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Research Article

Chaotic Behavior of Traffic-Flow Evolution with Two Departure Intervals in Two-Link Transportation Network

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In this study, the influence of traveler's departure time choice in day-to-day dynamic evolution of traffic flow in a transportation network is investigated. Combining historical information and real-time information, a dynamic evolution model of traffic flow with a study period divided into two intervals is proposed for a simple two-link network. Then, the evolution of network traffic flow is investigated using numerical experiments. Three types of information are considered: (1) only historical information, (2) only real-time information, and (3) both historical and real-time information. The results show that the dynamic evolution of network traffic flow under the three types of information is similar. However, the possibility of chaos occurrence under both historical and real-time information is smaller than that under two individual types of information. When chaos occurs, the chaotic behavior in traffic-flow evolution under only real-time information is relatively less complex than that under the other two types of information.

1. Introduction

Both traditional static and dynamic traffic assignments only focus on the equilibrium solution and its solution algorithm. The research on the evolution of network traffic flow focuses on finding whether the equilibrium of network flows exists and has been extensively analyzed. Such research does not focus on network flows equilibrium but on the dynamic evolution process of network traffic flow, exploring whether equilibrium exists in network flows and how the equilibrium is reached.

The dynamic evolution of network traffic flow is the macroscopic outcome of a large number of traveler's route choices. Therefore, from the view of individual travelers, different assumptions about traveler's choice behavior may lead to different results of network traffic-flow evolution. Nakayama et al. [1] assumed that drivers' cognitive ability is limited. A model system of drivers' cognition, learning, and route choice was formulated to determine the dynamic characteristics of a driver-network system through microsimulation. Kim et al. [2] used a day-to-day evolution approach and developed agent-based simulation models to investigate how three assumptions of the user equilibrium (UE) principle (perfect information, rationality, and homogeneity) influence network traffic flows. Wei et al. [3] proposed a day-to-day route choice model based on reinforcement learning to analyze the effects of traveler's memory, learning rate, and experience cognition on the evolution of traffic flow using multiagent simulation.

The preceding research was carried out using microscopic simulation. However, some scholars have simulated day-to-day route choice behavior at microlevel using experimental methods. Avineri et al. [4] discussed the influence of travel time information on the traveler's route choice using two-route choice laboratory experiments under uncertainty. Selten et al. [5] reported laboratory route choice experiments between a main route and a side route, showing that the mean numbers on both roads tend to be very near equilibrium, and the fluctuation of network flows under perfect information is smaller than that under imperfect information. Rapoport et al. [6] designed two experiments to study whether the paradox is behaviorally realized in two simulated traffic networks that differ from each other in their topology, and the results proved the existence of Braess paradox. Meneguzzer et al. [7] reported an experiment to analyze day-to-day route choice dynamics in a simple three-route network.

On the other hand, other scholars studied the evolution of network traffic flow directly from aggregate-perspective. Smith [8] proposed a dynamic system converging to the Wardrop equilibrium solution by using the Lyapunov method under the assumption that the cost-flow function is monotone. Friesz et al. [9] analyzed the influence of information quality provided by traveler's information system on the day-to-day adjustment processes of network flows. They found that the dynamic adjustment process can eventually achieve equilibrium under the conditions of complete and incomplete information. Guo et al. [10] proposed a dynamic system that modeled the day-to-day adjustment process of traffic flow with link-flow variables, whose stationary state was equivalent to the UE state. Xiao et al. [11] showed that the network flow dynamics, which was analogous to a damped oscillatory system, would approach the equilibrium state with minimum total potential energy and zero kinetic energy. The preceding studies were carried out with continuous-time dynamic models, although discrete-time models may be more plausible in reality. Horowitz [12] proposed a discrete dynamic system model based on a two-link network and investigated the stability of stochastic equilibrium for discrete-time deterministic process. More recently, Guo et al. [13] formulated the day-to-day evolution of link flows in traffic network using a discrete dynamic system model, and the equilibrium state would be in either a deterministic or a stochastic user equilibrium state. Rambha et al. [14] developed a dynamic day-to-day pricing model to minimize the expected total system travel time, contributing to a significant reduction in expected total system travel time compared with the no-toll case.

The evolution of network traffic flow is a discrete dynamic system that has attracted the interest of many researchers who used nonlinear dynamics to analyze this phenomenon. Cantarella et al. [15, 16] proposed a day-to-day dynamic model and used nonlinear dynamics to deduce the equilibrium condition. Dendrinos [17] found that short-term traffic flow had nonlinear chaos involving fast Fourier transform of urban traffic-flow time series. Zhang and Jarrett [18] investigated the dynamic behavior of traffic flow in a network using dynamic gravity model, in which equilibria, oscillations, periodic doubling, and chaos were found. Lan et al. [19, 20] investigated the existence of chaotic behavior for short-term traffic flow and developed a parsimony procedure to determine whether chaotic phenomena exist in traffic-flow dynamics. Xu and Gao [21, 22] further used UE assignment model to estimate origin-destination (OD) cost and found that chaos still existed even in a two-dimensional system. The authors also presented a discrete dynamic model for the day-to-day adjustment process of route choice and found oscillations and chaos of network traffic flow when travelers were sensitive to travel cost and demand. Guo and Huang [23] proposed a discrete dynamic traffic assignment model in the case of some travelers with imperfect information. Oscillations and chaos were observed when model parameters exceeded certain values. Liu et al. [24, 25] presented the day-to-day dynamic traffic-flow evolution based on nonlinear dynamics and formulated a similar model with elastic demand. Li et al. [26] used nonlinear dynamics to analyze day-to-day evolution characteristics of traffic flow under bounded rational and analyzed the relationship between traveler's rationality and system stability.

The preceding research on the evolution of network traffic flow only considers the influence of route choice, but does not consider the influence of departure time choice. It assumes that traveler's route choice is based on a single departure interval. However, traveler's departure time choice behavior is also an important factor that influences the evolution of network traffic flow. It is reasonable to relax this assumption because travelers will travel at different departure times to reduce their travel times. Therefore, it is necessary to analyze the evolution of network traffic flow under the combined influence of traveler's departure time and route choice behavior. The departure time is not fixed, and travelers adjust the departure time according to the congestion level of the network. Based on this concept, Cascetta and Cantarella [27] presented a day-to-day and within-day dynamic assignment models. They focused on the analysis of network daily flow fluctuations, but ignored the day-to-day traffic-flow evolution at different periods. Ziliaskopoulos and Rao [28] proposed a simulation-based model for equilibrium on dynamic networks when travelers simultaneously optimize their departure times and route choices, but did not consider the instability of traffic-flow evolution. Srinivasan and Guo [29] investigated the day-to-day dynamics in an urban traffic network induced by departure time dynamics in commuter decisions, but did not consider traveler's route choice behavior. In addition, numerous studies, such as Ben-Elia et al. [30], Han et al. [31], and Mahmassani et al. [32], have shown that travel information has an impact on traveler's choice behavior.

