Research Article

Permanence and Existence of Periodic Solutions for Nicholson's Blowflies Model with Feedback Control and Delay on Time Scales

Lin-Lin Wang and Yong-Hong Fan

School of Mathematics and Statistics Science, Ludong University, Yantai, Shandong 264025, China

Correspondence should be addressed to Yong-Hong Fan; fanyh.1993@sina.com

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Sufficient conditions are established for the permanence in a delayed Nicholson's blowflies model with feedback control on time scales. Our investigation confirms that the bounded feedback terms do not have any influence on the permanence of this system.

1. Introduction

In 2008, we considered the following discrete Nicholson's blowflies model with feedback control (see [1]):

$$x(n+1) = x(n) \exp \{-\delta(n) + p(n) \exp \{-\alpha(n)x(n)\} - c(n)U(n)\}, \quad (1)$$

$$\Delta U(n) = -a(n)U(n) + b(n)x(n-m),$$

where δ, α, p, b , and $c: \mathbb{Z}$ (integer number set) $\rightarrow \mathbb{R}^+$ (nonnegative real number set) are all bounded sequences, $a: \mathbb{Z} \rightarrow (0, 1)$ is also a bounded sequence and $\inf a > 0$, m is a nonnegative integer, and Δ is the first-order forward difference operator and obtained the following.

Theorem 1. Assume that

$$\inf_{n \in [0, +\infty) \cap \mathbb{Z}} (p(n) - \delta(n)) > 0 \quad (2)$$

hold true; then system (1) is permanent.

The continuous and discrete systems always appear separately, until in 1988, the theory of time scales, which has recently received much attention, was initiated by Hilger [2] in his Ph.D. thesis to unify both difference and differential calculus in a consistent way. Since then many

authors have investigated the dynamic equations on time scales (see [3–6]). This theory provides a powerful tool for applications to economics, population models, quantum physics, among others. In fact, the progressive field of dynamic equations on time scales contains, links, and extends the classical theory of differential and difference equations.

For the origin of mathematical model for Nicholson's blowflies, one can see [7]. For further study on equations with feedback control, we refer to [8–14] and references therein. And for investigation on delay differential equations, we refer to [15, 16] and so on.

In this paper, we will discuss the permanence of the following system:

$$x^\Delta(t) = -\delta(t) + p(t) \exp \{-\alpha(t) \exp \{x(t - \tau(t))\} - c(t)y(t - \theta(t)), \quad (3)$$

$$y^\Delta(t) = -a(t)y(t) + b(t) \exp \{x(t - \eta(t))\},$$

on time scales $\mathbb{T}$, where Δ stands for the delta-derivative and

$$1 - \mu(t)a(t) > 0, \quad (4)$$

in which $\mu(t) = \sigma(t) - t$ and $\sigma(t)$ is the forward jump operator on $\mathbb{T}$. To make (3) meaningful, we suppose that, for any $t \in \mathbb{T}$,

$$\begin{aligned} t - \tau(t) &\in \mathbb{T}, \\ t - \theta(t) &\in \mathbb{T}, \\ t - \eta(t) &\in \mathbb{T}. \end{aligned} \quad (5)$$

We assume that $\sup \mathbb{T} = \infty$, and, without loss of generality, suppose $0 \in \mathbb{T}$. In view of the biological significance, we also assume that the coefficient functions $\delta, \alpha, p, b, c, \tau, \theta$, and $\eta: \mathbb{T} \rightarrow \mathbb{R}^+$ are all bounded rd-continuous and $\inf_{t \in \mathbb{T}} a > 0$ (the definition of rd-continuous function will be given in Section 2).

When $\mathbb{T} = \mathbb{R}$, let $N(t) = \exp\{x(t)\}$, $U(t) = y(t)$, and then (3) can be rewritten as

$$\begin{aligned} N'(t) &= -\delta(t) N(t) \\ &\quad + p(t) N(t) \exp\{-\alpha(t) N(t - \tau(t))\} \\ &\quad - c(t) U(t - \theta(t)), \\ U'(t) &= -a(t) U(t) + b(t) N(t - \eta(t)). \end{aligned} \quad (6)$$

When $\mathbb{T} = \mathbb{Z}$, if we let $N(t) = \exp\{x(t)\}$, then (3) can be rewritten as

$$\begin{aligned} N(n+1) &= N(n) \exp\{-\delta(n) \\ &\quad + p(n) \exp\{-\alpha(n) x(n - \tau(n))\} \\ &\quad - c(n) U(n - \theta(n))\}, \\ \Delta U(n) &= -a(n) U(n) + b(n) x(n - \eta(n)). \end{aligned} \quad (7)$$

Obviously, (7) includes (1).

