Research Article

Exponential Stability and Robust H_∞ Control for Discrete-Time Time-Delay Infinite Markov Jump Systems

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In this paper, exponential stability and robust H_∞ control problem are investigated for a class of discrete-time time-delay stochastic systems with infinite Markov jump and multiplicative noises. The jumping parameters are modeled as an infinite-state Markov chain. By using a novel Lyapunov-Krasovskii functional, a new sufficient condition in terms of matrix inequalities is derived to guarantee the mean square exponential stability of the equilibrium point. Then some sufficient conditions for the existence of feedback controller are presented to guarantee that the resulting closed-loop system has mean square exponential stability for the zero exogenous disturbance and satisfies a prescribed H_∞ performance level. Numerical simulations are exploited to validate the applicability of developed theoretical results.

1. Introduction

During the past decades, Markov jump systems have been the subject of a great deal of research since they have been used extensively both in theory and in applications. Markov jump systems are hybrid dynamical systems composed of subsystems with the transitions determined by a Markov chain. A number of results that focused on Markov jump systems have been published ranging from filtering, stability, observability, and control to engineering application; see, for example, [1–15] and the references therein.

Note that most of the theoretical works related to Markov jump systems in the literatures concentrated on the case where the state space of the Markov chain is finite. However, it may be more appropriate to characterize abrupt changes in many real plants via an infinite-state Markov chain. As far as applications are concerned, infinite Markov jump systems are critical in some physics plants, such as solar thermal receiver, aircraft, and robotic manipulator systems. Theoretically, finite Markov jump systems are fundamentally different from those governed by infinite-state space. The work in [14] studied exponential almost sure stability of random jump systems. The work in [16] considered the definition and computation of an H_2 -type norm for stochastic

systems with infinite Markov jump and periodic coefficients. LQ-optimal control problem has been dealt with for discrete-time infinite Markov jump systems in [17]. The work in [18] demonstrated the inequivalence between stochastic stability and mean square exponential stability in discrete-time case. With this motivation, infinite Markov jump systems have stirred widespread research interests.

Time-delay is one of the inherent features of many practical systems and also is the big source of instability and poor performances in systems [19]. Moreover, stochastic modeling has had extensive applications. Hence, dynamical time-delay stochastic systems deserve our consideration. Stability analysis and controller design of time-delay Markov jump systems have been investigated by many authors [15, 20, 21]. Unfortunately, the literature about these issues for infinite Markov jump case is less developed. And, to the best of our knowledge, only a few results have been presented so far [18, 22, 23], let alone the problem involving time-delay. Actually, [18, 23] investigated the exponential stability and infinite horizon H_2/H_∞ control problem for discrete-time infinite Markov jump systems with multiplicative noises, respectively, but they neglected the effects of time-delay. Meanwhile, the authors in [22] considered time-delay, when discussing the stabilization problem for linear stochastic delay differential

equations with infinite Markovian switching, but it was hard for the obtained stability results to deal with control problem. As mentioned above, stability and control for time-delay stochastic systems with infinite Markov jump and multiplicative noises have not received enough attention despite their importance in practical applications, which motivates us for the present research.

We aim to address the exponential stability and H_∞ control problem for a class of discrete-time time-delay stochastic systems with infinite Markov jumps and multiplicative noises in this paper. The main contributions of this paper are as follows: First of all, we investigate exponential stability of the equilibrium point for the considered systems by employing a novel Lyapunov-Krasovskii functional. Further, a sufficient condition is established to ensure exponential stability with a given H_∞ performance index of the closed-loop system. And we introduce the slack matrix to decouple the Lyapunov matrices, which makes the H_∞ controller design feasible. Moreover, some numerical examples are provided to show the effectiveness of the proposed design approaches.

The remaining part of this paper is constructed as follows. In Section 2, we formulate the system model and recall some definitions and lemmas. In Section 3, we present our main results, where we derive some sufficient conditions for exponential stability with a given H_∞ performance index. Two numerical examples and their simulations are given to illustrate the effectiveness of the obtained results in Section 4. Conclusions are made in Section 5.

For convenience, we fix some notations that will be used throughout this paper. The n -dimensional real Euclidean space is denoted by $\mathcal{R}^n$. $\mathcal{R}^{m \times n}$ stands for the linear space of all m by n real matrices. Let $\|\cdot\|$ be the Euclidean norm of $\mathcal{R}^n$ or the operator norm of $\mathcal{R}^{m \times n}$. By S_n and $I(0)$ we denote the set of all $n \times n$ symmetric matrices and the identity (zero) matrix, respectively. A' denotes the transpose of a matrix (or vector) A . We say that A is positive (semipositive) definite if $A > 0$ (≥ 0). $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) represent the maximum (minimum) eigenvalue of A . $\delta_{(\cdot)}$ is called the Kronecker function. $\mathbf{Z}_+ := \{0, 1, \dots\}$. $\mathcal{D} := \{1, 2, \dots\}$. $l^2(\mathbf{Z}_+; \mathcal{R}^m) := \{\zeta \in \mathcal{R}^m \mid \zeta \text{ is } \mathcal{F}_t\text{-measurable, and } (\sum_{t=0}^{\infty} E\|\zeta(t)\|^2)^{1/2} < \infty\}$.

