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Research Article

Improved DEA Cross Efficiency Evaluation Method Based on Ideal and Anti-Ideal Points

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A new model is introduced in the process of evaluating efficiency value of decision making units (DMUs) through data envelopment analysis (DEA) method. Two virtual DMUs called ideal point DMU and anti-ideal point DMU are combined to form a comprehensive model based on the DEA method. The ideal point DMU is taking self-assessment system according to efficiency concept. The anti-ideal point DMU is taking other-assessment system according to fairness concept. The two distinctive ideal point models are introduced to the DEA method and combined through using variance ration. From the new model, a reasonable result can be obtained. Numerical examples are provided to illustrate the new constructed model and certify the rationality of the constructed model through relevant analysis with the traditional DEA model.

1. Introduction

Data envelopment analysis (DEA) is an effective nonparametric statistical method for processing evaluation problems of multiple inputs and outputs. The first DEA model, CCR model, was created in 1978, which has been widely used over these years. People use the DEA method that has been innovated and improved based on its original model to evaluate decision making units (DMU). The formal point is using self-assessment system and getting the best principle which benefits itself to get the efficiency. It is found that many problems occurred in the process of researching the CCR model [1]. Typical ones are as follows: (1) It has low recognition in DMUs whose efficiency value is 1. (2) Evaluations for the same DMU may have multiple weights and efficiency values. (3) Its self-assessment system excessively increases DMUs' outputs and narrows the inputs in order to maximize self-benefits without considering overall efficiency. To solve these problems, many methods have been proposed. The most typical one is called cross efficiency evaluation method, which was put forward by Sexton. The main idea of this method is using general self-assessment and other-assessment to eliminate the problem of easily enlarging its own advantages in the traditional DEA method. A reasonable combination of self-assessment and other-assessment makes the result relatively fair and reasonable. It is verified that the results are more realistic in practical application.

However, the cross efficiency is still not perfect. Scholars have focused on two aspects: one is the selection of competition and cooperation; the other is the aggregation of cross efficiency matrix. Paths for the former one are as follows: (a) Take the original logical relationship system as reference such as benevolent type, aggressive type, and neutral type. (b) Build a new reference system, such as new ideal point. The latter one is aggregated with multiattribute evaluation methods. In practice, different methods should be taken into consideration.

Wu et al. (2009) proposed a neutral cross efficiency model to analyze the relation between decision making units from a neutral perspective. In their neutral model, the efficiency was obtained by maximizing the ratio of the input indexes between one DMU and the combination of other DMUs [2]. Ramón et al. (2010) proposed a two-stage model. It was used to restrain weights to eliminate effects of the weights with nonpractical situation. To avoid the unreasonable weights of cross-efficiency evaluation, they extend the multiplier bound approach to the assessment of efficiency in order to guarantee nonzero weights. In particular, this approach allows inefficient DMUs to make a choice that prevents them from using unrealistic weighting schemes [3]. Wang et al.

(2011), in their study of DEA cross efficiency, combined the ideal point method with the decision makers' preference to research DEA input and output weights from a distance perspective, avoiding the original decision modes of aggressive or benevolent evaluation. Therefore, this new DEA decision making evaluation model is more neutral and logical [4]. Y. M. Wang and S. Wang (2013) found that most cross efficiency evaluations focused on the independence of input and output weights, while few people paid attention to the importance of the combination of cross efficiency and simple polymerization. In their study, they found that calculation with the polymerized cross efficiency method was necessary and the optimal DMU was affected by the optimal cross efficiency weight [5]. Ramón et al. (2014) considered DEA weights of all DMUs to obtain a cross efficiency value. This method prevented selecting from the second-class target level. Each unit had its own cross efficiency. A stable cross efficiency value can be obtained by calculating DEA weights [6]. Cook and Zhu (2014) used a multiplicative DEA model to solve the problem that DEA calculation weights were different and the problem that cost models were not independent and benefit models were not linear. This method efficiently restrained the second-class target level demand [7]. Barzegarinegad et al. (2015) introduced a new view according to ideal and anti-ideal points concept. The new model suggested a comprehensive ranking of DMUs and sought a set of common weights for DMUs to make them fully ranked [8]. Lim and Zhu (2015) used CRS and VRS models to obtain the optimal weight ratio and analyzed with the Cartesian System; then they proposed a cross efficiency evaluation system based on the VRS assumption [9]. Ramalho et al. (2010) reported that, in the general two-stage model, the DEA model was usually combined with the regression analysis. In this book, it mentioned that the traditional method cannot calculate precise date. The author proposed a method of combining part regression analysis and the DEA model. It was proved by combining the two-stage model [10]. Khodabakhshi and Aryavash (2017) presents cross efficiency method using weaknesses and strengths of DMUs by optimistic–pessimistic information, which can fully rank DMUs without any secondary goal [11]. Nasen and Kiaei (2017) introduce a new method which uses the weights resulting from the evaluation of ideal and anti-ideal virtual DMUs, and this method exhibits a new secondary goal that possibly prevents the existence of multiple weights in cross efficiency evaluation [12]. Rakhshan (2017) proposes TOPSIS-DEA for ranking efficient units which include the benefits of both data envelopment analysis and TOPSIS, and it also solves the issues that appear in DEA [13].