In what follows we shall use the notations

$$\begin{aligned} f^u &= \sup_{t \in \mathbb{T}} f(t), \\ f^l &= \inf_{t \in \mathbb{T}} f(t), \end{aligned} \quad (8)$$

where f is a bounded rd-continuous function in $\mathbb{T}$. Throughout this paper, we assume that

$$\begin{aligned} a^l &> 0, \\ \delta^l &> 0. \end{aligned} \quad (9)$$

In this case, for any $t \in \mathbb{T}$,

$$\mu(t) \leq \left(\frac{1}{a(t)} \right)^u =: L. \quad (10)$$

2. Preliminary

Before giving our main result, first we list some basic properties about time scales which could be found in ([2, 17, 18]).

Definition 2. A time scale is an arbitrary nonempty closed subset $\mathbb{T}$ of the real number $\mathbb{R}$.

Definition 3. For $t \in \mathbb{T}$ we define the forward jump operator $\sigma: \mathbb{T} \rightarrow \mathbb{T}$ and the backward jump operator $\rho: \mathbb{T} \rightarrow \mathbb{T}$, by

$$\begin{aligned} \sigma(t) &:= \inf \{s \in \mathbb{T} : s > t\}, \\ \rho(t) &:= \sup \{s \in \mathbb{T} : s < t\}, \end{aligned} \quad (11)$$

respectively.

Definition 4. Define the interval $[a, b]$ in $\mathbb{T}$ by

$$[a, b] := \{t \in \mathbb{T} : a \leq t \leq b\}. \quad (12)$$

Other types of intervals are defined similarly.

Definition 5 (Definition 1.58 in [18], P22). A function $f: \mathbb{T} \rightarrow \mathbb{R}$ is called rd-continuous provided that it is continuous at right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at left-dense points in $\mathbb{T}$. The set of rd-continuous functions $f: \mathbb{T} \rightarrow \mathbb{R}$ is denoted by

$$C_{rd} = C_{rd}(\mathbb{T}) = C_{rd}(\mathbb{T}, \mathbb{R}). \quad (13)$$

Definition 6. Assume $f: \mathbb{T} \rightarrow \mathbb{R}$ and let $t \in \mathbb{T}^k$, where

$$\mathbb{T}^k = \begin{cases} \mathbb{T} \setminus (\rho(\sup \mathbb{T}), \sup \mathbb{T}], & \text{if } \sup \mathbb{T} < \infty, \\ \mathbb{T}, & \text{if } \sup \mathbb{T} = \infty. \end{cases} \quad (14)$$

Then we define $f^\Delta(t)$ to be the number (provided it exists) with property that given any $\varepsilon > 0$, there is a neighborhood U of t such that

$$|[f(\sigma(t)) - f(s)] - f^\Delta(t)[\sigma(t) - s]| \leq \varepsilon |\sigma(t) - s|, \quad (15)$$

for all $s \in U$. We call $f^\Delta(t)$ the delta (or Hilger) derivative of $f(t)$ and it turns out that f^Δ is the usual derivative if $\mathbb{T} = \mathbb{R}$ and is the usual forward difference operator if $\mathbb{T} = \mathbb{Z}$.

Lemma 7 (Theorem 1.76 in [18], P28). *If $f^\Delta \geq 0$, then f is increasing.*

Definition 8 (Definition 2.25 in [18], P58). We say that a function $p: \mathbb{T} \rightarrow \mathbb{R}$ is regressive provided that

$$1 + \mu(t)p(t) \neq 0 \quad \text{for all } t \in \mathbb{T}^k \quad (16)$$

holds. The set of all regressive and rd-continuous functions $f: \mathbb{T} \rightarrow \mathbb{R}$ will be denoted by

$$\mathfrak{R} = \mathfrak{R}(\mathbb{T}) = \mathfrak{R}(\mathbb{T}, \mathbb{R}). \quad (17)$$

Definition 9. If $p \in \mathfrak{R}$, we define the exponential function by

$$e_p(t, s) = \exp \left(\int_s^t \xi_{\mu(\tau)}(p(\tau)) \Delta \tau \right), \quad \text{for } s, t \in \mathbb{T}, \quad (18)$$

where the cylinder transformation $\xi_h(z) = (1/h)\log(1 + zh)$, for $h > 0$.

