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Research Article

A Queuing Network Based Optimization Model for Calculating Capacity of Subway Station

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Capacity of subway station is an important factor to ensure the safety and improve the transportation efficiency. In this paper, based on the M/G/C/C state-dependent queuing model, a probabilistic selection optimization model is proposed to assess the capacity of the station. The goal of the model is to maximize the output rate of the station, and the decision variables of the model are the selection results of the passengers. Finally, this paper takes a subway station of Shanghai Metro as a case study and calculates the optimal selection probability. The proposed model could be used to analyze the average waiting time, congestion probability, and other evaluation indexes; at the same time, it verifies the validity and practicability of the model.

1. Introduction

Subway station plays an important role in gathering and switching passengers. Especially during peak hours, passengers may occupy the station in high densities, which has become normal phenomenon recently. As we known, high density may cause unpredictable accident, such as stampede. Therefore, how to define and calculate the capacity of subway station (CSS) has attracted more and more scholars' attention.

Over the past few decades, there have been many literatures related to CSS. The research can be classified into two categories according to the method: simulation and mathematic model. Simulation is used to describe passenger's route choice behavior under different scenarios. For example, Kaakai et al. [1] proposed a hybrid Petri nets based simulation model to discuss evaluation procedures. Ashel and Shields [2] developed a simulation model to analyze the evacuation capacity under the scenario that passengers cannot receive any information. Moreover, some researchers focus on the capacity of single facility. Crowding at a subway station's staircase was studied by Jiang et al. [3], who compared evacuation time performed by EXODUS software with real observation. Lam and Cheung [4] proposed a simulation model to analyze the relationship between pedestrian walking speed and passenger density for walking facilities in Hong Kong. These studies focus on how to simulate the features of passengers and evaluate the capacity of subway station in a microscopic aspect. However, little detailed attention has been paid to the optimal routing which could maximize the inbound capacity of subway station.

Mathematic model is usually designed from macroscopic aspect. Most of the existing literatures consider inner structure of subway station as a crucial factor to the capacity. For example, Xu [5] developed an optimization analytic model based on a given station topology. Pedestrian flow characteristics based on facility structure are studied by Yang [6]. Kittelson and Associates [7] assessed the evacuation ability of the stairs and corridors of the subway station based on the queuing model. He et al. [8] proposed a gray decentralized clustering based passenger flow routing optimization model for rail transit station. Shan et al. [9] constructed a congestion intensity discriminant model based on cumulative logistic regression. These studies define the subway station's capacity as the sum of each element (facility). However, these facilities are not mutually independent. In fact, passenger density in subway station is not balanced. For example, some escalators are crowded by passengers, while other escalators are occupied by few passengers. Therefore, it

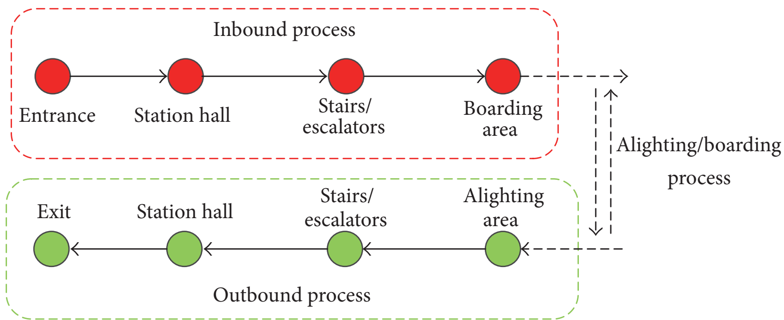

Figure 1: Inbound and outbound process at the subway station.

is obvious to note that the capacity of subway station is not only related to the layout of facilities and equipment, but also closely related to the passenger organization strategy.

Thus, in order to analyze the capacity of subway station, the relationship between passenger route choice and inner structure of subway station was firstly determined. This paper presents a novel definition of CSS and a queuing network based optimization model was proposed. Finally, a case study using real data from Shanghai Metro is developed to verify the validity and feasibility of this method.

The rest of this paper is organized as follows. Section 2 provides a detailed analysis of inbound and outbound process of subway station. And a definition of CSS is proposed. Section 3 presents a queuing network based optimization model. In Section 4, a case study with real data is provided to testify the feasibility and efficiency of the proposed model. Section 5 presents conclusion and future research topics.

