

Research Article

Optimal Scheme for Process Quality and Cost Control by Integrating a Continuous Sampling Plan and the Process Yield Index

Chunzhi Li, Shurong Tong 🕞, and Keqin Wang

School of Management, Northwestern Polytechnical University, No. 1, Dongxiang Road, Chang'an District, Xi'an 710129, China

Correspondence should be addressed to Shurong Tong; stong@nwpu.edu.cn

Received 28 April 2018; Revised 2 July 2018; Accepted 30 October 2018; Published 19 November 2018

Academic Editor: Lu Zhen

Copyright © 2018 Chunzhi Li et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The single level continuous sampling plan (CSP-1) is an in-line process control tool that has been commonly adopted in various manufacturing industries. However, CSP-1 is designed for only satisfying the quality constraint. At the same time, CSP-1 has disadvantages with the high probabilities of both Type I and Type II errors due to its inherent deficiency coming from the operating procedure. In this work, an optimal scheme for process quality and cost control is proposed to monitor the process cost and improve the process quality. The CSP-1 and the process yield index (S_{pk}) are integrated in the present scheme, which work independently and complementarily. The four parameters (clearance number, inspecting fraction, sample size, and critical value) are designed in the proposed scheme under simultaneously considering the quality and cost constraints. The sole feasible inspection scheme in CSP-1 under the two constraints is found and used for controlling the process quality. The probabilities of Type I and Type II errors are concurrently controlled at the stipulated level with the risk control scheme, which is constructed with two nonlinear inequation based on the accurate distribution of the index S_{pk} . A case study is illustrated to validate the effectiveness and practicality of the proposed scheme.

1. Introduction

With the advances in manufacturing and control technology, process quality can be constant at required level for the incontrol process. Multiple constraints, such as quality, cost, risk, and environmental adaptability, are imposed on the process control. The constraints are originated from the intensifying market competition and the demand of sustainable development. Many kinds of process control tools are developed to monitor and improve the process quality under various constraints for the in-control process. As far as we know, the process control tool is designed for satisfying only one control constraint. For example, the continuous sampling plan (CSP) is commonly adopted in process control; various CSPs are presented for meeting only the quality constraint. The process yield index (S_{pk}) is proposed to only express the process capability. No process control tool is developed for solving the optimal problem of multiple constraints.

Various CSPs are developed for satisfying the quality requirement and used to improve and control the process

quality. The current literature on CSP can be generally classified into three categories. Various CSPs with different inspecting procedures were presented in the first kind of literature. The single level CSP was designated as CSP-1 and was widely adopted for process quality control in manufacturing [1]. To meet different demands in process control, some reduced CSPs were presented with the objective of decreasing the number of units inspected when the probability of nonconformity for the process was very low [2–5]. Some tightened CSPs have been designed to guarantee that the outgoing quality meets stringent quality requirements stipulated by the customer [6-8]. Nevertheless, CSPs designed under quality constraint have not taken other constraint, such as cost and risk constraints, into account. The category II literature has investigated the influence of the change of the inspection scheme in CSPs on the inspection cost. From a cost perspective, it was demonstrated that implementing a CSP for a stable production process was inappropriate [9]. Considering the economic objective, the inspection scheme in CSP-1 could work most effectively when the probability of nonconformity was roughly two-thirds of the value of the average outgoing quality limit (AOQL) [10]. Moreover, the different methods have been proposed for optimizing the parameters of inspection schemes in CSP-1 in a view of cost [11–14]. However, there is no agreed conclusion about how to identify the inspection scheme in CSPs whose inspection cost is the minimum. The third kind of literature has proposed the integrated control scheme between CSP and other process control tools, such as preventive maintenance and specification limit [15–21]. The integrated schemes were designed with the objective of minimizing the cost function. Nevertheless, no integrating process control scheme is presented for simultaneously meeting the quality and cost constraints by combining the CSP and other process control tools.

The in-control process is commonly regarded as stochastic process. There are two kinds of risk when process control tools are adopted in process control. One risk is the probability of making type I error that an in-control process with good quality is rejected. The other is the probability of making type II error that an in-control process with bad quality is accepted. CSPs have an inherent deficiency that the two risks are high [22–25]. The process yield index S_{pk} is an effective performance measure for reflecting the influence of the machining centre drift and process deviation fluctuation on the probability of nonconformity. The index S_{pk} had a one-to-one relationship with the probability of conformity and nonconformity [26]. It has been demonstrated that the natural estimator of \hat{S}_{pk} was asymptotically normally distributed [27]. The accuracy of the natural estimator has been investigated with a simulation technique [28]. The process yields for some specified cases, such as the imprecise sample data, circular profiles, autocorrelation between linear profiles, and multiple stream processes, have been analyzed in some literature [29–32]. In the above researches, the index S_{pk} was used to only reflect the process capability. In recent years, some variable sampling plans based on the process capability indices have been proposed with the objective of building the determination rules for the acceptance or rejection of product lots [33-36]. Two risks are simultaneously taken into consideration in the proposed variable sampling plans by utilizing the inference property of the natural estimator S_{pk} . Nevertheless, no literature has been devoted to constructing the risk control strategy with the index S_{pk} under quality and cost constraints for in-line process control.

Distinguished from the objective of minimizing cost in the sampling inspection for lot acceptance decision [34], the in-line process control aims to reach the control goal of minimizing total cost. The total cost constraint restricts the in-constant process to keep constant at right capability level. Thus, the reasonable inspection cost is permitted in process control, which is called cost constraint. The integrated process control scheme needs to be presented with simultaneously considering the quality constraint and the inspection cost constraint. The inspection cost constraint can be translated into the maximum affordable inspected fraction for the inline process control. In this work, an optimal scheme for process quality and cost control by combining CSP-1 and the index S_{pk} is presented. CSP-1 is adopted in the integrated scheme due to its simplicity and practicability in operation. In the proposed plan, there are four parameters (the clearance number, the sampling fraction, the sampling size, and the critical value of the index S_{pk}) under the quality and cost constraints. The two parameters (the clearance number and the sampling fraction) in CSP-1 are utilized to guarantee that the average outgoing quality is conforming for the in-control process. The index S_{pk} is used to construct the risk control scheme. The two risks are concurrently controlled at the stipulated level by constructing two nonlinear inequations with the accurate distribution of the estimator \hat{S}_{pk} .