In China, scholars also perform the DEA cross efficiency recently. Wu and Liang (2006) analyzed the defects when using the final average cross efficiency to evaluate DMUs. They improved the cross efficiency method with the final cross efficiency weight coefficient by using the cooperative games theory and the coalition games theory [14]. Wang (2009) put forward the cross efficiency evaluation method based on super efficiency DEA model to solve the nonunique problem that the general efficiency evaluation value is 1. The result obtained by this method is the mean value of super efficiency DEA evaluation value, and the comprehensive efficiency evaluation value distribution is more reasonable, which can realize the total ordering of the DMUs [15]. Wang and Wei (2010), based on cross efficiency concept and the principles of self and other-assessment, used different weighting strategies achieving DMU cross efficiency value of three-parameter interval described as the optimal efficiency value, the worst efficiency value, and the most likely efficiency value. Then they used expectation value sorting method of the triangular fuzzy numbers to sort the efficiency value [16]. Yang et al. (2011) proposed a competitive cooperation cross efficiency evaluation method. This method maximized the total efficiency of its allies and minimized the total efficiency of its opponents with the premise that its maximum efficiency is ensured [17]. Zongsheng et al. (2012) confirmed the standard weight of all DMUs by maximum deviation. He obtained a cross efficiency matrix through the cross efficiency calculation and calculated the standard weight of each evaluation unit in the matrix with maximum deviation [18]. Cheng and Yang (2013) combined matrix network unit with the DEA model and proved that the necessary and sufficient condition of the weak DEA efficient DMUs is that all subsystems are weak DEA efficient units. It made up for the internal efficiency defects that the traditional DEA cannot reflect and provided a new way to evaluate the efficiency of complex systems [19]. In order to improve the problem that traditional DEA model generated different performance evaluation results, Xue et al. (2014) proposed the combination of the Gini Principle and DEA model. He used the improved model to accurately confirm the weight and then got the objective and the only comprehensive efficiency. Meanwhile, the model was combined with evaluations from the perspective of evaluators' preference and knowledge to make it more reasonable and effective [20]. Jiasen (2014) proposed a model based on weight balance and the present weight value of effective DMUs to solve the problem that the efficiency values of the same DMU in the cross efficiency model are multiple [21]. The DEA model can only be used to deal with accurate date while it is difficult to obtain direct information and data in the real life. Fang et al. (2015) used error propagation and entropy to process date and obtained the error distribution of the global efficiency of each DMU by using the cross efficiency; then the result was obtained by ranking all DMUs with a directed distance [22]. Liu et al. (2017) regard a DMU as an evaluator and propose the weighted average cross efficiency evaluation model by taking the reliability level as the weight [23]. Liu et al. (2017) proposes a new cross efficiency model in which weights are determined by the two neutral models [24]. Zhang and Gong (2017) propose a multiobjective DEA game cross efficiency model, which is the DEA game cross efficiency model for dynamic change of competitive and cooperative relationships among decision making units [25]. Wu et al. (2017) propose an approach to rank candidates based on DEA game cross efficiency model, in which each candidate is viewed as a player, and the game cross efficiency score is obtained when the DMU's own maximized efficiencies are averaged [26].

After summarizing the study above, this paper uses ideal points and multiattribute evaluations, combining ideal and anti-ideal points as an external reference system in the other-assessment stage to play a supporting role for the efficiency evaluation of all DMUs. The ideal point is based on efficiency and the anti-ideal point is based on fairness. The self-assessment is called upper level system and the model with two ideal points which is the core of the other-assessment is called second-level system. Crucially, the efficiency is still taken as the core of the combination of the DEA and two ideal point models. To achieve the sense of the other-assessment, efficiency and fairness should be focused on the second-level evaluations simultaneously, and we need to aggregate the second-level reference units based on positive ideal and negative anti-ideal points.