2. Definition of CCS

Currently, there is no unified definition of CSS. Different organizations have different interpretations. For example, CSS defined by Transportation Research Board (TRB) of United States is “the maximum number of passengers passing or occupying the equipment under normal circumstances” [10]. Moreover, according to China’s “Code for design of Metros,” CSS is defined as “the maximum passing capacity, that is, regardless of the passenger characteristics and the safe area to be distinguished, only the maximum passing capacity is considered.”

As we know, CSS is related to three subprocesses: inbound process, outbound process, and alighting/boarding process (as shown in Figure 1). The inbound process contains three key facilities: station hall, stairs (or escalators), and platform. Passengers enter the entrance gates and go through the station hall, stairs, and platform, and the end is when they wait at the boarding area. Outbound process is similar to the inbound process but different in direction. Note that alighting/boarding process obeys “first alighting, then boarding” principle.

The number of passengers that can be served by subway station contains two categories: inbound passengers and outbound passengers. Therefore, the number of passengers passing through the subway station in a unit time t can be expressed as

$$Z = \sum_t \left(N_I^t + N_o^t\right), \tag{1}$$

where N_I^t indicates the number of inbound passengers by unit time t and N_o^t is the number of outbound passengers by unit time t. Time period t is generally one hour, half an hour, fifteen minutes, or five minutes, and so on.

When travelling in the station, passengers will be affected by the layout of facilities, passenger flow organization, station capacity, and other factors. Therefore, it is necessary to clarify the various constraints. First of all, the number of passengers passing per unit time cannot exceed the station’s demand.

$$\begin{gathered}\sum_t N_I^t \leq \sum_t D_I^t, \\ \sum_t N_O^t \leq \sum_t D_O^t,\end{gathered} \tag{2}$$

where D_I^t, D_O^t indicate inbound demand and outbound demand, respectively. According to China’s urban rail transit passenger flow organization principle, passengers who are leaving will not be affected by any factors. That is, the number of outbound passengers served by subway station is equal to the demand:

$$\sum_t N_T^t \approx \sum_t D_T^t. \tag{3}$$

Therefore, this paper defined CSS as the maximum inbound passengers that can be served by one subway station in a given time period without any unpredicted incident such as train delay and fire.

3. Model Formulation

3.1. Assumptions. To facilitate problem formulation, we make the following assumptions:

(1) Alighting-boarding process in the station obeys “first-alighting-then-boarding” principle.

(2) There are inbound passengers at stairs, escalators, and other facilities as an M/G/C/C state-dependent queue model.

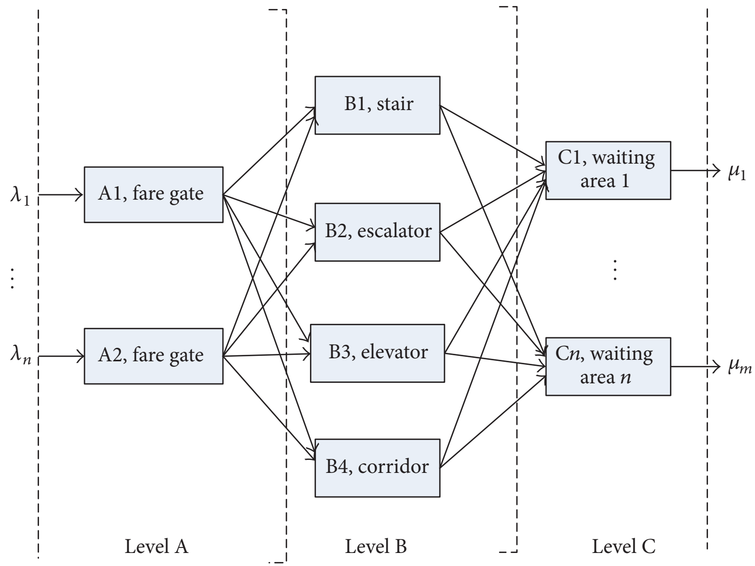

Figure 2: The queuing network structure of passenger inbound process.

(3) There is no passenger control strategy in the station.

(4) All passengers could board the train which is arriving at the platform.

(5) The order of inbound process must be followed.

3.2. Model. According to the definition of CSS in the previous section, it can be seen that CSS is related to inner structure and arrangement of the facilities. Therefore, subway station is modeled as a queuing network system that contains n inputs and m outputs (as shown in Figure 2). Input represents passenger arrival rate, and output represents passenger boarding rate. Note that total input rate is different from output rate because of the congestion and imbalance phenomenon. Therefore, the objective function can be constructed as follows:

$$\max Z = \sum_{i=1}^{m} \mu_i, \tag{4}$$

where μ_i represents the output rate of the ith terminal facility.