The rest of this paper is organized as follows. In Section 2, the concept and estimator of the index S_{pk} are introduced. In Section 3, the optimal scheme for process quality and cost control and the operating procedure are presented. The method of identifying the plan parameters is provided. In Section 4, the values of the parameters of the optimal scheme for three different quality constraints are tabulated for practical purposes. Comparisons of the operating characteristic (OC) curves between CSP-1 and the optimal scheme are given to present the advantages of the integrated scheme. In Section 5, an example of application is provided to validate the effectiveness and practicality of the integrated control plan. Finally, Section 6 concludes the paper.

2. Process Fraction Nonconforming and Process Yield Index

The process fraction nonconforming, p, is a crucial performance measure for in-line process quality control. For a process that is well controlled, the value of p can be taken as a constant. However, the index of p is an unknown variable and needs to be estimated. Let $F(\cdot)$ be the cumulative distribution function (CDF) of the quality characteristic interested, so p = 1 - [F(USL) - F(LSL)] for the in-control process with a two-sided specification limits, where USL is the upper specification limit and LSL is the lower specification limit. If the quality characteristic follows a normal distribution, we get

$$p = 1 - \left\{ \Phi\left(\frac{USL - \mu}{\sigma}\right) - \Phi\left(\frac{LSL - \mu}{\sigma}\right) \right\}$$
(1)

where μ and σ are the process mean and the process standard deviation, respectively, and $\Phi(\cdot)$ is the CDF of the standard normal distribution, N(0, 1). Unfortunately, there is no literature devoted to a study of the distribution properties of the index of p.

Boyles [13] proposed the use of the process yield index, S_{pk} , to obtain an exact measure of the process yield for a process with a normal distribution. There is a one-to-one relationship between S_{pk} and the process yield. The proposed index S_{pk} is defined as

$$S_{pk} = \frac{1}{3}\Phi^{-1}\left\{\frac{1}{2}\Phi\left(\frac{USL-\mu}{\sigma}\right) + \frac{1}{2}\Phi\left(\frac{\mu-LSL}{\sigma}\right)\right\}$$
(2)

where $\Phi^{-1}(\cdot)$ is the inverse function of the CDF, $\Phi(\cdot)$, of the standard normal distribution. Let $C_p = (USL - LSL)/(6\sigma)$ and $C_a = 1 - |\mu - M|/d$, where *M* is the middle point of the whole tolerance range, M = (USL + LSL)/2, and *d* is half the

tolerance range, d = (USL - LSL)/2. Equation (2) can also be expressed as

$$S_{pk} = \frac{1}{3}\Phi^{-1} \left\{ \frac{1}{2}\Phi \left(3C_p C_a \right) + \frac{1}{2}\Phi \left(3C_p \left(2 - C_a \right) \right) \right\}$$
(3)

The formula for the relationship between p and S_{pk} can be obtained from (1) and (2):

$$p = 2 - 2\Phi\left(3S_{pk}\right), \quad S_{pk} > 0 \tag{4}$$

Table 1 shows the one-to-one correspondence between process yield, process fraction nonconforming, and process yield index.

The process mean μ and the process standard deviation σ are usually unknown variables and need to be estimated using the sample mean \overline{x} and the sample standard deviation s, where $\overline{x} = (1/n) \sum_{i=1}^{n} x_i$ and $s = \sqrt{(1/(n-1)) \sum_{i=1}^{n} (x_i - \overline{x})^2}$. Thus, the estimator of S_{pk} can be written as follows:

$$\widehat{S}_{pk} = \frac{1}{3} \Phi^{-1} \left[\frac{1}{2} \Phi \left(\frac{USL - \overline{x}}{s} \right) + \frac{1}{2} \Phi \left(\frac{\overline{x} - LSL}{s} \right) \right]$$

$$= \frac{1}{3} \Phi^{-1} \left\{ \frac{1}{2} \Phi \left(3\widehat{C}_p \widehat{C}_a \right) + \frac{1}{2} \Phi \left(3\widehat{C}_p \left(2 - \widehat{C}_a \right) \right) \right\}$$
(5)

The estimator \hat{S}_{pk} is such a complicated function that it is impossible to obtain its exact cumulative distribution function and probability density function. Lee et al. [14] furnished a useful approximation to the distribution of \hat{S}_{pk} under a normal distribution as

$$\widehat{S}_{pk} \approx S_{pk} + \frac{1}{6\sqrt{n}} \frac{W}{\phi(3S_{pk})}$$
(6)

where ϕ is the probability density function of the standard normal distribution N(0, 1) and

W

$$= \begin{cases} \frac{\sqrt{n}}{2} \left[\frac{a\left(s^2 - \sigma^2\right)}{\sigma} \right] - \sqrt{n} \frac{b\left(\overline{x} - \mu\right)}{\sigma} & \text{for } \mu < M, \\ \frac{\sqrt{n}}{2} \left[\frac{a\left(s^2 - \sigma^2\right)}{\sigma} \right] + \sqrt{n} \frac{b\left(\overline{x} - \mu\right)}{\sigma} & \text{for } \mu > M, \end{cases}$$
(7)

where *a* and *b* are defined as functions of μ and σ (or C_p and C_a):

$$a = \frac{1}{\sqrt{2}} \left\{ \frac{USL - \mu}{\sigma} \phi \left(\frac{USL - \mu}{\sigma} \right) + \frac{\mu - LSL}{\sigma} \phi \left(\frac{\mu - LSL}{\sigma} \right) \right\}$$
$$= \frac{1}{\sqrt{2}} \left\{ 3C_p \left(2 - C_a \right) \phi \left(3C_p \left(2 - C_a \right) \right) + 3C_p C_a \phi \left(3C_p C_a \right) \right\}$$

TABLE 1: The process yield, process fraction nonconforming, and corresponding value of the process yield index S_{pk} .