The main purpose of this paper is introducing a model to evaluate the DEA efficiency of DMUs. In the new model we tend to suggest the ideal and anti-ideal points. We will propose two point view models, combine them into a new synthesis model, and then calculate the examples by using the new synthesis model of efficiency cross. The rest of this paper is organized as follows. Section 2 briefly introduces the common model of cross efficiency. In Section 3, a new view will be proposed with DEA model and ideal points, in which we will get a comprehensive evaluation model. Section 4 compares the proposed model with the traditional DEA model using two numerical examples. The paper is concluded in the final section.

2. The Model of Cross Efficiency

2.1. The Common CCR Model. DEA is an efficient nonparametric evaluation method for processing evaluation problems of multiple inputs and outputs. After the construction of the first DEA model, CCR model, the DEA method is verified to be an effective efficiency evaluation method.

Assume that there are n DMUs to be evaluated; each DMU_j $(j = 1, 2, \ldots, n)$ has m input indexes $X = (x_1, x_2, \ldots, x_m) \in R_+^m$ and s output indexes $Y = (y_1, y_2, \ldots, y_s) \in R_+^s$, where R_+^m and R_+^s are two sets of nonnegative numbers.

$$\max \quad E_{dd} = \sum_{r=1}^{s} u_{rd} y_{rd} \tag{1}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{i=1}^{m} v_{id} x_{id} = 1 \\ & \sum_{i=1}^{m} v_{id} x_{id} - \sum_{r=1}^{s} u_{rd} y_{rd} \geq 0, \quad j = 1, 2, \ldots, n \\ & u_{rd} \geq 0, \ v_{id} \geq 0, \ r = 1, 2, \ldots, s; \ i \\ & = 1, 2, \ldots, m, \end{aligned} \tag{2}$$

where v_{id} $(i = 1, 2, \ldots, m)$ and u_{rd} $(r = 1, 2, \ldots, s)$ stand for weights of x_{ij} and Y_{ij} in model (2).

2.2. The Cross Efficiency Model. CCR model, as a basic model, can be used to get an optimal target value, but the nonunique optimal target weight will lead to the nonunique model efficiency value. To cover this shortage, there are aggressive and benevolent types of cross efficiency models. The model can be expressed as follows:

$$\theta = \mathrm{opt} \sum_{r=1}^{s} \left(u_{rd} \sum_{j=1, j \neq d}^{n} y_{rj} \right) \tag{3}$$

$$\begin{aligned} \text{s.t.} \quad & \sum_{i=1}^{m} v_{id} \left(\sum_{j=1, j \neq 1}^{n} x_{ij} \right) = 1 \\ & \sum_{r=1}^{s} u_{rd} y_{rj} - E_{dd}^{*} \sum_{i=1}^{m} v_{id} x_{id} = 0 \\ & \sum_{i=1}^{m} v_{id} x_{ij} - \sum_{r=1}^{s} u_{rd} y_{rj} \geq 0 \\ & j = 1, 2, \ldots, n; \ j \neq d \\ & u_{rd} \geq 0, \ v_{id} \geq 0, \ r = 1, 2, \ldots, s; \\ & i = 1, 2, \ldots, m. \end{aligned} \tag{4}$$

The benevolent cross efficiency model is to achieve the maximum objective function; that is,

$$\theta = \max \quad \sum_{r=1}^{s} \left(u_{rd} \sum_{j=1, j \neq d}^{n} y_{rj} \right). \tag{5}$$

The aggressive cross efficiency model is to achieve the minimum objective function; that is,

$$\theta = \min \quad \sum_{r=1}^{s} \left(u_{rd} \sum_{j=1, j \neq d}^{n} y_{rj} \right). \tag{6}$$

Aggressive and benevolent efficiency models have the same constraints (model (4)). However, the optimal objective functions are opposite to each other (model (5) and model (6)), which shows that the evaluation strategies of these two models are different. Aggressive efficiency evaluation model chooses the weights which give the remaining $n - 1$ group evaluation units the minimum efficiency with the premise that its own efficiency value is ensured. Benevolent cross efficiency evaluation model chooses the weights which give the efficiency value of the remaining evaluation units the maximum weight under the premise that it ensures itself the maximum efficiency value. Therefore, the final efficiency values of these two methods will be different to a certain extent.