Note that the research previously conducted by the authors to investigate chaotic behavior [24, 25] has focused on day-to-day network traffic-flow evolution and considered only route choice behavior (departure time behavior was not considered). Therefore, the purpose of this paper is to analyze day-to-day dynamic evolution of network traffic flow considering both route choice and departure time. In addition, the influence of historical and real-time travel information on traveler's departure time and route choice is evaluated. A day-to-day dynamic evolution model considering the departure time and route choice is proposed and the dynamic evolution characteristics of traffic flow under the influence of different travel information are evaluated with a focus on the characteristics of chaos. The paper considers a two-link network which has been the main focus of researchers [5, 12, 15] and two intervals for the traveler's departure time choice. The presented two-interval formulation lays the foundation for the study of traffic-flow evolution in complex networks with multiple intervals.

2. Proposed Model

A simple road network is used in this study, as shown in Figure 1. The network consists of two parallel links (1 and 2)

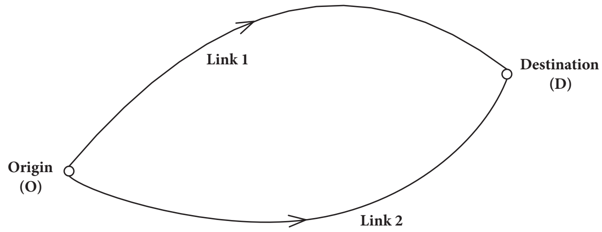


Figure 1: Two-link road network considered in the study.

in the same direction that are connected to an origin (O) and a destination (D).

The evolution of network traffic flow in a two-link network with two intervals is illustrated in Figure 2. First, travel demands in the two intervals on day n-1 are determined based on historical travel information under the assumption that daily traffic demand is fixed. Then, travel demand in interval 1 is assigned to two links based on travel information. Thus, the travel time of the links in interval 1 is obtained. Next, travel demand in interval 2 is assigned according to historical information and the travel time information in interval 1. The travel times of the links in interval 2 are also obtained. When all travelers arrive at their destination on day n - 1, the travel time information of the two links in the two intervals becomes new historical travel information. Finally, the distribution of network traffic flow on day n is updated according to the new historical travel information.

The study period is divided into two intervals. The OD demand is fixed. It is assumed that link travel time is related to link traffic volume. The travel time function is expressed as

$$\overline{C}_p^{n,t} = g_p\left(f_p^{n,t}\right), \quad t = 1, 2; \; p = 1, 2 \tag{1}$$

where $f_1^{n,t}$ and $f_2^{n,t}$ are the traffic volumes of links 1 and 2, respectively, in interval t on day n and $\overline{C}_1^{n,t}$ and $\overline{C}_2^{n,t}$ are the actual travel costs of links 1 and 2, respectively, in interval t on day n.

The evolution process of network flows is shown in Figure 2. The perceived travel cost on day n, which should be updated before the trip, is determined using the perceived and actual travel costs on day (n -1), as follows:

$$C_p^{n,t} = \varphi C_p^{n-1,t} + (1-\varphi)\overline{C}_p^{n-1,t}, \quad t = 1, 2; \; p = 1, 2 \tag{2}$$

where φ is the weight coefficient ($0 \le \varphi \le 1$) which reflects the traveler's dependence on the perceived travel cost of the previous day, where travel cost is determined based on travel time.

According to the perceived travel cost on day n, the probabilities of departure in two intervals can be determined. The utility of link p in interval t is defined as

$$U_{p,t}^n = V_{p/t}^n + V_t^n + \varepsilon_{p/t}^n + \varepsilon_t^n \tag{3}$$

where $V_{p/t}^n$ and $\varepsilon_{p/t}^n$are systematic and random components that change with the combination (t, p) in the utility of link p in interval t, respectively, V_t^n and ε_t^n are systematic and random components, which only change with t in the utility of link p in interval t, respectively. Note that $V_{p/t}^n = -\theta C_p^{n,t}$, $V_t^n = -c_t^n = -c_t$.

Assume that (a) ε_t^n and $\varepsilon_{p/t}^n$ are independent for all p (p = 1, 2) and t (t = 1, 2) and (b) $\varepsilon_{p/t}^n$ are independent and identically Gumbel distributed [33] with parameters (0, 1) for fixed t. Then, the probability of choosing departure time in interval t (=1,2) on day n is given by [34]

$$P^{n,1} = \frac{1}{1 + e^{\lambda((-c_2+\ln(e^{-\theta C_1^{n,2}}+e^{-\theta C_2^{n,2}}))-(-c_1+\ln(e^{-\theta C_1^{n,1}}+e^{-\theta C_2^{n,1}})))}} \tag{4}$$

$$\begin{aligned} P^{n,2} &= 1 - P^{n,1} \\ &= \frac{1}{1 + e^{\lambda((-c_1+\ln(e^{-\theta C_1^{n,1}}+e^{-\theta C_2^{n,1}}))-(-c_2+\ln(e^{-\theta C_1^{n,2}}+e^{-\theta C_2^{n,2}})))}} \end{aligned} \tag{5}$$

where λ and θ are related to traveler characteristics.

