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Research Article

Finite-Time Bounded Synchronization of the Growing Complex Network with Nondelayed and Delayed Coupling

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The objective of this paper is to discuss finite-time bounded synchronization for a class of the growing complex network with nondelayed and delayed coupling. In order to realize finite-time synchronization of complex networks, a new finite-time stable theory is proposed; effective criteria are developed to realize synchronization of the growing complex dynamical network in finite time. Moreover, the error of two growing networks is bounded simultaneously in the process of finite-time synchronization. Finally, some numerical examples are provided to verify the theoretical results established in this paper.

1. Introduction

In recent years, as finite-time control is an important indicator of test control of complex networks, which not only has important theoretical significance, but also has important practical value in practical engineering, finite-time control of complex networks has become one of the most interesting subjects in control theory of complex networks. To get fast convergence speed, much finite-time stabilization theory is proposed to investigate finite-time control of complex networks [1–10]. For example, in [11], Shen et al. investigated finite-time synchronization control of uncertain Markov jump neural networks by input constraints. In [12], Zhang et al. discussed finite-time stabilization of time-varying nonlinear systems by state feedback control. In [13], Wang et al. discussed finite-time global synchronization of Markovian jump complex networks by partially unknown transition rates with feedback control. In [14], Liu et al. considered finite-time synchronization of Markovian switching neutral complex networks based on pinning controller. In [15], Shi studied finite-time control of linear systems with time-varying sampling, and so on. It is noticed that most of the studies on finite-time synchronization of dynamical network have been mainly focused on static networks.

In addition, as bounded control of complex system is relevant in practical applications, for example, power networks cannot achieve complete synchronization, so it is desirable to obtain conditions within given bound such that the rotor phase differences between the generators remain [16]. In recent years, bounded control of complex dynamical network concerns the existence of bounded synchronization region globally stabilizing the complex system, which has grown very quickly, and it is one of the important issues in control theory and applications [17–21].

In [1–17], the authors studied the numbers of the nodes in the networks invariant about synchronization of complex dynamical networks. As the structures of many complex systems are typically dynamic, some new nodes can enter the network as time goes on [22]. Therefore, how to achieve synchronization of a growing dynamical network with bounded error is a very interesting and indeed important subject for research. Motivated by the existing works, the aim of this paper is to discuss finite-time synchronization of the growing complex dynamical network with nondelayed and delayed coupling. A new finite-time stabilization theory is proposed to investigate finite-time bounded synchronization of complex dynamical networks. Finite-time stabilization theory presented in this paper extends the conclusions of literatures [11–15] using finite-time stabilization theory.

The rest of this paper is organized as follows: Section 2 presents model and preliminaries. Section 3 gives the sufficient conditions of finite-time bounded synchronization. Section 4 presents an example and relates simulation results. Section 5 gives the conclusions of this paper.

2. Model and Preliminaries

Firstly, we introduce the network model and give some useful mathematical preliminaries.

Considering the growing complex dynamical network consisting of N_t identical coupled nodes at time t, with each node being an n-dimensional dynamical system. The state equations of complex dynamical network can be described by

$$\frac{dx_i(t)}{dt} = f(x_i(t),t) + c_1\sum_{j=1}^{N_t} a_{ij}(t)\,x_j(t) + c_2\sum_{j=1}^{N_t} b_{ij}(t)\,x_j(t-\tau), \quad i = 1,2,\ldots,N_t, \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T$, $f(x_i(t),t)$ standing for the activity of an individual subsystem is a vector value function, and c_1 and c_2 are two coupling strengths of the network. If there exists a link from node i to j $(i \neq j)$ at time t, and $a_{ii}(t) = -\sum_{j=1,j\neq i}^{N_t} a_{ij}(t)$, $b_{ii}(t) = -\sum_{j=1,j\neq i}^{N_t} b_{ij}(t)$.