For $h = 0$, we define $\xi_0(z) = z$. In this case, $\mathbb{T} = \mathbb{R}$, $\mu(t) = 0$, and $e_p(t, s) = \exp(\int_s^t p(\tau) \Delta \tau)$.

When $\mathbb{T} = h\mathbb{Z} = \{hk : k \in \mathbb{Z}\}$, we know that $\mu(hk) = h(k+1) - hk = h > 0$, and then for $s < t$,

$$\begin{aligned} e_p(t, s) &= \exp \left\{ \int_s^t \frac{1}{h} \log_e (1 - ha(\tau)) \Delta \tau \right\} \\ &= \exp \left\{ \sum_{k=s/h}^{t/h-1} \log_e (1 - ha(hk)) \right\} \\ &= \prod_{k=s/h}^{t/h-1} \{1 - ha(hk)\}. \end{aligned} \quad (19)$$

Definition 10. If $p \in \mathfrak{R}$, the function $\Theta(p)$ is defined by

$$\Theta(p)(t) = -\frac{p(t)}{1 + \mu(t)p(t)}, \quad \text{for } t \in \mathbb{T}^k. \quad (20)$$

Lemma 11. Suppose $p \in \mathfrak{R}$; then

- (1) $\Theta(\Theta(p)) = p$;
- (2) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t, s)$;
- (3) $1/e_p(t, s) = e_{\Theta(p)}(t, s)$;
- (4) $(1/e_p(\cdot, s))^\Delta = -p(\cdot)/e_p^\sigma(\cdot, s)$;
- (5) $(e_p(\cdot, s))^\Delta = p(\cdot)e_p(\cdot, s)$.

Lemma 12 (Theorem 2.44 in [18], P66). Assume that $p \in \mathfrak{R}$ and $t_0 \in \mathbb{T}$. If $1 + \mu(t)p(t) > 0$ on $\mathbb{T}^k$, then $e_p(t, t_0) > 0$ for all $t \in \mathbb{T}$.

Lemma 13 (Theorem 2.77 in [18], P77). Suppose $p \in \mathfrak{R}$ and $f : \mathbb{T} \rightarrow \mathbb{R}$ is rd-continuous. Let $t_0 \in \mathbb{T}$ and $y_0 \in \mathbb{R}$. The unique solution of the initial value problem

$$\begin{aligned} y^\Delta &= p(t)y + f(t), \\ y(t_0) &= y_0 \end{aligned} \quad (21)$$

is given by

$$y(t) = e_p(t, t_0)y_0 + \int_{t_0}^t e_p(t, \sigma(s))f(s)\Delta s. \quad (22)$$

In order to give our main result, we also need to establish the following definitions and lemmas. The first definition is the generalized version of the semicycle in discrete situation [19].

Definition 14. Let m be a constant and $f : \mathbb{T} \rightarrow \mathbb{R}$; a **positive semicycle relative to m** of f is defined as follows, as it consists of a “string” of terms:

$$\{f(t), t \in [s, t], s, t \in \mathbb{T}\}, \quad (23)$$

all greater than or equal to m , and a **negative semicycle relative to m** of f is defined as follows, as it is a “string” of terms:

$$\{f(t), t \in [s, t], s, t \in \mathbb{T}\}, \quad (24)$$

all less than or equal to m .

The following two lemmas could be found in [20].