2. Preliminaries

Consider the following discrete-time time-delay stochastic system with infinite Markov jump parameter and multiplicative noises:

$$\begin{aligned} x(t+1) &= C_0(s_t)x(t) + D_0(s_t)x(t-d) + R_0(s_t) \\ &\quad \cdot u(t) + H_0(s_t)v(t) + \sum_{k=1}^r [C_k(s_t)x(t) \\ &\quad + D_k(s_t)x(t-d) + R_k(s_t)u(t) + H_k(s_t)v(t)] \\ &\quad \cdot w_k(t), \\ 5pt y(t) &= L(s_t)x(t) + L_0(s_t)x(t-d) + N(s_t)u(t) \\ &\quad + E(s_t)v(t), \end{aligned}$$

$$x(t_0) = \phi(t_0),$$

$$t_0 = -\tilde{d}, -\tilde{d}+1, \dots, -1, 0, s(0) = s_0 \in \mathcal{D}, t \in \mathbf{Z}_+, \quad (1)$$

where $x(t) \in \mathcal{R}^n$ represents the system state, $u(t) \in \mathcal{R}^{n_u}$ is the control input, $v(t) \in \mathcal{R}^{n_v}$ denotes the disturbance, and $y(t) \in \mathcal{R}^{n_z}$ is the system output. $w(t) = \{w(t) \mid w(t) = (w_1(t), w_2(t), \dots, w_r(t))', t \in \mathbf{Z}_+\}$ is a sequence of independent random vectors defined on a given complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, which satisfies $E(w(t)) = 0$ and $E(w(t)w(s)') = I_r \delta_{(t-s)}$. $\phi(t_0)$ is a vector-valued initial condition. d is the bounded constant delay with $0 \leq d \leq \tilde{d}$. Markov chain $\{s_t\}_{t \in \mathbf{Z}_+}$ takes values in a countably infinite set $\mathcal{D}$ with transition probability matrix $\mathbf{P} = [p(i, j)]$, where $p(i, j) = P(s_{t+1} = j \mid s_t = i)$, and $\mathbf{P}$ is nondegenerate, $P(s_0 = i) > 0$ for all $i \in \mathcal{D}$. Assume $\{w_t\}_{t \in \mathbf{Z}_+}$ and $\{s_t\}_{t \in \mathbf{Z}_+}$ are mutually independent, and $\mathcal{F}_t = \{s_k, w_s \mid 0 \leq k \leq t, 0 \leq s \leq t-1\}$, $\mathcal{F}_0 = \sigma(s_0)$. Assume $v(t)$ belongs to $l^2(\mathbf{Z}_+; \mathcal{R}^{n_v})$.

We introduce the Banach spaces $\mathbb{A}_1^{m \times n} = \{A \mid A = (A(1), A(2), \dots), A(i) \in \mathcal{R}^{m \times n}, \|A\|_1 = \sum_{i=1}^{\infty} \|A(i)\| < \infty\}$ and $\mathbb{A}_{\infty}^{m \times n} = \{A \mid A = (A(1), A(2), \dots), A(i) \in \mathcal{R}^{m \times n}, \|A\|_{\infty} = \sup_{i \in \mathcal{D}} \|A(i)\| < \infty\}$. The notations $\mathbb{A}_1^{m \times n}(\mathbb{A}_{\infty}^{m \times n})$ will be written as $\mathbb{A}_1^n$ (resp., $\mathbb{A}_{\infty}^n$) and $\mathbb{A}_1^{n+}$ (resp., $\mathbb{A}_{\infty}^{n+}$) if and only if $m = n$ and $A(i) \in S_n$, $A(i) \geq 0$, $i \in \mathcal{D}$, respectively. When $Y, Z \in \mathbb{A}_1^{n+}$, $Y \leq Z$ means that $Y(i) \leq Z(i)$, $i \in \mathcal{D}$. Therefore, we have $\|Y\|_1 \leq \|Z\|_1$. For all coefficients of the considered systems, we suppose they have a finite norm $\|\cdot\|_{\infty}$.

Definition 1 (see [10, 18]). System (1) with $u(t) = 0$ and $v(t) = 0$ is called mean square exponential stability if there exist $\lambda \geq 1$ and $\tau \in (0, 1)$ such that

$$E\{\|x(t)\|^2\} \leq \lambda \tau^t \sup_{-\tilde{d} \leq l \leq 0} E\{\|\phi(l)\|^2\} \quad (2)$$

for all $t \in \mathbf{Z}_+$, $i \in \mathcal{D}$ and $x_0 \in \mathcal{R}^n$. Further, system (1) with $v(t) = 0$ is called exponential stabilizable if there exists a sequence $\{K(s_t)\}_{t \in \mathbf{Z}_+} \in \mathbb{A}_{\infty}^{n_u \times n}$ such that the closed-loop system