Then the controlled dynamical network can be

$$\frac{dy_i(t)}{dt} = f(y_i(t),t) + c_1\sum_{j=1}^{N_t} a_{ij}(t)\,y_j(t) + c_2\sum_{j=1}^{N_t} b_{ij}(t)\,y_j(t-\tau) + u_i(t). \tag{2}$$

So, the error equation is

$$\frac{de_i(t)}{dt} = f(y_i(t),t) - f(x_i(t),t) + c_1\sum_{j=1}^{N_t} a_{ij}(t)\,e_j(t) + c_2\sum_{j=1}^{N_t} b_{ij}(t)\,e_j(t-\tau) + u_i, \tag{3}$$

where $e_i(t) = (e_{i1}(t), e_{i2}(t), \ldots, e_{in}(t))^T$, $e_i(t) = y_i(t) - x_i(t)$.

We give the following preliminaries for obtaining the main result.

Assumption 1. We assume that f is Lipschitz with respect to its argument; that is,

$$\| f(y_i(t)) - f(x_i(t)) \| \leq \mu (y_i(t) - x_i(t)), \quad \mu \in R^+. \tag{4}$$

Lemma 2 (see [23]). *The following matrix inequality holds: $2x^Ty \leq x^TQx + y^TQ^{-1}y$ for any vectors $x, y \in R^m$ and positive-definite matrix $Q \in R^{m\times m}$. If not specified otherwise, inequality $Q > 0$ $(Q < 0,\ Q \geq 0,\ Q \leq 0)$ means Q is a positive (or negative, or semipositive, or seminegative) definite matrix.*

Lemma 3 (see [24]). *Let $a_1, a_2, \ldots, a_n > 0$ and $0 < r < p$. Then $(\sum_{i=1}^n a_i^p)^{1/p} \leq (\sum_{i=1}^n a_i^r)^{1/r}$.*

Lemma 4 (see [25]). *Given the following dynamical system*

$$\dot{x} = f(x(t)), \quad f(0) = 0,\ x \in R^n,\ x(0) = x_0. \tag{5}$$

Assume that a continuous and positive-definite $V(x)$ satisfies the following differential inequality:

$$\dot{V}(x) \leq -lV(x) - kV^{\eta}(x), \quad l > 0,\ k > 0,\ 0 < \eta < 1. \tag{6}$$

Then, the origin of the dynamical system (5) is finite-time stable. The settling time $T_x(x_0)$ satisfies

$$T_x(x_0) \leq \frac{1}{l(1-\eta)} \ln\left(1 + \frac{l}{k}V^{1-\eta}(x_0)\right). \tag{7}$$

In this paper, to illustrate finite-time bounded stability further, we present the following theorem.

Theorem 5. *Suppose that there exist continuous and positive-definite $V(x)$, $l > 0$, $0 < \eta < 1$, $k > 0$, and $0 < \beta < \infty$ such that*

$$\dot{V}(x) \leq -lV(x) - kV^{\eta}(x) + \beta. \tag{8}$$

Then, the trajectory of system (5) is finite-time stable, and

(i) *if $V^{\eta}(x) > \beta/(1-\theta)k$, then $T \leq (1/l(1-\eta))\ln(1 + (l/\theta_0 k)V^{1-\eta}(x_0))$, $0 < \theta \leq 1$, $0 < \theta_0 < 1$;*

(ii) *if $V(x) > \beta/(1-\theta)l$, then $T \leq (1/l\theta_0(1-\eta))\ln(1 + (l\theta_0/k)V^{1-\eta}(x_0))$, $0 < \theta \leq 1$, $0 < \theta_0 < 1$.*

Proof. (i) Obviously, there are $0 < \theta \leq 1$ such that inequality (8) can be

$$\dot{V}(x) \leq -lV(x) - \theta kV^{\eta}(x) - (1-\theta)kV^{\eta}(x) + \beta. \tag{9}$$

When $V^{\eta}(x) > \beta/(1-\theta)k$, $\dot{V}(x) \leq -lV(x) - \theta kV^{\eta}(x)$. According to Lemma 4, the decrease of $V(x)$ in finite time drives the trajectories of the dynamical system into $V^{\eta}(x) \leq \beta/(1-\theta)k$. So, the trajectories of the dynamical system are bounded in finite time as

$$\lim_{\theta\to\theta_0} x \in \left(V^{\eta}(x) \leq \frac{\beta}{(1-\theta)k}\right), \quad 0 < \theta_0 < 1. \tag{10}$$