Yield (%)	fraction nonconforming (%)	S_{pk}
99.3066052	0.6933948	0.90
99.5628077	0.4371923	0.95
99.7300204	0.2699796	1.00
99.8367295	0.1632705	1.05
99.9033152	0.0966848	1.10

$$b = \phi\left(\frac{USL - \mu}{\sigma}\right) - \phi\left(\frac{\mu - LSL}{\sigma}\right) = \phi\left[3C_p\left(2 - C_a\right)\right] - \phi\left(3C_pC_a\right)$$
(8)

The estimator \hat{S}_{pk} approximately follows a normal distribution $N(S_{pk}, (a^2 + b^2)/36n(\phi(3S_{pk}))^2)$. Thus, the probability density function of \hat{S}_{pk} can be obtained as

$$f_{\hat{S}_{pk}}(x) = \sqrt{\frac{18n}{\pi}} \\ \cdot \frac{\phi(3S_{pk})}{\sqrt{a^2 + b^2}} \exp\left[-\frac{18n(\phi(3S_{pk}))^2}{a^2 + b^2}(x - S_{pk})^2\right], \quad (9)$$

 $-\infty < x < +\infty$

3. The Optimal Scheme for Process Quality and Cost Control

3.1. Quality and Cost Constraints. Two constraints, the average outgoing quality limit (AOQL) and the inspection capability limit (AFI_{I}) , are predetermined by the practitioner. AFI is the average fraction inspected. AFI_{I} is the cost constraint and represents the maximum affordable inspection workload. The value of AFI_L is converted from the planned inspection cost. The value of AOQL is generally stipulated by the product designer or the customer. For the in-control process, the proportion of nonconformance p can be taken as constant. Obviously, there exists a specified in-control process, named limit quality process with quality limit p_{IQL} , for which conforming outgoing quality can be achieved only under the inspection capability limit AFI_L. This means that $AFI_L = 1 - AOQL/p_{IQL}$ for the limit quality process with the quality limit p_{IQL} . It needs to be noted that generally $p_{IOL} > AOQL.$

3.2. Determination of Parameters for the Quality Control Scheme in CSP-1. The quality control scheme in CSP-1 should guarantee that the two constraints, AOQL and AFI_L , are simultaneously satisfied when p ranges from 0 to p_{IQL} . According to the performance formulas in CSP-1 proved by



FIGURE 1: Comparisons of the curves of (a) AOQ and (b) AFI between (i_L, f_L) and other two types of the AOQL contour schemes.

Yang [37], the set of inequalities can be constructed as follows for the quality and cost constraints, respectively,

$$\frac{(1-f)pq^{i}}{(f+(1-f)q^{i})} \le \text{AOQL}$$
(10)

$$\frac{f}{\left(f + \left(1 - f\right)q^{i}\right)} \le AFI_{L} \tag{11}$$

where $p \le p_{IQL}$, q = 1 - p, 0 < f < 1, *i* is the clearance number and only takes positive integer, and *f* is the inspection fraction.

Obviously, all AOQL contour schemes can meet inequality (10) when *p* ranges from 0 to 1. Figure 1 shows the curves of the performance measures AOQ and AFI for all AOQL contour schemes under the given constraints AOQL and AFI_L, and AOQ is the average outgoing quality. The inspection schemes (i_2, f_2) represent the type of schemes with $p_L > p_{IQL}$, (i_3, f_3) with $p_L > p_{IQL}$, (i_L, f_L) with $p_L = p_{IQL}$, p_L is the point where max(AOQ) = AOQL occurs. There are infinite schemes, respectively, included in the two types of the inspection schemes (i_2, f_2) and (i_3, f_3) . The inspection scheme (i_L, f_L) with $p_L = p_{IQL}$ has only one included.

It can be seen from Figure 1(b) that only the inspection scheme (i_L, f_L) can meet the inequality (11) when $p \le p_{IQL}$. Thus, the inspection scheme (i_L, f_L) is the sole scheme which can simultaneously satisfy the two inequalities (10) and (11), named the optimal scheme.

It can be concluded from Figure 1 that the value of AOQ increases gradually to the quality constraint AOQL and the inspection workload also increases gradually to the inspection capability limit AFI_L when the process quality is close to the quality limit p_{IOL} for the optimal scheme (i_L, f_L) .

The scheme (i_L, f_L) can achieve the conforming outgoing quality AOQ < AOQL with a smaller inspection workload than AFI_L for an in-control process with $p < p_{IQL}$ and AOQ = AOQL under the inspection capability limit AFI_L for an in-control process with $p = p_{IQL}$.

The values of the parameters i_L and f_L cannot be easily achieved with inequalities (10) and (11). Li et al. [38] proposed the formulas to solve the parameters of the specified AOQL contour scheme for the specified p. Thus, the parameters i_L and f_L can be obtained as follows:

$$i_L = \frac{q_{IQL}}{p_{IQL} - \text{AOQL}} \tag{12}$$

$$f_L = \frac{q_{IQL}^i}{q_{IQL}^i + p_{IQL}iq_{IQL}^{-1} - 1}$$
(13)

where $q_{IQL} = 1 - p_{IQL}$.

Table 2 shows various inspection schemes (i_L, f_L) under three quality constraints and various cost constraints. The values of i_L decrease and the values of f_L increase when the values of the quality limit p_{IQL} become greater under the given quality requirement AOQL.

In addition, it can be observed from Figure 1(b) that, for the process with $p > p_{IQL}$, the minimum inspection workload demanded for meeting the quality constraint exceeds the inspection capability limit AFI_L ; the production should be stopped. Nevertheless, both the probabilities of making the type I and II errors are high when the process is controlled with the AOQL contour scheme in CSP-1. The risk control scheme for controlling concurrently the two risks in the optimal scheme should be redesigned under the quality and cost constraints.

		f_L	0.1218	0.3145	0.4498	0.5437	0.6117	0.6629	0.7028	0.7347
	= 0.0122	i_L	80	39	26	19	15	12	11	6
	AOQL =	$p_{\rm IOL}$	0.0244	0.0366	0.0488	0.0610	0.0732	0.0854	0.0976	0.1098
and $AFI_L(p_{IQL})$		AFI_L	0.5000	0.6667	0.7500	0.8000	0.8333	0.8571	0.8750	0.8889
specified AOQL		f_L	0.1195	0.3092	0.4425	0.5351	0.6022	0.6527	0.6920	0.7235
scheme under a	0.00143	i_L	697	348	232	174	139	115	66	86
) in the optimal :	AOQL = ($p_{\rm IOL}$	0.00286	0.00429	0.00572	0.00715	0.00858	0.01001	0.01144	0.01287
rol scheme (i_L , f_L		AFI_L	0.5000	0.6667	0.7500	0.8000	0.8333	0.8571	0.8750	0.8889
LE 2: The quality cont		f_L	0.1192	0.3086	0.4417	0.5342	0.6011	0.6515	0.6908	0.7222
TABI	00018.	$i_{\rm L}$	5554	2776	1851	1388	1110	925	793	693
	AOQL = ($p_{\rm IOL}$	0.00036	0.00054	0.00072	0.00090	0.00108	0.00126	0.00144	0.00162
		AFI_L	0.5000	0.6667	0.7500	0.8000	0.8333	0.8571	0.8750	0.8889

3.3. Determination of Parameters for the Risk Control Scheme. The in-control process with $p \leq AOQL$ should be accepted with a higher probability than $1 - \alpha$; α is the probability of making the type I error that the process with high quality level is rejected. The in-control process with $p > p_{IQL}$ should be rejected with a higher probability than $1 - \beta$; β is the probability of making the type II error that the process with low quality level is accepted. However, the two types of risks cannot be simultaneously controlled with the AOQL contour schemes. Thus, the risk control scheme based on the index S_{pk} is designed under the quality and cost constraints.