In addition, according to different levels in the subway station, this queuing network model can be divided into several submodels. For example, as shown in Figure 2, facilities A1 and A2 consist of level A which corresponds to submodel A. For the same reason, submodel B is corresponding to level B. Moreover, it is important to note that output of submodel A is input of submodel B and output of submodel B is input of submodel A. In the following, how to develop the queuing network model is discussed.

In this paper, a traditional single queuing model is extended to a network queuing model. Firstly, topology of the subway station is modeled. Then, connection structure among the fare gates, escalators, stairs, corridors, platforms, and other facilities is established.

The process between entering the fare gate and boarding the train is called delivery process. Passengers in this process will go through several levels of selection process. For example, passengers can choose stairs, escalators, or elevator after entering the station. After they arrive at the platform, they could choose waiting area 1 or 2 and so on.

According to assumption 5, passengers will not return to the station hall from platform. The sum of output selection probabilities of each facility is 1; for example, the sum of the probabilities of selecting facilities B1, B2, B3, and B4 from A1 in level A is 1. This can be formulated as follows:

$$\sum_{i} p_i^r = 1, \tag{5}$$

where p_i^r indicates the probability that the passenger who has been served in facility r will select the ith facility of the next level, and the value range is $[0, 1]$.

For any facility in the network structure model, during the process of walking or queuing, the speed of the passenger is affected by the gathering density. Therefore, this paper uses the state-dependent M/G/C/C queuing model to describe the queuing process of the nodes of the facilities. The probability of having n passengers in a node of the queuing system can be expressed as

$$P_n = \left\{ \frac{[\lambda E(T_1)]^n}{n! f(n) f(n-1) \cdots f(1)} \right\} P_0, \tag{6}$$

where λ is the passenger arrival rate and $E(T_1)$ indicates the expected service time for a single passenger, that is, the time spent by a passenger in the service of facility. $f(n)$ means the passenger service rate; it refers to the ratio of the passing speed of n passengers in the facility to the speed of the individual. P_0 represents the probability of no passengers in

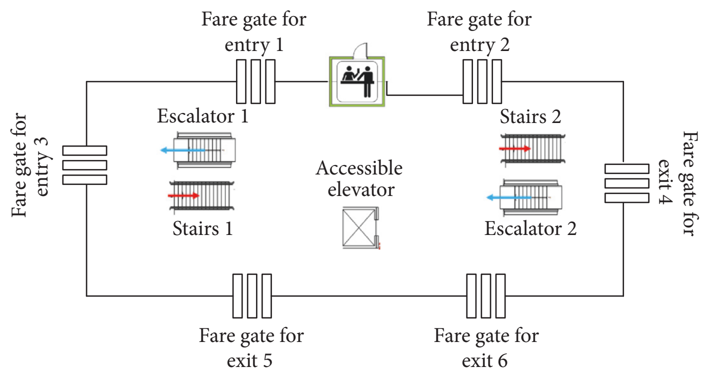

FIGURE 3: The layout of the station hall in QiBao station.

the facility, which can be calculated according to the following formula:

$$P_0 = \left[1 + \sum_{n}^{C} \frac{[\lambda E(T_1)]}{n! f(n) \cdots f(1)}\right]^{-1}, \quad (7)$$

where C represents the capacity of the facility. For stairs or escalators, the capacity can be obtained by being multiplied by the maximum density. For the fare gates and elevator, the maximum queue length can be set in advance.

In crowded conditions, the relationship between passenger speed and density is closely related to the factors such as travel destination, personal attributes, and travel time. In general, the greater the passenger flow density, the smaller the speed of movement. When the density reaches a certain limit, the passenger flow will not be able to move. This paper has analyzed the video data of the Shanghai subway station; it is found that pedestrian moving speed-density feature is similar to the classical BPR model in road traffic. Therefore, this paper uses the exponential speed-density function to describe the data:

$$V_n = V_1 \exp\left[-\left(\frac{n-1}{\beta}\right)^{\gamma}\right]. \quad (8)$$

In this formula, V_1 is free flow velocity of the passenger, V_n is average moving speed of the n passengers in the facilities, and γ and β can be calculated, respectively, by using the following formula:

$$\gamma = \frac{\ln\left[\ln\left(V_a/V_1\right)/\ln\left(V_b/V_1\right)\right]}{\left[\ln\left((a-1)/(b-1)\right)\right]},$$
$$\beta = \frac{(a-1)}{\left[\ln\left(V_1/V_a\right)\right]^{1/\gamma}}, \quad (9)$$

where V_a represents the passengers speed with maximum passing capacity and V_b is twice V_a, $a = S \cdot V_a$, and $b = S \cdot V_b$.