And the time needed to reach (10) is bounded as

$$T \leq \frac{1}{l(1-\eta)} \ln\left(1 + \frac{l}{\theta_0 k}V^{1-\eta}(x_0)\right). \tag{11}$$

(ii) Similarly to (i), there are $0 < \theta \leq 1$ such that inequality (8) can be

$$\dot{V}(x) \leq -\theta lV(x) - kV^{\eta}(x) - (1-\theta)lV(x) + \beta. \tag{12}$$

When $V(x) > \beta/(1-\theta)l$, $\dot{V}(x) \leq -\theta l V(x) - kV^{\eta}(x)$. Therefore, the decrease of $V(x)$ in finite time drives the trajectories of the dynamical system into $V(x) \leq \beta/(1-\theta)l$, and the trajectories of the dynamical system are bounded in finite time as

$$\lim_{\theta\to\theta_0} x \in \left(V(x) \leq \frac{\beta}{(1-\theta)l}\right), \quad 0 < \theta_0 < 1. \tag{13}$$

So, the time needed to reach (13) is bounded as

$$T \leq \frac{1}{l\theta_0(1-\eta)} \ln\left(1 + \frac{l\theta_0}{k} V^{1-\eta}(x_0)\right). \tag{14}$$

□

Remark 6. Let $V(x(t)) = (1/2)e^T(x(t))e(x(t))$, based on Theorem 5, we have the following conclusion for the settling time $T_x(x_0)$: (i) $V^{\eta}(x(t)) = ((1/2)\|e(x(t))\|^2)^{\eta} \leq \beta/(1-\theta_0)k$; that is, $\|e(x(t))\| \leq \sqrt{2}[\beta/(1-\theta_0)k]^{1/2\eta}$, so the error $e(x(t))$ is bounded; (ii) $V(x(t)) = (1/2)\|e(x(t))\|^2 \leq \beta/(1-\theta_0)l$, that is, $\|e(x(t))\| \leq \sqrt{2\beta/(1-\theta_0)l}$, so the error $e(x(t))$ is also bounded.

Remark 7. When $\beta = 0$, Theorem 5 is reduced to Lemma 4.

Remark 8. In [26], the authors present new finite-time stability by turning $\dot{V}(t) \leq lV(t) - kV^{\eta}(t)$ into $\dot{V}(t) \leq l(t)V(t) - k(t)V^{\eta}(t)$; the obtained results have less conservatism than the existing one. In this paper, as $\ln(1 + (l/\theta_0 k)V^{1-\eta}(x_0)) \neq \ln(1 + (l/k)V^{1-\eta}(x_0))$, so $T \neq T_x$, that is, the convergence time of Theorem 5 can be different from Lemma 4 by turning $\dot{V}(x) \leq -lV(x) - kV^{\eta}(x)$ into $\dot{V}(x) \leq -lV(x) - kV^{\eta}(x) + \beta$, and β is adjustable parameter.

Remark 9. In [27], the authors present new inequality theorems by turning $\dot{V}(t) \leq -l(t)V(t) + f(t)$ into $\lim_{t\to\infty} V(t) \leq b/a_2$, and $V(t)$ is bounded for $t \to \infty$. In this paper, $V(t)$ is bounded for $t \to T$ by using $\dot{V}(x) \leq -lV(x) - kV^{\eta}(x) + \beta$; T is finite-time.

3. Finite-Time Bounded Synchronization of Complex Networks

In this section, we study finite-time bounded synchronization between two networks. We can give the following main result.

Theorem 10. *If Assumption 1 holds, and there exists*

$$\gamma > \mu + \frac{l}{2} + \frac{c_1 N_t\left(\mu_1^2 \lambda_{\max}\{Q_1\} + \lambda_{\max}\{Q_1^{-1}\}\right)}{2} + \frac{c_2 N_t \mu_2^2 \lambda_{\max}\{Q_2\}}{2}, \quad c_2 = \frac{1}{\phi\left(\|e(t-\tau)\|^2 + \alpha\right)}, \tag{15}$$

where $\alpha > 0$, $c_1 > 0$, $\phi > 0$, $e(t-\tau) = ((e_1(t-\tau))^T, \ldots, (e_{N_t}(t-\tau))^T)^T$, $|a_{ij}(t)| \leq \mu_1 \in R$, and $|b_{ij}(t)| \leq \mu_2 \in R$.

