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Research Article

The Two-Stage Model of Entrepreneurs Financing Based on the Entry/Exit Decision

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Normally entrepreneur would raise fund from angel investors during the initial round. If the venture program was by then successful, the entrepreneur would then continue the fund-raising process from venture capitalist. By adopting the convertible preferred stock, we managed to construct the two-stage angel investment decision process. This research reveals the following: (1) The probability of the first stage's success has negative relationships with levels of priority dividend in both first and second stages, as well as with the venture capitalist's proportion of shares. (2) The probability of the second stage's success has negative relationships with the venture capitalist's proportion of shares and the dividend level of both first and second stage funding. (3) There has been a threshold of dividend distribution, which belongs to angel investor. While the level of angel investor's shares is higher than the threshold, AN would decide to join the second phase of the program; otherwise, AN would exit the project at the end of the first stage.

1. Introduction

Venture entrepreneur (EN), due to a lack of internal financial sources at early stages, depends crucially on outside financing and two potential sources include (1) angel investor (AN) and (2) venture capitalist (VC). Among the two sources of external financing, the AN is a perfect match for start-ups. This is because the VCs are simply not available for ENs at early stages. Li et al. [1] claimed that AN investment, as a relatively mature informal supporting scheme for start-ups, was the dominant capital source for the ENs at the seed-stage and the start-stage and effectively improved the development of the start-ups. Currently, the ANs market is gathering momentum in China. For instance, compared to 2014, Chinese angel institutions absorbed 124 new funds with disclosed amount of 20.357 billion RMB and invested in 2,075 cases with the disclosed amount of more than 10.188 billion RMB, showing a remarkable speed of development of Chinese angel investment market (Researching Center of Zero2IPO Group, 2016). These findings further illustrate that angel investment is a substantial source of financing for the start-ups.

Recent literature on angel investments is based on the following two aspects:

Firstly, the characteristics of AN. Shi [2] analyzed the characteristics, motivations, transaction structures, investing process, and postinvestment management of angel investment in China and proposed anticipation of evolution of AN. Wang et al. [3] analyzed the Chinese angel investment market in terms of development trend and countermeasures, based on first-hand research data and secondary sources. They found the following: (1) Chinese angel investment is still developing rapidly. (2) Problems still exist in investor recognition, standardization, and codification. Rodriguez [4] pointed out that, because the advantages and attractiveness of the projects cannot be easily revealed at the development phase and start-up phase, some ENs start from getting private loans from their friends or relatives, which is the original form of angel investments. Based on empirical results on 121 AN in 1038 new-type ventures, Wiltbank et al. [5] found the impacts of AN's predictive control strategies on the results of ventures. Through empirical studies, Hoberg et al. [6] concluded that EN prefers VC rather than the combination of AN and VC and will obtain better return in this way.

Secondly, moral hazard of EN. Kerr et al. [7] stated the fact that ventures funded by two successful angel groups experience superior outcomes to rejected ventures: they have higher survival rate, exits, employment, patenting, Web traffic, and financing. Kerr et al. [7] confirmed the positive effects for venture operations, with qualitative support for a higher likelihood of successful exits. Chemmanur and Chen [8] develop a theoretical analysis of an entrepreneur's choice between venture capital and angel financing at various stages in a private firm's life and characterize the dynamic evolution of the firm's contract with its financier (VC or AN). Johnson and Sohl [9] recommended that angel investors and ENs carry out different stages of phased investment when faced with ENs with completely external financing plans. Elitzur and Gavious [10] studied the relationship among ENs, AN, and VC by analyzing the connections and the relevant contract designs. They also took moral hazard into consideration and argued that an EN without a powerful helmsman tends to have an inefficient operation. Guo and Wang [11] introduced a signaling model to analyze the role of AN in screening of high-quality projects.

The two aspects above mainly focus on characteristics of angel financing and provisions of contract before investment realization. However, the process from entry to exit should be complete, and thus we should not ignore the exit of angel capital. The approaches that the capital financed from AN exits are related to financial instrument agreements when the angel capital enters. Convertible preferred stock is the preferred stock that allows owner to convert it to another type of stock under certain conditions. Yao [12] studied efficient exit of VC through convertible preferred stocks. He showed that the time of preferred stock conversion influences VC's exit decision. When incomes obtained from corporation's growth are greater than a specified interest, VC has an incentive to convert preferred shares into ordinary shares in order to gain benefits from the firm's growth. Generally speaking, VC's preferred stock is automatically converted into common shares when the company goes public. Therefore, the flexibility of convertible preferred stocks allows VC to exit more efficiently. Yao [12] also showed that the allocation of control power between the two parties is a problem that needs to be solved in the process of investment. Through conversion of preferred stock, EN and VC reallocate income and control so that the VC can have an optimal exit and the social benefit can be maximized. Zhang and Yang [13] studied the optimal exit decision with partaking convertible preferred stocks under certain conditions; VC can always achieve the optimal exit by using partaking convertible preferred stocks. Moreover, financier's gains from distribution of EN's surplus value, reputation, and corporate private interests also impact the control power of exit of VC. Cumming and Johan [14] studied the relation between venture capital contracts and exit. Flix and Gulamhussen [15] analyzed the exit decision in the European venture capital market and studied when to exit and how to interact with the exit form. The study of Medin [16] suggested that PE-backed firms demonstrate superior operating performance after exit, which to a large extent is driven by superior performance of VC-backed firms. BO-backed firms, however, did not demonstrate superior postexit operating performance. Zheng, et al. [17] thought that the strategies of VC and EN in the venture capital market may form an exit dilemma; therefore, the analysis of their strategy selection and entanglement using quantum game is given.

These studies mainly paid attention to the exit of venture capital but rarely considered the impact of convertible preferred stock owned by angel. Carpentier and Suret [18] analyzed exit-related perceptions of the members of a large, well-structured Canadian angel group; these angels should consider the initial public offering (IPO) as a possible exit mode. Securities regulation was also perceived as a major impediment to exit onto the stock market. Mahapatra [19] discussed the exit phase of the individual angels, angel syndicates, and corporate angels. His work tried to overcome this gap and examined only the exit phase of the angel syndicates and the corporate investors. The exit phase of the individual investors was reexamined in the light of their reported professionalism and evolution. Hellmann and Thiele [20] developed a theory of how angel and venture capital markets interact.