For any facility in the network structure diagram, when its service number exceeds the system service capacity, congestion will occur. And the probability of the occurrence can be calculated by using the following formula:

$$p^r = p\{N = c\}, \quad \forall r, i. \quad (10)$$

p^r is the probability of occurring congestion in facility r. For any facility r in the system, its output rate can be expressed as

$$\mu^r = \lambda\left(1 - p^r\right). \quad (11)$$

Considering the service strength of each facility in the network, the number of passengers expected to be served at any node must be within a reasonable range:

$$\sum_{i=1}^{C} i p_i^r \le U^r, \quad (12)$$

where U^r indicates the maximum number of people waiting for service r.

In this model, formula (4) is objective function, which is to maximize the sum of the output rate. p_i^r is a decision variable which indicates the probability that the passengers exiting any facility r select ith facility of next level. Formulae (5)–(12) are the constraints of the model. It can be seen that this model is a typical nonlinear programming model, and traditional algorithm is difficult to calculate. This paper uses C# and makes use of CPLEX toolkit to solve it.

4. Case Study

The proposed case study is a typical station (QiBao station) in Shanghai subway system located in line 9. Figure 3 shows the layout of the station hall, which contains three fare fates for entry, two escalators, two stairs, and one elevator. Figure 4 shows the layout of the platform. The yellow area represents the passenger's waiting area. For the sake of convenience, we only consider six waiting areas at up direction. Each fare gate has five devices, and the passenger volume for entry at peak hours is shown in Table 1. In this example, the width, length, and lifting speed of the escalator are set as 1 m, 10 m, and 0.7 m/s, respectively. The width and length of the stairs are set as 3 m and 15 m, respectively. Service rate of the elevator is 0.8/s, and the maximum number of people who are waiting for service is set as 200.

Figure 5 shows the queuing network of QiBao station. Passengers arriving at the platform will choose the appropriate waiting area to wait for the train. This paper assumes

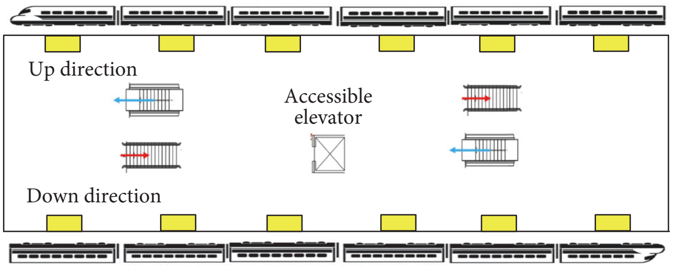

Figure 4: The layout of the platform in QiBao station.

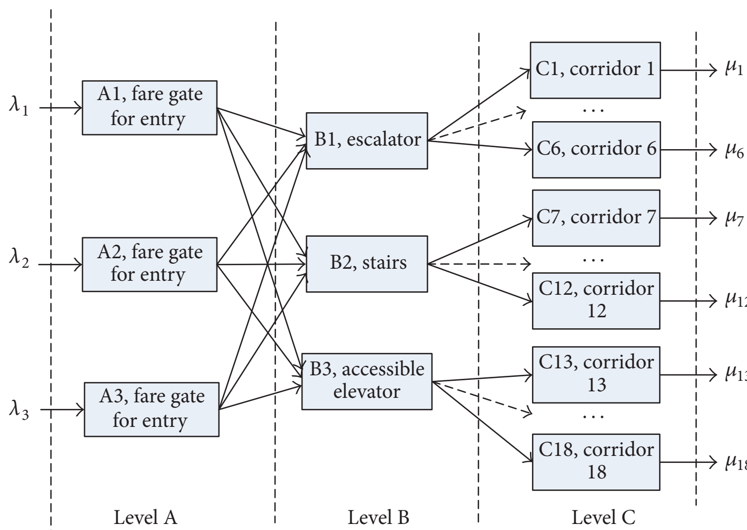

Figure 5: Queuing network of QiBao station.

that the transport capacity of the train can fully meet the need of passengers, which means that, in the waiting area, the passengers can board the train the first time without waiting for the next. Since the passengers arrive at each of the waiting areas of layer B of different travel distance, we assume that the passengers arrive at this kind of area through the "virtual channel" of different lengths. The geometrical dimensions of the channel are shown in Table 2.