3. The Proposed Model Based on Ideal and Anti-Ideal Points

DEA cross efficiency is a way to evaluate self and others. It introduces an external reference system in order to prevent

DMUs from excessively paying attention to its own advantages and avoiding disadvantages. The commonly used external reference system is the ideal point method. Chinglai and Kwangsun first put forward the idea of Technique for Order Preference by Similarity to Ideal Solution (TOPSIS), which is the basis of the ideal point model. The center part of the ideal point model is that the best choice from all units is the one which is closer to the ideal point and further from the anti-ideal point [27]. From the TOPSIS, different reference systems may cause different evaluation value orientations. The positive ideal point model, which will be introduced into the aggressive cross efficiency model, is built with the view-point of efficiency. Another new DMU is taken in the negative anti-ideal point model, which will be combined with benevolent efficiency model with the view-point of fairness. Considering both the modified aggressive and benevolent cross efficiency model and combining them together, DMUs may find their own advantage and fully develop themselves. Meanwhile, it will focus on their weakness and narrow the development gap among internal DMUs to get a balanced evaluation. In this paper, we will build a new model combining ideal points models and DEA model and take both efficiency and fairness into consideration.

There are many scholars studying the DEA cross efficiency together with the ideal and anti-ideal points; in addition, they get different method to combine the cross efficiency model and ideal and anti-ideal points models. One of the combination approaches is firstly using cross efficiency model to get the calculation results and then combining the results with ideal and anti-ideal points models, which is just taking the result of ideal and anti-ideal points models, respectively, put forward by Wang and Chin [28]. Barzegarinegad et al. suggest the other approach, which is firstly combining ideal and anti-ideal points and then taking the combination model into cross efficiency model. In this paper, we firstly combine ideal and anti-ideal points with cross efficiency model severally, and then, according to different concept, we form a comprehensive model into the matrixes of the ideal and anti-ideal points model for final evaluation. It is the combination of the evaluation matrixes.

3.1. A New Decision Making Unit. When performing cross efficiency analysis for different decision making units based on the principle of ideal and anti-ideal points, we invent another new DMU to revise its input and output with some characters, and with the different principle we get model (6) and model (7).

Fictitious Decision Making Unit Based on the Ideal Point

$$\mathrm{DMU}_m^+ = \left\{\min^+\left(x_{ij}\right), \max^+\left(y_{rj}\right), j = 1, 2, \ldots, n\right\}. \quad (7)$$

Therein, $X^+ = \{\min(x_{1j}), \min(x_{2j}), \ldots, \min(x_{mj})\}$, $Y^+ = \{\max(y_{1j}), \max(y_{2j}), \ldots, \max(y_{sj})\}$.

Fictitious Decision Making Unit Based on the Anti-Ideal Point

$$\mathrm{DMU}_m^- = \left\{\max^-\left(x_{ij}\right), \min^-\left(y_{rj}\right), j = 1, 2, \ldots, n\right\}. \quad (8)$$

Therein, $X^- = \{\max(x_{1j}), \max(x_{2j}), \ldots, \max(x_{mj})\}$, $Y^- = \{\min(y_{1j}), \min(y_{2j}), \ldots, \min(y_{sj})\}$.

3.2. Ideal and Anti-Ideal Points Model. Through analyzing similarities and differences of aggressive and benevolent cross efficiency models and according to model (4) and the core features of model (5) and model (5), we can get two new models based on the ideal and anti-ideal points.

According to efficiency concept, the efficiency of ideal point can be defined as

$$\begin{aligned}
\min \quad & \theta_+^* = \sum_{r=1}^{s}\left(u_{rd}Y^+\right) \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_{id}X^+ = 1 \\
& \sum_{r=1}^{s} u_{rd}y_{rj} - E_{dd}^*\sum_{i=1}^{m} v_{id}x_{id} = 0 \\
& \sum_{i=1}^{m} v_{id}x_{ij} - \sum_{r=1}^{s} u_{rd}y_{rj} \geq 0 \\
& j = 1, 2, \ldots, n;\ j \neq d \\
& u_{rd} \geq 0,\ v_{id} \geq 0,\ r = 1, 2, \ldots, s; \\
& i = 1, 2, \ldots, m.
\end{aligned} \quad (9)$$