Lemma 15. Assume that $A, B > 0$ and $x_1(0) = x_0 > 0$; furthermore suppose that

(1)

$$x_1^\Delta(t) \leq B - A \exp\{x_1(t)\}, \quad \text{for } t \geq 0, \quad (25)$$

and then

$$\lim_{t \rightarrow \infty} \sup x_1(t) \leq BL + \ln \frac{B}{A}, \quad (26)$$

(2)

$$x_1^\Delta(t) \geq B - A \exp\{x_1(t)\}, \quad \text{for } t \geq 0, \quad (27)$$

and there exists a constant $M > 0$, such that $\lim_{t \rightarrow \infty} \sup x_1(t) < M$; then

$$\lim_{t \rightarrow \infty} \inf x_1(t) \geq (B - A \exp\{M\})L + \ln \frac{B}{A}. \quad (28)$$

Lemma 16. Assume that $C(t), D(t) > 0$ are bounded rd-continuous functions and $C^l > 0$; furthermore suppose that the Δ -differentiable function $y_1(t)$ satisfies

(1)

$$y_1^\Delta(t) + C(t)y_1(t) \leq D(t), \quad \text{for } t \geq T_0, \quad (29)$$

and $D(t)$ is Δ -differentiable; then there exists a constant $T_2 > T_0$, such that for $t > T_2$, we have

$$y_1(t) \leq y_1(T_2)e_{(-C)}(t, T_2) + \frac{D(t)}{C^l}. \quad (30)$$

Especially, if $D(t)$ is bounded above ultimately with respect to K_1 , then

$$\lim_{t \rightarrow \infty} \sup y_1(t) \leq \frac{K_1}{C^l}. \quad (31)$$

(2)

$$y_1^\Delta(t) + C(t)y_1(t) \geq D(t), \quad \text{for } t \geq T_0, \quad (32)$$

and there exists a constant $T_3 > T_0$, such that, for $t > T_3$, we have

$$y_1(t) \geq \left(y_1(T_3) - \frac{D(T_3)}{C^u} \right) e_{(-C)}(t, T_3) + \frac{D(T_3)}{C^u}. \quad (33)$$

Especially, if $D(t)$ is bounded below ultimately with respect to k_1 , then

$$\lim_{t \rightarrow \infty} \inf y_1(t) \geq \frac{k_1}{C^u}. \quad (34)$$

Before giving our main result, we list the definition of uniform ultimate boundedness.

Definition 17. Solutions for system (3) are said to be uniformly ultimate bounded if there exist two constants λ_1 and λ_2 such that, for any initial condition

$$\begin{aligned} (\phi(t), \psi(t))^T &\in C_{rd}([- \lambda, 0], \mathbb{R} \times \mathbb{R}^+), \\ \lambda &= \max\{\tau^u, \eta^u, \theta^u\}, \end{aligned} \quad (35)$$

we have

$$\begin{aligned} \lambda_1 &\leq \liminf_{t \rightarrow \infty} x(t) \leq \limsup_{t \rightarrow \infty} x(t) \leq \lambda_2, \\ \lambda_1 &\leq \liminf_{t \rightarrow \infty} y(t) \leq \limsup_{t \rightarrow \infty} y(t) \leq \lambda_2, \end{aligned} \quad (36)$$

where λ_1 and λ_2 are independent of $(\phi(t), \psi(t))^T$.

3. Main Results

First, we give a lemma which will be useful for our further discussion.

Lemma 18. Let $(x(t), y(t))$ be any solution of system (3) with initial condition (35); then

$$\begin{aligned} \exp\{x(t)\} &> 0, \\ y(t) &> 0 \\ &\text{for all } t \in \mathbb{T}. \end{aligned} \quad (37)$$

Proof. The exponential form $\exp\{x(t)\}$ ensures that $\exp\{x(t)\} > 0$ for all $t \in \mathbb{T}$. Now we consider the second equation of system (3); by Lemma 13 and (4), we have

$$\begin{aligned} y(t) &= e_{-a}(t, 0) y_0 \\ &+ \int_0^t e_{-a}(t, \sigma(s)) b(s) \exp\{x(s - \eta(s))\} \Delta s \\ &> 0 \quad \text{for } t \geq 0. \end{aligned} \quad (38)$$

The proof is complete. $\square$

In the sequel, we assume that

$$(p(n) - \delta(n))^l > 0. \quad (39)$$

Theorem 19. Assume that (39) holds true; then system (3) is uniformly ultimate bounded.

We now prove the following result before proving Theorem 19. In fact, the two theorems are equivalent to each other.