$$\begin{aligned} x(t+1) &= [C_0(s_t) + R_0(s_t)K(s_t)]x(t) + D_0(s_t)x(t-d) \\ &\quad + H_0(s_t)v(t) \\ &\quad + \sum_{k=1}^r \{[C_k(s_t) + R_k(s_t)K(s_t)]x(t) \\ &\quad + D_k(s_t)x(t-d) + H_k(s_t)v(t)\}w_k(t), \\ y(t) &= [L(s_t) + N(s_t)K(s_t)]x(t) + L_0(s_t)x(t-d) \\ &\quad + E(s_t)v(t), \\ x(t_0) &= \phi(t_0), \\ t_0 &= -\tilde{d}, -\tilde{d}+1, \dots, -1, 0, s(0) = s_0 \in \mathcal{D}, t \in \mathbf{Z}_+, \end{aligned} \quad (3)$$

with $v(t) = 0$ has mean square exponential stability, where $u(t) = K(s_t)x(t)$ is called exponentially stabilizing feedback.

Definition 2. Closed-loop system (3) is said to have an H_∞ noise disturbance attenuation level $\gamma > 0$, if under zero initial value the following condition is satisfied:

$$\sum_{t=0}^{\infty} E \{ \|y(t)\|^2 \} < \gamma^2 \sum_{t=0}^{\infty} \{ E \|v(t)\|^2 \} \quad (4)$$

for any $v(t) \in l^2(\mathbb{Z}_+; \mathcal{R}^{n_v})$.

Lemma 3 (see [22]). We denote $\tilde{\mathbb{A}}_\infty^{n_+} = \{A \mid A \in \mathbb{A}_\infty^{n_+}, \text{ their exists } \varepsilon > 0 \text{ not depending upon } i \text{ such that } A(i) \geq \varepsilon I_n \text{ for all } i \in \mathcal{D}\}$. Let

$$B = \left\{ \begin{bmatrix} B_{11}(i) & B_{12}(i) \\ B_{12}(i)' & B_{22}(i) \end{bmatrix} \right\}_{i \in \mathcal{D}}. \quad (5)$$

Assume that $B_{22}(i) \geq \varepsilon I_{n_2} > 0$ for all $i \in \mathcal{D}$ for some $\varepsilon > 0$. Then, $B \in \tilde{\mathbb{A}}_\infty^{n_+}$ if and only if $B \mid B_{22} \in \tilde{\mathbb{A}}_\infty^{n_1+}$, where $n = n_1 + n_2$ and $B \mid B_{22} = \{B_{11}(i) - B_{12}(i)B_{22}(i)^{-1}B_{12}(i)'\}_{i \in \mathcal{D}}$ is called the Schur complement of B_{22} in B .

Remark 4. Lemma 3 is the infinite-dimensional version of Schur complements (see [24]).

3. Main Results

Firstly, stability will be analyzed, and a sufficient condition is obtained for system (1) with $u(t) = 0$ and $v(t) = 0$ to have mean square exponential stability.

Theorem 5. System (1) with $u(t) = 0$ and $v(t) = 0$ is exponentially mean square stable, if we can find matrices $P \in \tilde{\mathbb{A}}_\infty^{n_+}$, $Q \in \tilde{\mathbb{A}}_\infty^{n_+}$ such that the following matrix inequality holds:

$$\begin{aligned} & E [V_1(x(t+1), s_{t+1}) - V_1(x(t), s_t) \mid \mathcal{F}_t, s_t = i] \\ &= \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}' \begin{bmatrix} \sum_{k=0}^r C_k(i)' \mathcal{E}_i(P) C_k(i) - P(i) & \sum_{k=0}^r C_k(i)' \mathcal{E}_i(P) D_k(i) \\ \sum_{k=0}^r D_k(i)' \mathcal{E}_i(P) C_k(i) & \sum_{k=0}^r D_k(i)' \mathcal{E}_i(P) D_k(i) \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}, \end{aligned} \quad (9)$$

and

$$\begin{aligned} & E [V_2(x(t+1), s_{t+1}) - V_2(x(t), s_t) \mid \mathcal{F}_t, s_t = i] \\ &= x(t)' Q(i) x(t) - x(t-d)' Q(s_{t-d}) x(t-d) \\ &+ \sum_{\alpha=t-\tilde{d}}^{t-1} x(\alpha)' Q(s_\alpha) x(\alpha) \\ &- \sum_{\alpha=t-\tilde{d}}^{t-1} x(\alpha)' Q(s_\alpha) x(\alpha) \end{aligned}$$

$$\begin{bmatrix} -\mathcal{P} & C_x(i) & D_d(i) \\ * & -P(i) + (\tilde{d}+1)Q(i) & 0 \\ * & * & -Q(q) \end{bmatrix} < 0 \quad (6)$$

uniformly with respect to $(i, q) \in \mathcal{D} \times \mathcal{D}$, where

$$\begin{aligned} C_x(i) &= [C_0(i)', C_1(i)', \dots, C_r(i)']', \\ D_d(i) &= [D_0(i)', D_1(i)', \dots, D_r(i)']', \\ \mathcal{P} &= \text{diag} \left\{ \underbrace{\mathcal{E}_i(P)^{-1}, \dots, \mathcal{E}_i(P)^{-1}}_{r+1} \right\}, \\ \mathcal{E}_i(P) &= \sum_{j=1}^{\infty} p(i, j) P(j). \end{aligned} \quad (7)$$

