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Research Article

Implementation Flexibility of Multiperiod Rail Line Design with Consideration of Uncertainties in Population Distribution

Kunpeng Zhang,[1] Binglei Xie,[1] and Ding Liu[2]

[1] *Shenzhen Key Laboratory of Urban Planning and Decision Making Simulation, Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen 518055, China*
[2] *College of Transport and Communications, Shanghai Maritime University, Shanghai 201306, China*

Correspondence should be addressed to Ding Liu; stunker@126.com

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This paper aims to investigate the implementation flexibility of multiperiod rail line design in a linear monocentric city. Three alternatives (fast-tracking, deferring, and do-nothing-alternative (DNA) of a candidate rail line project) are examined, based on an in-depth uncertainties analysis of the demand side for this candidate rail line project. Conditions for the three alternatives of fast-tracking, deferring, and DNA are analytically explored and an illustrative example is given to demonstrate the application of the proposed models. Insightful findings are reported on the interrelationship between the rail line length and spatial and temporal correlation of population distribution as well as the implication of the correlation in practice. Sensitivity analyses are carried out in several scenarios in another numerical example to show the proposed conditions of three alternatives.

1. Introduction

In traditional transportation planning models, population distribution at each residential location was assumed to follow independence of irrelevant alternatives (IIA) (see, e.g., [1–4]). This assumption was generally acceptable since the traditional transportation planning models were commonly static and proposed for European and American cities with relatively low population densities.

The IIA assumption of population distribution, however, may result in inaccurate results and the ignorance of the correlation between random variables (Shao et al., 2012). Particularly, for cities with relatively high population densities, like Shanghai and Hong Kong in China, the correlation of travel demand was found to play a major role in road network expected total travel time [5]. Zhao and Kockelman [6] concluded that ignorance of correlation of travel demand would ultimately affect policy-making and infrastructure decisions. Yip et al. [7] confirmed the existence of the correlation of travel demand using the data in Hong Kong. Travel demand between two towns and urban areas in a two-5-year period, 1996–2001 and 2001–2006, was deployed for investigation in their study.

With the IIA assumption of population distribution, uncertainties of population distribution cannot be fully explored. Uncertainties of population distribution can be classified into variations of population distribution year by year and spatial-temporal correlation of population distribution [8–10]. The year-by-year variations of population distribution can be captured by a stochastic variable of annual population growth rate, with its mean value and standard deviation [9, 11, 12]. The spatial-temporal correlation of population distribution can be described by the spatial and temporal correlation coefficient of population densities [8, 9, 13].

Uncertainties of population distribution directly affect the travel demand of rail service and further the implementation of a candidate rail transit line. For instance, while the population distribution is higher than the forecasting result of the candidate rail transit line in original feasible report, the candidate rail transit line can be fast-tracked. While the spatial-temporal correlation of population distribution is

Table 1: Comparison between some closely concerned models with the model proposed in this paper.

Citation	Variation of population distribution	Spatial-temporal correlation of population distribution	Multi-period NDP	Implementation flexibility
Vuchi and Newell (1968)	No	No	No	No
Chien and Qin (2004)	No	No	No	No
Szeto and Lo [19]	Yes	No	Yes	No
Ukkusuri and Patil (2009)	Yes	No	Yes	Yes
Ma and Lo [20]	Yes	No	Yes	No
Li et al. [2]	No	No	No	No
Shao et al. [9]	Yes	Yes	No	No
Liu [14]	Yes	Yes	No	No
Peng et al. [22]	Yes	No	No	No
This paper	Yes	Yes	Yes	Yes

very closely, the candidate rail transit line may be fast-tracked [8, 14].

The implementation flexibility problem of a candidate rail transit line is investigated in this paper. Specifically, the following questions are explored with the proposed model:

(i) Under which condition, a rail line project should be fast-tracked or deferred?

(ii) Hong long this rail line should be built and what are the mean and standard deviation of the rail line length?

(iii) What effect exists between spatial and temporal correlation of population distribution and the rail line length?

The implementation flexibility problem of a candidate rail transit line is essentially a network design problem (NDP). In terms of time dimensions, NDP can be classified into two types of single-period NDP and multiperiod NDP. An NDP in most previous studies is typically examined in a single specific period. Lo and Szeto [15] extended a single-period NDP into a multiperiod NDP. Szeto and Lo [16, 17] incorporated time-dependent tolling into a multiperiod NDP. Szeto and Lo [18] took into account equity for multiperiod NDP, whereas Lo and Szeto [19] examined a multiperiod NDP with cost-recovery constraints. Ma and Lo [20] investigated time-dependent integrated transport supply and demand strategies and their impact on land use patterns.

Ukkusuri and Patil (2009) introduced flexibility into a multiperiod NDP, in which future investment could be deferred or abandoned. Flexibility is defined as the ability of the system to adapt to external changes, while maintaining satisfactory system performance [21]. Flexibility gives authorities and/or operators to fast-track or defer the future investment in a rail line system for several years, if necessary.

Table 1 compares some closely concerned models with the model proposed in this paper, with respect to variation of population distribution, spatial-temporal correlation of population distribution, multiperiod NDP, and implementation flexibility. In contrast with other studies, variation of population distribution and spatial-temporal correlation of population distribution are both considered in this paper. To explore the implementation flexibility of the candidate rail transit line, a multiperiod model is proposed in this paper.

Three major extensions to the related literature are proposed in this paper: (1) the over-year uncertainties of population distribution are considered while conducting the implementation flexibility analysis of the candidate rail transit line; (2) the spatial-temporal correlation of population distribution is incorporated into the proposed model; (3) the benefit of fast-tracking a project and the penalty for deferring a project for several years are considered analytically. The proposed model has the potential to help authorities and/or operators implementing candidate rail line projects in an appropriate year, in accordance with the yearly varied travel demand of the rail service and time-dependent construction cost of the rail line projects.

The reminder of this paper is organized as follows: basic considerations are presented in Section 2. The conditions of fast-tracking or deferring the rail line project for several years are then explored in Section 3. Section 4 gives two numerical examples to show the contributions of the proposed models. Conclusions and further work are given in Section 5.

2. Basic Considerations

As shown in Figure 1, x_1 and x_2 represent the residential locations. $L(t_1)$ and $L(t_2)$ are the length of a candidate rail line in year t_1 and t_2, which can be determined endogenously with the proposed model. $\widetilde{P}(x,t)$ is the yearly varied population density distributed in residential location x in year t, with $\forall x \in \{x_1, x_2\}$ and $\forall t \in \{t_1, t_2\}$.

To facilitate the presentation of the essential ideas, the notations are summarized as follows. The notations are partitioned into the two types of deterministic variables and stochastic variables.

(i) Deterministic Variables

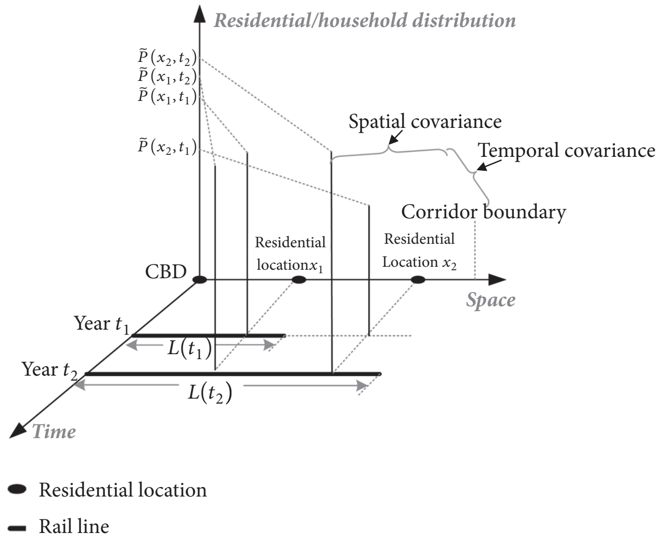

FIGURE 1: Configuration of a candidate rail line over years in a linear monocentric city.

$c(x,t)$: the yearly nominal generalized travel cost for traveling from residential location x to the CBD in year t.

$D_s(t)$: average station spacing of the candidate rail transit line in year t.

f_0: fixed component of fare for using the rail service.

f_1: variable component of fare per unit distance for using the rail service.

$h(q(x,t))$: headway of train operation during morning peak hour in year t.

$H(x,t)$: the nominal housing supply at residential location x in year t.

$L(t)$: rail length in year t.

$P(x,t)$: nominal population distribution at location x in year t.

$p(x,t)$: the probability of choosing to live at residential location x in year t.

$q(x,t)$: nominal travel demand of rail service at location x in year t.

$r(x,t)$: the nominal housing rent at residential location x in year t.

$U(x,t)$: the nominal disutility for residential location x in year t.