There are two key points, (AOQL, α) and (p_{IOL} , β), that need to be considered at the same time in the risk control scheme. The OC curve should pass through the two designated points to meet the two constraints AOQL and AFI_L. Let S_{AOQL} be the value of the process yield index corresponding to the quality level p = AOQL and let S_{IQL} be the value of the process yield index corresponding to the quality level $p = p_{IOL}$. Therefore, the two key points (AOQL, α) and (p_{IQL}, β) for the OC function can be designated as (S_{AOQL}, α) and (S_{IQL}, β) . It means that if the estimator of \hat{S}_{pk} for the incontrol process is greater than the given value of S_{AOQL} , the probability of accepting the in-control process will be greater than $1 - \alpha$, and if the estimator of \hat{S}_{pk} for the in-control process is lower than the fixed value of S_{IQL} , the probability of accepting the in-control process will be less than the given value of β .

For the quality characteristic following a normal distribution and having a lower specification limit *LSL* and upper specification limit *USL*, the OC function with $S_{pk} = S'$ can be given as

$$P\left(\widehat{S}_{pk} \ge s_0 \mid S_{pk} = S'\right) = \int_{s_0}^{\infty} \sqrt{\frac{18n}{\pi}}$$
$$\cdot \frac{\phi\left(3S'\right)}{\sqrt{a^2 + b^2}} \exp\left[-\frac{18n\left(\phi\left(3S'\right)\right)^2}{a^2 + b^2}\right]$$
(14)

$$\times (x-S')^2 dx, -\infty < x < +\infty$$

Thus, the two nonlinear inequalities specified by the two risks can be obtained as follows:

$$P_{1}\left(\widehat{S}_{pk} \geq s_{0} \mid S_{pk} = S_{AOQL}\right) = \int_{s_{0}}^{\infty} \sqrt{\frac{18n}{\pi}}$$

$$\cdot \frac{\phi\left(3S_{AOQL}\right)}{\sqrt{a_{AOQL}^{2} + b_{AOQL}^{2}}} \exp\left[-\frac{18n\left(\phi\left(3S_{AOQL}\right)\right)^{2}}{a_{AOQL}^{2} + b_{AOQL}^{2}}\right]$$
(15)
$$\times \left(x - S_{AOQL}\right)^{2} dx \geq 1 - \alpha$$

$$P_{2}\left(\widehat{S}_{pk} \geq s_{0} \left| S_{pk} = S_{IQL} \right. \right) = \int_{s_{0}}^{\infty} \sqrt{\frac{18n}{\pi}}$$

$$\cdot \frac{\phi\left(3S_{IQL}\right)}{\sqrt{a_{IQL}^{2} + b_{IQL}^{2}}} \exp\left[-\frac{18n\left(\phi\left(3S_{IQL}\right)\right)^{2}}{a_{IQL}^{2} + b_{IQL}^{2}}\right]$$

$$\times \left(x - S_{IQL}\right)^{2} dx \leq \beta$$
(16)

The two parameters, the critical value s_0 and the sample size *n*, must simultaneously satisfy the two nonlinear inequalities (15) and (16). There are infinite values of the combination (n, s_0) which can meet inequalities (15) and (16). The specified value of (n, s_0) which can simultaneously satisfy (17) and (18) is the boundary of all the feasible combinations (n, s_0) . It means that the combination of (n, s_0) meeting (17) and (18) can satisfy inequalities (15) and (16).

$$\int_{s_0}^{\infty} \sqrt{\frac{18n}{\pi}} \\ \cdot \frac{\phi \left(3S_{AOQL}\right)}{\sqrt{a_{AOQL}^2 + b_{AOQL}^2}} \exp \left[-\frac{18n \left(\phi \left(3S_{AOQL}\right)\right)^2}{a_{AOQL}^2 + b_{AOQL}^2}\right) \right] (17) \\ \times \left(x - S_{AOQL}\right)^2 dx = 1 - \alpha \\ \int_{s_0}^{\infty} \sqrt{\frac{18n}{\pi}} \frac{\phi \left(3S_{IQL}\right)}{\sqrt{a_{IQL}^2 + b_{IQL}^2}} \exp \left[-\frac{18n \left(\phi \left(3S_{IQL}\right)\right)^2}{a_{IQL}^2 + b_{IQL}^2}\right) \\ \times \left(x - S_{IQL}\right)^2 dx = \beta$$

$$(18)$$

The solution (n, s_0) for (17) and (18) is sole. For example, under two constraints, AOQL = $0.00018(S_{AOQL} = 1.2485)$ and $p_{IQL} = 0.0009(S_{IQL} = 1.1067)$, and two given risks, $\alpha = 0.1$ and $\beta = 0.1$, the sole solution $(n, s_0) = (225, 1.1730)$ can be obtained from (17) and (18). It implies that the value of the estimator \hat{S}_{pk} is calculated with the 225 sample data. If $\hat{S}_{pk} \ge 1.1730$, the process is judged to be controllable with the current inspection scheme (i_L, f_L) under the quality and cost constraints, and the current inspection scheme (i_L, f_L) will continue. If $\hat{S}_{pk} < 1.1730$, the inspection with the inspection scheme (i_L, f_L) will become uneconomic under the given constraints AOQL and AFI_L , and the production should be stopped or the maintenance will be triggered.

3.4. Operation Procedure. The roles of CSP-1 and the index S_{pk} in the optimal scheme are independent and complementary when considering simultaneously the two constraints AOQL and AFI_L for the process quality and cost control. The objective of employing an inspection scheme (i_L, f_L) is to reduce the proportion of nonconformance and achieve the conforming outgoing quality. The combination of (n, s_0) is used to control simultaneously the two types of risks and can

be regarded as a new stopping rule for CSP-1. The operating procedure for the optimal scheme is as follows.