Proof. Construct the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(x(t), s_t) &= x(t)' P(s_t) x(t) \\ &+ \sum_{m=t-d}^{t-1} x(m)' Q(s_m) x(m) \\ &+ \sum_{\alpha=t-\tilde{d}+1}^0 \sum_{\beta=t-1+\alpha}^{t-1} x(\beta)' Q(s_\beta) x(\beta) \\ &= V_1(x(t), s_t) + V_2(x(t), s_t) \\ &+ V_3(x(t), s_t). \end{aligned} \quad (8)$$

By the assumption that $w(t)$ is independent of the Markov chain $\{s_t\}_{t \in \mathbb{Z}_+}$ and $E[w(t)] = 0$, besides $\mathcal{F}_{t-\tilde{d}} \subset \mathcal{F}_t$, we have

$$\begin{aligned} & \leq x(t)' Q(i) x(t) - x(t-d)' Q(s_{t-d}) x(t-d) \\ &+ \sum_{\alpha=t-\tilde{d}}^{t-1} x(\alpha)' Q(s_\alpha) x(\alpha), \end{aligned} \quad (10)$$

and

$$\begin{aligned} & E [V_3(x(t+1), s_{t+1}) - V_3(x(t), s_t) \mid \mathcal{F}_t, s_t = i] \\ &= \tilde{d} x(t)' Q(i) x(t) - \sum_{\alpha=t-\tilde{d}}^{t-1} x(\alpha)' Q(s_\alpha) x(\alpha). \end{aligned} \quad (11)$$

Thus, combining (8) with (9)-(11), we get

$$\begin{aligned} E[V(x(t+1), s_{t+1}) - V(x(t), s_t) | \mathcal{F}_t, s_t = i] \\ \leq a(t)' R_{iq}(P) a(t), \end{aligned} \quad (12)$$

where

$$R_{iq}(P) = \begin{bmatrix} \sum_{k=0}^r C_k(i)' \mathcal{E}_i(P) C_k(i) - P(i) + (\tilde{d}+1)Q(i) & \sum_{k=0}^r C_k(i)' \mathcal{E}_i(P) D_k(i) \\ \sum_{k=0}^r D_k(i)' \mathcal{E}_i(P) C_k(i) & \sum_{k=0}^r D_k(i)' \mathcal{E}_i(P) D_k(i) - Q(q) \end{bmatrix} \quad (13)$$

with $q = s_{t-\tilde{d}}$ and $a(t)$ is defined as $a(t) = [x(t)' \ x(t-d)']'$. Applying Lemma 3 to (6) leads to

$$\begin{aligned} \text{diag}\{-P(i) + (\tilde{d}+1)Q(i), -Q(q)\} \\ + \mathcal{M}(i)' \mathcal{P}^{-1} \mathcal{M}(i) < 0, \end{aligned} \quad (14)$$

where $\mathcal{M}(i) = [C_x(i) \ D_d(i)]$. Further, we have $R_{iq}(P) < 0$. It is clear from $R_{iq}(P) < 0$ that there exists a sufficiently small scalar $\varepsilon > 0$ such that $R_{iq}(P) < -\varepsilon I_n$. Therefore, it follows that

$$\begin{aligned} E[V(x(t+1), s_{t+1}) - V(x(t), s_t)] \\ < -\varepsilon E[\|x(t)\|^2]. \end{aligned} \quad (15)$$

On the other hand, by using (8), we deduce that

$$\begin{aligned} E[V(x(t), s_t)] \leq \theta_1 E[\|x(t)\|^2] \\ + (\tilde{d}+1)\theta_2 \sum_{\beta=t-\tilde{d}}^{t-1} E[\|x(\beta)\|^2], \end{aligned} \quad (16)$$

where

$$\begin{aligned} \theta_1 &= \max_{l \in \mathcal{D}} \lambda_{\max}(P(l)), \\ \theta_2 &= \max_{p \in \mathcal{D}} \lambda_{\max}(Q(p)). \end{aligned} \quad (17)$$

Noting (15) and (16), for any constant $\kappa > 1$, we obtain that

$$\begin{aligned} \kappa^{t+1} E[V(x(t+1), s_{t+1})] - \kappa^t E[V(x(t), s_t)] \\ = \kappa^{t+1} E[V(x(t+1), s_{t+1}) - V(x(t), s_t)] \\ + \kappa^t (\kappa - 1) E[V(x(t), s_t)] \\ \leq [-\kappa\varepsilon + (\kappa - 1)\theta_1] \kappa^t E[\|x(t)\|^2] \\ + (\kappa - 1)\theta_3 \sum_{\beta=t-\tilde{d}}^{t-1} \kappa^t E[\|x(\beta)\|^2], \end{aligned} \quad (18)$$

where $\theta_3 = (\tilde{d}+1)\theta_2$. By taking summation from 0 to $T-1$ on both sides of (18), for $T \geq \tilde{d}+1$, it implies that