(ii) Stochastic Variables

$\widetilde{P}(x,t)$: yearly varied population distribution at location x in year t.

$\widetilde{q}(x,t)$: yearly varied travel demand of rail service at location x in year t.

$\widetilde{U}(x,t)$: yearly varied disutility for residential location x in year t.

To facilitate the presentation of the essential ideas, without loss of generality, the following assumptions are made in this paper:

A1: the candidate rail line project is assumed to be linear and start from the CBD and then be built along a linear monocentric city, as shown in Figure 1 [1, 2, 23]. The candidate rail line project in each period is assumed to finish on time and the rail service is expected to be provided at the end of each period [19].

A2: the standard deviation (SD) of population distribution is assumed to be an increasing function with respect to its mean value. This function is referred to as the stochastic population distribution function. Besides, the stochastic population distribution function is assumed as a nondecreasing function with respect to its mean value [9, 14].

A3: travelers' responses to the quality of rail service are measured by a generalized travel cost that is a weighted combination of in-vehicle time, access time, waiting time, and the fare [24]. Travelers are assumed to be homogeneous and have the same preferred arrival time at workplace located in the CBD. This study focuses mainly on travelers' home-based work trips, which are compulsory activities, and thus the number of trips is not affected by a variety of factors, such as income level [2].

A4: a rail project is worthy of investment, if benefit is not less than the penalty in its planning and operation horizon. The study period is assumed to be a one-hour period, for instance, the morning peak hour, which is usually the most critical period in the day [25].

3. Model Formulation

Population distribution is closely related to the planning procedure of a rail line project. This data and its growth rate over years are used to make strategic decisions including whether to introduce, defer, or fast-track new rail lines; how long the rail should be built; and how to determine effective train operation parameters, such as the number of carriages in each train, the headway between trains, and the fares.

3.1. Uncertainties of Population Distribution. To allow for the yearly uncertainty of population distribution, it is assumed that there exists a perturbation in the population distribution. The yearly varied population distribution $\widetilde{P}(x,t)$ is given by the following equation [11]:

$$\widetilde{P}(x,t) = P(x,t) + \varepsilon, \tag{1}$$

where $P(x,t)$ is the nominal population distribution, $E[\widetilde{P}(x,t)] = P(x,t)$; ε is a random term, such that $E[\varepsilon] = 0$. Note that the nominal population distribution $P(x,t)$ is deterministic.

In previous studies, population distribution was assumed to follow a Poisson distribution (e.g., [26]), normal distribution (e.g., Fu et al., 2012), or multivariate normal (MVN) distribution (e.g., [27]). For the assumption of Poisson distribution, equality of mean and variance was very rigid and may be inappropriate in practice [28]. The normal distribution was commonly assumed to maintain tractability of the model (Fu et al., 2012). Following the assumption of MVN distribution, it was important to propose efficient algorithms to avoid excessive computational costs [27, 29].

In terms of *A2*, the SD of population distribution can be expressed as [9, 14]

$$\sigma_p(x,t) = \sqrt{\operatorname{var}[\widetilde{P}(x,t)]} = \sqrt{\operatorname{var}[\varepsilon]} = \varphi(P(x,t)), \tag{2}$$

where $\varphi(\cdot)$ is defined as the stochastic population distribution function, which represents the functional relationship between the mean value and the variance of the stochastic population distribution.

To take spatial and temporal correlation of population distribution into account, the following spatial and temporal covariance is defined as [8]

$$\begin{aligned}\sigma_p(x_1,t_1;x_2,t_2) &= \operatorname{cov}[\widetilde{P}(x_1,t_1),\widetilde{P}(x_2,t_2)] \\ &= \rho_{x_1,t_1}^{x_2,t_2}\varphi(P(x_1,t_1))\varphi(P(x_2,t_2)),\end{aligned} \tag{3}$$

where $\rho_{x_1,t_1}^{x_2,t_2}$ $(-1 \le \rho_{x_1,t_1}^{x_2,t_2} \le 1)$ is the correlation coefficient, which is an important measurement reflecting the statistical correlation between $\widetilde{P}(x_1,t_1)$ and $\widetilde{P}(x_2,t_2)$. There are three correlation coefficient cases: negative, positive, or zero, representing negative, positive statistical dependence or statistical independence of population distribution. Specifically, with $x_1 = x_2$, and $t_1 = t_2$, the spatial and temporal covariance becomes the SD value.

The spatial and temporal correlation of population distribution cannot be taken into account under the assumptions of independent Poisson distribution and normal distribution. For instance, under independent normal distribution, the probability density function (PDF) of population distribution is given by $f(P(x,t)) = (1/\sqrt{2\pi}\sigma_p(x,t))\exp(-(\widetilde{P}(x,t)-P(x,t))^2/2(\sigma_p(x,t))^2)$. Under MVN distribution, the PDF of population distribution is $f(\mathbf{P}(\mathbf{x},\mathbf{t})) = (1/(\sqrt{2\pi})^{|\mathbf{R}|/2}|\sum^{\mathbf{P}(\mathbf{x},\mathbf{t})}|^{1/2})\exp(-(1/2)(\widetilde{\mathbf{P}}(\mathbf{x},\mathbf{t}) - \mathbf{P}(\mathbf{x},\mathbf{t}))^T(\sum^{\mathbf{P}(\mathbf{x},\mathbf{t})})^{-1}(\widetilde{\mathbf{P}}(\mathbf{x},\mathbf{t}) - \mathbf{P}(\mathbf{x},\mathbf{t})))$, where $\mathbf{P}(\mathbf{x},\mathbf{t}) = (P(x_1,t_1),P(x_1,t_2),\cdots P(x_n,t_m))$ (n is the residential location number, m is the planning time horizon) is a $|\mathbf{R}|$-vector, $|\sum^{\mathbf{P}(\mathbf{x},\mathbf{t})}|$ is the determinant of $\sum^{\mathbf{P}(\mathbf{x},\mathbf{t})}$, and $\sum^{\mathbf{P}(\mathbf{x},\mathbf{t})}$ is the covariance matrix of $\mathbf{P}(\mathbf{x},\mathbf{t})$.

To consider the yearly uncertainty of rail service travel demand, it is assumed that a perturbation exists in this travel demand, and the yearly perturbed travel demand $\widetilde{q}(x,t)$ is given by the following equation:

$$\widetilde{q}(x,t) = q(x,t) + \varepsilon_1, \tag{4}$$

where $q(x,t)$ is the nominal travel demand at location x in year t and ε is a random term, with $E[\widetilde{q}(x,t)] = q(x,t)$ and $E[\varepsilon_1] = 0$.

The nominal travel demand of rail service is assumed to be a function of population distribution and generalized travel cost from residential locations to the CBD destination in terms of *A3*. Without loss of generality, an exponential function is used as follows [30]:

$$\begin{aligned}q(x,t) = P(x,t)\exp(-\theta_1 c(x,t)), \\ \forall x \in [0,L], \ t \in [0,T],\end{aligned} \tag{5}$$

where $q(x,t)$, $P(x,t)$, and $c(x,t)$ are the yearly nominal travel demand of rail service, population distribution, and generalized travel cost, respectively; θ_1 is sensitivity parameter in travel demand function.

Population distribution $P(x,t)$ is closely concerned with the disutility perceived by travelers residing at residential location x in year t. Because of the spatial and temporal correlation of population distribution, population distributed at different locations and years are not independent or irrelevant. To facilitate the computational advantage of the use of a closed-form analytical expression, the C-Logit model proposed by Cascetta et al. [31] was applied, owing to its relatively low levels of calibration influences and its rational behaviour consistent with random utility theory [32].

The C-Logit model of population distribution can be stated as follows [32]:

$$P(x,t) = P^t p(x,t), \tag{6}$$

where P^t is the total population along the rail line system in year t. $p(x,t)$ is the probability of choosing to live at residential location x in year t:

$$p(x,t) = \frac{\exp[-\theta_2(U(x,t)+CF(x,t))]}{\int_0^L \exp[-\theta_2(U(w,t)+CF(w,t))]\,\mathrm{d}w}, \tag{7}$$

where θ_2 is a dispersion parameter of population distribution. If θ_2 is large, population distribution along the candidate rail line is then assumed to be a decentralized type. If θ_2 is small, population is centrally distributed along the candidate rail line. $CF(x,t)$ is the commonality factor for each residential location.

Several functional commonality factor forms have been proposed by Cascetta et al. [31]. These forms can be classified into two types: flow-dependent and flow-independent. The flow-dependent functional form is used in this study as follows [11]:

$$CF(x,t) = \alpha_2 \ln \frac{U(x,t)}{\int_0^L U(w,t)\,dw}, \quad \forall t \in [0,T], \tag{8}$$

where α_2 indicates a constant parameter. Examples of the effect of changes to this parameter are given in the work of Cascetta et al. [31] and Prashker and Bekhor [33].