The travel demand in the two intervals can be calculated according to the probabilities of departure choice as follows.

$$d^{n,1} = \frac{D}{1 + e^{\lambda((-c_2+\ln(e^{-\theta C_1^{n,2}}+e^{-\theta C_2^{n,2}}))-(-c_1+\ln(e^{-\theta C_1^{n,1}}+e^{-\theta C_2^{n,1}})))}} \tag{6}$$

$$d^{n,2} = \frac{D}{1 + e^{\lambda((-c_1+\ln(e^{-\theta C_1^{n,1}}+e^{-\theta C_2^{n,1}}))-(-c_2+\ln(e^{-\theta C_1^{n,2}}+e^{-\theta C_2^{n,2}})))}} \tag{7}$$

Let

$$\begin{aligned} &P\left(C_1^{n,1}, C_2^{n,1}, C_1^{n,2}, C_2^{n,2}\right) \\ &\quad = \frac{1}{1 + e^{\lambda((-c_2+\ln(e^{-\theta C_1^{n,2}}+e^{-\theta C_2^{n,2}}))-(-c_1+\ln(e^{-\theta C_1^{n,1}}+e^{-\theta C_2^{n,1}})))}}. \end{aligned} \tag{8}$$

Traffic assignment is carried out in two intervals after determining travel demand. Travelers update their perceived travel costs based on historical and real-time travel information in the chosen departure time. Therefore, the updated perceived travel cost is determined based on the perceived travel cost on day n and the actual travel cost at the previous interval and is expressed as

$$\widehat{C}_p^{n,t} = \alpha C_p^{n,t} + (1-\alpha)\overline{C}_p^{n,t-1}, \quad t = 1, 2; \; p = 1, 2 \tag{9}$$

where α is the weight coefficient ($0 \le \alpha \le 1$) which reflects the traveler's dependence on the perceived travel cost on day n and $\overline{C}_p^{n,0}$ is the real-time information of the network in interval 1 on day n, which is the free-flow link travel time.

It is worth noting that for $\alpha = 1$ the traveler's perceived travel cost does not change. That is, the traveler selects the route based on historical travel information. For $\alpha = 0$, the traveler's perceived travel cost depends on actual travel cost in the previous interval on the same day. That is, the traveler selects the route based on real-time travel information. The traveler's perceived travel cost for $\alpha = 1$ is expressed as

$$\widehat{C}_p^{n,t} = C_p^{n,t}, \quad t = 1, 2; \; p = 1, 2 \tag{10}$$

The traveler's perceived travel costs for $\alpha = 0$ is expressed as follows.

$$\widehat{C}_p^{n,t} = \overline{C}_p^{n,t-1}, \quad t = 1, 2; \; p = 1, 2 \tag{11}$$

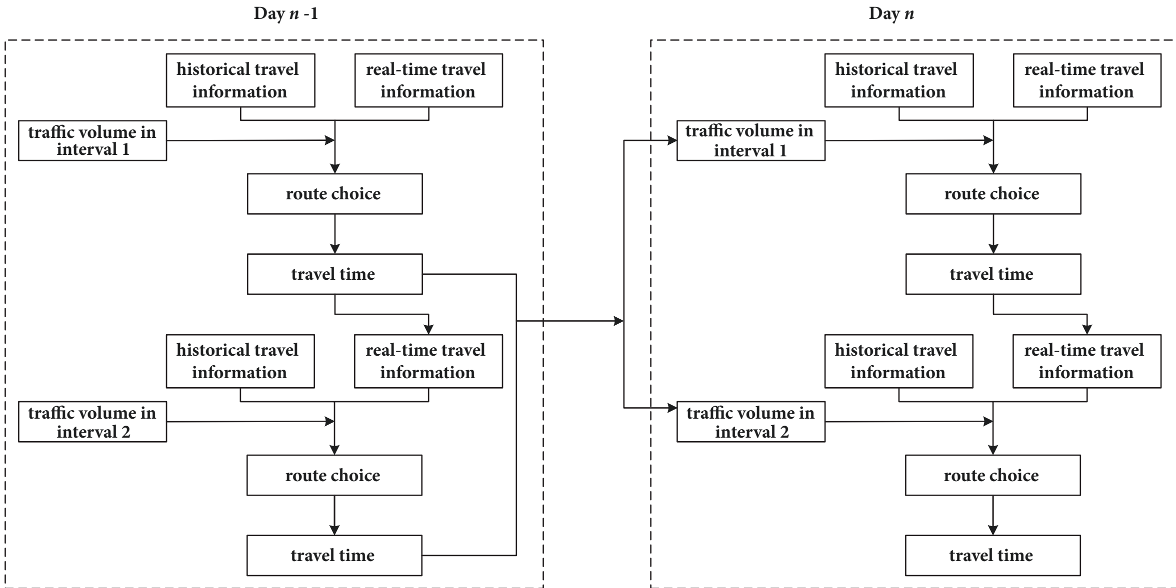


Figure 2: Evolution of network traffic flow on two-link network with a study period divided into two intervals.

The utility of link p ($p = 1, 2$) in interval t ($t = 1, 2$) contains systematic utility component $-\theta\widehat{C}_p^{n,t}$ and random utility component ξ_p^t. It is assumed that the random utility component ξ_p^t is Gumbel distributed with parameters (0, 1). The probability of route choice in interval t ($t = 1, 2$) on day n is given by the following.

$$\begin{aligned} p_1^{n,t} &= P\left(-\theta\widehat{C}_1^{n,t} + \xi_1^t \geq -\theta\widehat{C}_2^{n,t} + \xi_2^t\right) \\ &= \frac{1}{1 + e^{\theta(\widehat{C}_1^{n,t} - \widehat{C}_2^{n,t})}} \end{aligned} \tag{12}$$

$$p_2^{n,t} = 1 - p_1^{n,t} = \frac{1}{1 + e^{-\theta(\widehat{C}_1^{n,t} - \widehat{C}_2^{n,t})}} \tag{13}$$

Route flows in interval 1 can be assigned according to the probability of route choice in interval 1 on day n. That is,

$$f_1^{n,1} = \frac{d^{n,1}}{1 + e^{\theta(\widehat{C}_1^{n,1} - \widehat{C}_2^{n,1})}} \tag{14}$$

$$f_2^{n,1} = \frac{d^{n,1}}{1 + e^{-\theta(\widehat{C}_1^{n,1} - \widehat{C}_2^{n,1})}}. \tag{15}$$

Let

$$p_1\left(C_1^{n,1}, C_2^{n,1}\right) = \frac{1}{1 + e^{\theta(\widehat{C}_1^{n,1} - \widehat{C}_2^{n,1})}}. \tag{16}$$

Then, traffic assignment in interval 2 is given by the following.

$$f_1^{n,2} = \frac{d^{n,2}}{1 + e^{\theta(\widehat{C}_1^{n,2} - \widehat{C}_2^{n,2})}} \tag{17}$$

$$f_2^{n,2} = \frac{d^{n,2}}{1 + e^{-\theta(\widehat{C}_1^{n,2} - \widehat{C}_2^{n,2})}} \tag{18}$$

Let

$$p_2\left(C_1^{n,1}, C_2^{n,1}, C_1^{n,2}, C_2^{n,2}\right) = \frac{1}{1 + e^{\theta(\widehat{C}_1^{n,2} - \widehat{C}_2^{n,2})}}. \tag{19}$$