Then complex dynamical networks (1)-(2) can realize finite-time bounded synchronization based on the following controller:

$$u_i = -\gamma e_i(t) - \rho \operatorname{sign}(e_i(t)) |e_i(t)|^{\xi}, \tag{16}$$

where $\rho = k/2^{\eta}$, $\eta = (\xi+1)/2$.

Proof. According to Theorem 5, we only need to prove $\dot{V}(x) + lV(x) + kV^{\eta}(x) \leq \beta$.

Considering the following nonnegative function:

$$V(t) = \frac{1}{2}\sum_{i=1}^{N_t} (e_i(t))^T e_i(t), \tag{17}$$

we have

$$\begin{aligned}
\dot{V}(x) + lV(x) + kV^{\eta}(x) &= \sum_{i=1}^{N_t} (e_i(t))^T \Bigg[f(y_i(t),t) - f(x_i(t),t) + c_1 \sum_{j=1}^{N_t} a_{ij}(t) e_j(t) \\
&\quad + c_2 \sum_{j=1}^{N_t} b_{ij}(t) e_j(t-\tau) - \gamma e_i(t) - \rho \operatorname{sign}(e_i(t)) |e_i(t)|^{\xi} \Bigg] + \frac{l}{2}\sum_{i=1}^{N_t} (e_i(t))^T e_i(t) \\
&\quad + k\left(\frac{1}{2}\sum_{i=1}^{N_t} (e_i(t))^T e_i(t)\right)^{\eta} = \sum_{i=1}^{N_t} (e_i(t))^T \cdot [f(y_i(t),t) - f(x_i(t),t)] \\
&\quad + c_1 \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} a_{ij}(t)(e_i(t))^T e_j(t) + c_2 \sum_{i=1}^{N_t}\sum_{j=1}^{N_t} b_{ij}(t)(e_i(t))^T e_j(t-\tau) - \rho \sum_{i=1}^{N_t} (e_i(t))^T \\
&\quad \cdot \operatorname{sign}(e_i(t)) |e_i(t)|^{\xi} - \gamma \sum_{i=1}^{N_t} (e_i(t))^T e_i(t) + \frac{l}{2} \cdot \sum_{i=1}^{N_t} (e_i(t))^T e_i(t) + k\left(\frac{1}{2}\sum_{i=1}^{N_t} (e_i(t))^T e_i(t)\right)^{\eta}.
\end{aligned} \tag{18}$$

In (18), by using Lemma 2, obviously,

$$\begin{aligned}
&\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} a_{ij}(t)[e_i(t)]^T e_j(t) \\
&\quad \leq \frac{1}{2}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} a_{ij}^2(t)[e_i(t)]^T Q_1 e_i(t) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} (e_j(t))^T Q_1^{-1} e_j(t)
\end{aligned}$$

$$\begin{aligned}
&\leq \frac{N_t\mu_1^2}{2}\sum_{i=1}^{N_t}\left[e_i(t)\right]^T Q_1 e_i(t) \\
&\quad + \frac{N_t}{2}\sum_{j=1}^{N_t}\left(e_j(t)\right)^T Q_1^{-1} e_j(t), \\
&\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} b_{ij}(t)\left[e_i(t)\right]^T e_j(t-\tau) \\
&\leq \frac{1}{2}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t} b_{ij}^2(t)\left[e_i(t)\right]^T Q_2 e_i(t) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left(e_j(t-\tau)\right)^T Q_2^{-1} e_j(t-\tau) \\
&\leq \frac{N_t\mu_2^2}{2}\sum_{i=1}^{N_t}\left[e_i(t)\right]^T Q_2 e_i(t) \\
&\quad + \frac{N_t}{2}\sum_{j=1}^{N_t}\left(e_j(t-\tau)\right)^T Q_2^{-1} e_j(t-\tau),
\end{aligned} \tag{19}$$

where $|a_{ij}(t)| \leq \mu_1 \in R$, $|b_{ij}(t)| \leq \mu_2 \in R$.