The population conservation equation can be expressed by

$$\int_0^L P(w,t)\,dw = P^t, \tag{9}$$

where P^t is the total population number along the candidate rail transit line in year t. To response the year-by-year change of the total population, a constant yearly growth rate is assumed [19]:

$$P^t = (1+\gamma(t))\,P^{t-1}, \tag{10}$$

where $\gamma(t)$ is the average yearly growth rate of the total population number after year t. As $\gamma(t)$ is positive, the implication is that the number of total population along the candidate rail transit line increases and vice versa.

In this paper, the disutility of travelers mainly consists of travel costs from residential location to CBD and housing rent. It is assumed that travelers put equal weighting on generalized travel cost and housing rent. Mathematically, it can be expressed as [2]

$$\widetilde{U}(x,t) = U(x,t) + \varepsilon_2(x,t), \tag{11}$$

$$U(x,t) = c(x,t) + r(x,t), \tag{12}$$

where $U(x,t)$ is nominal disutility, $c(x,t)$ is the nominal generalized travel cost, $r(x,t)$ is the nominal housing rent at residential location x in year t, and $\varepsilon_2(x,t)$ is the associated random terms, where $E[\varepsilon_2(x,t)] = 0$.

The nominal generalized travel cost consists of fare, access cost for travelers from residential locations to rail stations, waiting cost for rail service at stations, and in-vehicle cost from rail stations to CBD, shown as follows [30]:

$$c(x,t) = f(x,t) + \mu_c t_c + \mu_w t_w + \mu_i t_i, \tag{13}$$

where $\mu_c/\mu_w/\mu_i$ are values of access time, waiting time, and in-vehicle time, respectively; $f(x,t)$ is distance-based fare for rail service, t_c is average access time for travelers from residential locations to the rail station, t_w is the average waiting time for rail service at stations, and t_i is average in-vehicle cost from rail stations to CBD. The distanced-based fare $f(x,t)$ is given by

$$f(x,t) = f_0 + f_1 x, \tag{14}$$

where f_0 is the fixed fare component and f_1 is the variable fare component per kilometer. Waiting time t_w is closely concerned with travel demand and supply of the rail service. For long-term planning, this value can be estimated using the following function:

$$t_w = \alpha h(q(x,t)), \tag{15}$$

where α is a calibration parameter which depends on the distribution of train headway and travelers arrival time, and $h(q(x,t))$ is the average headway [34].

The nominal rent for a house rises in proportion to the number of families wanting to live in that house. For instance, the landlord can increase the rent in accordance with the demand. Hence, in theory of the housing supply matches demand, the nominal rent is likely to be reduced. In response to this factor, housing rent is assumed to be given by the following function [35]:

$$r(x,t) = \alpha_1 \left(1 + \beta_1 \frac{P(x,t)}{H(x,t) - P(x,t)}\right), \tag{16}$$

where α_1 and β_1 are parameters and $H(x,t)$ is the nominal housing supply. Calibration of the housing function parameters is indispensable.

3.2. Payoff of Fast-Tracking or Deferring a Candidate Rail Project. A candidate rail transit line project can be implemented in three different ways:

(1) It can be implemented as planned (do-nothing-alternative (DNA)).
(2) It can be fast-tracked several years.
(3) It can be deferred several years.

Which alternative should be used depends on the payoff with respect to benefit and penalty, as shown in Table 2.

For the DNA, benefit comes from the convenience of rail service supplied to travelers during the operation horizon of this rail line system. This benefit can be measured by customer surplus. Mathematically, it can be expressed as (Ukkusuri and Patil, 2009):

$$\begin{aligned}\breve{\pi}(\gamma(t)) &= \sum_{t=0}^{T} \int_0^{L(t)} \left[\int_0^{q(x,t)} (q(w,t))^{-1}\,dw \right. \\ &\quad \left. - q(x,t)\,c(x,t) \right] dx = \sum_{t=0}^{T} \int_0^{L(t)} \frac{q(x,t)}{\theta_1}\,dx.\end{aligned} \tag{17}$$

Since $q(x,t)$ is an increasing function of $P(x,t)$ and $P(x,t)$ is an increasing function of γ, $\breve{\pi}(\gamma(t))$ is an increasing function of $\gamma(t)$.

The daily average penalty of the DNA is comprised by construction cost, operation cost, and maintenance cost [36]. This penalty can be expressed as

$$\begin{aligned}&\widehat{\pi}(i(t)) \\ &\quad = \frac{C_l L(t) + C_s L(t)/D_s(t) + C_{om} PWF(i(t),T)}{365},\end{aligned} \tag{18}$$

where C_l is the construction cost for rail line per kilometer if this project is implemented as schedule, C_s is the construction

Table 2: The benefit and penalty of each alternative.

Alternative	Project start time	Interest rate	Growth factor of population density per year	Benefit	Penalty
DNA	0	$i(t)$	$\gamma(t)$	$\breve{\pi}(\gamma(t))$	$\widehat{\pi}(i(t))$
Fast-tracked	$-t_1$	$i_f(t)$	$\gamma_f(t)$	$\breve{\pi}_f(\gamma_f(t))$	$\widehat{\pi}_f(i_f(t))$
Deferred	t_2	$i_d(t)$	$\gamma_d(t)$	$\breve{\pi}_d(\gamma_d(t))$	$\widehat{\pi}_d(i_d(t))$

Notes:
(1) The original project start time as schedule is set as year 0.
(2) The planning and operation horizon is assumed to be same for the three alternatives.
(3) Normally, the following equations are held:
$\gamma_f(t) > \gamma(t) > \gamma_d(t)$ or $i_d(t) > i(t) > i_f(t)$.

cost of each rail station, $D_s(t)$ is the average station spacing of this rail transit line, and C_{om} is the annual operation and maintenance cost of this rail transit line project. The number "365" is used to convert the annual average penalty to daily average penalty.

If the alternative option of fast-tracking this rail line project is chosen, travelers would enjoy the benefits of rail service earlier. Meanwhile, the total cost of construction, operation, and maintenance costs of the system would increase compared with those of the original plan, within the same project planning and operation horizon. Mathematically, the benefit can be expressed as

$$\breve{\pi}_f\left(\gamma_f(t)\right) = \sum_{t=-t_1}^{T-t_1} \int_0^{L(t)} \frac{q(x,t)}{\theta_1} \mathrm{d}x, \tag{19}$$

and the daily average penalty for this alternative is expressed as follows:

$$\widehat{\pi}_f\left(i_f(t)\right) = \frac{C_l CAF\left(i_f(t), t_1\right) L(t) + C_s CAF\left(i_f(t), t_1\right) L(t)/D_s(t) + C_{om} CAF\left(i_f(t), t_1\right) PWF\left(i_f(t), T\right)}{365}, \tag{20}$$

where $CAF(i_f(t), t_1)$ is compound-amount factor, defined as $(1+i_f(t))^{t_1}$. This factor takes into account the time cost of each construction or operation cost component of rail project in terms of average interest rate $i_f(t)$ and fast-tracking period t_1. The first term $C_l CAF(i_f(t), t_1)L(t)/365$ represents the daily construction cost of rail line, as the rail transit line project is fast-tracked t_1 years. Accordingly, the second term in (21) represents the daily construction cost of all rail stations, and the third term in (21) represents the daily operation and maintenance cost of this rail transit line project.

If the alternative of deferring this rail transit line project is chosen, the total cost of construction, operation, and maintenance costs would decrease compared with those of the original plan, within the same project planning and operation horizon. However, the travelers would suffer the penalty of traffic crowding until a better rail service is supplied. Mathematically, the daily average benefit is calculated by

$$\breve{\pi}_d\left(\gamma_d(t)\right) = \frac{C_l PWF'\left(i_d(t), t_2\right) L(t) + C_s PWF'\left(i_d(t), t_2\right) L(t)/D_s(t) + C_{om} PWF'\left(i_d(t), t_2\right) PWF\left(i_d(t), T\right)}{365}, \tag{21}$$

where $PWF'(i_d(t), t_2)$ is present-worth factor to obtain present value of a future value, defined as $1/(1+i(t))^t$. The first term $C_l PWF'(i_d(t), t_2)L(t)/365$ represents the daily construction cost of rail line, as the rail transit line project is deferred t_2 years. Accordingly, the second term in (22) represents the daily construction cost of all rail stations, and the third term in (22) represents the daily operation and maintenance cost of this rail transit line project.