$$\begin{aligned} \kappa^T E[V(x(T), s_T)] - E[V(x(0), s_0)] &\leq [-\kappa\varepsilon \\ &+ (\kappa - 1)\theta_1] \sum_{t=0}^{T-1} \kappa^t E[\|x(t)\|^2] + (\kappa - 1) \\ &\cdot \theta_3 \sum_{t=0}^{T-1} \sum_{\beta=t-\tilde{d}}^{t-1} \kappa^t E[\|x(\beta)\|^2] \leq [-\kappa\varepsilon + (\kappa - 1)\theta_1] \\ &\cdot \sum_{t=0}^{T-1} \kappa^t E[\|x(t)\|^2] + (\kappa - 1)\theta_3 \\ &\cdot \left\{ \tilde{d} \sum_{\beta=-\tilde{d}}^{-1} \kappa^{\beta+\tilde{d}} E[\|x(\beta)\|^2] \right. \\ &+ \tilde{d} \sum_{\beta=0}^{T-1-\tilde{d}} \kappa^{\beta+\tilde{d}} E[\|x(\beta)\|^2] \\ &+ \tilde{d} \sum_{\beta=T-1-\tilde{d}}^{T-1} \kappa^{\beta+\tilde{d}} E[\|x(\beta)\|^2] \left. \right\} \leq [-\kappa\varepsilon \\ &+ (\kappa - 1)\theta_1] \sum_{t=0}^{T-1} \kappa^t E[\|x(t)\|^2] + (\kappa - 1)\theta_3 \\ &\cdot \left\{ \tilde{d} \kappa^{\tilde{d}} \max_{-\tilde{d} \leq \beta \leq 0} E[\|\phi(\beta)\|^2] \right. \\ &+ \tilde{d} \kappa^{\tilde{d}} \sum_{\beta=0}^{T-1} \kappa^{\beta} E[\|x(\beta)\|^2] \left. \right\} = [-\kappa\varepsilon + (\kappa - 1)\theta_1] \\ &+ (\kappa - 1)\theta_3 \tilde{d} \kappa^{\tilde{d}} \sum_{t=0}^{T-1} \kappa^t E[\|x(t)\|^2] + (\kappa - 1) \\ &\cdot \theta_3 \tilde{d} \kappa^{\tilde{d}} \max_{-\tilde{d} \leq \beta \leq 0} E[\|\phi(\beta)\|^2]. \end{aligned} \quad (19)$$

Recalling (8) and (16), denoting $\theta_0 = \min_{l \in \mathcal{D}} \lambda_{\min}(P(l))$ and $\theta = \max\{\theta_1, (\tilde{d} + 1)\theta_2\}$, we have

$$E[V(x(T), s_T)] \geq \theta_0 E[\|x(T)\|^2], \quad (20)$$

and

$$E[V(x(0), s_0)] \leq \theta \max_{-\tilde{d} \leq \beta \leq 0} E[\|\phi(\beta)\|^2], \quad (21)$$

respectively. Furthermore, it suffices to show that there exists a constant $\kappa_0 > 1$ such that

$$[-\kappa_0 \varepsilon + (\kappa_0 - 1)\theta_1] + (\kappa_0 - 1)\theta_3 \tilde{d} \kappa_0^{\tilde{d}} = 0. \quad (22)$$

Actually, letting $f(\kappa) = [-\kappa \varepsilon + (\kappa - 1)\theta_1] + (\kappa - 1)\theta_3 \tilde{d} \kappa^{\tilde{d}}$, then we have $f'(\kappa) > 0$ and $f(1) < 0$. Therefore, (22) has a unique solution $\kappa_0 > 1$. By substituting (20)-(22) into (19), we obtain

$$E[\|x(T)\|^2] \leq \lambda_0 \left(\frac{1}{\kappa_0} \right)^T \max_{-\tilde{d} \leq \beta \leq 0} E[\|\phi(\beta)\|^2], \quad (23)$$

where $\lambda_0 = (\theta + (\kappa_0 - 1)\theta_3 \tilde{d} \kappa_0^{\tilde{d}})/\theta_0$. This indicates that system (1) with $u(t) = 0$ and $v(t) = 0$ has mean square exponential stability. The proof is completed. $\square$

Remark 6. Due to the consideration of an infinite-state Markov chain, the infinite dimension Banach spaces have been introduced. Furthermore, it should be pointed out that a novel Lyapunov-Krasovskii functional (8) has been constructed to analyze the mean square exponential stability for system (1) with $u(t) = 0$ and $v(t) = 0$.

Next, we prove that system (1) with $u(t) = 0$ verifies the H_∞ performance disturbance attenuation γ .

Theorem 7. System (1) has mean square exponential stability for $u(t) = 0$ and $v(t) = 0$ with a prescribed H_∞ performance γ for $u(t) = 0$, if we can find matrices $P \in \tilde{\mathbb{A}}_\infty^{n+}$, $Q \in \tilde{\mathbb{A}}_\infty^{n+}$ such that the following matrix inequality holds:

$$\begin{bmatrix} -\mathcal{P} & C_x(i) & D_d(i) & H_v(i) & 0 \\ * & -P(i) + (\tilde{d} + 1)Q(i) & 0 & 0 & L(i)' \\ * & * & -Q(q) & 0 & L_0(i)' \\ * & * & * & -\gamma^2 I & E(i)' \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (24)$$

uniformly with respect to $(i, q) \in \mathcal{D} \times \mathcal{D}$, where

$$H_v(i) = [H_0(i)', H_1(i)', \dots, H_r(i)']'. \quad (25)$$

Proof. It is well established that (24) implies (6). Applying Theorem 5 one obtains that system (1) has mean square exponential stability for $u(t) = 0$ and $v(t) = 0$.