According to this model, we can get an evaluation matrix:

$$\theta_+^* = \begin{bmatrix} \theta_{11}^{+*} & \theta_{12}^{+*} & \cdots & \theta_{1n}^{+*} \\ \vdots & \vdots & \vdots & \vdots \\ \theta_{n1}^{+*} & \theta_{n2}^{+*} & \cdots & \theta_{nn}^{+*} \end{bmatrix}. \quad (10)$$

According to fairness concept, the efficiency of anti-ideal point can be defined as

$$\begin{aligned}
\max \quad & \theta_-^* = \sum_{r=1}^{s}\left(u_{rd}Y^-\right) \\
\text{s.t.} \quad & \sum_{i=1}^{m} v_{id}X^- = 1 \\
& \sum_{r=1}^{s} u_{rd}y_{rj} - \theta_{dd}^*\sum_{i=1}^{m} v_{id}x_{id} = 0 \\
& \sum_{i=1}^{m} v_{id}x_{ij} - \sum_{r=1}^{s} u_{rd}y_{rj} \geq 0 \\
& j = 1, 2, \ldots, n;\ j \neq d \\
& u_{rd} \geq 0,\ v_{id} \geq 0,\ r = 1, 2, \ldots, s; \\
& i = 1, 2, \ldots, m.
\end{aligned} \quad (11)$$

Table 1: Data for seven DMUs with three inputs and three outputs.

DMU	$X1$	$X2$	$X3$	$Y1$	$Y2$	$Y3$
1	12	400	20	60	35	17
2	19	750	70	139	41	40
3	42	1500	70	225	68	75
4	15	600	100	90	12	17
5	45	2000	250	253	145	130
6	18	730	50	132	45	45
7	41	2350	600	305	159	97

We can get another evaluation matrix:

$$\theta_{-}^{*} = \begin{bmatrix} \theta_{11}^{-*} & \theta_{12}^{-*} & \cdots & \theta_{1n}^{-*} \\ \vdots & \vdots & \vdots & \vdots \\ \theta_{n1}^{-*} & \theta_{n2}^{-*} & \cdots & \theta_{nn}^{-*} \end{bmatrix}. \tag{12}$$

The above-mentioned θ is efficiency value; m and s are input and output indexes, respectively; n is the number of decision making units; v and u are input and output weight coefficients.

3.3. Aggregation Methods considering Both Efficiency and Fairness. The ideal point model discussed in this text pursues the maximum output under the minimum investment, which is achieved by improving the aggressive type based on the core of efficiency. The anti-ideal point model is created for companies to avoid pursuing excessive outputs by considering the improved benevolent type on a fair and equitable basis and for restraining them with the maximum input index and the minimum output index. A conventional assessment method is to separate them apart. In this paper, we combine them together, considering both efficiency and fairness to achieve an efficient assessment which does not violate the efficient assessment orientation of the DEA method.

There are many ways to aggregate date. Calculation of the result of the positive ideal and negative anti-ideal points and the general efficient calculation are mentioned in this article. In the general calculation process, there are two methods: weighted arithmetic mean and arithmetic–geometric mean. The notion of weighted mean refers to the fact that the observed value can have a strong impact on the evaluation result; the larger the value is, the deeper the impact will be. And it possesses the complementary or typical values of a set of numbers by using the product of their values. Geometric mean is a tape of mean or average, which indicated the equilibrium tendency of the whole calculation. With a minimized index value, the mean value of the system may equal zero. Features of these two kinds of mean are consistent with the start point and features of positive ideal and negative anti-ideal point. Functionality and equilibrium are the two features of a system operation. By combining these two different ways of calculations with the model calculation, an assessment closer to reality can be easily accepted by people. In summary, an overall efficiency value calculation model can be obtained:

$$\theta_{i\pm}^{*} = \varphi_1 \sum_{j=1}^{n} \frac{\theta_{ij}^{+*}}{n} + \varphi_2 \sqrt[n]{\prod_{j=1}^{n} \theta_{ij}^{-*}}. \tag{13}$$

According to the above-mentioned model, we can obtain the optimal efficiency evaluation vector:

$$\theta^{*} = \left(\theta_1^{*}, \theta_2^{*}, \theta_3^{*}, \ldots, \theta_n^{*}\right). \tag{14}$$

The evaluation recognition can be enhanced by using the ration of each variance and the sum of the variances which determines the weight, so we have

$$\begin{aligned} \varphi_1 &= \frac{\sigma\left(\theta_{+}^{*}\right)}{\sigma\left(\theta_{+}^{*}\right) + \sigma\left(\theta_{-}^{*}\right)}, \\ \varphi_2 &= \frac{\sigma\left(\theta_{-}^{*}\right)}{\sigma\left(\theta_{+}^{*}\right) + \sigma\left(\theta_{-}^{*}\right)}. \end{aligned} \tag{15}$$

From analyzing the problems of the traditional CCR model and researching into the ideal point model, we can achieve the assumption of the new improved model by combining cross efficiency value under the condition of ideal point with that of anti-ideal point.