In current literature, most scholars mainly studied optimal exit of VC. As an approach of venture financing, besides "love fund" and VC, AN is relatively rarely discussed with VC and the incentive mechanism, and exit mechanism of AN is rarely explained from multistage dynamic angles. AN differs from the traditional sense of VC in that the former will choose the optimal time to exit. When start-up is mature or bankrupt, and AN will find ways to sell equity for funds; then the exit phase is particularly important for AN. These are the differences between this article and other studies.

This paper, in the aspect of AN, considers convertible preferred stock as an incentive mechanism of venture capital investment and analyzes the mechanism of angel investment from the multistage dynamic investment process.

The innovation is as follows: the operation of the project is divided into two stages: EN finances from AN in the first stage, EN finances from AN and VC in the second stage; we study how to design the contract and when to quit in the cases, with and without AN. The exit decision is given by AN numerical calculation.

2. Model Assumption and Establishment

EN has a project while its own capital is zero. In such case, EN needs external financing. Considering the uncertainty and riskiness of innovative project, AN finances the project with the following method. At the first stage of project, AN provides capital $I_1(I_1 > 0)$. After the end of first phase of the project, if the project performs badly and fails, then AN liquidates and exists. On the other hand, if the project has a good operation condition, AN transmits signal to the outside world and begins the second round of financing. At this time, VC provides capital $I_2(I_2 > 0)$. Due to the asymmetry of information, EN as an agent of this project will try to maximize his own benefit. In order to enhance the probability of project's success, AN not only needs to invest capital $I_1(I_2)$ but also would provide value-added services (like participation in decision making and management of strategy, providing consulting services, assisting in public relation, doing product promotion, and increasing market share). Considering these factors, the profit of the project not only relates to EN's effort $e_{11}(e_{11} > 0)$ but also relates to AN's effort $e_{21}(e_{21} > 0)$. That is the same case in the second phase of the project. EN and AN choose effort, respectively, to maximize their own benefit. The same for VC, he would provide the project with effort $e_{32}(e_{32} > 0)$.

The success probability of the first phase is related to the effort of AN and EN in the first stage. Thus we assume success probability of the first phase is

$$p_1 = r_1 e_{11} + r_2 e_{21}, \quad \left(0 \le r_1 e_{11} + r_2 e_{21} \le 1\right). \tag{1}$$

The success probability of the second phase is related to AN and EN's second stage effort; at the same time the effort of first stage will also continue affecting the success probability of the second phase. Thus we assume the success probability of second phase is

$$\begin{aligned} p_2 &= \theta_1 e_{12} + \theta_2 e_{22} + \theta_3 e_{32} + \rho\left(e_{11} + e_{21}\right), \\ &\quad \left(0 \le \theta_1 e_{12} + \theta_2 e_{22} + \theta_3 e_{32} + \rho\left(e_{11} + e_{21}\right) \le 1\right). \end{aligned} \tag{2}$$

$\rho(\rho < \theta_1, \theta_2, \theta_3)$ is the coefficient representing the effort of the first stage, which affects the second stage's probability. The magnitude of ρ is lower than θ_1, θ_2, and θ_3, because the first stage effort has little influence on the probability of the second stage.

This article assumes the success probability of project is a linear function of participant i's effort. This means that the level of each participant's effort is substitute rather than complementary. The reasons for this assumption are as follows.

Firstly, Ma [21] points out that there is symmetry information between EN and VC. EN has lots of uncertainties, for example, the uncertainty of research and technology, the uncertainty of management ability, and the uncertainty of employees. It is hard to separate their contribution of effort. In such case, it is meaningful; VC as a principle motivates EN to work harder.

Secondly, Casamatta [22] states EN and VC face a double moral hazard. The consulting services they provide to project are indivisible. In order to encourage each other to work hard, EN and VC must have the right to share the benefit from project. In addition, in this article, the efforts of both sides are unobservable. EN provides the project with technical and innovative services while VC provides the project with management service.

Supposing the cost of effort can be measured by money and cost function satisfies $c'(\cdot) > 0$, $c''(\cdot) > 0$, we assume the cost of various stages with EN's effort is $c(e_{11}) = (1/2)e_{11}^2$, $c(e_{12}) = (1/2)e_{12}^2$.

And the cost of various stages with AN's effort is $c(e_{21}) = (1/2)e_{21}^2$, $c(e_{22}) = (1/2)e_{22}^2$,

The cost of VC's effort is $c(e_{32}) = (1/2)e_{32}^2$.

The income of project is a random variable R. The input of EN decreases marginally with the increase of input. Bascha and Walz [23] assume the density function of random income R is $g(R) = \lambda e^{-\lambda R}$.

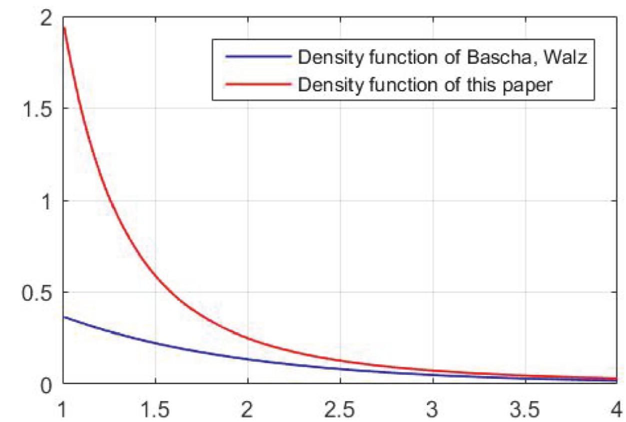

Figure 1: Comparison of two density function.

In order to simplify the computation, this paper constructs a new density function $g(R) = 2a_2^2/R^3$ (Figure 1 shows the image of the two density functions).

Newly constructed density function has consistent characteristic of density function of Bascha and Walz [23], so the density this paper uses is feasible.

The variables and symbols are summarized as Table 1.

3. Two-Stage Game Model of AN Continued Investment

In this section, we consider the second stage investment which AN can do. The game order is as follows:

Phase 0: AN analyzes the quality of project and chooses the high-quality project to invest.

Phase 1: As the equity holder, AN will design the contract to get the best return. AN will not only supply funds but also assist in the management which the firm needs.