Theorem 20. Assume that (39) holds true; then there exist two positive constants λ_1 and λ_2 such that

$$\begin{aligned} \lambda_1 &\leq \liminf_{t \rightarrow \infty} \exp\{x(t)\} \leq \limsup_{t \rightarrow \infty} \exp\{x(t)\} \\ &\leq \lambda_2, \end{aligned} \quad (40)$$

and

$$\lambda_1 \leq \liminf_{t \rightarrow \infty} y(t) \leq \limsup_{t \rightarrow \infty} y(t) \leq \lambda_2, \quad (41)$$

for any solutions $(x(t), y(t))$ of (3) with initial condition (35).

Proof. We divided the proof into four claims.

Claim 1. There exists a positive constant K_1 such that

$$\lim_{t \rightarrow \infty} \sup x(t) \leq K_1, \quad (42)$$

where K_1 can be chosen as

$$K_1 = \frac{\exp\{(p^u - \delta^l)\tau^u\}}{\alpha^l} \ln \frac{p^u}{\delta^l} + (p^u - \delta^l)l. \quad (43)$$

Proof. From the first equation of system (3) and the positivity of $y(t)$, we have

$$\begin{aligned} x^\Delta(t) &\leq -\delta(t) + p(t) \exp\{-\alpha(t) \exp\{x(t - \tau(t))\}\} \\ &\leq -\delta^l + p^u \exp\{-\alpha^l \exp\{x(t - \tau(t))\}\}. \end{aligned} \quad (44)$$

From the first equation of (3), we have

$$x^\Delta(t) \leq p^u - \delta^l, \quad \text{for all } t \in \mathbb{T}, \quad (45)$$

thus

$$x(t) - x(t - \tau(t)) \leq (p^u - \delta^l)\tau(t) \leq (p^u - \delta^l)\tau^u, \quad (46)$$

which implies that

$$\exp\{x(t - \tau(t))\} \geq \exp\{-(p^u - \delta^l)\tau^u\} \exp\{x(t)\}, \quad (47)$$

and by (44), we have

$$\begin{aligned} x^\Delta(t) &\leq -\delta^l \\ &+ p^u \exp\{-\alpha^l \exp\{-(p^u - \delta^l)\tau^u\} \exp\{x(t)\}\}, \end{aligned} \quad (48)$$

and for simplicity, we set

$$A_1 = \alpha^l \exp\{-(p^u - \delta^l)\tau^u\}, \quad (49)$$

obviously, $A_1 > 0$. In the following, we divided the proof into three cases:

(1) There exists some $N_1 \in \mathbb{T}$ such that $x(t) \leq (1/A_1) \ln(p^u/\delta^l)$ for all $t > N_1$, and then we have

$$\lim_{t \rightarrow \infty} \sup x(t) \leq \frac{1}{A_1} \ln \frac{p^u}{\delta^l} < K_1. \quad (50)$$

(2) There exists some $N_2 \in \mathbb{T}$ such that $x(t) > (1/A_1) \ln(p^u/\delta^l)$ for all $t > N_2$, and then from (48), we have

$$x^\Delta(t) \leq 0 \quad \text{for } t > N_2 + \tau^u, \quad (51)$$

which implies that $\lim_{t \rightarrow +\infty} x(t)$ exists and if we denote $\lim_{t \rightarrow +\infty} x(t) = \lambda$, and then

$$\lambda \geq \frac{1}{A_1} \ln \frac{p^u}{\delta^l}, \quad (52)$$

and thus (48) shows that

$$\{-\delta^l + p^u \exp \{-A_1 \lambda\}\} \geq 0, \quad (53)$$

that is,

$$\lambda \leq \frac{1}{A_1} \ln \frac{p^u}{\delta^l}, \quad (54)$$

while from (52) and (54), we get

$$\lambda = \frac{1}{A_1} \ln \frac{p^u}{\delta^l}; \quad (55)$$

therefore

$$\lim_{t \rightarrow \infty} \sup x(t) = \frac{1}{A_1} \ln \frac{p^u}{\delta^l} < K_1. \quad (56)$$