The daily average penalty of deferring the rail transit line project mainly comes from the travel inconvenience of travelers. For instance, traffic congestion cannot be eliminated while rail service is not supplied on time. This daily average penalty can be measured by consumer surplus of travelers, expressed as

$$\widehat{\pi}_d\left(\gamma_d(t)\right) = \sum_{t=0}^{t_2} \int_0^{L(t)} \frac{q(x,t)}{\theta_1} \mathrm{d}x. \tag{22}$$

where t_2 is the deferred period and the integral term represents daily consumer surplus of travelers, namely, daily average penalty of deferring the rail project.

3.3. Conditions for Fast-Tracking or Deferring a Candidate Rail Project. In terms of $A4$, a rail transit line project is worthy of investment, if benefit $\breve{\pi}(\gamma(t))$ is not less than the penalty $\widehat{\pi}(i(t))$ in its planning and operation horizon $t \in [0, T]$. Specifically, this rail project is break-even, if benefit $\breve{\pi}(\gamma(t))$ is equal to penalty $\widehat{\pi}(i(t))$, namely,

$$\breve{\pi}(\gamma(t)) = \widehat{\pi}(i(t)), \tag{23}$$

In (23), given interest rate $i(t)$, rail length $L(t)$, and average station spacing $D_s(t)$, only the growth rate of total population density $\gamma(t)$ is unknown in terms of (1)–(21). By solving (23), a break-even growth rate $\gamma^*(t)$ of total population along the candidate rail system can be obtained. As the growth rate of the total population in the candidate rail system is greater than $\gamma^*(t)$, this rail project is worthy of investment. Similarly, given the growth rate of total population $\gamma(t)$, rail length $L(t)$, and average station spacing $D_s(t)$, only the interest rate $i(t)$ is unknown. By solving (23), the break-even interest rate $i^*(t)$ can be determined. This $i^*(t)$ is the internal rate of return (IRR), which makes this project just break-even. When the actual interest rate is lower than this IRR $i^*(t)$, the project is worthy of investment. With a given interest rate $i^*(t)$, a growth rate of total population density $\gamma^*(t)$, and average station spacing $D_s(t)$, the rail length $L^*(t)$, which makes this project break-even, can be determined year by year by solving (23).

As previously stated, the rail transit line project can be fast-tracked or deferred, while interest rate or growth rate of total population varies over years. Different scenarios of interest rate and growth rate of total population over years are investigated here. For each scenario, the increased benefit and increased penalty or the consumer surplus loss and capital cost saving should be firstly compared, so as to determine the suitable alternative. A detailed results summary is given in Table 3. The expressions of benefit and penalty in Table 3 are given by (18)-(23).

The values of $\gamma_f(t) > \gamma(t) > \gamma_d(t)$ and $i_d(t) > i(t) > i_f(t)$ in Table 3 are assumed to be given. The conditions for fast-tracking or deferring the rail project are sufficient conditions. If the values of $\gamma_f(t) > \gamma(t) > \gamma_d(t)$ and $i_d(t) > i(t) > i_f(t)$ are unknown, the necessary conditions for fast-tracking or deferring the rail projects are required. Propositions 3 and 4 present necessary conditions for fast-tracking or deferring the rail project. The values of $\gamma_f^*(t)$, $i_f^*(t)$, $\gamma_d^*(t)$, and $i_d^*(t)$ can be determined endogenously.

Proposition 1. *Necessary condition for fast-tracking the rail project is summarized as follows.*

A rail project is worthy of fast-tracking, only if the increased benefit $(\partial\breve{\pi}_f(\gamma_f(t))/\partial\gamma_f(t))(\gamma_f(t) - \gamma^*(t))$ is more than the increased penalty $(\partial\widehat{\pi}_f(i_f(t))/\partial i_f(t))(i^*(t) - i_f(t))$; namely,

$$\frac{\partial\breve{\pi}_f(\gamma_f(t))}{\partial\gamma_f(t)}\left(\gamma_f(t) - \gamma^*(t)\right) > \frac{\partial\widehat{\pi}_f(i_f(t))}{\partial i_f(t)}\left(i^*(t) - i_f(t)\right). \tag{24}$$

where

$$\frac{\partial\breve{\pi}(\gamma_f(t))}{\partial\gamma_f(t)} = \frac{1}{\theta_1}\sum_{t=-t_1}^{T-t_1}\int_0^L \left(\left((t-1)(1+\gamma^*(t))^{t-2}P^0 p(x,t)\right)(1-\theta_1 c(x,t))\right)dx.$$

$$\begin{aligned}\frac{\partial\widehat{\pi}_f(i_f(t))}{\partial i_f(t)} &= \left(C_l L(t) + \frac{C_s L(t)}{D_s(t)}\right)\frac{t_1\left(1+i_f(t)\right)^{t_1-1}}{365} \\ &\quad + C_{om}\frac{(T+t_1)i_f(t)\left(1+i_f(t)\right)^T - \left[\left(1+i_f(t)\right)^T - 1\right]\left(1+i_f(t)+i_f(t)T\right) - t_1 i_f(t)}{(365)\,i_f^2(t)\left(1+i_f(t)\right)^{T-t_1+1}}.\end{aligned} \tag{25}$$

Proof. The first-order derivative of benefit $\breve{\pi}_f(\gamma_f(t))$ with respect to growth rate $\gamma_f(t)$ is derived as follows:

$$\frac{\partial\breve{\pi}_f(\gamma_f(t))}{\partial\gamma_f(t)} = \frac{1}{\theta_1}\sum_{t=-t_1}^{T-t_1}\int_0^L \frac{\partial q(x,t)}{\partial\gamma_f(t)}dx,$$

$$\frac{\partial q(x,t)}{\partial\gamma_f(t)} = \exp(-\theta_1 c(x,t))\left[\frac{\partial P(x,t)}{\partial\gamma_f(t)} - \theta_1 P(x,t)\frac{\partial c(x,t)}{\partial\gamma_f(t)}\right],$$

$$\frac{\partial P(x,t)}{\partial\gamma_f(t)} = \frac{\partial P^t}{\partial\gamma_f(t)}p(x,t) + P^t\frac{\partial p(x,t)}{\partial\gamma_f(t)},$$

Table 3: Alternative choice with respect to different scenarios of growth factor of population density over years and interest rate ($\gamma_f(t) > \gamma(t) > \gamma_d(t)$ and $i_d(t) > i(t) > i_f(t)$).

	Interest rate	Growth factor	Results
Scenario 1	$i_f(t)$	$\gamma_d(t)$	$(\partial\breve{\pi}(\gamma_d(t))/\partial\gamma_d(t))(\gamma^*(t)-\gamma_d(t)) < (\partial\widehat{\pi}(i_f(t))/\partial i_f(t))(i^*(t)-i_f(t))$, fast-track $(\partial\breve{\pi}(\gamma_d(t))/\partial\gamma_d(t))(\gamma^*(t)-\gamma_d(t)) = (\partial\widehat{\pi}(i_f(t))/\partial i_f(t))(i^*(t)-i_f(t))$, DNA $(\partial\breve{\pi}(\gamma_d(t))/\partial\gamma_d(t))(\gamma^*(t)-\gamma_d(t)) > (\partial\widehat{\pi}(i_f(t))/\partial i_f(t))(i^*(t)-i_f(t))$, defer
Scenario 2	$i_f(t)$	$\gamma(t)$	Fast-track
Scenario 3	$i_f(t)$	$\gamma_f(t)$	Fast-track
Scenario 4	$i(t)$	$\gamma_d(t)$	Defer
Scenario 5	$i(t)$	$\gamma(t)$	DNA
Scenario 6	$i(t)$	$\gamma_f(t)$	Fast-track
Scenario 7	$i_d(t)$	$\gamma_d(t)$	Defer
Scenario 8	$i_d(t)$	$\gamma(t)$	Defer
Scenario 9	$i_d(t)$	$\gamma_f(t)$	$(\partial\breve{\pi}(\gamma_f(t))/\partial\gamma_f(t))(\gamma_f(t)-\gamma^*(t)) > (\partial\widehat{\pi}(i_d(t))/\partial i_d(t))(i_d(t)-i^*(t))$,fast-track $(\partial\breve{\pi}(\gamma_f(t))/\partial\gamma_f(t))(\gamma_f(t)-\gamma^*(t)) = (\partial\widehat{\pi}(i_d(t))/\partial i_d(t))(i_d(t)-i^*(t))$,DNA $(\partial\breve{\pi}(\gamma_f(t))/\partial\gamma_f(t))(\gamma_f(t)-\gamma^*(t)) < (\partial\widehat{\pi}(i_d(t))/\partial i_d(t))(i_d(t)-i^*(t))$, defer

$$\frac{\partial P^t}{\partial \gamma_f(t)} = (t-1)\left(1+\gamma(t)\right)^{t-2} P^0,$$