Phase 2: If the project succeeds, in the second round of investment, VC is introduced to control project. As a result, VC becomes the equity holder of the project. If AN does not exit, EN and AN will become the agents. As the principal, VC will design the contract to get the best return and will supply funds as well as assisting in management.

Then, we design the contract according to backward induction.

3.1. The Second Stage Game Model of Contract. In the second stage, VC invests in the project. EN/VC provides a convertible preferred stock contract; see Figure 2.

Explanation of Figure 2. In the first stage, if the project is successful and AN can execute the convertible preferred stock, then y is a share of AN and $1 - y$ is a share of EN. If AN does not execute the convertible preferred stock, then D_1 is bond yield, γ is a share of AN from the residue $p_2 R - D_1$ of the income, and $1 - \gamma$ is a share of EN from the residue $p_2 R - D_2$ of the income. In the second stage, if the project is successful, if VC can execute the convertible preferred stock, then φ_1 is a share of VC, φ_2 is a share of AN, and the share

Table 1: The variables and symbols.

Symbol	Specification
e_{ij}	The effort of i at the j -stage, $i = EN, AN, VC$, j=1,2
$c(e_{ij}) = (1/2)e_{ij}^2$	The effort cost of i at the stage j, $i = EN, AN, VC$, j=1,2
Π_{EN1}, Π_{AN1}	The expected income of EN and AN in the first stage
$\Pi_{EN2}, \Pi_{AN2}, \Pi_{VC2}$	The expected income of EN, AN, and VC at the second stage
I_1	The financing amount of AN at the first stage
I_2	The financing amount of VC at the second stage
δ	Discount factor of income of project
a_1	The project residual at the first stage
a_2	The project residual at the second stage with the participation of AN
$\tilde{a}_2$	The residual of project at the second stage without the participation of AN
φ_1	In the second stage, VC transfers the shares to the EN, when AN can participate
$\tilde{\varphi}_1$	In the second stage, VC transfers the shares to the EN, when AN does not participate
φ_2	In the second stage, VC transfers the shares to the AN, when AN can participate
$\tilde{\varphi}_2$	In the second stage, VC transfers the shares to the AN, when AN does not participate
γ	In the second stage, a share of AN from the rest $p_2R - D_2$ of the income, when AN can participate
$\tilde{\gamma}$	In the second stage, a share of AN from the rest $p_2R - D_2$ of the income, when AN does not participate
y	In the first stage, AN transfers the shares to the EN
D_2	In the second stage, VC's bond yields, when AN can participate
$\widetilde{D}_2$	In the second stage, VC's bond yields, when AN does not participate
D_1	In the first stage, AN's bond yields, when AN can participate
$\widetilde{D}_1$	In the first stage, AN's bond yields, when AN does not participate
k_1	Risk free interest rate of capital I_1
k_2	Risk free interest rate of capital I_2

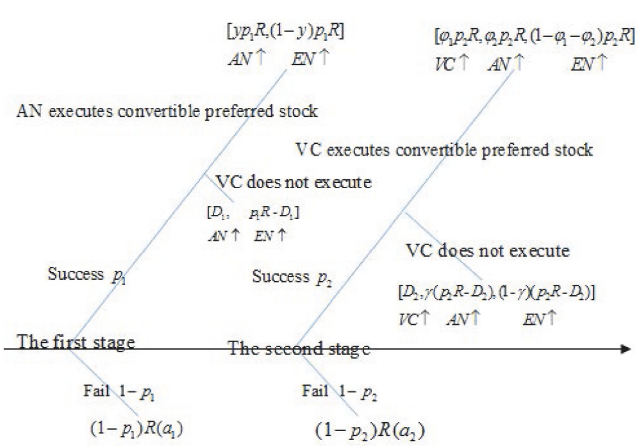

Figure 2: AN participates, the two-stage contract design.

of EN is $1 - \varphi_1 - \varphi_2$, if VC does not execute the convertible preferred stock, then D_2 is bond yield.

According to the analysis above, the optimal financing problem of VC is

$$\max_{D_2,\varphi_1,\varphi_2,\gamma} \Pi_{VC2} = \max_{D_2,\varphi_1,\varphi_2,\varphi_3,\gamma} \int_{a_2}^{D_2} (1-p_2) R \frac{2a_2^2}{R^3} dR + \int_{D_2}^{D_2/\varphi_1} p_2 D_2 \frac{2a_2^2}{R^3} dR + \int_{D_2/\varphi_1}^{+\infty} \varphi_1 p_2 R \frac{2a_2^2}{R^3} dR - \frac{1}{2} e_{32}^2 - I_2. \quad (3)$$

$$st: \quad e_{12}^* \in \arg\max_{e_{12}} \Pi_{EN2} = \max_{e_{12}} \int_{D_2}^{D_2/\varphi_1} (1-\gamma) p_2 (R - D_2) \frac{2a_2^2}{R^3} dR + \int_{D_2/\varphi_1}^{+\infty} (1-\varphi_1-\varphi_2) p_2 R \frac{2a_2^2}{R^3} dR - \frac{1}{2} e_{12}^2 \quad (4)$$

$$e_{22}^* \in \arg\max_{e_{22}} \Pi_{AN2} = \max_{e_{22}} \int_{D_2}^{D_2/\varphi_1} p_2 (R - D_2) \frac{2a_2^2}{R^3} dR + \int_{D_2/\varphi_1}^{+\infty} \varphi_2 p_2 R \frac{2a_2^2}{R^3} dR - \frac{1}{2} e_{22}^2 \quad (5)$$

$$e_{32}^{*} \in \arg\max_{e_{32}} \Pi_{VC2} = \max_{e_{32}} \int_{a_2}^{D_2} (1-p_2) R \frac{2a_2^2}{R^3} dR + \int_{D_2}^{D_2/\varphi_1} p_2 D_2 \frac{2a_2^2}{R^3} dR + \int_{D_2/\varphi_1}^{+\infty} \varphi_1 p_2 R \frac{2a_2^2}{R^3} dR - \frac{1}{2} e_{32}^2 - I_2. \quad (6)$$

Equation (3) is VC's objective function. VC gets the best return by determining the $D_2, \varphi_1, \varphi_2, \gamma$ in the contract. $\int_{D_2}^{D_2/\varphi_1} D_2 (2a_2^2/R^3) dR$ is the profit of VC when the project fails and the possibility is $1 - p_2$. $\int_{D_2}^{D_2/\varphi_1} D_2 (2a_2^2/R^3) dR$, $\int_{D_2/\varphi_1}^{+\infty} \varphi_1 R (2a_2^2/R^3) dR$ are the profit of VC in two conditions: does not convert or convert. And the possibility is p_2.