So,

$$\begin{aligned}
&\dot{V}(x) + lV(x) + kV^{\eta}(x) \\
&\leq \sum_{i=1}^{N_t}\left(e_i(t)\right)^T\left[f\left(y_i(t),t\right) - f\left(x_i(t),t\right)\right] \\
&\quad + \frac{c_1 N_t\mu_1^2}{2}\sum_{j=1}^{N_t}\left[e_i(t)\right]^T Q_1 e_i(t) + \frac{c_1 N_t}{2} \\
&\quad \cdot \sum_{j=1}^{N_t}\left(e_j(t)\right)^T Q_1^{-1} e_j(t) + \frac{c_2 N_t\mu_2^2}{2} \\
&\quad \cdot \sum_{i=1}^{N_t}\left[e_i(t)\right]^T Q_2 e_i(t) + \frac{c_2 N_t}{2}\sum_{j=1}^{N_t}\left(e_j(t-\tau)\right)^T Q_2^{-1} \\
&\quad \cdot e_j(t-\tau) - \rho\sum_{i=1}^{N_t}\left(e_i(t)\right)^T \operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\xi} \\
&\quad - \gamma\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) + \frac{l}{2}\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) \\
&\quad + k\left(\frac{1}{2}\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t)\right)^{\eta} \leq \mu\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) \\
&\quad + \frac{c_1 N_t\mu_1^2}{2}\sum_{i=1}^{N_t}\left[e_i(t)\right]^T Q_1 e_i(t) + \frac{c_1 N_t}{2} \\
&\quad \cdot \sum_{j=1}^{N_t}\left(e_j(t)\right)^T Q_1^{-1} e_j(t) + \frac{c_2 N_t\mu_2^2}{2} \\
&\quad \cdot \sum_{i=1}^{N_t}\left[e_i(t)\right]^T Q_2 e_i(t) + \frac{c_2 N_t}{2}\sum_{j=1}^{N_t}\left(e_j(t-\tau)\right)^T Q_2^{-1} \\
&\quad \cdot e_j(t-\tau) - \rho\sum_{i=1}^{N_t}\left(e_i(t)\right)^T \operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\xi} \\
&\quad - \gamma\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) + \frac{l}{2}\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) \\
&\quad + k\left(\frac{1}{2}\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t)\right)^{\eta} \leq \mu\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) \\
&\quad + \frac{c_1 N_t\mu_1^2\lambda_{\max}\{Q_1\}}{2}\sum_{i=1}^{N_t}\left[e_i(t)\right]^T e_i(t) \\
&\quad + \frac{c_1 N_t\lambda_{\max}\{Q_1^{-1}\}}{2}\sum_{j=1}^{N_t}\left(e_j(t)\right)^T e_j(t) \\
&\quad + \frac{c_2 N_t\mu_2^2\lambda_{\max}\{Q_2\}}{2}\sum_{i=1}^{N_t}\left[e_i(t)\right]^T e_i(t) \\
&\quad + \frac{c_2 N_t\lambda_{\max}\{Q_2^{-1}\}}{2}\sum_{j=1}^{N_t}\left(e_j(t-\tau)\right)^T e_j(t-\tau) \\
&\quad - \rho\sum_{i=1}^{N_t}\left(e_i(t)\right)^T \operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\xi} \\
&\quad - \gamma\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) + \frac{l}{2}\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t) \\
&\quad + k\left(\frac{1}{2}\sum_{i=1}^{N_t}\left(e_i(t)\right)^T e_i(t)\right)^{\eta}.
\end{aligned} \tag{20}$$

In (20), obviously,

$$\begin{aligned}
&\sum_{i=1}^{N_t}\left(e_i(t)\right)^T \operatorname{sign}\left(e_i(t)\right)\left|e_i(t)\right|^{\xi} \\
&= \sum_{i=1}^{N_t}\left(\left|e_i(t)\right|^{\xi}\right)^T \operatorname{sign}\left(e_i(t)\right) e_i(t) \\
&= \sum_{i=1}^{N_t}\left(\left|e_i(t)\right|^{\xi}\right)^T \left|e_i(t)\right| = \sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left|e_{ij}(t)\right|^{\xi}.
\end{aligned} \tag{21}$$