$$\frac{\partial p(x,t)}{\partial \gamma_f(t)} = \frac{-\theta_2 \exp\left[-\theta_2\left(U(x,t)+CF(x,t)\right)\right]\left(\partial U(x,t)/\partial\gamma_f(t)+\partial CF(x,t)/\partial\gamma_f(t)\right)}{\int_0^L \exp\left[-\theta_2\left(U(w,t)+CF(w,t)\right)\right]\mathrm{d}w} + \frac{\theta_2 \exp\left[-\theta_2\left(U(x,t)+CF(x,t)\right)\right]\int_0^L \exp\left[-\theta_2\left(U(w,t)+CF(w,t)\right)\right]\left(\partial U(w,t)/\partial\gamma_f(t)+\partial CF(w,t)/\partial\gamma_f(t)\right)\mathrm{d}w}{\left[\int_0^L \exp\left(-\theta_2\left(U(w,t)+CF(w,t)\right)\right)\mathrm{d}w\right]^2}, \tag{26}$$

$$\frac{\partial U(x,t)}{\partial \gamma_f(t)} = \frac{\partial c(x,t)}{\partial \gamma_f(t)} + \frac{\partial r(x,t)}{\partial \gamma_f(t)},$$

$$\frac{\partial CF(x,t)}{\partial \gamma_f(t)} = \alpha_2 \frac{\int_0^L \sqrt{U(w,t)}\mathrm{d}w}{U(x,t)}\left[\frac{\partial U(x,t)}{\partial \gamma_f(t)}\frac{1}{\int_0^L \sqrt{U(w,t)}\mathrm{d}w} - \frac{U(x,t)\int_0^L\left(1/\sqrt{U(w,t)}\right)\left(\partial U(w,t)/\partial\gamma_f(t)\right)\mathrm{d}w}{2\left(\int_0^L\sqrt{U(w,t)}\mathrm{d}w\right)^2}\right], \tag{27}$$

$$\frac{\partial c(x,t)}{\partial \gamma_f(t)} = \alpha\frac{\partial h\left(q(x,t)\right)}{\partial q(x,t)}\frac{\partial q(x,t)}{\partial \gamma_f(t)} = 0,$$

$$\frac{\partial r(x,t)}{\partial \gamma_f(t)} = \frac{\beta H(x,t)}{\left(H(x,t)-P(x,t)\right)^2}\frac{\partial P(x,t)}{\partial \gamma_f(t)} = 0,$$

Thus, we have

$$\frac{\partial \breve{\pi}\left(\gamma_f(t)\right)}{\partial \gamma_f(t)} = \frac{1}{\theta_1} \cdot \sum_{t=-t_1}^{T-t_1} \int_0^L \left(\frac{\partial P(x,t)}{\partial \gamma_f(t)} \left(1-\theta_1 P(x,t)\right) \frac{\partial c(x,t)}{\partial \gamma_f(t)} \right) \mathrm{d}x = \frac{1}{\theta_1} \sum_{t=-t_1}^{T-t_1} \int_0^L \left(\left((t-1)\left(1+\gamma^*(t)\right)^{t-2} P^0 p(x,t) \right) \cdot \left(1-\theta_1 c(x,t)\right) \right) \mathrm{d}x. \quad (28)$$

The first-order derivative of penalty $\widehat{\pi}_f(i_f(t))$ with respect to interest rate $i_f(t)$ is derived as

$$\begin{aligned}\frac{\partial \widehat{\pi}_f\left(i_f(t)\right)}{\partial i_f(t)} &= \frac{1}{365} \frac{\partial\left(C_l CAF\left(i_f(t), t_1\right) L(t) + C_s CAF\left(i_f(t), t_1\right) L(t)/D_s(t) + C_{om} CAF\left(i_f(t), t_1\right) PWF\left(i_f, T\right)\right)}{\partial i_f} \\ &= \left(C_l L(t) + \frac{C_s L(t)}{D_s(t)}\right) \frac{t_1\left(1+i_f(t)\right)^{t_1-1}}{365} \\ &\quad + C_{om} \frac{\left(T+t_1\right) i_f(t)\left(1+i_f(t)\right)^T - \left[\left(1+i_f(t)\right)^T - 1\right]\left(1+i_f(t)+i_f(t)T\right) - t_1 i_f(t)}{(365)\, i_f^{\,2}(t)\left(1+i_f(t)\right)^{T-t_1+1}}.\end{aligned} \quad (29)$$

Given the values of $i^*(t), i_f(t), \gamma^*(t), D_s(t)$ and rail length $L^*(t)$, solving the equation of $(\partial\breve{\pi}_f(\gamma_f(t))/\partial\gamma_f(t))(\gamma_f(t)-\gamma^*(t)) = (\partial\widehat{\pi}_f(i_f(t))/\partial i_f(t))(i^*(t)-i_f(t))$, a threshold value $\gamma_f^*(t)$ could be obtained. Once the expected growth factor of the total population density is larger than $\gamma_f^*(t)$, the project is worth fast-tracking. Similarly, given a value of $i^*(t), \gamma^*(t), \gamma_f(t), D_s(t)$ and rail length $L^*(t)$, a threshold value $i_f^*(t)$ could be obtained. Once the interest rate is lower than $i_f(t)$, the project is worth fast-tracking.

Given the values of $i^*(t), \gamma^*(t), D_s(t)$, the growth rate of $\gamma_f(t)$, and interest rate $i_f(t)$, the year-by-year rail length $L_f(t)$ can also be determined, shown as Corollary 2. □

Corollary 2. *Given the values of $i^*(t), \gamma^*(t), D_s(t)$, the growth rate of $\gamma_f(t)$, and interest rate $i_f(t)$, the year-by-year rail length $L_f(t)$ for fast-tracking the rail project t_1 years is given by*

$$\begin{aligned} L(t) &= \frac{365 \sum_{t=-t_1}^{T-t_1} \int_0^L \left(\left((t-1)\left(1+\gamma^*(t)\right)^{t-2} P^0 p(x,t)\right)\left(1-\theta_1 c(x,t)\right)\right) dx}{\theta_1\left(C_l + C_s/D_s(t)\right) t_1 \left(1+i_f(t)\right)^{t_1-1}} \\ &\quad - \frac{C_{om}}{\left(C_l + C_s/D_s(t)\right)} \frac{\left(T+t_1\right) i_f(t)\left(1+i_f(t)\right)^T - \left[\left(1+i_f(t)\right)^T - 1\right]\left(1+i_f(t)+i_f(t)T\right) - t_1 i_f(t)}{t_1 i_f^{\,2}(t)\left(1+i_f(t)\right)^T}. \end{aligned} \quad (30)$$

Because the values of $p(x,t)$ are a stochastic value, the year-by-year rail length $L_f(t)$ is also a stochastic value. The expression of its standard deviation is given by

$$\sigma(L(t)) = \frac{365 \int_{-t_1}^{T-t_1} \int_{-t_1}^{T-t_1} \int_0^{L(t)} \int_0^{L(t)} \sigma_p\left(x_1, t_{11}; x_2, t_{12}\right) \sigma\left(\left(t_{11}-1\right)\left(1+\gamma^*\left(t_{11}\right)\right)^{t_{11}-2} P\left(x_1, t_{11}\right)\right) \times \sigma\left(\left(t_{12}-1\right)\left(1+\gamma^*\left(t_{12}\right)\right)^{t_{12}-2} P\left(x_2, t_{12}\right)\right) dx_1 dx_2 dt_{11} dt_{12}}{\theta_1\left(C_l + C_s/D_s(t)\right) t_1 \left(1+i_f(t)\right)^{t_1-1}}, \quad (31)$$

where $((t_{11}-1)(1+\gamma^*(t_{11}))^{t_{11}-2}P(x_1,t_{11}))$ *and* $\sigma_p(x_1,t_{11};x_2,t_{12})$ *are given by (3) and (4).*

Proof. In terms of Proposition 1, let $(\partial\breve{\pi}_f(\gamma_f(t))/\partial\gamma_f(t))(\gamma_f(t)-\gamma^*(t))=(\partial\breve{\pi}_f(i_f(t))/\partial i_f(t))(i^*(t)-i_f(t))$; we have

$$L(t)=\frac{365\sum_{t=-t_1}^{T-t_1}\int_0^L\left(\left((t-1)(1+\gamma^*(t))^{t-2}P^0p(x,t)\right)(1-\theta_1c(x,t))\right)\mathrm{d}x}{\theta_1(C_l+C_s/D_s(t))t_1\left(1+i_f(t)\right)^{t_1-1}}-\frac{C_{om}}{(C_l+C_s/D_s(t))}\frac{(T+t_1)i_f(t)\left(1+i_f(t)\right)^T-\left[\left(1+i_f(t)\right)^T-1\right]\left(1+i_f(t)+i_f(t)T\right)-t_1i_f(t)}{t_1i_f{}^2(t)\left(1+i_f(t)\right)^T}. \tag{32}$$