Let us now show that system (1) with $u(t) = 0$ satisfies a prescribed H_∞ performance level. To this end, constructing

the same Lyapunov-Krasovskii functional $V(x(t), s_t)$ as in Theorem 5 and under the zero initial condition, the following index is introduced:

$$\begin{aligned} J^T &= \sum_{t=0}^T E \{ \|y(t)\|^2 - \gamma^2 \|v(t)\|^2 \} = \sum_{t=0}^T E \{ \|y(t)\|^2 \\ &\quad - \gamma^2 \|v(t)\|^2 + [V(x(t+1), s_{t+1}) - V(x(t), s_t)] \} \\ &\quad - V(x(T+1), s_{T+1}) \leq \sum_{t=0}^T E \{ \|y(t)\|^2 - \gamma^2 \|v(t)\|^2 \\ &\quad + [V(x(t+1), s_{t+1}) - V(x(t), s_t)] \} \leq E \{ b(t)' \\ &\quad \cdot [\mathcal{A}(s_t, s_{t-d}) - \mathcal{B}(s_t)' \mathcal{C}(s_t)^{-1} \mathcal{B}(s_t)] b(t) \}, \end{aligned} \quad (26)$$

where

$$\begin{aligned} \mathcal{A}(s_t, s_{t-d}) &= \text{diag} \{ -P(s_t) + (\tilde{d} + 1)Q(s_t), -Q(s_{t-d}), -\gamma^2 I \}, \\ \mathcal{B}(i) &= \begin{bmatrix} C_x(s_t) & D_d(s_t) & H_v(s_t) \\ L(s_t) & L_0(s_t) & E(s_t) \end{bmatrix}, \\ \mathcal{C}(s_t) &= \text{diag} \{ -\mathcal{P}, -I \}, \end{aligned} \quad (27)$$

and $b(t)$ is defined as $b(t) = [x(t)' \ x(t-d)' \ v(t)']'$. The last ' $\leq$ ' in (26) holds as a result of the similar line with (12). Then, by using Lemma 3 in (24), we obtain that $\mathcal{A}(s_t, s_{t-d}) - \mathcal{B}(s_t)' \mathcal{C}(s_t)^{-1} \mathcal{B}(s_t) < 0$. Thus, $J^T < 0$. Taking the limit $T \rightarrow \infty$ in (26), we have

$$\sum_{t=0}^{\infty} E[\|y(t)\|^2] < \gamma^2 \sum_{t=0}^{\infty} E[\|v(t)\|^2]. \quad (28)$$

This ends the proof. $\square$

Combining Theorem 5 with Theorem 7, the following corollary can be easily derived for closed-loop system (3).

Corollary 8. Let the feedback control gain $K(i)$, $i \in \mathcal{D}$, be given. Then closed-loop system (3) has mean square exponential stability for $v(t) = 0$ with a prescribed H_∞ performance γ if there exist two matrices $P \in \tilde{\mathbb{A}}_\infty^{n+}$ and $Q \in \tilde{\mathbb{A}}_\infty^{n+}$, such that

$$\begin{bmatrix} -\mathcal{P} & \bar{C}_x(i) & D_d(i) & H_v(i) & 0 \\ * & -P(i) + (\tilde{d} + 1)Q(i) & 0 & 0 & \bar{L}(i)' \\ * & * & -Q(q) & 0 & L_0(i)' \\ * & * & * & -\gamma^2 I & E(i)' \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (29)$$

uniformly with respect to $(i, q) \in \mathcal{D} \times \mathcal{D}$, where

$$\begin{aligned}\bar{C}_x(i) &= \left[(C_0(i) + R_0(i) K(i))' \right. \\ &\quad \left. (C_1(i) + R_1(i) K(i))', \dots, (C_r(i) + R_r(i) K(i))' \right]', \quad (30) \\ \bar{L}(i) &= L(i) + N(i) K(i).\end{aligned}$$

Below, based on Corollary 8, we are ready to present the H_∞ controller design for system (1).