4. Numerical Example

We now illustrate the applications of the proposed DEA models and ideal and anti-ideal points model using two numerical examples. One is a simple DEA efficiency-rating example, and the other is a complicated performance rating case with China's metal manufacturing industry.

Verification Case. Consider a DEA efficiency evaluation problem with seven departments (DMUs) in a university, each DMU with three inputs and three outputs. The real value is taken from Wang et al. and it is shown in Table 1.

As can be seen from the rating results of Table 1, the CCR model identifies DMU_1 through DMU_3 and DMU_5 through DMU_7 as DEA efficient units, which means they perform equally well. However, it cannot discriminate among them any further. In order to rank the six DEA efficient units, now, we use the proposed DEA models to reevaluate these seven

Table 2: Efficiency rating and the proposed model values for the seven DMUs.

DMU	CCR		Benevolent		Aggressive		Ideal and anti-ideal points	
	Efficiency	Rank	Efficiency	Rank	Efficiency	Rank	Efficiency	Rank
1	1	1	0.9123	3	0.7457	3	0.7473	4
2	1	1	0.9182	2	0.7834	2	0.8399	2
3	1	1	0.7793	6	0.6983	5	0.7210	5
4	0.8197	7	0.5778	7	0.5212	7	0.5769	7
5	1	1	0.8895	5	0.6918	6	0.7157	6
6	1	1	1	1	0.8661	1	0.9095	1
7	1	1	0.8901	4	0.7360	4	0.7666	3

Table 3: DEA efficiency correlation test.

Spearman's rho	Benevolent	Aggressive	Ideal & anti-ideal points
Benevolent			
Correlation	1.000	.964	.929
Sig. (bilateral)		.000	.003
Aggressive			
Correlation	.964	1.000	.964
Sig. (bilateral)			.000
Ideal & anti-ideal points			
Correlation	.929	.964	1.000
Sig. (bilateral)	.003	.000	

DMUs. We calculate the ranking of the respective models under the ideal and anti-ideal points and the comprehensive ranking. The resulting efficiency rating and the ranking are presented in Table 2. By calculation, $\varphi_1 = 0.6758$, $\varphi_2 = 0.3242$.

It is clear from Table 2 that the DEA model based on both ideal and anti-ideal point can evaluate the original six DEA efficient units DMU_1 through DMU_3 and DMU_5 through DMU_7 to be not completely the same. In addition, we take the ration of each variance and the sum of the variances getting the comprehensive efficiency. Based on the approach, the following ranking order can be obtained.

$DMU_6 \succ DMU_2 \succ DMU_7 \succ DMU_1 \succ DMU_3 \succ DMU_5 \succ DMU_4$, where the symbol "$\succ$" means "performs better than." The DEA model based on ideal point assesses that $DMU_5 \succ DMU_3$, however, the DEA model based on anti-ideal point assesses that $DMU_3 \succ DMU_5$. The proposed model is considered both efficient (ideal point) and fair (anti-ideal point); the final ranking is $DMU_3 \succ DMU_5$. All of the three DEA models, which are benevolent DEA model, aggressive DEA model, and ideal and anti-ideal points DEA model, agree that DMU_6 is the best DMU and DMU_4 is the worst DMU.

Through Table 2, we can see that, after the Spearman test, the test values of correlations among benevolent efficiency value, aggressive efficiency value, and general value of ideal and anti-ideal points are all above 0.9. The results of the test can be seen in Table 3.

Application Case. Consider a complicated performance rating case with China's metal manufacturing industry, including twenty-nine companies. It selects the enterprise total assets (X_1), asset-liability ratio (X_2), financial cost ratio (X_3), and financial expense (X_4) as inputs for measuring the efficiency of the metal manufacturing industry in our country finance investment; it takes return on net assets (Y_1), revenue growth rate (Y_2), total assets turnover ratio (Y_3), earnings per share (Y_4), and current ratio (Y_5) as outputs. The raw data is presented in Table 4.