Then, the dynamic system model is translated into the following equations.

$$\begin{aligned} C_1^{n,1} &= \varphi C_1^{n-1,1} + (1-\varphi)\, g_1\Big(D \\ &\quad \cdot P\left(C_1^{n-1,1}, C_2^{n-1,1}, C_1^{n-1,2}, C_2^{n-1,2}\right) \\ &\quad \cdot p_1\left(C_1^{n-1,1}, C_2^{n-1,1}\right)\Big) \end{aligned} \tag{20}$$

$$\begin{aligned} C_2^{n,1} &= \varphi C_2^{n-1,1} + (1-\varphi)\, g_2\Big(D \\ &\quad \cdot P\left(C_1^{n-1,1}, C_2^{n-1,1}, C_1^{n-1,2}, C_2^{n-1,2}\right) \\ &\quad \cdot \left(1 - p_1\left(C_1^{n-1,1}, C_2^{n-1,1}\right)\right)\Big) \end{aligned} \tag{21}$$

$$\begin{aligned} C_1^{n,2} &= \varphi C_1^{n-1,2} + (1-\varphi)\, g_1\Big(D \\ &\quad \cdot \left(1 - P\left(C_1^{n-1,1}, C_2^{n-1,1}, C_1^{n-1,2}, C_2^{n-1,2}\right)\right) \\ &\quad \cdot p_2\left(C_1^{n-1,1}, C_2^{n-1,1}, C_1^{n-1,2}, C_2^{n-1,2}\right)\Big) \end{aligned} \tag{22}$$

$$\begin{aligned} C_2^{n,2} &= \varphi C_2^{n-1,2} + (1-\varphi)\, g_2\Big(D \\ &\quad \cdot \left(1 - P\left(C_1^{n-1,1}, C_2^{n-1,1}, C_1^{n-1,2}, C_2^{n-1,2}\right)\right) \\ &\quad \cdot \left(1 - p_2\left(C_1^{n-1,1}, C_2^{n-1,1}, C_1^{n-1,2}, C_2^{n-1,2}\right)\right)\Big) \end{aligned} \tag{23}$$

Note that (12) to (23) describe day-to-day dynamic evolution process of route flow in each of the two intervals, unlike those of Cantarella et al. [16] which reflect the day-to-day dynamic evolution process of route flow in only one interval. Whether the evolution of network traffic flow is chaotic can be determined according to the Lyapunov exponent of the dynamic system. The Lyapunov exponent of n-dimensional discrete dynamic system is calculated as [35]

$$L_i = \lim_{k \longrightarrow \infty} \frac{1}{k} \log \left| u_i \left[J_k \cdots J_1 \right] \right| \tag{24}$$

where J_i is the Jacobian matrix and u_i is the eigenvalue of the matrix. In this paper, the maximum Lyapunov exponent is used to judge whether the evolution of network traffic flow is chaotic. If the exponent is greater than 0, the evolution of network traffic flow is considered chaotic.

3. Numerical Investigation

The traffic network shown in Figure 1 was used in this numerical investigation. The travel time for each link is calculated using the following Bureau of Public Roads formula:

$$t = t_0 \left[1 + 0.15 \left(\frac{f}{Q} \right)^4 \right] \tag{25}$$

where t is the travel time to traverse a link, t_0 is the link free-flow travel time, f is the link traffic volume, and C is the link capacity. The following data were assumed for the experiments: for link 1, free-flow travel time (t_{10}) is 22 min and capacity (Q_1) is 1500 veh/h, and for link 2, free-flow travel time (t_{20}) is 25 min and capacity (Q_2) is 2000 veh/h. It is assumed that the inherent disutility in interval 1 (c_1) is 5 and that in interval 2 (c_2) is 3. The OD demand in the study period is 3000 veh and the study period is 2 hours, which is divided into two intervals. The travelers make route choice in the two-link network and can depart from the origin in any one of the two intervals.

3.1. Effect of θ on Network Flow Evolution for $\alpha = 1$. For $\alpha = 1$, the traveler's perceived travel cost is unchanged, indicating that the traveler selects the route based on only historical travel information. The influence of traveler's dependence on the perceived travel cost in previous day on the evolution of the network flows in different situations is discussed next.

The bifurcation diagram formed by traffic-flow evolution of link 1 in interval 1, where φ changes as θ increases, is shown in Figure 3. When θ is small, no matter what φ is, the evolution of network traffic flow will not become chaotic but will always maintain stability, as shown in Figure 3(a) and in the right parts of Figures 3(b) and 3(c). Then, as θ increases, the periodic phenomena appear in the evolution of traffic flow, as shown in the left parts of Figures 3(b) and 3(c). The results show that the chaotic phenomena appear when θ exceeds a certain value ($\theta \geq 1.09$). Figure 3(d) shows an example of the start of chaos for $\alpha = 1$. When chaos occurs in traffic-flow evolution, the chaotic region is shifted from small to larger φ as θ increases, as shown in Figures 3(e) and 3(f).

The results show that the evolutionary processes of traffic demand and traffic flow are similar. The evolution characteristics of traffic demand are shown in Figure 4, which are similar to those of Figure 3. In the evolution of traffic demand and traffic flow, chaotic behaviors appear simultaneously.

3.2. Effect of λ or θ on Network Flow Evolution for Different α. The different states of traffic-flow evolution with different α(1,0.5 and 0) are shown in Figure 5. For $\alpha = 1$, to analyze the chaotic behavior of traffic-flow evolution with θ and φ, the different states of the evolution with θ and φ are plotted using numerical experiments for $\lambda = 0.4$, as shown in the left figure of Figure 5(a). The evolution characteristics of traffic flow in Figure 3 can be further verified from the left figure of Figure 5(a). The system will not be in chaotic state when θ is small. On the contrary, the evolution will be chaotic when θ exceeds 1.09. In the case of chaos in the evolution of network traffic flow, the chaotic region gradually shifts to the side of larger φ as θ increases. In addition, the states of the evolution with parameters λ and φ when $\theta = 3$ are shown in right figure of Figure 5(a), which is similar to that in the left figure of Figure 5(a). For $0 < \alpha < 1$, the traveler's perceived travel cost is determined using both historical and real-time travel information. Taking $\alpha = 0.5$ as an example, the system states for different θ and φ, $\lambda = 0.4$, and $\alpha = 0.5$ are shown in the left figure of Figure 5(b). As noted, the nonchaotic region is larger than the chaotic region. When θ exceeds 1.35, the evolution appears chaotic. In addition, the states of evolution for different λ and φ, $\theta = 3$, and $\alpha = 0.5$ are shown in the right figure of Figure 5(b), which is also similar to that in the left figure of Figure 5(b).