Theorem 9. For system (1), a state feedback controller can be designed such that closed-loop system (3) has mean square exponential stability for $v(t) = 0$ and a given H_∞ performance γ can be ensured if there exist matrices $\hat{P} \in \bar{\mathbb{A}}_{\infty}^{n+}$, $\hat{Q} \in \bar{\mathbb{A}}_{\infty}^{n+}$, $\hat{K} \in \mathbb{A}_{\infty}^{n_u \times n}$, and $F \in \mathcal{R}^{n \times n}$ such that

$$\begin{bmatrix} \Phi(i) & \Gamma(i) & \Upsilon(i) \\ * & -I & 0 \\ * & * & -\widehat{\mathcal{P}} \end{bmatrix} < 0, \quad (31)$$

uniformly with respect to $(i, q) \in \mathcal{D} \times \mathcal{D}$, where

$$\begin{aligned}\Phi(i) &= \begin{bmatrix} \hat{P}(i) + (\tilde{d} + 1)\hat{Q}(i) - F' - F & 0 & 0 \\ * & -\hat{Q}(q) & 0 \\ * & * & -\gamma^2 I \end{bmatrix}, \\ \Gamma(i) &= [L(i)F + N(i)\hat{K}(i), L_0(i)F, E(i)]', \\ \Upsilon(i) &= [C_x(i)F + R_u(i)\hat{K}(i), D_d(i)F, H_v(i)]', \quad (32) \\ R_u(i) &= [R_0(i)', R_1(i)', \dots, R_r(i)']', \\ \widehat{\mathcal{P}} &= \text{diag} \left\{ \underbrace{\mathcal{E}_i(\hat{P})^{-1}, \dots, \mathcal{E}_i(\hat{P})^{-1}}_{r+1} \right\}, \\ \mathcal{E}_i(\hat{P}) &= \sum_{j=1}^{\infty} p(i, j) \hat{P}(j)^{-1}.\end{aligned}$$

Moreover, if matrix inequalities (31) are feasible, then an exponentially stabilizing feedback gain can be given by

$$K(i) = \hat{K}(i) F^{-1}. \quad (33)$$

Proof. Via Lemma 3, we conclude that (29) is equivalent to the following matrix inequality:

$$\begin{bmatrix} -P(i) + (\tilde{d} + 1)Q(i) & 0 & 0 & \bar{L}(i)' & \bar{C}_x(i)' \\ * & -Q(q) & 0 & L_0(i)' & D_d(i)' \\ * & * & -\gamma^2 I & E(i)' & H_v(i)' \\ * & * & * & -I & 0 \\ * & * & * & * & -\mathcal{P} \end{bmatrix} < 0. \quad (34)$$

Premultiply $\text{diag}\{F', I, I, I\}$ and postmultiply $\text{diag}\{F, F, I, I, I\}$ with (34), and let

$$\begin{aligned}\hat{P}(i) &= P(i)^{-1}, \\ \hat{Q}(i) &= F' Q(i) F, \\ \hat{K}(i) &= K(i) F.\end{aligned} \quad (35)$$

By a tedious calculation, one can rewrite (34) as

$$\begin{bmatrix} \widehat{\Phi}(i) & \Gamma(i) & \Upsilon(i) \\ * & -I & 0 \\ * & * & -\widehat{\mathcal{P}} \end{bmatrix} < 0, \quad (36)$$

where

$$\begin{aligned}\widehat{\Phi}(i) &= \begin{bmatrix} -F' \hat{P}(i)^{-1} F + (\tilde{d} + 1)\hat{Q}(i) & 0 & 0 \\ * & -\hat{Q}(q) & 0 \\ * & * & -\gamma^2 I \end{bmatrix}.\end{aligned} \quad (37)$$

According to Corollary 8 and the fact that

$$(\hat{P}(i) - F)' \hat{P}(i)^{-1} (\hat{P}(i) - F) \geq 0, \quad (38)$$

namely,

$$-F' \hat{P}(i)^{-1} F \leq \hat{P}(i) - F - F', \quad (39)$$

the desired result is derived. $\square$

Remark 10. The work in [20] presented a necessary and sufficient condition for the existence of the mixed H_2/H_∞ control by four coupled matrix Riccati equations (CMREs). Note that CMREs are hardly solved in practice, and this motivates us to find a new sufficient condition in terms of matrix inequalities that can be easily solved to guarantee that the resulting closed-loop system has mean square exponential stability for the zero exogenous disturbance and satisfies a prescribed H_∞ performance level.

Remark 11. With the introduction of a slack matrix F , a sufficient condition is obtained in Theorem 9, in which the Lyapunov matrices are not involved in any product with system matrices. This makes the H_∞ controller design feasible and can be easily carried out by solving corresponding matrix inequalities.

Remark 12. It is worth noting that the obtained results can be extended to discrete-time time-delay infinite Markov jump stochastic systems with time-varying delays. Assume that the time-varying delay $d(t)$ satisfies $d_m \leq d(t) \leq d_M$; then by

similar procedures to the above and choosing the following Lyapunov-Krasovskii function

$$\begin{aligned} V(x(t), s_t) &= x(t)' P(s_t) x(t) \\ &+ \sum_{m=t-d(t)}^{t-1} x(m)' Q(s_m) x(m) \\ &+ \sum_{\alpha=-d_M+2}^{-d_m+1} \sum_{\beta=t-1+\alpha}^{t-1} x(\beta)' Q(s_\beta) x(\beta), \end{aligned} \quad (40)$$

the corresponding results can be derived.

4. Illustrative Example

In this section, some illustrative examples are presented to demonstrate the effectiveness of the developed method.