The traditional CCR model evaluates 7 of 29 DMUs to be DEA efficient and cannot distinguish them further, so many DMUs are being rated as DEA efficient. The original intention of ranking the DMUs cannot be realized. The DEA models with benevolent and aggressive are chosen to reevaluate the performance of the 29 DMUs, and we also still use the ideal and anti-ideal point model to get the efficiency of the 29 DMUs. The efficiency of the 29 DMUs with four models is shown in Table 6. Because the CCR model aims to maximize its own efficiency, the results are higher than other models. The efficiency values obtained by the ideal and anti-ideal points model is mostly between the benevolent efficiency and the aggressive efficiency, which shows that the model is efficient and fair. The ideal and anti-ideal points index shows that DMU_{13} has the best overall performance, which is followed by DMU_7, DMU_{10}, and DMU_{27}, while DMU_1 has the worst performance followed by DMU_3, DMU_{28}, and DMU_{25}. The rankings of the 29 DMUs with four models

Table 4: Data for twenty-nine DMUs with three inputs and three outputs.

DMU	X_1	X_2	X_3	X_4	Y_1	Y_2	Y_3	Y_4	Y_5
1	1.0000	0.8615	0.3778	0.6999	0.5401	0.1636	0.3994	0.5530	0.1115
2	0.1325	0.9033	0.4627	0.1934	0.4929	0.3918	0.4147	0.1805	0.1232
3	0.5260	0.8275	0.4582	1.0000	0.3989	0.1566	0.6249	0.1963	0.1138
4	0.1199	0.7958	0.6509	0.1992	0.3455	0.2025	0.3143	0.1000	0.1147
5	0.1653	0.5995	0.5093	0.2311	0.3936	0.1955	0.3140	0.1558	0.1240
6	0.1118	0.2704	0.1000	0.1348	0.4850	0.2541	0.2797	0.2383	0.2412
7	0.1574	0.4337	0.2593	0.1287	0.2352	0.3100	1.0000	1.0000	0.1832
8	0.1103	0.5008	0.3008	0.1525	0.5701	0.2599	0.6326	0.1614	0.1656
9	0.1185	0.4528	0.6124	0.2038	0.4761	0.2053	0.3702	0.1292	0.1595
10	0.1023	0.3989	0.1882	0.1408	0.6753	0.2155	0.6769	0.2640	0.1675
11	0.1256	0.4072	0.3392	0.1645	0.4029	0.2114	0.4979	0.1870	0.1523
12	0.1276	0.5038	0.7720	0.2113	0.4390	0.1738	0.2313	0.1099	0.1267
13	0.1022	0.2820	0.3069	0.1507	0.7561	0.1564	0.4748	0.2319	0.2255
14	0.1065	0.4393	0.4164	0.1575	0.6612	0.2986	0.3056	0.1082	0.1515
15	0.1269	0.6168	0.2408	0.1440	0.4388	0.6869	0.2748	0.3282	0.1115
16	0.1533	0.7862	0.4799	0.2251	0.4298	0.3047	0.4377	0.1163	0.1175
17	0.1179	0.5188	0.2765	0.1484	0.4556	0.2488	0.5382	0.3218	0.1702
18	0.1179	0.3204	0.3163	0.1583	0.4300	0.1782	0.6388	0.2062	0.1769
19	0.1415	0.3499	0.1166	0.1000	0.2643	0.2928	0.3847	0.3925	0.1831
20	0.1257	0.5399	0.5380	0.1764	0.4225	0.5014	0.2477	0.2126	0.1306
21	0.1281	0.3000	0.2011	0.1273	0.3879	0.1333	0.4839	0.4246	0.1924
22	0.1151	0.4305	0.2607	0.1475	0.4632	1.0000	0.2472	0.1998	0.2056
23	0.1066	0.3780	0.3058	0.1502	0.6645	0.3584	0.2691	0.1998	0.1363
24	0.1547	0.8324	0.6759	0.2691	0.3882	0.1245	0.2970	0.5145	0.1226
25	0.1291	0.4833	0.5727	0.1820	0.4311	0.3336	0.2041	0.1403	0.1357
26	0.1244	0.6089	0.3394	0.1611	0.4079	0.2191	0.4133	0.2062	0.1262
27	0.1022	0.3801	0.2214	0.1448	0.7779	0.2486	0.5813	0.2191	0.1490
28	0.1100	0.8030	0.6977	0.2066	0.7037	0.2658	0.6461	0.1741	0.1161
29	0.1835	0.7399	0.4916	0.2834	0.1000	0.3468	0.5062	0.7585	0.1381

are shown in Table 5, from which it can be found that 29 DMUs are all ranked and distinguished according to their performances.