Equation (4) means the incentive compatible constraint of EN. EN gets the best profit by determining the effort level. When the project fails, the profit of EN is 0, and $\int_{D_2}^{D_2/\varphi_1} (1-\gamma)(R-D_2)(2a_2^2/R^3) dR$ and $\int_{D_2/\varphi_1}^{+\infty} (1-\varphi_1-\varphi_2) p_2 R (2a_2^2/R^3) dR$ are the profit of EN in the two different conditions: the preferred stock does not convert or convert. The possibility is also p_2.

Equation (5) means the incentive compatible constraint of AN. AN gets the best profit by determining the effort level. When the project fails, the profit of AN is 0, and $\int_{D_2}^{D_2/\varphi_1} p_2 (R - D_2)(2a_2^2/R^3) dR$ and $\int_{D_2/\varphi_1}^{+\infty} \varphi_2 p_2 R (2a_2^2/R^3) dR$ are the profit of EN in the two different conditions: the preferred stock does not convert or convert. The possibility is still p_2.

Equation (6) means the incentive compatible constraint of VC. VC gets the best profit by determining the effort level. Solving the first-order of (4), (5), and (6), the optimal effort of EN, AN, and CV are as follows:

$$\begin{aligned} e_{12}^{*} &= (1-\gamma)\theta_1 \left(\frac{a_2^2}{D_2} - \frac{2a_2^2\varphi_1}{D_2} + \frac{2a_2^2\varphi_1^2}{D_2} \right) + (1-\varphi_1-\varphi_2)\theta_1 \frac{2a_2^2\varphi_1}{D_2}. \\ e_{22}^{*} &= \gamma\theta_2 \left(\frac{a_2^2}{D_2} - \frac{2a_2^2\varphi_1}{D_2} + \frac{2a_2^2\varphi_1^2}{D_2} \right) + \varphi_2\theta_2 \frac{2a_2^2\varphi_1}{D_2}. \\ e_{32}^{*} &= \theta_3 \left(-2a_2 - \frac{3a_2^2}{D_2} + \frac{2a_2^2\varphi_1^2}{D_2} \right). \end{aligned} \quad (7)$$

3.2. The First Stage Game Model of Contract. In the first stage of the project, AN finances the project; according to the assumption in Section 1, the success probability of the first stage is given as $p_2 = r_1 e_{11} + r_2 e_{21}$, and the income of the project is stochastic, whose density function is given as $g(R) = 2a_1^2/R^3$.

AN and EN make a contract of the convertible preferred stock and the mechanism of the contract; see Figure 2.

AN and EN need to make sure that their efforts in the first stage maximize their total income in the first and second stage; optimal financing problem is as follows:

$$\max_{D_1,\varphi} \Pi_{AN} = \Pi_{AN1}(e_{11}, e_{21}) + \delta \Pi_{AN2}^{*}(e_{11}, e_{21}, e_{12}^{*}, e_{21}^{*}, e_{32}^{*}) \quad (8)$$

$$\text{S.T.} \quad \max_{e_{11}} \Pi_{EN} = \Pi_{EN1}(e_{11}, e_{21}) + \delta \Pi_{EN2}^{*}(e_{11}, e_{21}, e_{12}^{*}, e_{21}^{*}, e_{32}^{*}) \quad (9)$$

$$\max_{e_{21}} \Pi_{AN} = \Pi_{AN1}(e_{11}, e_{21}) + \delta \Pi_{AN2}^{*}(e_{11}, e_{21}, e_{12}^{*}, e_{21}^{*}, e_{32}^{*}), \quad (10)$$

of which

$$\Pi_{EN1} = \int_{D_1}^{D_1/\varphi} (r_1 e_{11} + r_2 e_{21})(R - D) \frac{2a_1^2}{R^3} dR + (1-\varphi) \int_{D_1/\varphi}^{+\infty} (r_1 e_{11} + r_2 e_{21})(R - D) \frac{2a_1^2}{R^3} dR - \frac{1}{2} e_{11}^2. \quad (11)$$

$$\begin{aligned} \Pi_{EN2}^{*} &= \int_{D_2}^{D_2/\varphi_1} (1-\gamma) \cdot \left[\theta_1 e_{12}^{*} + \theta_2 e_{22}^{*} + \theta_3 e_{32}^{*} + \rho(e_{11} + e_{21}) \right] (R - D_2) \cdot \frac{2a_2^2}{R^3} dR + (1-\varphi_1-\varphi_2) \int_{D_2/\varphi_1}^{+\infty} (1-\varphi_1-\varphi_2) \\ &\quad \cdot \left[\theta_1 e_{12}^{*} + \theta_2 e_{22}^{*} + \theta_3 e_{32}^{*} + \rho(e_{11} + e_{21}) \right] R \frac{2a_2^2}{R^3} dR - \frac{1}{2} e_{12}^2. \end{aligned} \quad (12)$$

$$\Pi_{AN1}^{*} = \int_{a_1}^{D_1} (1 - r_1 e_{11} - r_2 e_{21}) R \frac{2a_1^2}{R^3} dR \int_{D_1}^{D_1/\varphi} D_1 \cdot \frac{2a_2^2}{R^3} dR + \varphi \int_{D_1/\varphi_1}^{+\infty} (r_1 e_{11} + r_2 e_{21}) R \frac{2a_1^2}{R^3} dR - \frac{1}{2} e_{21}^2 - I_1. \quad (13)$$

$$\Pi_{AN2}^{*} = \int_{D_2}^{D_2/\varphi_1} \gamma \left[\theta_1 e_{12}^{*} + \theta_2 e_{22}^{*} + \theta_3 e_{32}^{*} + \rho(e_{11} + e_{21}) \right] \cdot (R - D_2) \frac{2a_2^2}{R^3} dR$$

$$+\varphi_2\int_{D_2/\varphi_1}^{+\infty}\left[\theta_1 e_{12}^*+\theta_2 e_{22}^*+\theta_3 e_{32}^*+\rho\left(e_{11}+e_{21}\right)\right] R \cdot \frac{2a_2^2}{R^3}dR-\frac{1}{2}e_{22}^2. \quad (14)$$

Formula (9) is AN's object function; AN decides D_1, φ in the contract to maximize his income.

