty} x_1(k) &\ge \frac{K_1 + \alpha_1 m_1^{x_2}}{1 + m_1^{x_2}}, \\ \liminf_{k\to+\infty} \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s) &\ge \frac{K_1 + \alpha_1 m_1^{x_2}}{1 + m_1^{x_2}}, \\ V_2 = \liminf_{k\to+\infty} x_2(k) &\ge \frac{K_2 + \alpha_2 m_1^{x_1}}{1 + m_1^{x_1}}, \\ \liminf_{k\to+\infty} \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s) &\ge \frac{K_2 + \alpha_2 m_1^{x_1}}{1 + m_1^{x_1}}. \end{aligned} \tag{39}$$

For the above $\varepsilon > 0$, it follows from (39) that there is an integer $k_4 > k_3$ such that, for all $k > k_4$,

$$\begin{aligned} x_1(k) &> \frac{K_1 + \alpha_1 m_1^{x_2}}{1 + m_1^{x_2}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{x_1}, \\ x_2(k) &> \frac{K_2 + \alpha_2 m_1^{x_1}}{1 + m_1^{x_1}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{x_2}, \\ \sum_{s=0}^{+\infty} J_1(s)\, x_1(k-s) &> \frac{K_1 + \alpha_1 m_1^{x_2}}{1 + m_1^{x_2}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{x_1}, \\ \sum_{s=0}^{+\infty} J_2(s)\, x_2(k-s) &> \frac{K_2 + \alpha_2 m_1^{x_1}}{1 + m_1^{x_1}} - \frac{\varepsilon}{2} \overset{\text{def}}{=} m_2^{x_2}. \end{aligned} \tag{40}$$

Noting that

$$\frac{K_1+\alpha_1 m_1^{x_2}}{1+m_1^{x_2}} > K_1, \quad \frac{K_2+\alpha_2 m_1^{x_1}}{1+m_1^{x_1}} > K_2, \quad i=1,2. \tag{41}$$

Then from (33) and (40) we have

$$m_2^{x_i} > m_1^{x_i}, \quad i=1,2. \tag{42}$$

Continuing the above steps, we can get four sequences $\{M_k^{x_1}\}$, $\{M_k^{x_2}\}$, $\{m_k^{x_1}\}$, and $\{m_k^{x_2}\}$ such that

$$\begin{aligned} M_k^{x_1} &= \frac{K_1+\alpha_1 M_{k-1}^{x_2}}{1+M_{k-1}^{x_2}} + \frac{\varepsilon}{k}, \\ M_k^{x_2} &= \frac{K_2+\alpha_2 M_{k-1}^{x_1}}{1+M_{k-1}^{x_1}} + \frac{\varepsilon}{k}; \\ m_k^{x_1} &= \frac{K_1+\alpha_1 m_{k-1}^{x_2}}{1+m_{k-1}^{x_2}} - \frac{\varepsilon}{k}, \\ m_k^{x_2} &= \frac{K_2+\alpha_2 m_{k-1}^{x_1}}{1+m_{k-1}^{x_1}} - \frac{\varepsilon}{k}. \end{aligned} \tag{43}$$

Clearly, we have

$$\begin{aligned} m_k^{x_1} &< V_1 \le U_1 < M_k^{x_1}, \\ m_k^{x_2} &< V_2 \le U_2 < M_k^{x_2}, \\ & k=0,1,2,\ldots. \end{aligned} \tag{44}$$

Now, we will prove $\{M_k^{x_i}\}(i=1,2)$ is monotonically decreasing and $\{m_k^{x_i}\}(i=1,2)$ is monotonically increasing by means of inductive method.

First of all, from (37) and (42) we have $M_2^{x_i} < M_1^{x_i}$, $m_2^{x_i} > m_1^{x_i}(i=1,2)$. For $k \ge 2$, we assume that $M_k^{x_i} < M_{k-1}^{x_i}$ and $m_k^{x_i} > m_{k-1}^{x_i}$, $i=1,2$, holds, and then from the strictly increasing of function $g_i(x)=(K_i+\alpha_i x)/(1+x)$, $i=1,2$, it immediately follows that