(3) For simplicity, set $\beta = (1/A_1) \ln(p^u/\delta^l)$. Assume that $x(t)$ oscillates about β . By (48), we know that $u(t) \geq \beta$ implies that $x^\Delta(t) \leq 0$. Thus, by Lemma 7, if we let $\{x(t_l)\}$ be the first element of a l th positive semicycle relative to β of $\{x(t)\}$, then

$$\lim_{t \rightarrow \infty} \sup x(t) = \lim_{t \rightarrow \infty} \sup x(t_l). \quad (57)$$

Now we divided the proof into two cases: t_l is left-dense and t_l is left-scattered. If the former holds, then

$$x(t_l) = \beta. \quad (58)$$

If the latter holds, then $x(\rho(t_l)) \leq \beta$, by (45), we have

$$\begin{aligned} \int_{\rho(t_l)}^{t_l} x^\Delta(s) \Delta s &\leq \int_{\rho(t_l)}^{t_l} (p^u - \delta^l) \Delta s \\ &= (p^u - \delta^l)(t_l - \rho(t_l)) \leq (p^u - \delta^l)L, \end{aligned} \quad (59)$$

and therefore

$$x(t_l) \leq x(\rho(t_l)) + (p^u - \delta^l)l \leq \beta + (p^u - \delta^l)L, \quad (60)$$

while from (58) and (60), we have

$$\lim_{t \rightarrow \infty} \sup x(t_l) \leq \beta + (p^u - \delta^l)L = K_1. \quad (61)$$

By (50), (56), and (61), we complete the proof of Claim 1.

Claim 2. There exists a positive constant K_2 such that

$$\lim_{t \rightarrow \infty} \sup y(t) \leq K_2. \quad (62)$$

Proof. From (42), we know that, for any arbitrary positive number ε , there exists an N_3 such that $x(t) \leq K_1 + \varepsilon$ for all $t > N_3$, and then from the second equation of system (3), we have

$$\begin{aligned} y^\Delta(t) &\leq -a(t)y(t) + b^u \exp\{K_1 + \varepsilon\}, \\ &\text{for } t > N_3 + \eta^u, \end{aligned} \quad (63)$$

by Lemma 16 (1),

$$\lim_{t \rightarrow \infty} \sup y(t) \leq \frac{b^u \exp\{K_1 + \varepsilon\}}{a^l}. \quad (64)$$

Let $\varepsilon \rightarrow 0$, and then we can obtain

$$\lim_{t \rightarrow \infty} \sup y(t) \leq \frac{b^u \exp\{K_1\}}{a^l} := K_2. \quad (65)$$

The proof of Claim 2 is complete.

Claim 3. There exists a constant k_1 such that

$$\lim_{t \rightarrow \infty} \inf x(t) \geq k_1. \quad (66)$$

By Claims 1 and 2 and the first equation of system (3), we have

$$x^\Delta(t) \geq -\delta^u + p^l \exp\{-\alpha^u \exp\{K_1\}\} - c^u K_2 \geq -A_2, \quad (67)$$

for t sufficiently large, where $A_2 > 0$ is a constant. Then

$$\begin{aligned} x(t) - x(t - \eta(t)) &\geq -A_2 \eta(t) \geq -A_2 \eta^u, \\ x(t) - x(t - \tau(t)) &\geq -A_2 \tau(t) \geq -A_2 \tau^u. \end{aligned} \quad (68)$$

From the second equation of system (3), we have

$$y^\Delta(t) \leq -a(t)y(t) + b^u \exp\{A_2 \tau^u\} \exp\{x(t)\}, \quad (69)$$

and then Lemma 16 (1) implies that, for any $t > T_2$,

$$\begin{aligned} y(t) &\leq y(T_2) e_{(-a)}(t, T_2) \\ &\quad + \frac{b^u \exp\{A_2 \tau^u\} \exp\{x(t)\}}{a^l}, \end{aligned} \quad (70)$$

noting that

$$y(T_2) e_{(-a)}(t, T_2) \rightarrow 0, \quad \text{as } t \rightarrow +\infty, \quad (71)$$

and hence there exists a positive integer $K > T_2$ such that, for any solution $(x(t), y(t))$ of system (3), $y(T_2) e_{(-a)}(t, T_2) < \gamma/2$, as $t > K$, where $\gamma = \inf_{t \in \mathbb{T}} (p(t) - \delta(t))$. Fixing K , we get

$$\begin{aligned} y(t) &\leq y(T_2) e_{(-a)}(t, T_2) + \frac{b^u \exp\{A_2 \tau^u\} \exp\{x(t)\}}{a^l} \\ &\text{for } t > K. \end{aligned} \quad (72)$$