Step 1 (process quality control). Calculate the value of the parameters (i_L, f_L) under the given values of AOQL and AFI_L . Implement the inspection scheme (i_L, f_L) according to the procedure in CSP-1 for the process with the quality characteristics interested.

Step 2 (process risk control). Calculate the value of the parameters (n, s_0) under the given values of AOQL and AFI_L . Keep the latest number *n* of inspection data consecutively in the order of production when the inspection scheme (i_L, f_L) in CSP-1 is performed. Calculate the value of \hat{S}_{pk} with the latest number *n* of inspection data. The determination is carried out as follows:

- (i) continue Step 1 if $\hat{S}_{pk} \ge s_0$,
- (ii) stop production if $\widehat{S}_{pk} < s_0$.

where s_0 is the threshold used to guarantee that the two risks are controlled at the stipulated level. *n* is the number of the last inspection data recorded consecutively during inspection, including the screening inspection stage and the fraction inspection stage in CSP-1. The number *n* of inspection data is used to calculate the value of \hat{S}_{pk} .

4. Analyses and Comparisons

In the proposed optimal scheme for process quality and cost control, process quality control can be achieved by performing the inspection scheme (i_L, f_L) , and process risk control can be attained with the combination of (n, s_0) . The combination of (n, s_0) also plays the role of stopping rule in the inspection scheme (i_L, f_L) . The values of *n* and s_0 are different for various values of α and β for specified constraints AOQL and AFI_L . Tables 3–5 show the values of the combination of (n, s_0) for $\alpha = 0.01$, 0.05, and 0.1, and $\beta = 0.01$, 0.05, and 0.1 under three quality requirements, AOQL 0.00018, 0.00143 and 0.0122, and three inspection capability limits, $AFI_L = 0.6667$, 0.8 and 0.8571. According to the one-to-one correspondence between AFI_L , p_{IQL} , and S_{IQL} , three different process yield limits, $S_{IOL} = 1.1534$, 1.1067 and 1.0750 ($AFI_L = 0.6667$, 0.8 and 0.8571) under $S_{AOQL} =$ 1.2485 (AOQL = 0.00018,); S_{IQL} = 0.9520, 0.8966 and 0.8585 $(AFI_L = 0.6667, 0.8 \text{ and } 0.8571)$ under $S_{AOQL} = 1.0628$ (AOQL = 0.00143); and $S_{IQL} = 0.6967, 0.6245$, and 0.5734 $(AFI_L = 0.6667, 0.8 \text{ and } 0.8571) \text{ under } S_{AOQL} = 0.8354$ (AOQL = 0.0122), respectively, are considered to examine the behavior of (n, s_0) . It can be observed from Tables 3–5 that the value of the sample size *n* becomes smaller as the α or the β becomes larger. This phenomenon can be interpreted to mean that if the practitioner reduces the expected values at which high quality processes are rejected and/or low quality processes are accepted, the sample size for the judgement on the quality and capability of the processes will reduce. For a given S_{AOQL} , α , and β , the sample size becomes smaller as the process yield limit decreases (the value of S_{IQL} becomes smaller). For a fixed α , β , and S_{IOL} , the value of *n* becomes smaller when the value of AOQL becomes larger (the value of S_{AOQL} becomes smaller). We can explain these phenomena by saying that the determination of the quality and capability of the processes can be done easily using a smaller number of inspection data when the difference between the quality limit p_{IQL} and the quality constraint AOQL becomes larger. For example, for $S_{AOQL} = 1.2485$, $\alpha = 0.01$, and $\beta = 0.01$, the required number of inspection data is 1688 for $S_{IQL} = 1.1534$ and only 485 for $S_{IQL} = 1.0750$.

Figures 2-4 show the OC curves to depict a comparison of the optimal scheme with other two AOQL contour inspection schemes in CSP-1. It can be seen from Figures 2-4 that, for various given values of the quality constraint AOQL and various values of the process quality limit $p_{IOL}(S_{IOL})$ (which represents the cost constraint), the OC curves for the proposed integrated schemes are more ideal than the OC curves for the other two schemes in CSP-1. When α and β take larger values, for example, when $\alpha = 0.1$ and $\beta = 0.1$, the OC curves still have a more ideal shape than the curves for the other two schemes in CSP-1 at a higher yield level of S_{IOL} . When α and β are both given smaller values, for example, $\alpha = 0.01$ and $\beta = 0.01$, the OC curves are more ideal than the curves for CSP-1 at various quality limit levels of $p_{IOL}(S_{IOL})$. The OC curves for the optimal schemes move towards the right when the values of p_{IOL} become bigger (the values of S_{IOL} become smaller), which shows that the risk control scheme can supply the more rational probabilities of the acceptance and rejection for the quality control scheme in the optimal scheme than the AOQL contour scheme in CSP-1.

It can be noted that the inspection workload has not increased in the proposed integrated scheme because the number of inspection data *n* used to calculate the estimator of \hat{S}_{pk} is recorded during the CSP-1.

5. Example Application

In order to present the way in which the proposed optimal scheme can be applied in practice, the following example taken from a compressor manufacturing enterprise is considered. A cylinder is the key functioning part of an air conditioning compressor. Cylinder thickness is an important dimension for ensuring compressor performance. Based on the quality requirements given by the designer, the tolerance range of the cylinder thickness is set to 27.784 ± 0.002 and the quality requirement AOQL is set to 0.00018. The current inspection scheme, (i, f) = (1540, 0.5), is adopted in CSP-1 to control the outgoing quality. For using the optimal scheme, the values of the four constraints are specified as AOQL = $0.00018 (S_{AOQL} = 1.2485), AFI_L = 0.8571 (S_{IQL} = 1.0750),$ $\alpha = 0.05$, and $\beta = 0.05$. From Table 3, the optimal scheme can be found as $(i_L, f_L, n, s_0) = (925, 0.6515, 242, 1.1553).$ Using the proposed control scheme, the probability of accepting an in-control process with high quality level (e.g., the nonconforming fraction of the in-control process is lower than the value of AOQL = 0.00018) is greater than $1-\alpha = 0.95$. The probability of accepting an in-control process with low quality level (e.g., the value of the index \widehat{S}_{pk} is lower than the value of $S_{IQL} = 1.0750$) is less than the value of $\beta = 0.05$. Performing the quality control scheme $(i_L, f_L) = (925, 0.6515),$