For $\alpha = 0$, traveler's perceived travel cost is determined by the actual travel cost in the previous interval on the same day; in other words, travelers select route according to real-time travel information. The system's state with θ and φ for $\lambda = 0.4$ is shown in the left figure of Figure 5(c). As shown in the left figure of Figure 5(c), the evolution shows chaos when θ exceeds 1.35, and the chaotic region is mainly on the right. In addition, the states of the evolution with λ and φ for $\theta = 3$ are shown in the right figure of Figure 5(c), which is similar to that in the left figure of Figure 5(c). Based on the above analysis, it is concluded that the effects of λ and θ on network traffic-flow evolution are similar.

3.3. Comparison of Chaotic Region under Different Types of Information. The different states of traffic-flow evolution under three types of travel information are shown in Figure 6 for λ =0.3 and 0.8. As noted, the chaotic region under the combined influence of historical and real-time travel information is smaller than that under the individual influence of historical or real-time travel information. Therefore, the possibility of chaos occurrence in traffic-flow evolution is relatively small when the travelers choose the routes according to both types of travel information.

To investigate the effect of different types of information on traffic-flow evolution, Figure 7 shows the system states for different α and φ for specific values of ($\theta = 5, \lambda = 0.4$) and ($\theta = 3, \lambda = 0.8$). As noted, the chaotic region decreases first and then increases as α increases. Therefore, this result further verifies that traveler's route choice based on both historical

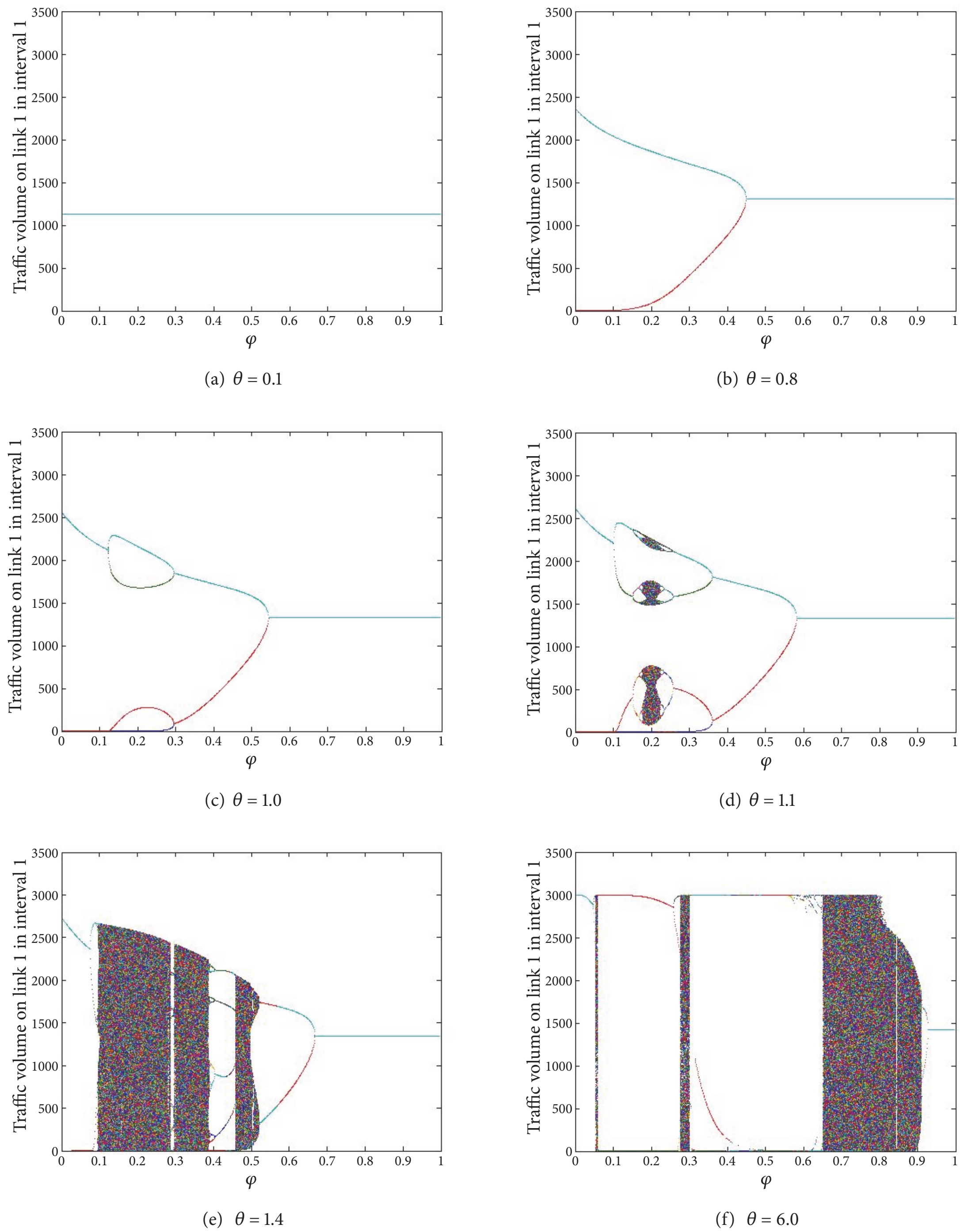
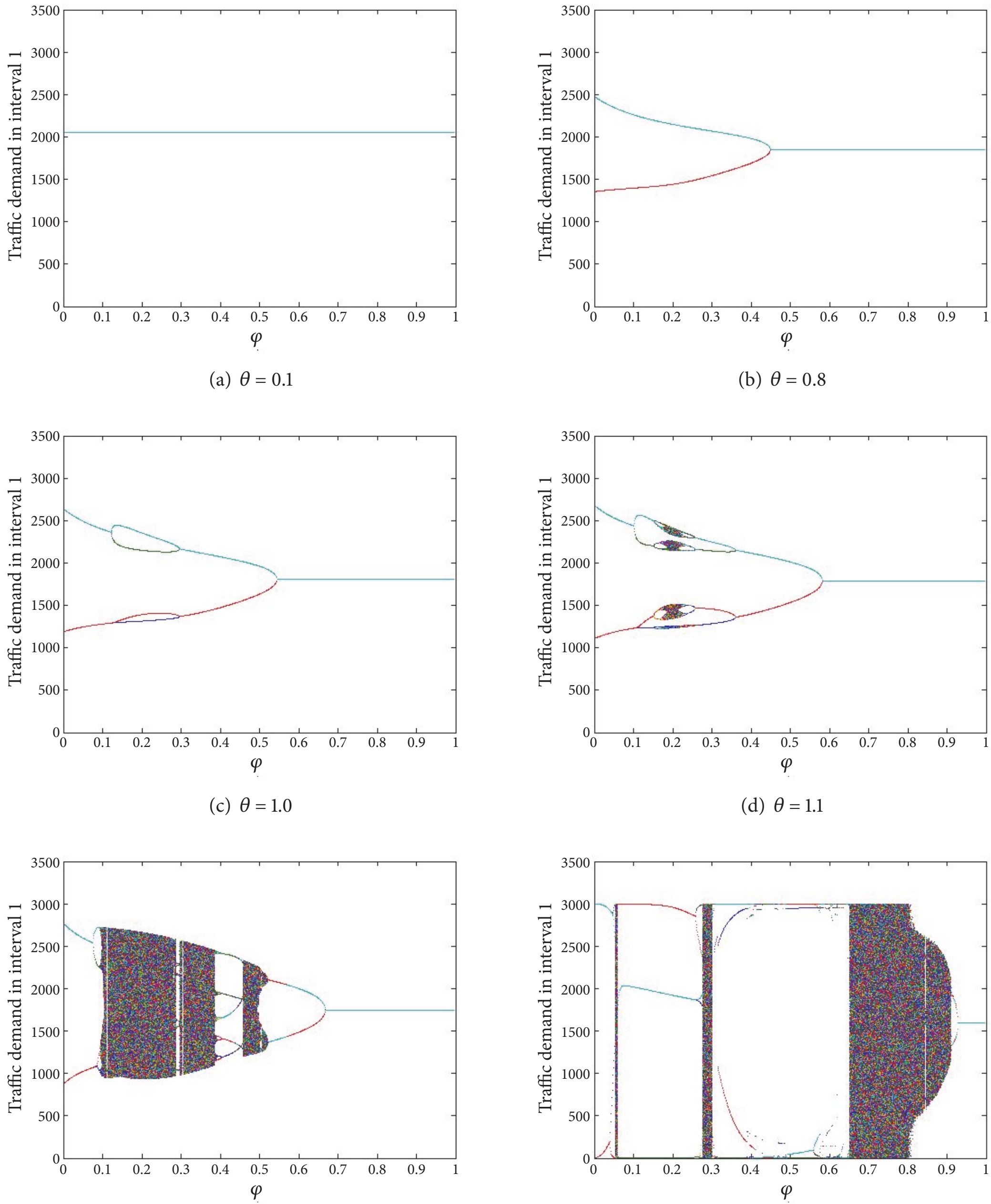