$$\begin{aligned} M_{k+1}^{x_1} &= \frac{K_1+\alpha_1 M_k^{x_2}}{1+M_k^{x_2}} + \frac{\varepsilon}{k+1} < \frac{K_1+\alpha_1 M_{k-1}^{x_2}}{1+M_{k-1}^{x_2}} + \frac{\varepsilon}{k} \\ &= M_k^{x_1}; \\ M_{k+1}^{x_2} &= \frac{K_2+\alpha_2 M_k^{x_1}}{1+M_k^{x_1}} + \frac{\varepsilon}{k+1} < \frac{K_2+\alpha_2 M_{k-1}^{x_1}}{1+M_{k-1}^{x_1}} + \frac{\varepsilon}{k} \\ &= M_k^{x_2}, \\ m_{k+1}^{x_1} &= \frac{K_1+\alpha_1 m_k^{x_2}}{1+m_k^{x_2}} - \frac{\varepsilon}{k+1} > \frac{K_1+\alpha_1 m_{k-1}^{x_2}}{1+m_{k-1}^{x_2}} - \frac{\varepsilon}{k} \\ &= M_k^{x_1}; \\ m_{k+1}^{x_2} &= \frac{K_2+\alpha_2 m_k^{x_1}}{1+m_k^{x_1}} - \frac{\varepsilon}{k+1} > \frac{K_2+\alpha_2 m_{k-1}^{x_1}}{1+m_{k-1}^{x_1}} - \frac{\varepsilon}{k} \\ &= M_k^{x_2}. \end{aligned} \tag{45}$$

Equations of (45) show that $\{M_k^{x_i}\}(i=1,2)$ is monotonically decreasing and $\{m_k^{x_i}\}(i=1,2)$ is monotonically increasing. Consequently, $\lim_{k\to+\infty}\{M_k^{x_i}\}$ and $\lim_{k\to+\infty}\{m_k^{x_i}\}$ $(i=1,2)$ both exist. Let

$$\lim_{k\to+\infty} M_k^{x_i} = \overline{X}_i, \quad \lim_{k\to+\infty} m_k^{x_i} = \underline{X}_i, \quad i=1,2. \tag{46}$$

From (43), we have

$$\begin{aligned} \overline{X}_1 &= \frac{K_1+\alpha_1\overline{X}_2}{1+\overline{X}_2}; \\ \overline{X}_2 &= \frac{K_2+\alpha_2\overline{X}_1}{1+\overline{X}_1}; \\ \underline{X}_1 &= \frac{K_1+\alpha_1\underline{X}_2}{1+\underline{X}_2}; \\ \underline{X}_2 &= \frac{K_2+\alpha_2\underline{X}_1}{1+\underline{X}_1}; \end{aligned} \tag{47}$$

Here, (47) shows that $(\overline{X}_1,\overline{X}_2)$ and $(\underline{X}_1,\underline{X}_2)$ are all solutions of system (7). However, system (7) has unique positive solution (x_1^*,x_2^*). Therefore

$$U_i = V_i = \lim_{k\to+\infty} x_i(k) = x_i^*, \quad i=1,2; \tag{48}$$

that is, $E_+(x_1^*,x_2^*)$ is globally attractive. The proof of the theorem is completed. □

4. Examples

In this section we shall give an example to illustrate the feasibility of the main result.

Example 1. Consider the following example:

$$\begin{aligned} x_1(k+1) &= x_1(k) \\ &\cdot \exp\left\{3\left[\frac{0.2+0.3\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_2(n-s)}{1+\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_2(n-s)} \right.\right. \\ &\left.\left. - x_1(k)\right]\right\}, \end{aligned}$$

$$x_2(k+1) = x_2(k) \cdot \exp\left\{0.5\left[\frac{0.5 + 1.5\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_1(n-s)}{1+\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_1(n-s)} - x_2(k)\right]\right\}. \quad (49)$$

Corresponding to system (1), we have $r_1 = 3$, $K_1 = 0.2$, $\alpha_1 = 0.3$, $r_2 = 0.5$, $K_2 = 0.5$, $\alpha_2 = 1.5$, and hence

$$\alpha_i > K_i, \quad i = 1, 2. \quad (50)$$

Also,

$$r_1\alpha_1 = 0.9 < 1, \qquad r_2\alpha_2 = 0.75 < 1. \quad (51)$$

Hence, all the conditions of Theorem 1 hold, and it follows from Theorem 1 that system (49) admits a unique globally attractive positive equilibrium. Figure 1 supports this assertion.