By Lemma 3, we have

$$\left(\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left|e_{ij}\right|^{\xi+1}\right)^{1/(\xi+1)} \geq \left(\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left|e_{ij}\right|^{2}\right)^{1/2}. \tag{22}$$

Hence,

$$\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left|e_{ij}(t)\right|^{\xi+1} \geq \left(\sum_{i=1}^{N_t}\sum_{j=1}^{N_t}\left|e_{ij}\right|^{2}\right)^{(\xi+1)/2} = \left(\sum_{i=1}^{N_t}(e_i(t))^T e_i(t)\right)^{(\xi+1)/2}. \tag{23}$$

So

$$\begin{aligned}
\dot{V}(x) + lV(x) + kV^{\eta}(x) \leq{}& \mu\sum_{i=1}^{N_t}(e_i(t))^T e_i(t) \\
&+ \frac{c_1 N_t \mu_1^2 \lambda_{\max}\{Q_1\}}{2}\sum_{i=1}^{N_t}[e_i(t)]^T e_i(t) \\
&+ \frac{c_1 N_t \lambda_{\max}\{Q_1^{-1}\}}{2}\sum_{j=1}^{N_t}(e_j(t))^T e_j(t) \\
&+ \frac{c_2 N_t \mu_2^2 \lambda_{\max}\{Q_2\}}{2}\sum_{i=1}^{N_t}[e_i(t)]^T e_i(t) \\
&+ \frac{c_2 N_t \lambda_{\max}\{Q_2^{-1}\}}{2}\sum_{j=1}^{N_t}(e_j(t-\tau))^T e_j(t-\tau) \\
&- \rho\left(\sum_{i=1}^{N_t}(e_i(t))^T e_i(t)\right)^{(\xi+1)/2} - \gamma\sum_{i=1}^{N_t}(e_i(t))^T e_i(t) \\
&+ \frac{l}{2}\sum_{i=1}^{N_t}(e_i(t))^T e_i(t) + k\left(\frac{1}{2}\sum_{i=1}^{N_t}(e_i(t))^T e_i(t)\right)^{\eta} \\
\leq{}& \sum_{i=1}^{N_t}\left[\mu + \frac{l}{2} + \frac{c_1 N_t\left(\mu_1^2\lambda_{\max}\{Q_1\} + \lambda_{\max}\{Q_1^{-1}\}\right)}{2}\right. \\
&\left. + \frac{c_2 N_t \mu_2^2 \lambda_{\max}\{Q_2\}}{2} - \gamma\right](e_i(t))^T e_i(t) \\
&- \rho\left(\sum_{i=1}^{N_t}(e_i(t))^T e_i(t)\right)^{(\xi+1)/2} + k\left(\frac{1}{2}\right. \\
&\left.\cdot\sum_{i=1}^{N_t}(e_i(t))^T e_i(t)\right)^{\eta} + \frac{c_2 N_t \lambda_{\max}\{Q_2^{-1}\}}{2} \\
&\cdot\sum_{j=1}^{N_t}(e_j(t-\tau))^T e_j(t-\tau).
\end{aligned} \tag{24}$$

By choosing appropriate $\gamma > \mu + l/2 + c_1 N_t(\mu_1^2\lambda_{\max}\{Q_1\} + \lambda_{\max}\{Q_1^{-1}\})/2 + c_2 N_t \mu_2^2 \lambda_{\max}\{Q_2\}/2$, $c_2 = 1/\phi(\|e(t-\tau)\|^2 + \alpha)$, $\rho = k/2^{\eta}$, $\eta = (\xi+1)/2$, then, $\dot{V}(x) + lV(x) + kV^{\eta}(x) \leq N_t\lambda_{\max}\{Q_2^{-1}\}\sum_{j=1}^{N_t}(e_j(t-\tau))^T e_j(t-\tau)/2\phi(\|e(t-\tau)\|^2+\alpha) \leq N_t\lambda_{\max}\{Q_2^{-1}\}/2\phi = \beta$. Based on Theorem 5 and Remark 6, the error system is finite-time bounded stability. The proof is completed. □