Figure 3: Flow bifurcation diagram with φ for $\alpha = 1$, $\lambda = 0.4$ for different θ.

and real-time information can reduce the probability of chaos occurrence.

3.4. Comparison of Chaotic Complexity under Different Types of Information. A phase diagram for each step of the system (i.e., projection of the system attractor on the plane with the two coordinates) is shown in Figure 8, where the x-axis is the flow of link 1 in interval 1 and the y-axis is the flow on link 1 in interval 2. In this figure, the chaotic behavior in traffic-flow evolution was analyzed under three different types of travel information. As noted, the complexity of the chaotic attractor under real-time information is smaller than that under only historical or combined historical and real-time information. Therefore, the traveler's route choice based on real-time information can make the chaos in traffic-flow evolution less complex.



(a) $\theta = 0.1$

(b) $\theta = 0.8$

(c) $\theta = 1.0$

(d) $\theta = 1.1$

(e) $\theta = 1.4$

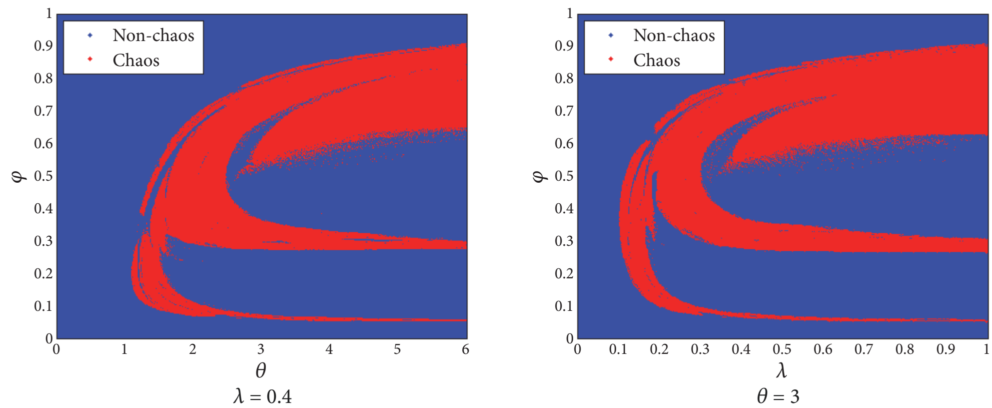
(f) $\theta = 6.0$

Figure 4: Demand bifurcation diagram with φ for $\alpha = 1$, $\lambda = 0.4$ for different θ.

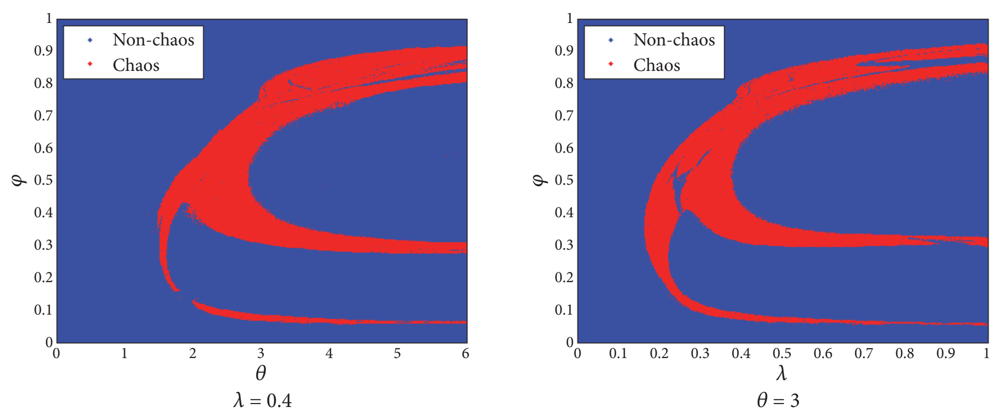
4. Conclusions

In this paper, a day-to-day dynamic evolution model of network traffic flow is formulated considering departure time choice in a two-link network with two-interval analysis period. Traffic-flow evolution under different types of information is investigated using numerical experiments by changing traveler characteristic parameters (φ, θ, and λ) and travel information parameter (α). Based on this study, the following comments are offered:

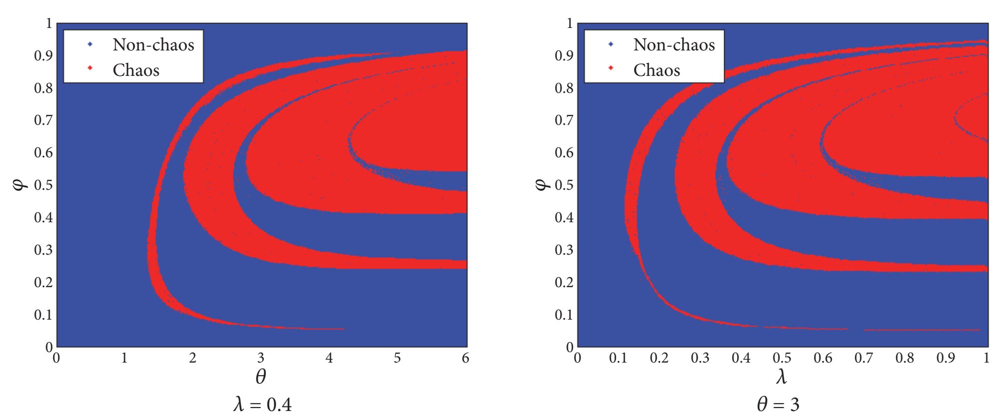
(1) The evolution of network traffic flow as θ or λ increases is similar for different types of travel information: chaos does not appear initially when θ or λ is small, but chaos is found when θ or λ exceeds a certain value. The explanation of this phenomenon is straightforward. The parameter θ reflects traveler's sensitivity to travel time during route choice. As θ increases, travelers become more sensitive to travel time. Thus, travelers tend to select the shortest route when θ is large, which makes network flows



(a) $\alpha = 1$



(b) $\alpha = 0.5$

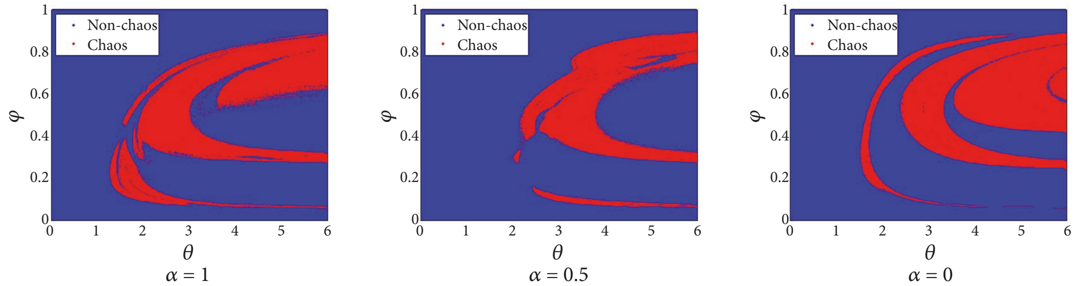


(c) $\alpha = 0$

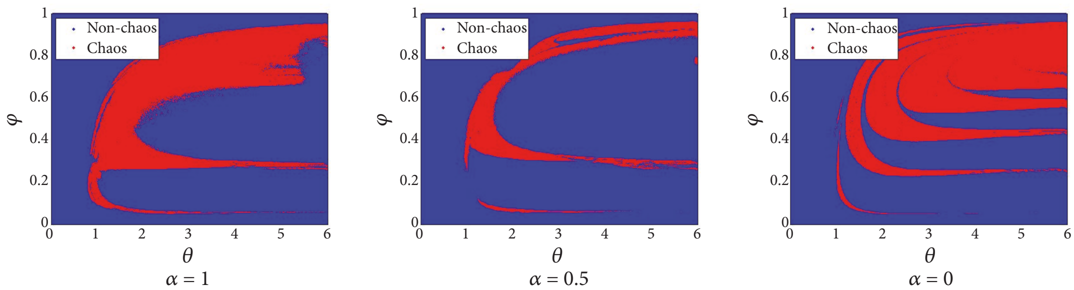
FIGURE 5: System state for different value of α.

unstable and the chaos more likely to happen. For the parameter λ, which reflects traveler's sensitivity to the perceived cost of the interval, as λ increases, travelers become more sensitive to the perceived cost of the interval. Similarly, chaos is more likely to occur when λ exceeds a certain value. Therefore, the traveler sensitivity to the perceived cost of the interval and the perceived route travel cost has similar effects on the evolution of network traffic flow considering traveler's departure time and route choice.

(2) Overall, the possibility of chaos occurrence is relatively small under the combined historical information and real-time information. However, in the case of chaos occurrence, the complexity of chaotic behavior is relatively small under real-time information alone. This is of great significance for the

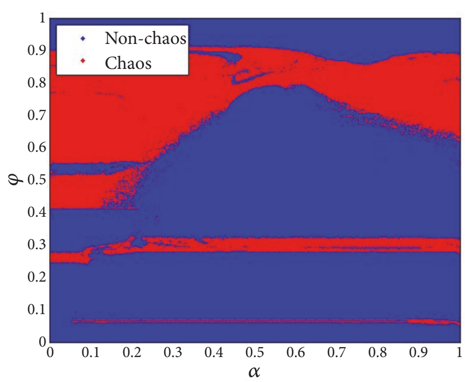


(a) $\lambda = 0.3$

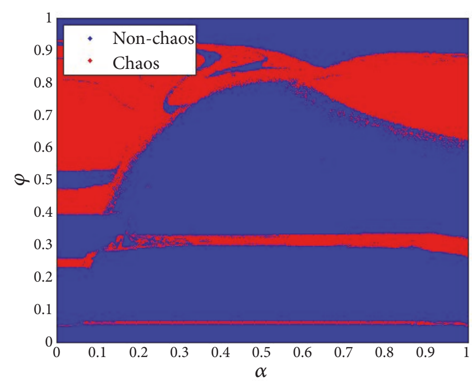


(b) $\lambda = 0.8$

FIGURE 6: System state for different θ and φ for λ(=0.3, 0.8) and α(=1, 0.5, 0).