According to Table 1, the input rates of the three fare gates are 1.30 persons/second, 0.91 persons/second, and 1.00 person/second. According to the input rate, the optimal probability can be found out. The calculation results are shown in Table 3

It can be seen from Table 3 that the probability that passengers who pass each facility or equipment in level A choose B2 in level B is more than 50%. In order to improve the capacity, the staircase capacity should be fully utilized. In the actual process of organizing passenger flow, for improving the capacity of the station, the staff should use the railings and markings to guide more passengers to choose the stairs. Table 4 shows the congestion probability of each facility and equipment, and it can be seen that the waiting time of the facility at level B is the longest, which is the "bottleneck" limiting the carrying capacity of the station. Therefore, during peak hours, it is recommended that station can increase the capacity by making a unified organization plan on escalators, stairs, and other facilities.

5. Conclusion

Based on the analysis of the characteristics of passenger receiving service from equipment in subway station, this paper constructs a network queuing model based on M/G/C/C. The model takes the maximum output rate of the station as the objective function, and the decision variable is the probability of selection of the service flow line by passengers. At the same time, it takes a case analysis of the Shanghai subway station as the background and calculates the optimal output selection probability of each station. Evaluation criteria such as average waiting time and congestion probability are analyzed, and the validity and practicability of the model are verified.

The study found that stairs and escalators are still the "bottlenecks" which determine the carrying capacity of subway stations. Adjusting the protection of flow line of the escalators, the stairs can be a good way to reduce the probability of equipment congestion and can reasonably cope with the impact of the tide of high passenger flow on the

TABLE 1: The arrival rate of passengers entering QiBao station in peak hours.

	Facility number	The number of people entering the station	Arrival rate
Fare gate for entrance 1	G101	992	1.3 persons/s
	G102	820	
	G103	982	
	G104	921	
	G105	991	
Fare gate for entrance 2	G201	753	0.91 persons/s
	G202	474	
	G203	692	
	G204	681	
	G205	691	
Fare gate for entrance 3	G301	873	1.00 person/s
	G302	621	
	G303	691	
	G304	721	
	G305	711	

TABLE 2: Geometric dimensions of the virtual corridors.

Virtual corridor	Length (m)	Width (m)
Corridor 1	10	1
Corridor 2	8	1
Corridor 3	12	1
Corridor 4	12	1
Corridor 5	8	1
Corridor 6	8	1
Corridor 7	5	1
Corridor 8	5	1
Corridor 9	5	1
Corridor 10	5	1
Corridor 11	12	1
Corridor 12	12	1
Corridor 13	9	1
Corridor 14	9	1
Corridor 15	14	1
Corridor 16	14	1
Corridor 17	6	1
Corridor 18	6	1

station. The proposed network model can provide an effective means for quantitative analysis of station carrying capacity. But if it is applied to the actual operation and management,

TABLE 3: Optimal selection probability.

Selection probability	Value
A1 → B1	0.231
A1 → B2	0.535
A1 → B3	0.234
A2 → B1	0.212
A2 → B2	0.637
A2 → B3	0.151
A3 → B1	0.231
A3 → B2	0.548
A3 → B3	0.221
B1 → C1	0.261
B1 → C2	0.101
B1 → C3	0.182
B1 → C4	0.11
B1 → C5	0.09
B1 → C6	0.256
B2 → C1	0.212
B2 → C2	0.15
B2 → C3	0.13
B2 → C4	0.15
B2 → C5	0.147
B2 → C6	0.211
B3 → C1	0.198
B3 → C2	0.158
B3 →C3	0.143
B3 → C4	0.149
B3 → C5	0.154
B3 → C6	0.198

Table 4: Calculation of facilities and equipment.

Facility	Average waiting time	Congestion probability
Fare gate for entry 1	2.066	0.195
Fare gate for entry 2	1.508	0.184
Fare gate for entry 3	1.908	0.167
Escalator	7.188	0.553
Stairs	3.098	0.350
Elevator	5.331	0.401
Corridor 1	1.501	0.095
Corridor 2	1.503	0.090
Corridor 3	1.511	0.085
Corridor 4	1.608	0.075
Corridor 5	1.548	0.070
Corridor 6	1.118	0.077
Corridor 7	1.981	0.074
Corridor 8	2.011	0.073
Corridor 9	1.999	0.078
Corridor 10	2.113	0.090
Corridor 11	1.430	0.091
Corridor 12	1.406	0.099
Corridor 13	1.876	0.089
Corridor 14	1.770	0.088
Corridor 15	1.734	0.081
Corridor 16	1.777	0.099
Corridor 17	1.234	0.093
Corridor 18	1.654	0.086

it is necessary to study its matching with the line transport capacity.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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