Remark 5. By Remark 6, if $V^{\eta}(x) > \beta/(1-\theta)k$, then $T \leq (1/l(1-\eta))\ln(1 + (l/\theta_0 k)V^{1-\eta}(x_0))$, $\|e(x(T))\| \leq \sqrt{2}[\beta/(1-\theta_0)k]^{1/2\eta}$; if $V(x) > \beta/(1-\theta)l$, then $T \leq (1/l\theta_0(1-\eta))\ln(1 + (l\theta_0/k)V^{1-\eta}(x_0))$, $\|e(x(T))\| \leq \sqrt{2\beta/(1-\theta_0)l}$.

Remark 6. For the finite time t_0, we can find the appropriate value β by adjusting ϕ value. (i) If $\phi \geq (1-\theta)kN_t\lambda_{\max}\{Q_2^{-1}\}/2V^{\eta}(t_0)$, then $\beta = N_t\lambda_{\max}\{Q_2^{-1}\}/2\phi$ and $V^{\eta}(t_0) > \beta/(1-\theta)k$. (ii) If $\phi \geq (1-\theta)lN_t\lambda_{\max}\{Q_2^{-1}\}/2V(t_0)$, then $\beta = N_t\lambda_{\max}\{Q_2^{-1}\}/2\phi$ and $V(t_0) > \beta/(1-\theta)l$.

Remark 7. In the network, when the number of nodes N_t increases, the value of γ will also increase, and the suitable γ value can always be found.

4. Illustrative Example

In this section, we present an example to illustrate the usefulness of Theorem 10 in this paper.

Consider the following Lü system [28, 29]:

$$\begin{aligned}
\dot{x}^i &= \begin{pmatrix} -36 & 36 & 0 \\ 0 & 20 & 0 \\ 0 & 0 & -3 \end{pmatrix}\begin{pmatrix} x_1^i \\ x_2^i \\ x_3^i \end{pmatrix} + \begin{pmatrix} 0 \\ -x_1^i x_3^i \\ x_1^i x_2^i \end{pmatrix} \\
&= Gx^i + h(x^i).
\end{aligned} \tag{25}$$

Obviously, $\|G\|_2 \approx 52.9843$, and

$$\begin{aligned}
&\left\|h(y^i) - h(x^i)\right\|_2 \\
&\leq \sqrt{(-y_3^i e_1^i - x_1^i e_3^i)^2 + (y_2^i e_1^i + x_1^i e_2^i)^2} \\
&\leq \sqrt{2M_1^2 + M_2^2 + M_3^2}\left\|e^i\right\|_2 \approx 64.6142\left\|e^i\right\|_2.
\end{aligned} \tag{26}$$

It is well known that Lü attractor is bounded. Here we suppose that all nodes are running in the given bounded region [30]. Our theoretical and numerical analyses show that there exist constants $M_1 = 25$, $M_2 = 30$, and $M_3 = 45$ satisfying $\|x_j^i\|, \|y_j^i\| \leq M_j$, $j = 1, 2, 3$, so

$$\left\|f(y^i) - f(x^i)\right\|_2 \leq \mu\left\|e^i\right\|_2 \approx 117.5985\left\|e^i\right\|_2. \tag{27}$$

In numerical simulation, let the initial values of the state variable be rand $[0, 3]$. When the time $0 < t \leq 0.04$, the coupling network with 6 nodes is described by the coupling matrices A, B,

$$A = \begin{pmatrix} -4+\sin t & 1 & 1 & 1 & 1 & -\sin t \\ 1 & -4 & 0 & 1 & 1 & 1 \\ 1 & 0 & -3 & 1 & 1 & 0 \\ 1 & 1 & 1 & -3 & 0 & 0 \\ 1 & 1 & 1 & 0 & -3-\sin t & \sin t \\ -\sin t & 1 & 0 & 0 & \sin t & -1 \end{pmatrix},$$

$$B = \begin{pmatrix} -3-\cos^2 t & 1 & 0 & 2 & 0 & \cos^2 t \\ 1 & -3 & 0 & 1 & 0 & 1 \\ 0 & 0 & -3 & 1 & 1 & 1 \\ 2 & 1 & 1 & -5 & 1 & 0 \\ 0 & 0 & 1 & 1 & -\sin^2 t - 2 & \sin^2 t \\ \cos^2 t & 1 & 1 & 0 & \sin^2 t & -3 \end{pmatrix}. \tag{28}$$