According to Table 5, we get Table 6, which shows the correlation among the efficiency values of the four DEA models. After the Spearman test, the test values of correlations among CCR efficiency value, benevolent efficiency value, aggressive efficiency value, and general value of ideal and anti-ideal points are all above 0.9. Therefore, it can be concluded that the proposed combining cross efficiency method of ideal and anti-ideal points is reasonable.

5. Conclusions and Prospect

Combining the benevolent and aggressive efficiency models of DEA cross efficiency with the ideal point method, based on two different aspects of efficiency and fairness, we improve the benevolent cross efficiency model and aggressive cross efficiency model, respectively. By combining these two models, we receive a general efficiency value. The improved method is more scientific and reasonable for evaluating the efficiency value of decision making units. It ensures a balanced development by promoting decision making units from both efficiency and fairness levels instead of blindly developing good projects. The validity and scientific nature of improved models and the improvement of cross efficiency evaluations can also be testified by verifying calculation examples and their relevant results.

Conflicts of Interest

There are no conflicts of interest regarding the publication of this manuscript.

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Table 5: Efficiency rating and rank for the twenty-nine DMUs.

DMU	CCR		Benevolent		Aggressive		Ideal and anti-ideal points	
	Efficiency	Rank	Efficiency	Rank	Efficiency	Rank	Efficiency	Rank
1	0.4015	26	0.1366	29	0.1212	29	0.1257	29
2	0.6486	21	0.5016	21	0.4643	21	0.4544	21
3	0.3635	28	0.1746	28	0.1559	28	0.1612	28
4	0.5169	24	0.3889	24	0.3630	23	0.3552	24
5	0.3912	27	0.3550	26	0.3190	26	0.3312	25
6	1.0000	1	0.7808	8	0.7115	6	0.7611	6
7	1.0000	1	0.9451	3	0.8517	3	0.8892	2
8	0.9120	10	0.7910	7	0.7109	7	0.7130	9
9	0.6623	20	0.5507	19	0.5015	19	0.5168	19
10	1.0000	1	0.9529	2	0.8543	2	0.8689	3
11	0.7085	18	0.6209	18	0.5512	18	0.5711	17
12	0.4663	25	0.3973	23	0.3629	24	0.3750	23
13	1.0000	1	0.9579	1	0.8709	1	0.9112	1
14	0.8048	16	0.6644	14	0.5961	14	0.6131	15
15	0.9029	11	0.6236	17	0.5578	17	0.5681	18
16	0.5227	23	0.4210	22	0.3791	22	0.3796	22
17	0.8377	14	0.7082	10	0.6465	10	0.6576	10
18	0.9678	8	0.7951	6	0.7030	8	0.7288	7
19	1.0000	1	0.6626	15	0.5928	15	0.6239	14
20	0.6769	19	0.5414	20	0.4825	20	0.5030	20
21	0.9523	9	0.7460	9	0.6748	9	0.7144	8
22	1.0000	1	0.8908	4	0.7706	5	0.8167	5
23	0.8716	12	0.7025	11	0.6244	12	0.6532	11
24	0.8221	15	0.6811	13	0.6130	13	0.6451	12
25	0.5776	22	0.3606	25	0.3421	25	0.3294	26
26	0.7363	17	0.6421	16	0.5786	16	0.6037	16
27	1.0000	1	0.8566	5	0.7845	4	0.8173	4
28	0.3012	29	0.2484	27	0.2274	27	0.2333	27
29	0.8457	13	0.7025	11	0.6338	11	0.6441	13

Table 6: DEA efficiency correlation test.

Spearmen's rho	CCR	Benevolent	Aggressive	Ideal and anti-ideal point
CCR				
Correlation	1.000	.936	.940	.940
Sig. (bilateral)		.000	.000	.000
Benevolent				
Correlation	.936	1.000	.997	.994
Sig. (bilateral)	.000		.000	.000
Aggressive				
Correlation	.940	.997	1.000	.995
Sig. (bilateral)	.000	.000		.000
Ideal and anti-ideal point				
Correlation	.940	.994	.995	1.000
Sig. (bilateral)	.000	.000	.000	

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