Example 1. Consider the following one-dimensional discrete-time time-delay stochastic system with infinite Markov jumps:

$$\begin{aligned} x(t+1) &= c_0(s_t) x(t) + d_0(s_t) x(t-d) \\ &+ \sum_{k=1}^r [c_k(s_t) x(t) + d_k(s_t) x(t-d)] w_k(t), \end{aligned} \quad (41)$$

where the transition probability is defined by $p(i, i) = 1/4$, $p(i, i+1) = 3/4$, $p(i, j) = 0$, $j \neq i, i+1$, $i, j \in \mathcal{D}$. Now take

$$\begin{aligned} c_0(i) &= \sqrt{\frac{i}{i+1}}, \\ c_1(i) &= \sqrt{\frac{1}{i+1}}, \\ c_k(i) &= 0, \\ &k = 2, 3, \dots, r, \quad i \in \mathcal{D}, \\ d_0(i) &= \sqrt{\frac{i}{i+1}}, \\ d_1(i) &= \sqrt{\frac{1}{i+1}}, \\ d_k(i) &= 0, \\ &k = 2, 3, \dots, r, \quad i \in \mathcal{D}. \end{aligned} \quad (42)$$

Let $P(i) = 4(i+1)/3i$, $Q(i) = 1/9i(i+1)$, and time-delay $d = 2$. By direct computation, (6) holds. According to Theorem 5, we deduce that system (41) has mean square exponential stability, and Figure 1 presents the state response of system (41) with initial conditions $\phi(t_0) = 0.5$ for $t_0 = -2, -1, 0$.

FIGURE 1: System state response in Example 1.

Example 2. Consider the following one-dimensional discrete-time time-delay stochastic system with infinite Markov jumps:

$$\begin{aligned} x(t+1) &= [c_0(s_t) + r_0(s_t) K(s_t)] x(t) + d_0(s_t) x(t-d) \\ &+ h_0(s_t) v(t) \\ &+ \sum_{k=1}^r \{ [c_k(s_t) + r_k(s_t) K(s_t)] x(t) \\ &+ d_k(s_t) x(t-d) + h_k(s_t) v(t) \} w_k(t), \end{aligned} \quad (43)$$

$$\begin{aligned} y(t) &= [l(s_t) + n(s_t) K(s_t)] x(t) + l_0(s_t) x(t-d) \\ &+ e(s_t) v(t), \end{aligned}$$

$$x(t_0) = \phi(t_0),$$

$$t_0 = -\tilde{d}, -\tilde{d}+1, \dots, -1, 0, \quad s(0) = s_0 \in \mathcal{D}, \quad t \in \mathbf{Z}_+,$$

where the transition probability is defined by $p(i, i) = 1/2$, $p(i, i+1) = 1/2$, $p(i, j) = 0$, $j \neq i, i+1$, $i, j \in \mathcal{D}$. The coefficients of system (43) are reset to be

$$\begin{aligned} c_0(i) &= -\frac{i}{(i+1)^2}, \\ c_1(i) &= \frac{1}{i+1}, \\ c_k(i) &= 0, \\ &k = 2, 3, \dots, r, \quad i \in \mathcal{D}, \\ d_0(i) &= -\frac{i}{2(i+1)}, \\ d_1(i) &= \frac{1}{i+1}, \\ d_k(i) &= 0, \\ &k = 2, 3, \dots, r, \quad i \in \mathcal{D}, \end{aligned}$$

FIGURE 2: System state response in Example 2.

FIGURE 3: System output response in Example 2.

$$\begin{aligned}
 r_0(i) &= -\frac{i}{i+1}, \\
 r_1(i) &= 1, \\
 r_k(i) &= 0, \\
 k &= 2, 3, \dots, r, \quad i \in \mathcal{D}, \\
 h_0(i) &= 1, \\
 h_1(i) &= 1, \\
 h_k(i) &= 0, \\
 k &= 2, 3, \dots, r, \quad i \in \mathcal{D}, \\
 l(i) &= \frac{1}{i+1}, \\
 l_0(i) &= 1, \\
 n(i) &= 1, \\
 e(i) &= 0.1.
 \end{aligned} \tag{44}$$

The purpose here is to design an H_∞ controller such that the closed-loop system has mean square exponential stability and with a given H_∞ norm bound $\gamma = 0.5$. Applying Theorem 9, the H_∞ controller can be designed as

$$K(i) = -\frac{1}{i+1}. \tag{45}$$

With the initial conditions $\phi(t_0) = 0.1$ for $t_0 = -2, -1, 0$ and the exogenous disturbance $v(t) = 2e^{-t} \sin t$, Figures 2 and 3 show the state and output responses, respectively.

5. Conclusions

In this paper, the issue of exponential stability and robust H_∞ control for a class of discrete-time time-delay stochastic

systems with infinite Markov jumps and multiplicative noises has been studied. Time-delay and infinite Markov jump are taken into consideration simultaneously. By using Lyapunov-Krasovskii functional and introducing slack matrix, a matrix inequality approach has been adopted to ensure the mean square exponential stability and satisfy a prescribed H_∞ performance level. Finally, some illustrative examples are given to demonstrate the usefulness of the proposed design methods. Further research directions would include the investigation on H_2/H_∞ control problem and asynchronous control problem for discrete-time time-delay stochastic systems with infinite Markov jumps.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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