Suppose that there are new nodes entering the network at time $0.04 < t \leq 0.08$, the coupling network with 7 nodes is described by the coupling matrices A', B',

$$A' = \begin{pmatrix} \cos t - 1 & -\cos t & 0 & 2 & 0 & -1 & 0 \\ -\cos t & \cos t & 4 & 0 & 0 & -2 & -2 \\ 0 & 4 & 1 & -2 & -3 & 0 & 0 \\ 2 & 0 & -2 & -4 & 3 & 0 & 1 \\ 0 & 0 & -3 & 3 & -2 & 2 & 0 \\ -1 & -2 & 0 & 0 & 2 & 1+\sin t & -\sin t \\ 0 & -2 & 0 & 1 & 0 & -\sin t & 1+\sin t \end{pmatrix}, \tag{29}$$

$$B' = \begin{pmatrix} -\sin t & \sin t & -2 & 0 & 2 & 0 & 0 \\ \sin t & -1 & 0 & 0 & 0 & 0 & 1-\sin t \\ -2 & 0 & 1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 2 & -4 & 0 & 2 & 0 \\ 2 & 0 & -1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & 0 & -3 & 1 \\ 0 & 1-\sin t & 0 & 0 & -1 & 1 & -1+\sin t \end{pmatrix}.$$

Let $k = l = 1, \alpha = 0.1, c_1 = 1, \tau = 0.05, Q_1 = Q_2 = I, \eta = 0.75$, $\xi = 0.5, c_2 = 1/100(\|e(t-\tau)\|^2 + 0.1), \gamma_1 > 196.0985 + 108c_2$ (network with 6 nodes), $\gamma_2 > 177.5985 + 63c_2$ (network with 7 nodes); let $\gamma = 300 + 108c_2$. Let $E_j(t) = \sqrt{\sum_{i=1}^{N_t} \|y_{ij}(t) - x_{ij}(t)\|^2 / N_t}$ $(j = 1, 2, 3)$, obviously, two networks achieve synchronization when $E_j(t)$ no longer increases. Figure 1 shows the variance of the synchronization errors. Figure 1 shows $E_j(t) \to 0$. Numerical simulation illustrates the effectiveness of Theorem 10.

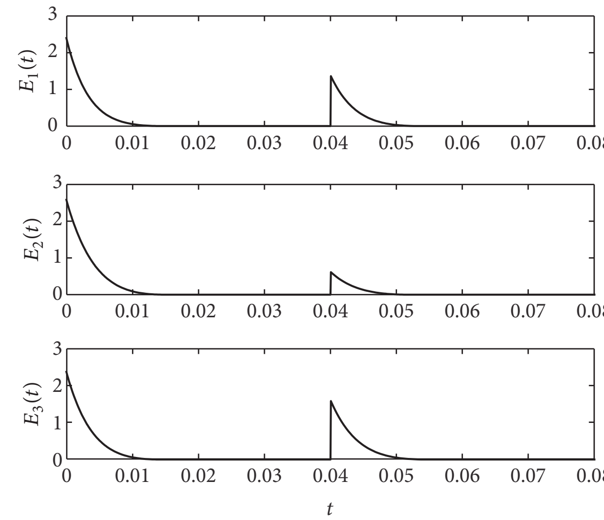

Figure 1: Finite-time bounded synchronization errors.

5. Conclusion

In this paper, finite-time synchronization for a class of the growing complex dynamical network with nondelayed and delayed coupling was investigated. A new finite-time bounded stable theory is proposed; synchronization criterion was derived to ensure the realization of finite-time bounded synchronization. Finally, numerical simulation was given to verify the effectiveness of the proposed schemes.

Competing Interests

The authors declare that they have no competing interests.

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