α β		$S_{IQL} =$	$S_{IQL} = 1.1534$		1.1067	$S_{IQL} = 1.0750$		
	(AFI_{I})		= 0.6667)	$(AFI_L$	= 0.8)	$(AFI_L =$	$(AFI_L = 0.8571)$	
		$(i_L, f_L) = (2$	2776, 0.3086)	$(i_L, f_L) = (1$	388, 0.5342)	$(i_L, f_L) = ($	925, 0.6515)	
		п	s_0	п	s ₀	п	s ₀	
0.01	0.01	1688	1.1985	740	1.1730	485	1.1553	
	0.05	1271	1.1895	551	1.1610	362	1.1406	
	0.10	1046	1.1850	459	1.1520	305	1.1308	
0.05	0.01	1229	1.2060	523	1.1850	345	1.1702	
	0.05	843	1.1985	370	1.1730	242	1.1553	
	0.10	673	1.1925	296	1.1640	195	1.1446	
0.1	0.01	996	1.2120	431	1.1940	279	1.1808	
	0.05	661	1.2045	290	1.1820	188	1.1661	
	0.10	512	1.1985	225	1.1730	147	1.1553	

TABLE 3: The values of (n, s_0) at $S_{AOQL} = 1.2485$ (AOQL = 0.00018) and three levels of $S_{IQL}(AFI_L)$ for $\alpha = 0.01, 0.05$, and 0.1 and $\beta = 0.01, 0.05$, and 0.1.

TABLE 4: The values of (n, s_0) at $S_{AOQL} = 1.2485$ (AOQL = 0.00018) and three levels of $S_{IQL}(AFI_L)$ for $\alpha = 0.01, 0.05$, and 0.1 and $\beta = 0.01, 0.05$, and 0.1.

αβ		$S_{IQL} =$	$S_{IQL} = 0.9520$		0.8966	$S_{IQL} = 0.8585$	
		$(AFI_L =$	$(AFI_L = 0.6667)$		(= 0.8)	$(AFI_L = 0.8571)$	
		$(i_L, f_L) = ($	348, 0.3092)	$(i_L, f_L) = ($	174, 0.5351)	$(i_L, f_L) = ($	115, 0.6527)
		n	s ₀	п	s ₀	п	s ₀
0.01	0.01	874	1.0035	371	0.9720	239	0.9498
	0.05	655	0.9945	281	0.9585	181	0.9327
	0.10	554	0.9885	236	0.9480	153	0.9214
0.05	0.01	639	1.0125	266	0.9870	168	0.9674
	0.05	437	1.0035	186	0.9720	120	0.9498
	0.10	358	0.9975	150	0.9615	97	0.9373
0.1	0.01	508	1.0200	214	0.9960	135	0.9799
	0.05	346	1.0110	144	0.9825	92	0.9624
	0.10	265	1.0035	113	0.9720	73	0.9498

TABLE 5: The values of (n, s_0) at $S_{AOQL} = 1.2485$ (AOQL = 0.00018) and three levels of $S_{IQL}(AFI_L)$ for $\alpha = 0.01, 0.05$, and 0.1 and $\beta = 0.01, 0.05$, and 0.1.

α β		$S_{IQL} =$	$S_{IQL} = 0.6967$		0.6245	$S_{IQL} = 0.5734$	
		$(AFI_L = 0.6667)$		(AFI_{L})	2 = 0.8)	$(AFI_L = 0.8571)$	
		$(i_L, f_L) = 0$	(39, 0.3145)	$(i_L, f_L) =$	(19, 0.5437)	$(i_L, f_L) =$	(12, 0.6629)
		п	s ₀	п	s ₀	п	s_0
0.01	0.01	324	0.7590	129	0.7140	78	0.6800
	0.05	246	0.7470	99	0.6960	61	0.6590
	0.10	206	0.7395	85	0.6855	52	0.6452
0.05	0.01	229	0.7710	89	0.7320	53	0.7026
	0.05	163	0.7590	65	0.7140	39	0.6800
	0.10	133	0.7500	53	0.7012	32	0.6647
0.1	0.01	187	0.7800	71	0.7455	42	0.7188
	0.05	127	0.7680	49	0.7267	29	0.6961
	0.10	100	0.7590	39	0.7140	24	0.6800



FIGURE 2: Comparisons of OC curves for the optimal schemes and inspection schemes in CSP-1 under AOQL = 0.00018 (a) at $AFI_L = 0.6667$, (b) at $AFI_L = 0.8$, and (c) at $AFI_L = 0.8571$.

242 sample items are recorded consecutively, as shown in Table 6. The normal probability plot of the 242 sample items is displayed in Figure 5. Figure 6 shows the histogram of the 242 observed data with the lower and upper specification limits. Obviously, based on the normality test in Figures 5 and 6, the in-control process is shown to be close to a normal distribution. Carrying out the calculation with the 242 inspection data, we get $\overline{x} = 27.7842$, s = 0.000517, $\hat{S}_{pk} = 1.2144$. Obviously, the estimator of $\hat{S}_{pk} = 1.2144$ is greater than the value of $s_0 = 1.1553$. The optimal scheme which can simultaneously meet the two constraints AOQL = 0.00018 and $AFI_L =$ 0.8571 will be continued. Table 7 shows a comparison of the