Formula (10) is EN's incentive compatibility constrain; EN decides the effort to maximize his income.

Formula (15) is AN's incentive compatibility constrains; AN decides the effort to maximize his income.

According to the above-mentioned model, we solve the optimal effort of AN and EN in the first stage:

$$\begin{aligned} e_{11}^* &= r_1\left(\frac{a_1^2}{D_1}-\frac{a_1^2\varphi^2}{D_1}\right) \\ &+\delta\rho\left[\left(1-\gamma\right)\left(\frac{a_2^2}{D_2}-\frac{2a_2^2\varphi^1}{D_2}+\frac{2a_2^2\varphi^1}{D_2}\right)\right. \\ &\left.+\left(1-\varphi_1-\varphi_2\right)\frac{2a_2^2\varphi^1}{D_2}\right]. \end{aligned} \quad (15)$$

$$\begin{aligned} e_{21}^* &= r_2\left(-2a_1+\frac{3a_1^2}{D_1}+\frac{a_1^2\varphi^2}{D_1}\right) \\ &+\delta\rho\left[\gamma\left(\frac{a_2^2}{D_2}-\frac{2a_2^2\varphi^1}{D_2}+\frac{2a_2^2\varphi^1}{D_2}\right)+\varphi_2\frac{2a_2^2\varphi^1}{D_2}\right]. \end{aligned} \quad (16)$$

According to the first stage optimal effort level, we can get the optimal income of AN in the second stage of the project:

$$\begin{aligned} \Pi_{AN}^* &= \Pi_{AN1}\left(e_{11}^*, e_{21}^*\right) \\ &+\delta\Pi_{AN2}^*\left(e_{11}^*, e_{21}^*, e_{12}^*, e_{21}^*, e_{32}^*\right). \end{aligned} \quad (17)$$

3.3. The Optimal Contract and Its Characteristics. According to (8), (9), (10), (15), and (16), we take the second stage optimal effort into the second stage success probability:

$$\begin{aligned} p_2^* &= \theta_1 e_{12}^*+\theta_2 e_{22}^*+\theta_3 e_{32}^*+\rho\left(e_{11}^*+e_{21}^*\right)=\theta_1 e_{12}^* \\ &+\frac{\delta\rho^2}{\theta_1}e_{12}^*+\theta_2 e_{22}^*+\frac{\delta\rho^2}{\theta_2}\left(e_{22}^*+\theta_3 e_{32}^*\right) \\ &+\rho\left[r_1\left(\frac{a_1^2}{D_1}-\frac{a_1^2\varphi^2}{D_1}\right)\right. \\ &\left.+r_2\left(-2a+\frac{3a_1^2}{D_1}+\frac{a_1^2\varphi^2}{D_1}\right)\right]. \end{aligned} \quad (18)$$

To simplify the model, we assume $\theta_1=\theta_2=\theta_3=\theta$, $r_1=r_2$; thus

$$\begin{aligned} p_2^* &= \delta\rho^2\left(\frac{a_2^2}{D_2}-\frac{a_2^2\varphi_1^2}{D_2}\right)+\theta^2\left(-2a_2+\frac{4a_2^2}{D_2}\right) \\ &+r\rho\left(-2a_2+\frac{4a_1^2}{D_1}\right). \end{aligned} \quad (19)$$

VC's optimal utility is as follows:

$$\begin{aligned} \Pi_{VC2}^* &= 2a_2-\frac{2a_2^2}{D_2}+\left[\delta\rho^2\left(\frac{a_2^2}{D_2}-\frac{a_2^2\varphi_1^2}{D_2}\right)\right. \\ &\left.+\theta^2\left(-2a_2+\frac{4a_2^2}{D_2}\right)+r\rho\left(-2a_1+\frac{4a_1^2}{D_1}\right)\right] \\ &\times\left(-2a_2+\frac{3a_2^2}{D_2}+\frac{a_2^2\varphi_1^2}{D_2}\right)-\frac{1}{2}\theta_1^2\left(-2a_2+\frac{3a_2^2}{D_2}\right. \\ &\left.+\frac{a_2^2\varphi_1^2}{D_2}\right)-I_2. \end{aligned} \quad (20)$$

In (20), we calculate the first-order derivatives of φ_1 and get

$$\begin{aligned} \frac{\partial\Pi_{VC2}^*}{\partial\varphi_1} &= \delta\rho^2\left(-\frac{2a_2^2\varphi_1}{D_2}\right)\left(-2a_2-\frac{3a_2^2}{D_2}+\frac{a_2^2\varphi_1^2}{D_2}\right) \\ &+\left[\frac{2a_2^2\varphi_1^2}{D_2}+\theta^2\left(-2a_2+\frac{4a_2^2}{D_2}\right)\right. \\ &\left.+r\rho\left(-2a_1+\frac{4a_1^2}{D_1}\right)\right]-\theta_1^2\left(-2a_2+\frac{3a_2^2}{D_2}+\frac{a_2^2\varphi_1^2}{D_2}\right) \\ &\cdot\frac{2a_2^2\varphi_1}{D_2}=0. \end{aligned} \quad (21)$$

According to formula (21), there is

$$\begin{aligned} \left(\varphi_1^*\right)^2 &= \frac{2\delta\rho^2a_2-2r\rho a_1+4r\rho a_1^2/D_1}{2\delta\rho^2a_2^2+\theta_1^2a_2^2}D_2 \\ &+\frac{-2\delta\rho^2a_2^2+\theta_1^2a_2^2}{2\delta\rho^2a_2^2+\theta_1^2a_2^2}. \end{aligned} \quad (22)$$

Guo and Zeng [24] thought that the income of VC from the initial contract should be no less than the external opportunity income; thus we assume that if the project is not liquidated, the income of VC should be no less than his external opportunity income. Risk-free income of venture capital I_2 is k_2I_2, among which, k_2 is risk-free rate of interest; thus we can determine the preferred stock coupon, VC gets $D_2=k_2I_2$; thus optimal contract of the second stage is

$$\begin{aligned} &D_2 = k_2I_2 \\ &\varphi_1^* \\ &= \sqrt{\frac{2\delta\rho^2a_2-2r\rho a_1+4r\rho a_1^2/D_1}{2\delta\rho^2a_2^2+\theta_1^2a_2^2}D_2^*+\frac{-2\delta\rho^2a_2^2+\theta_1^2a_2^2}{2\delta\rho^2a_2^2+\theta_1^2a_2^2}}. \end{aligned} \quad (23)$$

According to (21) and (22), there is Conclusion 1.