With given $i^*,\gamma^*,i_f,\gamma_f$, the standard deviation of $L(t)$, $\sigma(L(t))$ is calculated by

$$\sigma(L(t))=\frac{365}{\theta_1(C_l+C_s/D_s(t))t_1\left(1+i_f(t)\right)^{t_1-1}}\sigma\left(\sum_{t=-t_1}^{T-t_1}\int_0^{L(t)}\left(\left((t-1)(1+\gamma^*(t))^{t-2}P^0p(x,t)\right)(1-\theta_1c(x,t))\right)\mathrm{d}x\right)$$
$$=\frac{365\int_{-t_1}^{T-t_1}\int_{-t_1}^{T-t_1}\int_0^{L(t)}\int_0^{L(t)}\sigma_p(x_1,t_{11};x_2,t_{12})\,\sigma\left((t_{11}-1)(1+\gamma^*(t_{11}))^{t_{11}-2}P(x_1,t_{11})\right)\sigma\left((t_{12}-1)(1+\gamma^*(t_{12}))^{t_{12}-2}P(x_2,t_{12})\right)\mathrm{d}x_1\mathrm{d}x_2\mathrm{d}t_{11}\mathrm{d}t_{12}}{\theta_1(C_l+C_s/D_s(t))t_1\left(1+i_f(t)\right)^{t_1-1}} \tag{33}$$

From (31), it can be seen that $\sigma(L(t))$ is underestimated while both spatial and temporal covariance parameters are both positive and overestimated while both spatial and temporal covariance parameters are negative. □

Proposition 3. *Necessary condition for deferring the rail project is summarized as follows.*

A rail project is worth deferring by several years, if the loss of consumer welfare $(\partial\breve{\pi}_d(\gamma_d(t))/\partial\gamma_d(t))(\gamma^*(t)-\gamma_d(t))$ *is less than the saving in capital cost* $(\partial\breve{\pi}_d(i_d(t))/\partial i_d(t))(i_d(t)-i^*(t))$; *namely,*

$$\frac{\partial\breve{\pi}_d(\gamma_d(t))}{\partial\gamma_d(t)}(\gamma^*(t)-\gamma_d(t))<\frac{\partial\breve{\pi}_d(i_d(t))}{\partial i_d(t)}(i_d(t)-i^*(t)) \tag{34}$$

where

$$\frac{\partial\breve{\pi}_d(i_d(t))}{\partial i_d(t)}=\left(C_lL(t)+\frac{C_sL(t)}{D_s(t)}\right)\frac{-t_2}{365\cdot(1+i_d(t))^{t_2+1}}+C_{om}\frac{Ti_d(t)(1+i_d(t))^T+\left((1+i_d(t))^T-1\right)\left((1+i_d(t)+Ti_d(t))-t_2i_d(t)\right)}{(365)\,i_d^2\left(1+i_d(t)\right)^{T+t_s+1}}$$
$$\frac{\partial\breve{\pi}_d(\gamma_d(t))}{\partial\gamma_d(t)}=\frac{1}{\theta_1}\sum_{t=t_2}^{T+t_2}\int_0^L\left(\left((t-1)(1+\gamma^*(t))^{t-2}P^0p(x,t)\right)(1-\theta_1c(x,t))\right)dx. \tag{35}$$

Proof. The first-order derivatives of benefit $\breve{\pi}_d(i_d(t))$ and penalty $\widehat{\pi}_d(\gamma_d(t))$ with respect to interest $i_d(t)$ and growth rate $\gamma_d(t)$ are derived as follows, respectively:

$$\begin{aligned}
\frac{\partial \breve{\pi}_d(i_d(t))}{\partial i_d(t)} &= \frac{\partial\left(C_l PWF'(i_d(t),t_2)L(t) + C_s PWF'(i_d(t),t_2)L(t)/D_s(t) + C_{om}PWF'(i_d(t),t_2)PWF(i_d(t),T)\right)}{365\cdot\partial i_d(t)} \\
&= \left(C_l L(t) + \frac{C_s L(t)}{D_s(t)}\right)\frac{-t_2}{365\cdot(1+i_d(t))^{t_2+1}} \\
&\quad + C_{om}\frac{Ti_d(t)(1+i_d(t))^T + \left((1+i_d(t))^T - 1\right)\left((1+i_d(t)+Ti_d(t)) - t_2 i_d(t)\right)}{(365)\,i_d^2(1+i_d(t))^{T+t_s+1}} \\
\frac{\partial \widehat{\pi}_d(\gamma_d(t))}{\partial \gamma_d(t)} &= \frac{1}{\theta_1}\sum_{t=t_2}^{T+t_2}\int_0^L\left(\left((t-1)(1+\gamma^*(t))^{t-2}P^0 p(x,t)\right)(1-\theta_1 c(x,t))\right)\mathrm{d}x.
\end{aligned} \tag{36}$$

By solving the equation of $(\partial\breve{\pi}_d(\gamma_d(t))/\partial\gamma_d(t))(\gamma^*(t) - \gamma_d(t)) = (\partial\widehat{\pi}_d(i_d(t))/\partial i_d(t))(i_d(t) - i^*(t))$, with given values of $i^*(t), \gamma^*(t), D_s(t)$, rail length $L^*(t)$, and growing rate $\gamma_d(t)$ or interest rate $i_d(t)$, the other threshold value of $\gamma_d(t)$ or $i_d(t)$ can be determined. With given values of $i^*(t), \gamma^*(t), D_s(t), \gamma_d(t)$, and $i_d(t)$, the year-by-year rail length $L_d(t)$ can also be updated for the alternative of deferring this project as Corollary 2. The interrelationship between interest rate and growing rate of the total population distribution is explored, shown as the following Proposition 4. □

Proposition 4. *Given rail length $L(t)$ and average station spacing $D_s(t)$, IRR $i^*(t)$ is a strictly increasing function of the break-even growth rate of the total population $\gamma^*(t)$ in the linear monocentric city.*

Proof. Let $\breve{\pi}(\gamma(t)) = \widehat{\pi}(i(t))$, in terms of (18) and (19); we have

$$\begin{aligned}
\breve{\pi}(\gamma(t)) &= \sum_{t=0}^{T}\int_0^L \frac{q(x,t)}{\theta_1}\mathrm{d}x \\
&= \frac{1}{\theta_1}\sum_{t=0}^{T}(1+\gamma(t))^t P^0\int_0^L \frac{\exp\left[-\theta_2\left(U(x,t)+\alpha_0\ln\left(U(x,t)/\int_0^L\sqrt{U(w,t)}\mathrm{d}w\right)\right)\right]}{\int_0^L\exp\left[-\theta_2\left(U(w,t)+\alpha_0\ln\left(U(x,t)/\int_0^L\sqrt{U(w,t)}\mathrm{d}w\right)\right)\right]\mathrm{d}w}(1-\theta_1 c(x,t))\,\mathrm{d}x \\
&= \frac{1}{\theta_1}\sum_{t=0}^{T}(1+\gamma(t))^t P^0\int_0^L \frac{\exp(-\theta_2 U(x,t))\left((2/L)\sqrt{U(x,t)}\right)^{-\theta_2\alpha_0}}{\int_0^L\exp(-\theta_2 U(w,t))\left((2/L)\sqrt{U(x,t)}\right)^{-\theta_2\alpha_0}\mathrm{d}w}(1-\theta_1 c(x,t))\,\mathrm{d}x \\
&= \frac{1}{\theta_1}\sum_{t=0}^{T}(1+\gamma(t))^t P^0\int_0^L \frac{\exp(-\theta_2 U(x,t))U(x,t)^{-\theta_2\alpha_0/2}}{(1/f_1)\int_{U(0,t)}^{U(L,t)}\exp(-\theta_2 U(w,t))U(w,t)^{-\theta_2\alpha_0/2}\mathrm{d}U(w,t)}(1-\theta_1 c(x,t))\,\mathrm{d}x \\
&= \frac{1}{\theta_1}\sum_{t=0}^{T}(1+\gamma(t))^t P^0\int_0^L \frac{\exp(-\theta_2 U(x,t))U(x,t)^{-\theta_2\alpha_0/2}}{(1/\theta_2 f_1)\left(\sum_{i=0}^{n}U(w,t)^{-\theta_2\alpha_0/2-i}\right)\exp(-\theta_2 U(w,t))\Big|_0^L}(1-\theta_1 c(x,t))\,\mathrm{d}x \\
&= \frac{\theta_2 f_1}{\theta_1}\sum_{t=0}^{T}(1+\gamma(t))^t P^0\int_0^L\sum_{i=0}^{n}U(w,t)^i(1-\theta_1 c(w,t))\,\mathrm{d}w
\end{aligned} \tag{37}$$