(a) $\theta = 5$ and $\lambda = 0.4$



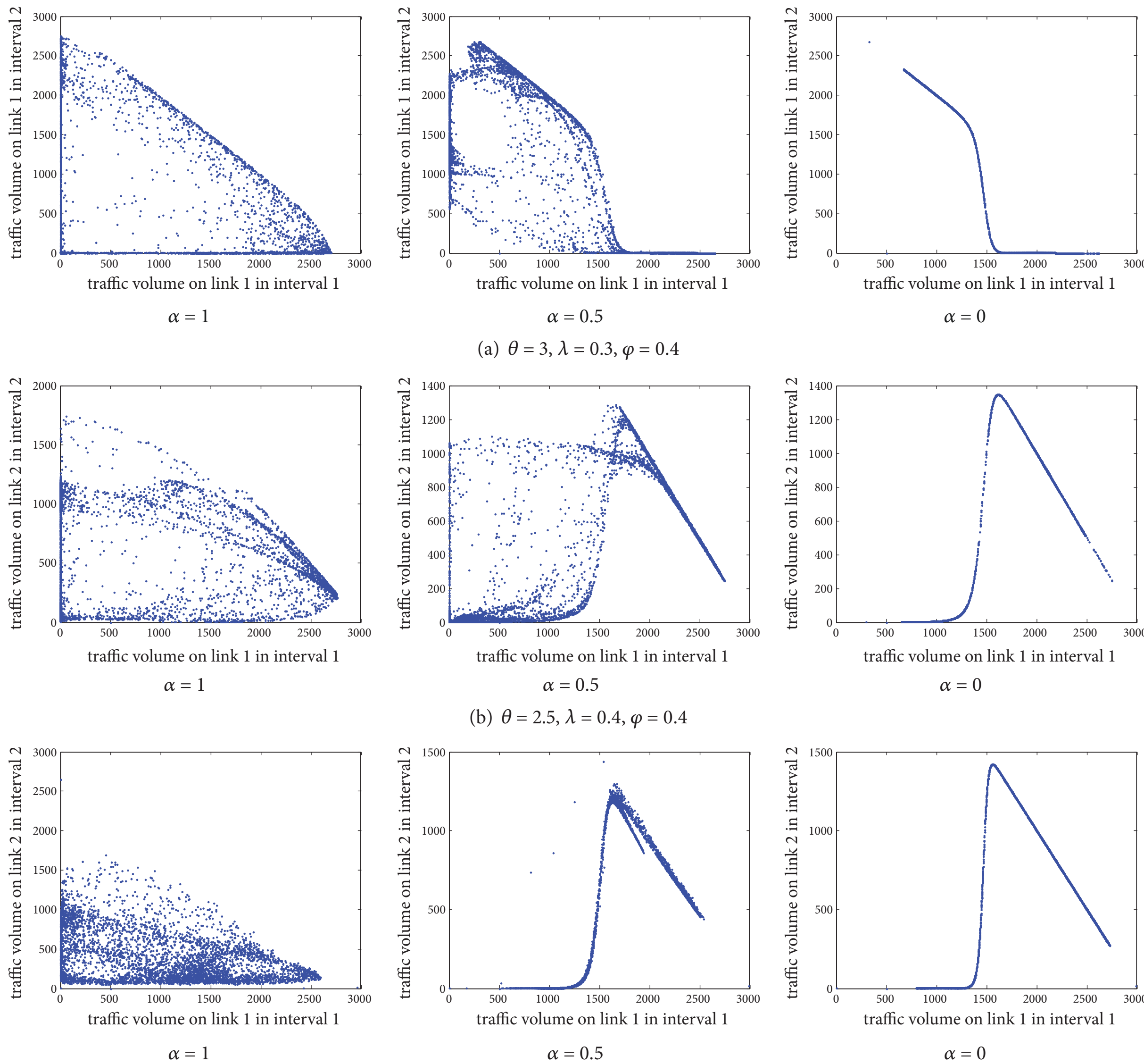
(b) $\theta = 3$ and $\lambda = 0.8$

FIGURE 7: The system's state for different α and φ for ($\theta = 5$, $\lambda = 0.4$) and ($\theta = 3$, $\lambda = 0.8$).

management and control of network traffic flow using travel information systems. The results show that both historical and real-time information should be used to guide network flows in normal circumstances. When the network traffic flow is unstable, especially when chaos occurs, real-time information should be used to regulate traffic flows.

(3) The evolution of network traffic flow considering traveler's route and departure time choices is modeled in this paper by introducing a learning mechanism based on travel time information from previous days and that from the same day provided by a real-time informative system. Under the basic framework of this model, other different behavioral assumptions may be adopted, such as bounded rationality. In addition, the proposed model can be used to simulate the conditions where different users are advised by different information systems. For example, one-half of travelers are influenced by historical information and the other half are advised by real-time information.

(4) The findings of this paper are only applicable to a two-link network with two-interval period. The research method of this paper would provide a useful background for analyzing traffic flows on more complex networks. The dynamic evolution characteristics of



(a) $\theta = 3$, $\lambda = 0.3$, $\varphi = 0.4$

(b) $\theta = 2.5$, $\lambda = 0.4$, $\varphi = 0.4$

(c) $\theta = 5$, $\lambda = 0.8$, $\varphi = 0.9$

Figure 8: Projection of chaotic attractor for α for specific θ, λ and φ.

network traffic flow are analyzed by establishing a larger dimensional nonlinear dynamic model. However, the chaos phenomenon may be more complicated, such as the occurrence of hyperchaos. Some of the preceding topics are currently explored by the authors.

(5) The research presented in this paper is still in the theoretical stage, and therefore its verification using actual data is warranted. If the chaos phenomenon of traffic flow is proved to exist using a large amount of field data, it will raise new research questions, for example, how chaos control can be carried out to make network traffic flow reach a stable state. This definitely would help traffic engineers and practitioners to effectively manage and control road traffic.

Data Availability

The numerical simulations data used to support the findings of this study were supplied by Wensi Chen under license and so cannot be made freely available. Requests for access to these data should be made to Wensi Chen's e-mail address: 496311795@qq.com.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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