Conclusion 1. The second stage success probability p_2 is inversely related to VC's share φ_1, is inversely related to the second stage convertible preferred stock coupon D_2, and is inversely related to the first stage preferred stock coupon D_1.

Practical significance of Conclusion 1 is that the success probability of the second stage is related to the contract designed by VC; lowering φ_1, D_2 can incentivize VC and EN to work harder, thus decreasing the agency cost and improving the success probability of the project. This is because modestly lowering φ_1, D_2 can increase the share of profit of AN and EN. Besides, the success probability p_2 of the second stage is related to the first stage convertible point D_1; lowering the convertible point can make EN and AN share the profit, thus giving both sides incentive to work hard.

Conclusion 2. The VC's share is a linear function of his preferred stock coupon, and when $D_1 > 2ra_1^2/(ra_1-\delta\rho a_2)$, φ_1^2 is an increasing function of D_2; when $D_1 < 2ra_1^2/(ra_1-\delta\rho a_2)$, φ_1^2 is an decreasing function of D_2.

Conclusion 2 shows that the linear relationship between φ_1^2 and D_2 is related to the preferred stock coupon in the first stage; according to Conclusion 1, a lower D_1 can incentivize EN and AN to work hard in the second stage, although increasing D_2 can adversely influence the hard work of EN and AN; a good outcome of the first stage can motivate EN and AN to go on working hard, and the success probability and the income of the project are guaranteed; thus VC can moderately decrease the share φ_1 to motivate EN and AN to go on working hard. Similarly, when D_1 is higher, EN and AN can slug in the second stage, while increasing D_2 will adversely affect the hard work of EN and AN; these two factors affect the project's success probability and income; thus VC can moderately increase share φ_1 to guarantee his own income. In reality, AN's preferred stock coupon should guarantee his investment capital; that is, $D_1 > I_1$, and $2ra_1^2/(ra_1-\delta\rho a_2) > 2ra_1^2/ra_1 = 2a_1$; in other words, only if the preferred stock coupon of AN satisfies the condition $D_1 > 2ra_1^2/(ra_1-\delta\rho a_2) > 2ra_1^2/ra_1 = 2a_1$, AN will make sure he will not lose.

Conclusion 3. The share of VC is the decreasing function of the residual value with the project a_2; when $a_1 > 0.25D_1$, VC's share is an increasing function of a_1; when $a_1 < 0.25D_1$, VC's share is a decreasing function of a_1.

Practical significance of Conclusion 3 is that the greater the residual value of the project in the second stage, the greater the income of VC when the project fails; thus VC can decrease his share in a high income scenario and incentivize AN and EN to work hard. The share of VC is also related to the residual value in the first stage; when $a_1 < 0.25D_1$, if D_1 is smaller, EN and AN will work hard, and the success probability of the project will rise.

According to (15) and (16), we substitute the optimal efforts into the probability of success in the first period; there is

$$p_1^* = r_1e_{11} + r_2e_{21}^*$$

$$D_1^* = k_2I_2$$

$$\begin{aligned}
&= \frac{\delta\rho r}{\theta_1}e_{12}^* + \frac{\delta\rho r}{\theta_1}e_{22}^* + r_1^2\left(\frac{a_1^2}{D_1} - \frac{a_1^2\varphi^2}{D_1}\right) \\
&\quad + r_2^2\left(-2a_1 + \frac{3a_1^2}{D_1} + \frac{a_1^2\varphi^2}{D_1}\right) \\
&= \frac{\delta\rho r}{\theta_1}e_{12}^* + \frac{\delta\rho r}{\theta_1}e_{22}^* + r^2\left(-2a_1 + \frac{4a_1^2}{D_1}\right) \\
&= \delta\rho r\left(\frac{a_2^2}{D_2} - \frac{a_2^2\varphi_1^2}{D_2}\right) + r^2\left(-2a_1 + \frac{4a_1^2}{D_1}\right)
\end{aligned} \tag{24}$$

so the optimal utility of AN in the first period is

$$\begin{aligned}
\Pi_{AN1}^* &= 2a_1 - \frac{2a_1^2}{D_1} \\
&\quad + \left[\delta\rho^2 r\left(\frac{a_2^2}{D_2} - \frac{a_2^2\varphi_1^2}{D_2}\right) + r^2\left(-2a_1 + \frac{4a_1^2}{D_1}\right)\right] \\
&\quad \cdot \left(-2a_1 + \frac{3a_1^2}{D_1} + \frac{a_1^2\varphi^2}{D_1}\right) \\
&\quad - \frac{1}{2}\left[r_1\left(-2a_1 + \frac{3a_1^2}{D_1} + \frac{a_1^2\varphi^2}{D_1}\right) + \frac{\delta\rho}{\theta_1}e_{22}^*\right]^2 - I_1.
\end{aligned} \tag{25}$$

The partial derivation of (25) with respect to φ is

$$\begin{aligned}
&\frac{\partial\Pi_{AN1}^*}{\partial\varphi} \\
&\quad = \left[\delta\rho r\left(\frac{a_2^2}{D_2} - \frac{a_2^2\varphi_1^2}{D_2}\right) + r^2\left(-2a_1 + \frac{4a_1^2}{D_1}\right)\right]\frac{2a_1^2\varphi}{D_1} \\
&\qquad - r_1\left[r_1\left(-2a_1 + \frac{3a_1^2}{D_1} + \frac{a_1^2\varphi^2}{D_1}\right) + \frac{\delta\rho}{\theta_1}e_{22}^*\right]\frac{2a_1^2\varphi}{D_1} \\
&\quad = 0.
\end{aligned} \tag{26}$$

From (25) and (26), we can get

$$(\varphi^*)^2 = \frac{r_1\delta\rho e_{12}}{a_1^2r_1^2}D_1 - 1. \tag{27}$$

On the basis of the assumptions of Guo and Zeng [24], if the project is not liquidated, the least return that the AN gains is equal to AN's external opportunity. If k_1I_1 is AN's return of external risk-free investments, the dividends that AN gains on prefer stocks must be $D_1 = k_1I_1$.