					in composition and	mas dour am mon				
27.7834	27.7844	27.7839	27.7834	27.7844	27.7838	27.7851	27.7845	27.7836	27.784	27.7842
27.7834	27.7842	27.7836	27.7832	27.7846	27.785	27.7844	27.7850	27.7837	27.7844	27.7840
27.7844	27.7844	27.7835	27.7834	27.7839	27.7852	27.7845	27.7848	27.7836	27.7841	27.7848
27.7844	27.7849	27.7835	27.7836	27.7838	27.7846	27.785	27.7846	27.7841	27.7845	27.7854
27.7849	27.7842	27.7837	27.7834	27.7846	27.7847	27.7844	27.7852	27.7844	27.7841	27.7845
27.7844	27.7841	27.7838	27.7830	27.7840	27.7843	27.7844	27.7843	27.7839	27.7844	27.7843
27.7847	27.7844	27.7844	27.783	27.7842	27.7845	27.7840	27.7852	27.7837	27.7843	27.7849
27.7849	27.7835	27.7846	27.7834	27.7839	27.7849	27.7839	27.7848	27.7844	27.7835	27.7845
27.7849	27.7836	27.7834	27.7844	27.7842	27.7846	27.7837	27.7844	27.784	27.7842	27.7846
27.7849	27.7836	27.7830	27.7842	27.7845	27.7844	27.7847	27.7846	27.7844	27.7843	27.7848
27.7847	27.7846	27.7831	27.7841	27.7844	27.7851	27.7836	27.785	27.7844	27.7842	27.7849
27.7846	27.7836	27.7832	27.7842	27.7843	27.7846	27.7836	27.7850	27.7842	27.7834	27.7843
27.7849	27.7836	27.7834	27.7838	27.7845	27.7845	27.7838	27.7851	27.7842	27.7834	27.7840
27.7844	27.7839	27.7832	27.7841	27.7847	27.7847	27.7841	27.7846	27.7842	27.7834	27.7840
27.7846	27.7841	27.7829	27.7842	27.7844	27.7849	27.7844	27.7845	27.7845	27.7834	27.7837
27.7842	27.7837	27.7835	27.7844	27.7846	27.7852	27.7846	27.7842	27.7843	27.7835	27.7844
27.7849	27.7839	27.7832	27.7844	27.7849	27.7849	27.7842	27.7844	27.7844	27.7835	27.7841
27.7844	27.7838	27.7834	27.7841	27.7844	27.7848	27.7848	27.7851	27.7844	27.7847	27.7843
27.7844	27.7834	27.7833	27.7844	27.7844	27.7847	27.7841	27.7847	27.7843	27.784	27.7844
27.7842	27.7841	27.7843	27.7844	27.7844	27.7837	27.7842	27.7846	27.7839	27.7844	27.7851
27.7836	27.7836	27.7836	27.7842	27.7842	27.7845	27.7845	27.7842	27.7844	27.7844	27.7843
27.7846	27.7840	27.7845	27.7842	27.7838	27.7836	27.7836	27.7836	27.7836	27.7836	27.7837

TABLE 6: The 242 observations collected consecutively from the inspection process in CSP-I.



FIGURE 3: Comparisons of OC curves for the optimal schemes and inspection schemes in CSP-1 under AOQL = 0.00143 (a) at $AFI_L = 0.6667$, (b) at $AFI_L = 0.8$, and (c) at $AFI_L = 0.8571$.

performances between the original inspection scheme and the optimal scheme, where L(p) represents the probability of acceptance. It can be seen from Table 7 that the performance *AFI* increases by 0.103172, *AOQ* decreases by 0.000028, and the value of the probability of acceptance increases by 0.062855.

6. Conclusions

An optimal scheme for the process quality and cost control is proposed to monitor the process capability and improve process quality. The CSP-1 and S_{pk} , which play independent and complimentary roles, are integrated in the optimal scheme.



FIGURE 4: Comparisons of OC curves for the optimal schemes and inspection schemes in CSP-1 under AOQL = 0.0122 (a) at AFI_L = 0.6667, (b) at AFI_L = 0.8, and (c) at AFI_L = 0.8571.

The sole feasible inspection scheme in CSP-1 for meeting concurrently the quality and cost constraints is one of the AOQL contour schemes in CSP-1, which occurs at the point of $p_L = p_{IQL}$. Two types of risk under quality and cost constraints are simultaneously controlled at the stipulated level with the risk control scheme. The risk control scheme is constructed with two nonlinear inequalities based on

the accurate distribution of the natural estimator \hat{S}_{pk} . The combination of the two risk control parameters (n, s_0) plays the role of the stopping rule in the inspection scheme (i_L, f_L) . There is a one-to-one correspondence between the values of the four parameters (i_L, f_L, n, s_0) and the given values of the four constraints (quality, cost, and the two risks). The proposed optimal scheme shows the advantages over the



FIGURE 5: The normal probability plot of the 242 observations.



FIGURE 6: The histogram of the 242 observations with double specification limits.

TABLE 7: Comparison of the performances of AFI, AOQ, and L(p).

	(i, f) = (1540, 0.5)	$(i_L, f_L, n, s_0) =$ (925, 0.6515, 242, 1.1553)
AFI	0.602509	0.705681
AOQ	0.000107	0.000079
L(p)	0.794982	0.857837

original process control tools in the measures of *AFI*, *AOQ*, and the probability of acceptance.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- H. F. Dodge, "A sampling inspection plan for continuous production," *Annals of Mathematical Statistics*, vol. 14, pp. 264–279, 1943.
- [2] K. Govindaraju and C. Kandasamy, "Design of generalized CSP-C continuous sampling plan," *Journal of Applied Statistics*, vol. 27, no. 7, pp. 829–841, 2000.
- [3] G. J. Lieberman and H. Solomon, "Multi-level continuous sampling plans," *Annals of Mathematical Statistics*, vol. 26, pp. 686– 704, 1955.
- [4] P. Guayjarernpanishk and T. Mayureesawan, "The design of two-level continuous sampling plan MCSP-2-C," *Applied Mathematical Sciences*, vol. 6, no. 89-92, pp. 4483–4495, 2012.