with

$$\frac{1}{f_1}\int_{U(0,t)}^{U(L,t)} \exp\left(-\theta_2 U(w,t)\right) U(w,t)^{-\theta_2\alpha_0/2}\, \mathrm{d}U(w,t) = \frac{1}{\theta_2 f_1}\left(\sum_{i=0}^{n} U(w,t)^{-\theta_2\alpha_0/2-i}\right)\exp\left(-\theta_2 U(w,t)\right)\Bigg|_0^L + \frac{1}{f_1}\int_{U(0,t)}^{U(L,t)} \exp\left(-\theta_2 U(w,t)\right) \cdot U(x,t)^{-\theta_2\alpha_0/2-n}\, \mathrm{d}U(x,t), \quad (38)$$

where n is a positive integer and big enough to confirm that

$$\frac{1}{f_1}\int_{U(0,t)}^{U(L,t)} \exp\left(-\theta_2 U(w,t)\right) U(x,t)^{-\theta_2\alpha_0/2-n}\, \mathrm{d}U(x,t) = 0 \quad (39)$$

holds. Then, we have

$$\sum_{t=0}^{T}(1+\gamma(t))^t \int_0^L \sum_{i=0}^{n} U(w,t)^i \left(1-\theta_1 c(w,t)\right) \mathrm{d}w = \frac{\theta_1}{\theta_2}\frac{C_l L(t) + C_s L(t)/D_s + C_{om} PWF(i(t),T)}{365 f_1 P^0}.$$

$$\frac{\mathrm{d}i(t)}{\mathrm{d}\gamma(t)} = \frac{\left(\sum_{t=0}^{T} t\left(1+\gamma(t)\right)^{t-1}\int_0^L \sum_{i=0}^{n} U(w,t)^i \left(1-\theta_1 c(w,t)\right)\mathrm{d}w\right)}{\left(\theta_1 C_{om}/\theta_2 f_1 P^0\right)\left(\left(Ti(t)(1+i(t))^T - \left[(1+i(t))^T - 1\right]\left[1+i(t)+Ti(t)\right]\right)/i^2(t)(1+i(t))^{T+1}\right)} > 0 \quad (40)$$

In other words, IRR $i^*(t)$ is a strictly increasing function of the break-even growth rate of the total population $\gamma^*(t)$. □

Proposition 4 shows that rail project is more worthy of investment with a high growth rate of total population.

4. Numerical Examples

To facilitate the presentation of the essential ideas and contributions of this study, two illustrative examples are employed. Example 1 presents the year-by-year design of a rail line with the proposed conditions of three alternatives, in terms of rail length. Sensitivity analyses are conducted on the spatial and temporal covariance of population distribution, to investigate its effect on the standard deviation of rail length. Example 2 presents the year-by-year conditions for fast-tracking or deferring a rail project in a given toy network, in terms of average break-even growth rate of the total population and average IRR.

4.1. Example 1. The rail configuration is shown in Figure 1. For simplicity without loss of generality, the average station spacing is 1.1km [30]. The mean and standard values of year-by-year break-even rail length are determined in this numerical example.

The stochastic residential/household distribution function is defined as

$$\varphi(P(x,t)) = \frac{P(x,t)}{100}. \quad (41)$$

Other input notation parameters are summarized in Table 4.

Two scenarios with different spatial and temporal covariance of population distribution are investigated to explore the effects of spatial and temporal covariance on the values of rail length. In this example, it is assumed that one has the following:

Scenario a:

$$\sigma_p(x_1,t_1;x_1,t_2) = 0.2,$$
$$\sigma_p(x_1,t_1;x_2,t_1) = 0.3, \quad (42)$$
$$\sigma_p(x_1,t_1;x_2,t_2) = 0.1$$

Scenario b:

$$\sigma_p(x_1,t_1;x_1,t_2) = 0.3,$$
$$\sigma_p(x_1,t_1;x_2,t_1) = 0.4, \quad (43)$$
$$\sigma_p(x_1,t_1;x_2,t_2) = 0.1$$

In each scenario, two alternatives are considered, shown as follows.

Alternative 5. Fast-tracking rail transit line project two years: the average IRR i^* is set as 10%, average break-even growing rate γ^* is 0.2, average total population growth rate for fast-tracking this rail transit line project γ_f by two year is 0.3, the average IRR for fast-tracking the rail project two years is 2%, and planning and operation horizon T are 3.

Alternative 6. Deferring rail transit line project three years: the average IRR i^* is set as 10%, average break-even growth rate γ^* is 0.2, average IRR for deferring this rail transit line project three years i_d is 31%, average growth rate of the total population for deferring this rail project three years γ_d is 0.1, and planning and operation horizon T are 3.

Figure 2 summarizes the results of year-by-year rail length under alternatives of fast-tracking it two years and deferring it three years. Both mean values and standard deviation

Table 4: Notations.

Symbol	Definition	Value
T	Planning and operation horizon (Years)	–
$i(t)$	Interest rate	–
$\gamma(t)$	Growing factor of total population density along the transportation corridor	–
x	Distance between residential location and CBD	–
$D_s(t)$	Average station spacing (HK\$)	10
f_0	Fixed component of fare for using the rail service (HK\$)	2.5
f_r	Variable component of fare per unit distance for using the rail service (HK\$/km)	0.4
$h(q(x,t))$	Headway of train operation during morning peak-hour in year t (Minutes)	–
$L(t)$	Rail length in year t (Km)	20
ε	Random term of perturbed population distribution function	–
ε_1	Random term of perturbed travel demand function	–
$\varepsilon_2(x,t)$	Random term of perturbed disutility function	–
θ_1	Sensitivity parameter in travel demand function	0.002
θ_2	Sensitivity parameter in probability function of choosing to live at residential locations	0.03
α	Parameter of waiting cost	0.5
α_1	Parameter of rent function	8
α_2	Commonality factor	0.5
β_1	Parameter of rent function	2
ρ	Average annual number of trips to the CBD per household	365
η	Average daily number of trips to the CBD per household	1.0
$\mu_c/\mu_w/\mu_i$	Parameters for travel cost function	80/100/50
C_l	construction cost for rail line per kilometer (million HK\$)	8.76
C_s	construction cost of each rail station (million HK\$)	13.14
C_{om}	Annual operation and maintenance cost of the rail transit line project (million HK\$)	137.97

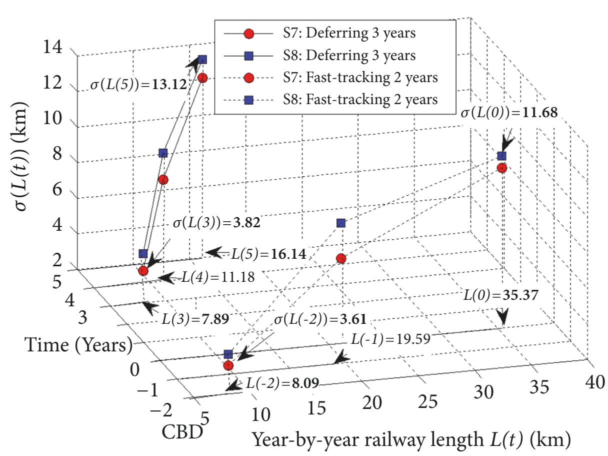

Figure 2: Year-by-year rail length with respect to alternatives of fast-tracking this rail transit line project two years and deferring it three years under Scenarios a and b.

values of rail length can be determined year by year with the proposed model.

It can be found that the standard rail length deviation values $\sigma(L(t))$ increase, as the spatial and temporal covariance of population distribution increases from Scenario a to Scenario b. This implies that while correlationship between the year-by-year population densities distributed along the candidate rail transit line strengthens, the standard deviation values of rail length increases.

The mean value of rail length under the alternative of fast-tracking two years is larger than that under the alternative of deferring three years. However, the corresponding standard deviation of rail length $\sigma(L(t))$ under the alternative of fast-tracking two years is smaller than that under alternative of

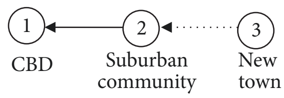

Figure 3: The proposed toy network.

deferring three years. For instance, $L(-2) = 8.09$ km is larger than $L(3)$=7.89 km, while $\sigma(L(-2)) = 3.61$ km is smaller than $\sigma(L(3)) = 3.82$ km. $L(0) = 35.37$ km is larger than $L(5)$=16.14 km, while $\sigma(L(0)) = 11.68$ km is smaller than $\sigma(L(5)) = 13.12$ km.

4.2. Example 2. A toy network with three locations, Central Business District (CBD), suburban community, and new town, is used to demonstrate the application of the proposed propositions, as shown in Figure 3. In the base year, a rail exists between locations CBD and suburban community, and a candidate rail extension project is to be implemented between locations suburban community and new town. The distances between locations CBD and suburban community and CBD and new town are 6km and 20km, respectively.