In the first period, the contract designed by AN and EN should be

$$\varphi_1^* = \sqrt{\frac{r_1 \delta \rho e_{12}}{a_1^2 r_1^2} D_1 - 1},$$
$$= \sqrt{\frac{r_1 \delta \rho \left[(1-\gamma)\theta_1 \left(a_2^2/D_2 - 2a_2^2\varphi_1/D_2 + a_2^2\varphi_1^2/D_2 \right) + (1-\varphi_1-\varphi_2)\theta_1 \left(2a_2^2\varphi_1/D_2 \right) \right]}{a_1^2 r_1^2} D_1^* - 1}. \quad (28)$$

From (24) (27) we have following conclusions.

Conclusion 4. The success probability p_1 in the first period is inversely proportional to the dividends D_1, D_2 converted on preferred stocks and inversely proportional to VC's shares φ_1.

Conclusion 5. The AN' stock share strictly monotonically increases with the increment of the dividends D_1 that AN gains on preferred stocks.

Practical significance of Conclusion 5 is that when D_1 goes up, EN's return will goes down if the project has a poor quality. As a result, the smaller the success probability of the project, the lower the profits that will come up when EN becomes slake and indolent. In this case, the only way for AN is to increase dividend φ. This suggests that a higher D_1 gives rise to a higher φ.

The formula $e_{22}^* = \gamma\theta_1(a_2^2/D_2 - 2a_2^2\varphi_1/D_2 + a_2^2\varphi_1^2/D_2) + \varphi_2\theta_1(2a_2^2\varphi_1/D_2)$ indicates that AN's efforts in the second period are a function of φ_1, D_2, γ.

On the basis of $D_1^*, \varphi^*, D_2^*, \varphi_1^*$, the optimal total utility of AN is as follows:

$$\begin{aligned}
\Pi_{AN}^* &= \Pi_{AN1}^* + \delta\Pi_{AN2}^* = 2a_1 - \frac{2a_1^2}{D_1^*} + \left[\delta\rho^2 r \left(\frac{a_2^2}{D_2^*} - \frac{a_2^2\varphi_1^2}{D_2^*} \right) + r^2 \left(-2a_1 + \frac{4a_1^2}{D_1^*} \right) \right] \left(-2a_1 + \frac{3a_1^2}{D_1^*} \right. \\
&\left. + \frac{a_1^2\varphi^2}{D_1^*} \right) - \frac{1}{2} \left[r_1 \left(-2a_1 + \frac{3a_1^2}{D_1^*} + \frac{a_1^2\varphi^2}{D_1^*} \right) + \frac{\delta\rho}{\theta_1} \cdot e_{22}^* \right]^2 - I_1 + \delta \left\{ \gamma \left[\delta\rho^2 \left(\frac{a_2^2}{D_2^*} - \frac{a_2^2\varphi_1^{*2}}{D_2^*} \right) \right. \right. \\
&+ \theta^2 \left(-2a_2 + \frac{4a_2^2}{D_2^*} \right) + r\rho \left(-2a_1 + \frac{4a_1^2}{D_1^*} \right) \bigg] \times \left(\frac{a_2^2}{D_2^*} - \frac{a_2^2\varphi_1^{*2}}{D_2^*} + \frac{a_2^2\varphi_1^{*2}}{D_2^*} \right) + \varphi_2 \left[\delta\rho^2 \left(\frac{a_2^2}{D_2^*} - \frac{a_2^2\varphi_1^{*2}}{D_2^*} \right) \right. \\
&\left. \left. + \theta^2 \left(-2a_2 + \frac{4a_2^2}{D_2^*} \right) + r\rho \left(-2a_1 + \frac{4a_1^2}{D_1^*} \right) \right] \frac{2a_2^2\varphi_1^{*2}}{D_2^*} - \frac{1}{2} e_{22}^{*2} \right\}.
\end{aligned} \quad (29)$$

4. The Game Model that AN Does Not Invest in the Second Period

We still use backward induction method to find the optimal contracts.

4.1. The Optimal Decision in the Second Period. If AN quits in the second stage and VC continues to invest in the project, now the success probability of project is $p_2 = \tilde{\theta}_1 e_{12} + \tilde{\theta}_3 e_{32} + \rho(e_{11} + e_{21})$. The profits of the project are random and the probability density is $g(R) = 2\tilde{a}_2/R^3$.

In this case, VC and EN sign a convertible preferred stock contract and the mechanism of the contract; see Figure 3.

Explanation of Figure 3: In the first stage, if the project is successful and AN executes the convertible preferred stock, then y is AN's a share and $1-y$ is a share of EN; if AN does not execute the convertible preferred stock, then $\widetilde{D}_1$ is bond yield, $\tilde{y}$ is a share of AN from the rest $p_2 R - D_1$ of the income, and $1-\tilde{y}$ is a share of EN from the rest $p_2 R - \widetilde{D}_1$ of the income. In the second stage, if the project is successful, if VC can execute the convertible preferred stock, then $\tilde{\phi}_1$ is a share of VC and the share of EN is $1-\tilde{\phi}_1$; if VC does not execute the convertible preferred stock, then $\widetilde{D}_2$ is bond yield.

The following is the optimal problem of VC:

$$\begin{aligned}
&\max_{\widetilde{D}_2, \varphi_1} \widetilde{\Pi}_{VC2} \\
&= \max_{\widetilde{D}_2, \tilde{\varphi}_1} \int_{\tilde{a}_2}^{\widetilde{D}_2} (1-p_2) R \frac{2\tilde{a}_2^2}{R^3} dR \\
&+ \int_{\widetilde{D}_2}^{+\infty} \tilde{\varphi}_1 p_2 R \frac{2\tilde{a}_2^2}{R^3} dR - \frac{1}{2} e_{32}^2 - I_2.
\end{aligned} \quad (30)$$

$$\begin{aligned}
\text{S.T.} \quad &\max_{e_{12}} \widetilde{\Pi}_{EN2} \\
&= \max_{e_{12}} \int_{\tilde{a}_2}^{\widetilde{D}_2} (1-p_2) R \frac{2\tilde{a}_1^2}{R^3} dR \\
&+ \int_{\widetilde{D}_2/\tilde{\varphi}_1}^{+\infty} (1-\tilde{\varphi}_2) p_2 R \frac{2\tilde{a}_2^2}{R^3} dR - \frac{1}{2} e_{12}^2.
\end{aligned} \quad (31)$$

$$\begin{aligned}
&\max_{e_{32}} \widetilde{\Pi}_{VC2} \\
&= \max_{e_{32}} \int_{\tilde{a}_2}^{\widetilde{D}_2} (1-p_2) R \frac{2\tilde{a}_1^2}{R^3} dR
\end{aligned}$$