Set

$$M_1 = \frac{b^u \exp\{A_2 \tau^u\}}{a^l}, \quad (73)$$

and then

$$y(t) \leq y(T_2) e_{(-a)}(t, T_2) + M_1 \exp\{x(t)\}. \quad (74)$$

Notice that $e^{-x} \geq 1 - x$, for $x > 0$, and then from the first equation of system (3), we have

$$\begin{aligned} x^\Delta(t) &\geq -\delta(t) + p(t)(1 - \alpha(t) \exp\{x(t - \tau(t))\}) \\ &\quad - c(t)y(t - \theta(t)) \\ &\geq p(t) - \delta(t) \\ &\quad - (p(t)\alpha(t))^u \exp\{A_2\tau^u\} \exp\{x(t)\} \\ &\quad - c(t)y(T_2)e_{(-a)}(t, T_2) \\ &\quad - M_1c(t) \exp\{x(t)\}, \end{aligned} \quad (75)$$

for $t > K$. According to the choosing of K , we have

$$\begin{aligned} x^\Delta(t) &\geq \frac{\gamma}{2} \\ &\quad - ((p(t)\alpha(t))^u \exp\{A_2\tau^u\} + M_1c^u) \exp\{x(t)\} \end{aligned} \quad (76)$$

for $t > K$.

By Lemma 15 (2), we have

$$\liminf_{t \rightarrow +\infty} x(t) \geq (B - A \exp\{K_1\})L + \ln \frac{B}{A}, \quad (77)$$

where

$$B = ((p(t)\alpha(t))^u \exp\{A_2\tau^u\} + M_1c^u), \quad A = \frac{\gamma}{2}, \quad (78)$$

and choosing

$$k_1 = (B - A \exp\{K_1\})L + \ln \frac{B}{A}, \quad (79)$$

this shows that the conclusion holds true.

Claim 4. There exists a positive constant k_2 such that

$$\liminf_{t \rightarrow +\infty} y(t) \geq k_2. \quad (80)$$

Notice that (66) implies that there exists an N_4 such that $x(t) \geq k_1/2$ for all $t > N_4$, and then from the second equation of system (3), we have

$$\begin{aligned} y^\Delta(t) &\geq -a(t)y(t) + b^l \exp\left\{\frac{k_1}{2}\right\}, \\ &\quad \text{for } t > N_4 + \eta^u, \end{aligned} \quad (81)$$

by Lemma 16 (2),

$$\liminf_{t \rightarrow +\infty} y(t) \geq \frac{b^l \exp\{k_1/2\}}{a^u} := k_2. \quad (82)$$

The proof of Claim 4 is complete.

Choose

$$\begin{aligned} \lambda_1 &= \min\{\exp\{k_1\}, k_2\}, \\ \lambda_2 &= \max\{\exp\{K_1\}, K_2\}, \end{aligned} \quad (83)$$

obviously,

$$\lambda_i > 0, \quad i = 1, 2. \quad (84)$$

And

$$\begin{aligned} \lambda_1 &\leq \liminf_{t \rightarrow \infty} \exp\{x(t)\} \leq \limsup_{t \rightarrow \infty} \exp\{x(t)\} \leq \lambda_2, \\ \lambda_1 &\leq \liminf_{t \rightarrow \infty} y(k) \leq \limsup_{t \rightarrow \infty} y(t) \leq \lambda_2. \end{aligned} \quad (85)$$

Thus we complete the proof of Theorem 20. $\square$

Theorem 21. *If the coefficient functions $\delta, \alpha, p, b, c, \tau, \theta$, and $\eta : \mathbb{T} \rightarrow \mathbb{R}^+$ are all ω -periodic rd-continuous functions and also satisfy $\bar{p} > \bar{\delta}$ and (9), then system (3) has at least an ω -periodic solution with strictly positive for its second component.*

Corollary 22. *If all the conditions in Theorem 21 hold, then (6) or (7) has at least a positive ω -periodic solution.*

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

All authors contributed equally to this work. All authors read and approved the final manuscript.

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