- [5] S. Balamurali and K. Subramani, "Modified CSP-C continuous sampling plan for consumer protection," *Journal of Applied Statistics*, vol. 31, no. 4, pp. 481–494, 2004.
- [6] K. Govindaraju and S. Balamurali, "Tightened single-level continuous sampling plan," *Journal of Applied Statistics*, vol. 25, no. 4, pp. 451–461, 1998.
- [7] C. Kandasamy and K. Govindaraju, "Selection of tightened twolevel continuous sampling plans," *Journal of Applied Statistics*, vol. 20, no. 2, pp. 271–284, 1993.
- [8] V. S. S. Kumar, "A tightened M-level continuous sampling plan for Markov-dependent production processes," *IIE Transactions*, vol. 16, no. 3, pp. 257–261, 1984.
- [9] C. R. Cassady, L. M. Maillart, I. J. Rehmert, and J. A. Nachlas, "Demonstrating deming's kp rule using an economic model of the CSP-1," *Quality Engineering*, vol. 12, no. 3, pp. 327–334, 2000.
- [10] A. J. Duncan, Quality control and industrial statistics, Revised ed, Richard D. Irwin, Inc., Homewood, Ill., 1959.
- [11] M. Eleftheriou and N. Farmakis, "Expected cost for continuous sampling plans," *Communications in Statistics—Theory and Methods*, vol. 40, no. 16, pp. 2969–2984, 2011.
- [12] H. F. Yu, W. C. Yu, and W. P. Wu, "A mixed inspection policy for CSP-1 and precise inspection under inspection errors and return cost," *Computers & Industrial Engineering*, vol. 57, no. 3, pp. 652–659, 2009.
- [13] A. Haji and R. Haji, "The optimal policy for a sampling plan in continuous production in terms of the clearance number," *Computers & Industrial Engineering*, vol. 47, no. 2-3, pp. 141–147, 2004.
- [14] C. H. Chen and C. Y. Chou, "Economic design of continuous sampling plan under linear inspection cost," *Journal of Applied Statistics*, vol. 29, no. 7, pp. 1003–1009, 2002.
- [15] W. Xu, Y. Y. Yu, and Q. S. Zhang, "An evaluation method of comprehensive product quality for customer satisfaction based on intuitionistic fuzzy number," *Discrete Dynamics in Nature Society*, vol. 2018, no. 1, p. 12, 2018.
- [16] B. Bouslah, A. Gharbi, and R. Pellerin, "Joint economic design of production, continuous sampling inspection and preventive maintenance of a deteriorating production system," *International Journal of Production Economics*, vol. 173, pp. 184–198, 2016.
- [17] J. Zhang, "Optimal control problem of converter steelmaking production process based on operation optimization method," *Discrete Dynamics in Nature and Society*, Art. ID 483674, 13 pages, 2015.
- [18] R. Mehdi, R. Nidhal, and C. Anis, "Integrated maintenance and control policy based on quality control," *Computers & Industrial Engineering*, vol. 58, no. 3, pp. 443–451, 2010.
- [19] C. H. Chen, C. Y. Chou, and T. S. Cheng, "Joint design of continuous sampling plans and specification limits," *The International Journal of Advanced Manufacturing Technology*, vol. 21, pp. 235–237, 2003.
- [20] C. H. Chen and C.-Y. Chou, "Joint design of economic manufacturing quantity, sampling plan and specification limits," *Economic Quality Control*, vol. 17, no. 2, pp. 145–153, 2002.
- [21] C. H. Chen and C. Y. Chou, "Design a continuous sampling plan based on quadratic quality loss function," Asia Pacific Management Review, vol. 6, no. 4, pp. 485–489, 2001.
- [22] S. Muthulakshmi, "Stopping rules to limit inspection effort in CSP-C continuous sampling plan," *International Journal of Mathematical Archive*, vol. 3, pp. 656–662, 2012.

- [23] Y. L. Fan, "Two more excellent stopping rules for continuous sampling plan," *Journal of Applied Statistics*, vol. 19, no. 6, pp. 24–30, 2000.
- [24] Y. L. Fan, "Continuous sampling plan CSP-1 with stopping inspection rule (n^{*}-1)," *Acta Mathematicae Applicatae Sinica*, vol. 19, no. 1, pp. 135–143, 1995.
- [25] R. B. Murphy, "Stopping rules with CSP-1 sampling plans," *Industrial Control*, vol. 16, no. 5, pp. 10–16, 1959.
- [26] R. A. Boyles, "Process capability with asymmetric tolerances," *Communications in Statistics—Simulation and Computation*, vol. 23, no. 3, pp. 615–643, 1994.
- [27] J. C. Lee, H. N. Hung, W. L. Pearn, and T. L. Kueng, "On the distribution of the estimated process yield index Spk," *Quality* and Reliability Engineering International, vol. 18, pp. 111–116, 2002.
- [28] C. W. Wu and M.-Y. Liao, "Estimating and testing process yield with imprecise data," *Expert Systems with Applications*, vol. 36, no. 8, pp. 11006–11012, 2009.
- [29] W. L. Pearn and C. C. Chuang, "Accuracy analysis of the estimated process yield based on Spk," *Quality and Reliability Engineering International*, vol. 20, no. 4, pp. 305–316, 2004.
- [30] F. K. Wang, "Process Yield for Multiple Stream Processes with Individual Observations and Subsamples," *Quality and Reliability Engineering International*, vol. 32, no. 2, pp. 335–344, 2016.
- [31] F. K. Wang, "Measuring the process yield for circular profiles," *Quality and Reliability Engineering International*, vol. 31, no. 4, pp. 579–588, 2015.
- [32] F. K. Wang and Y. Tamirat, "Process yield analysis for autocorrelation between linear profiles," *Computers & Industrial Engineering*, vol. 71, no. 1, pp. 50–56, 2014.
- [33] A. Lepore, B. Palumbo, and P. Castagliola, "A note on decision making method for product acceptance based on process capability indices C_{pk} and C_{pmk}," *European Journal of Operational Research*, vol. 267, no. 1, pp. 393–398, 2018.
- [34] M. S. Fallah Nezhad and S. Seifi, "Repetitive group sampling plan based on the process capability index for the lot acceptance problem," *Journal of Statistical Computation and Simulation*, vol. 87, no. 1, pp. 29–41, 2017.
- [35] Y. Tamirat and F. K. Wang, "Sampling Plan based on the Exponentially Weighted Moving Average Yield Index for Autocorrelation within Linear Profiles," *Quality and Reliability Engineering International*, vol. 32, no. 5, pp. 1757–1768, 2016.
- [36] I. Negrin, Y. Parmet, and E. Schechtman, "Developing a sampling plan based on Cpk-unknown variance," *Quality and Reliability Engineering International*, vol. 27, no. 1, pp. 3–14, 2011.
- [37] G. L. Yang and G. L. Yang, "A renewal-process approach to continuous sampling plans," *Technometrics*, vol. 25, no. 1, pp. 59–67, 1983.
- [38] C. Z. Li, S. R. Tong, and K. Q. Wang, "Optimal CSP-1 boundary scheme based on the estimator of the proportion of conformance for specified in-control process," *Quality Technology & Quantitative Management*, 2018.





International Journal of Mathematics and Mathematical Sciences





Applied Mathematics

Hindawi

Submit your manuscripts at www.hindawi.com



The Scientific World Journal



Journal of Probability and Statistics







International Journal of Engineering Mathematics

Complex Analysis

International Journal of Stochastic Analysis



Advances in Numerical Analysis



Mathematics



Mathematical Problems in Engineering



Journal of **Function Spaces**



International Journal of **Differential Equations**



Abstract and Applied Analysis



Discrete Dynamics in Nature and Society



Advances in Mathematical Physics