The mean value of population distribution at each location in the base year and the travel demand by rail from suburban community (denoted as location 2) and new town (denoted as location 3) to CBD (denoted as location 1) are set as

$$P(1,0) = 5000 \ \ (\text{persons}),$$
$$P(2,0) = 15000 \ \ (\text{persons}), \tag{44}$$
$$P(3,0) = 15000 \ \ (\text{persons}).$$

Other input notation parameters are same with example 1.

Six scenarios, presented below, are used to examine the conditions for fast-tracking or deferring the candidate rail transit line project shown in the Figure 3.

Scenario 1. The year-by-year average break-even growth rate $\gamma^*(t)$ with respect to different average interest rates $i(t)$ are given in Table 5. It can be seen that the year-by-year average break-even population growth rate $\gamma^*(t)$ increases, while the given average interest rate $i(t)$ increases. This result is in accordance with Proposition 4.

Scenario 2. The year-by-year average IRR $i^*(t)$, with respect to different population growth rates $\gamma(t)$, are given in Table 6. For instance, with an average population growth rate γ of 0.3, this rail transit line project is not worthy of investment in year 1, during which period the average interest rate is larger than 2.6%. As time goes by, investment in this rail transit line project in years 2 and 3 is worthwhile as the respective average interest rate is smaller than 7.9% and 13.8%.

Scenario 3. The year-by-year conditions for fast-tracking the rail project, in terms of break-even growth rate of the total population $\gamma_f(t)$, are given in Table 7. The average IRR $i^*(t)$ is set as 10%, the average break-even growth rate γ^* is 0.3, the average interest rate for fast-tracking this rail project i_f is 5%, and planning and operation horizon T are 3.

Table 5: Year-by-year average break-even growing factor of the total population with respect to different average interest rates.

Average interest rate i	Average break-even growth factor γ^* in each year		
	Year = 1	Year = 2	Year= 3
3%	**0.12**	**0.17**	0.26
6%	**0.14**	**0.27**	0.30
9%	**0.25**	**0.29**	0.41

Table 6: Average IRR of the rail transit line project with respect to different growth factor of the total population over years.

Average growth factor γ	Average IRR i^* in each year (%)		
	Year = 1	Year = 2	Year= 3
0.2	2.4%	7.3%	8.5%
0.3	**2.6%**	**7.9%**	**13.8%**
0.4	2.8%	12.8%	14.9%

Table 7: Conditions for fast-tracking this rail transit line project in Scenario 3.

Fast-tracking time (Year)	Conditions $\gamma_f(t)$ in each year t during planning and operation horizon T		
	$t = 1$	$t = 2$	$t = 3$
1	**0.2684**	**0.2878**	**0.2938**
2	0.3327	0.3168	0.3097
3	0.3821	0.3421	0.3243

Table 8: Conditions for fast-tracking the rail transit line project in Scenario 4.

Fast-tracking time (Year)	t Conditions $i_f(t)$ in each year during planning and operation horizon T		
	$t = 1$	$t = 2$	$t = 3$
1	16.12%	9.98%	**2.10%**
2	3.48%	**-10.34%**	**-20.76%**

Table 7 shows the results of these conditions. It can be seen that the year-by-year average population growth rate $\gamma_f(t)$ for fast-tracking this rail project 1 year is below 0.3 and increases year by year from 0.2684 to 0.2938 during the planning and operation horizon T. The break-even population growth rates $\gamma_f(t)$ for fast-tracking this rail project 2 or 3 years are above 0.3 and decreases year by year during the planning and operation horizon T.

Scenario 4. The year-by-year conditions for fast-tracking the rail transit line project, in terms of average IRR $i_f(t)$, are given in Table 8. The average IRR $i^*(t)$ is set as 10%, the average break-even growth rate $\gamma^*(t)$ is 0.2, the average population growth rate for fast-tracking this rail project γ_f is 0.3, and the planning and operation horizon T are 3.

Table 9: Conditions for deferring this railway project in Scenario 5.

Deferring time (Year)	Conditions $\gamma_d(t)$ in each year t during planning and operation horizon T		
	$t = 1$	$t = 2$	$t = 3$
1	**-0.2144**	-0.03023	0.05611
2	-0.0519	0.0601	0.1125
3	0.1121	0.1512	0.1695

Table 10: Conditions for deferring this railway project in Scenario 6.

Deferring time (Year)	Conditions $i_d(t)$ in each year t during planning and operation horizon T		
	$t = 1$	$t = 2$	$t = 3$
1	**68.74**%	72.98%	78.40%
2	**37.86**%	40.43%	43.64%
3	**26.22**%	28.22%	30.62%

It can be seen that year-by-year average IRR(s) for fast-tracking the rail transit line project 1 year in this scenario are all positive and decrease year by year from 16.12% to 2.10% during planning and operation horizon T. For instance, this rail transit line project is not worth fast-tracking 1 year, if the interest rate in year 3 is greater than 2.10%. Table 8 shows that fast-tracking this rail transit line project 2 years is not worthwhile, because the average IRR(s) in years 2 and 3 are negative.

Scenario 5. The year-by-year conditions for deferring the rail transit line project, in terms of average population growth rate $\gamma_d(t)$, are given in Table 9. The average IRR i^* is set as a 10%, average break-even growth rate γ^* is 0.3, average interest rate suffered for deferring this rail transit line project i_d is 20%, and planning and operation horizon T are 3.

Table 9 presents the results of Scenario 5. It shows that the break-even growth rate for deferring this rail project $\gamma_d(t)$ increases with the increase of deferring time. For instance, in year $t = 1$, $\gamma_d(t)$ increases from -0.2144 to 0.1121 as deferring time increases from 1 to 3.

Scenario 6. The year-by-year conditions for deferring this rail project, in terms of average IRR i_d, are given based on Proposition 4 shown in Table 10. The average IRR i^* is set as 10%, the average break-even growth rate γ^* is 0.2, the average population growth rate for deferring this rail project γ_d is 0.3, and planning and operation horizon T are 3.

Table 10 shows that the year-by-year average IRR(s) for fast-tracking this rail transit line project $i_d(t)$ are all larger than the average IRR $i^*(t)$ 10% and decrease year by year during the planning and operation horizon T. For instance, IRR(s) decreases from 68.74% to 26.22% in the first year of the planning and operation horizon.

5. Conclusions and Future Work

This paper proposes models for multiperiod flexible rail line design in a linear monocentric city. The proposed models consider the effects of year-by-year variation of the total population and spatial-temporal correlation of population distribution explicitly. In contrast with the traditional single-period NDP models for rail transit line, the proposed models have the following merits: the candidate rail project can be fast-tracked or deferred; conditions for fast-tracking or deferring the implemented rail project are analytically investigated.

The proposed models offer several insights. For example, the mean values of rail design variables are closely concerned with interest rates and the total population growth rate. The spatial and temporal covariance of population distribution only affects the standard deviation values of the rail length. The rail transit line project can be fast-tracked or deferred when actual interest rates or actual population growth rate match break-even IRR and break-even growth rate.

This paper provides a new avenue for the modeling and analysis of flexible rail transit line design in a linear monocentric city. Further research is needed in the following directions:

(1) The monocentric city is assumed in this paper, namely, only with one CBD and several other residential locations. The city boundary is not explicitly considered. Therefore, it is necessary to elaborate the city boundary so as to extend the polycentric CBD model in a further study.

(2) In this paper, population are assumed to be homogenous with trips commuting only from residences to CBD. However, previous studies have shown that income levels dominate residential location choices (Hartwick et al., 1976; Kwon, 2003). Therefore, the proposed model could be extended to incorporate household income levels over the years for determining residential location choices and population distribution.

(3) In this paper, the investigation of investment risk is based on interest rates and temporal and spatial population covariance and variations of population distribution year by year. However, there are many other investment risk sources, for instance, client investment risk related to private operators for allocated government projects. A detailed model should be developed to take into account projects with such contents, since the performance of related projects are influenced considerably.

(4) Only rail mode is considered in this paper. This assumption can be extended to a multimodal situation in further studies. With more travel modes being considered, travelers' travel mode choice behaviour can be incorporated into the extended models (Chowdhury and Chien, 2002; Li et al., 2006) [37].

(5) The decision to extend a rail line involves consideration of technological, social, and economic factors. The prime reason could be social or in other words a desire to make life more convenient as regards manoeuvrability for a specific set of people, namely, those living in the vicinity of the line and new stations to be constructed. However, only pressing economic factor is considered in this paper. More detailed social factors can be taken into account in further studies, for instance, appreciation of land value along the rail line.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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