Example 2. Consider the following example:

$$x_1(k+1) = x_1(k) \cdot \exp\left\{5\left[\frac{0.2 + 0.3\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_2(n-s)}{1+\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_2(n-s)} - x_1(k)\right]\right\},$$
$$x_2(k+1) = x_2(k) \cdot \exp\left\{1\left[\frac{0.5 + 1.5\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_1(n-s)}{1+\sum_{s=0}^{\infty}((e-1)/e)e^{-s}x_1(n-s)} - x_2(k)\right]\right\}. \quad (52)$$

Corresponding to system (1), we have $r_1 = 5$, $K_1 = 0.2$, $\alpha_1 = 0.3$, $r_2 = 1$, $K_2 = 0.5$, $\alpha_2 = 1.5$, and, obviously,

$$\alpha_i > K_i, \quad i = 1, 2. \quad (53)$$

However,

$$r_1\alpha_1 = 1.5 > 1, \qquad r_2\alpha_2 = 1.5 > 1. \quad (54)$$

Hence, condition (H_2) in Theorem 1 could not be satisfied, and Theorem 1 could not be applied to this example. However, numeric simulation (Figure 2) also shows that system (52) admits a unique globally attractive positive equilibrium.

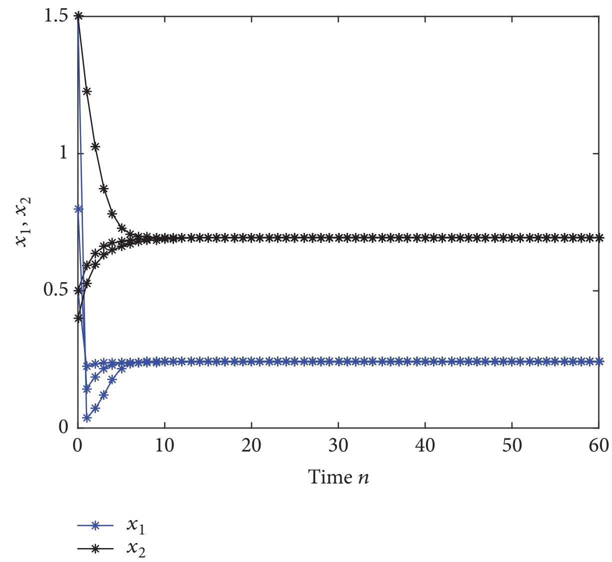

Figure 1: Dynamic behaviors of the solution $(x_1(n), x_2(n))$ of system (49), with the initial conditions $(x_1(s), x_2(s)) = (0.8, 0.4), (0.5, 0.5)$, and $(1.5, 1.5)$, $s = \cdots, -n, -n+1, \ldots, -1, 0$, respectively.

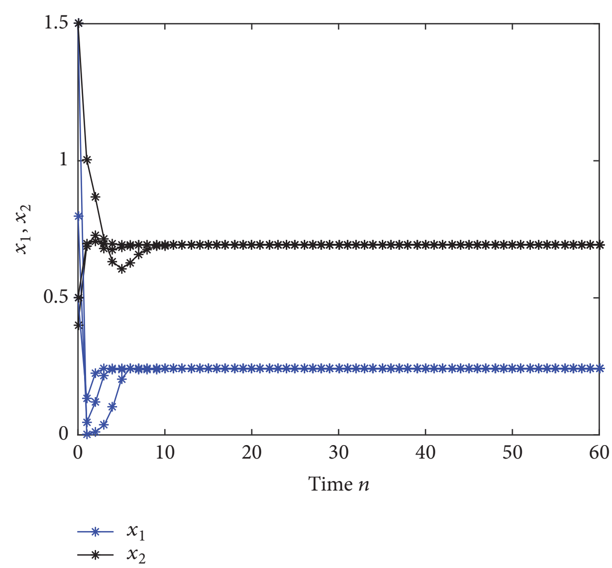

Figure 2: Dynamic behaviors of the solution $(x_1(n), x_2(n))$ of system (52), with the initial conditions $(x_1(s), x_2(s)) = (0.8, 0.4), (0.5, 0.5)$, and $(1.5, 1.5)$, $s = \cdots, -n, -n+1, \ldots, -1, 0$, respectively.

5. Discussion

In [6], Yang et al. proposed system (6); under the assumption $\alpha_i > K_i$, $i = 1, 2$, they showed that if $r_i\alpha_i \leq 1$, then the mutualism model admits a unique globally asymptotically stable positive equilibrium.

In this paper, we try to incorporate the infinite deviating arguments, and, by developing the analysis technique of Yang et al. [6] and using the difference inequality of Chen [7], we also obtain the sufficient conditions which ensure the global attractivity of the positive equilibrium. Example 1 shows the feasibility of our main result.

Since condition (H_2) is the most important restriction on the coefficients of the system, one interesting issue is whether the result of Theorem 3 could hold if (H_2) is not satisfied. Example 2 shows that our result (Theorem 3) still have room to improve. We leave this for future investigation.

Competing Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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