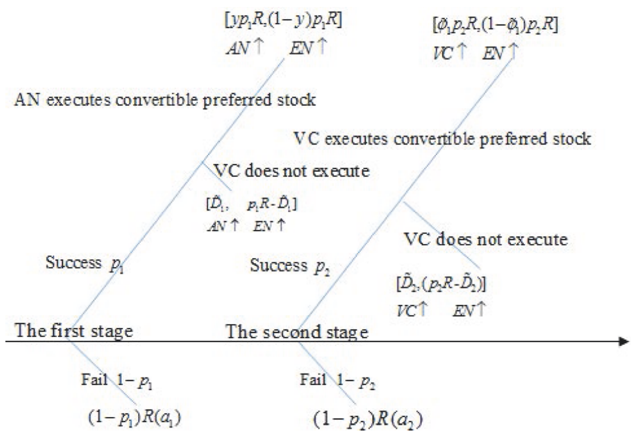

Figure 3: AN does not invest project in the second period.

$$+ \int_{\widetilde{D}_2}^{\widetilde{D}_2/\widetilde{\varphi}_1} p_2 \widetilde{D}_2 \frac{2\widetilde{a}_2^2}{R^3} dR$$
$$+ \int_{\widetilde{D}_2/\widetilde{\varphi}_1}^{+\infty} \widetilde{\varphi}_1 p_2 R \frac{2\widetilde{a}_2^2}{R^3} dR - \frac{1}{2} e_{32}^2 - I_2. \tag{32}$$

Formula (30) is the target function of VC. VC determines the $\widetilde{D}_2, \widetilde{\varphi}_1$ in the contracts to maximize his or her own returns.

Formula (31) is the incentive compatibility constraint of EN. EN determines his or her efforts to maximize his or her own returns.

Formula (32) is the incentive compatibility constraint of AN. AN determines his or her efforts to maximize his or her own returns.

From formula (31) and (32), the optimal efforts of VC and EN are as follows:

$$\widetilde{e}_{12}^* = \widetilde{\theta}_1 \left(\frac{\widetilde{a}_2^2}{\widetilde{D}_2} - \frac{2\widetilde{a}_2^2 \widetilde{\varphi}_1^2}{\widetilde{D}_2} + \frac{\widetilde{a}_2^2 \widetilde{\varphi}_1^2}{\widetilde{D}_2} \right) + \left(1 - \widetilde{\varphi}_1\right) \widetilde{\theta}_1 \frac{2\widetilde{a}_2^2 \widetilde{\varphi}_1^2}{\widetilde{D}_2}. \tag{33}$$

$$\widetilde{e}_{32}^* = \widetilde{\theta}_3 \left(-2\widetilde{a}_2 + \frac{3\widetilde{a}_2^2}{\widetilde{D}_2} + \frac{\widetilde{a}_2^2 \widetilde{\varphi}_1^2}{\widetilde{D}_2} \right). \tag{34}$$

4.2. The Optimal Decision in the First Period. In the first period, the success probability of project is $p_1 = r_1 e_{11} + r_2 e_{21}$. The profits of the project are still random and the probability density is $g(R) = 2\widetilde{a}_1/R^3$.

The following is the optimal problem of AN:

$$\max_{\widetilde{D}_1, \widetilde{\varphi}} \quad \widetilde{\Pi}_{AN} = \max_{\widetilde{D}_1, \widetilde{\varphi}} \Pi_a^{(1)} \left(e_{11}, e_(21)\right). \tag{35}$$

$$\text{S.T.} \quad \max_{e_{21}} \widetilde{\Pi}_a = \max_{e_{21}} \Pi_{AN1} \left(e_{11}, e_{21}\right). \tag{36}$$

$$\max_{e_{11}} \widetilde{\Pi}_{EN} = \max_{e_{11}} \widetilde{\Pi}_{EN1} \left(e_{11}, e_{21}\right) + \delta \widetilde{\Pi}_{EN2} \left(e_{11}, e_{21}, \widetilde{e}_{12}^*, \widetilde{e}_{32}^*\right). \tag{37}$$

$$\widetilde{\Pi}_{AN1} = \int_{\widetilde{a}_1}^{D_1} \left(1 - r_1 e_{11} - r_2 e_{21}\right) R \frac{2\widetilde{a}_1^2}{R^3} dR$$
$$+ \int_{\widetilde{D}_1}^{\widetilde{D}_1/\widetilde{\varphi}} \left(r_1 e_{11} + r_2 e_{21}\right) \widetilde{D}_1 R \frac{2\widetilde{a}_1^2}{R^3} dR$$
$$+ \widetilde{\varphi} \int_{\widetilde{D}_1/\widetilde{\varphi}}^{+\infty} \left(r_1 e_{11} + r_2 e_{21}\right) R \frac{2\widetilde{a}_1^2}{R^3} dR - \frac{1}{2} e_{21}^2 - K_1. \tag{38}$$

$$\widetilde{\Pi}_{EN1} = \int_{\widetilde{D}_1}^{\widetilde{D}_1/\widetilde{\varphi}} \left(r_1 e_{11} + r_2 e_{21}\right) \left(R - \widetilde{D}_1\right) R \frac{2\widetilde{a}_1^2}{R^3} dR$$
$$+ \left(1 - \widetilde{\varphi}\right) \int_{\widetilde{D}_1/\widetilde{\varphi}}^{+\infty} \left(r_1 e_{11} + r_2 e_{21}\right) R \frac{2\widetilde{a}_1^2}{R^3} dR - \frac{1}{2} e_{11}^2.$$

Formula (35) is the target function of VC. VC determines the $\widetilde{D}_1, \widetilde{\varphi}$ in the contracts to maximize his or her own returns.

Formula (36) is the incentive compatibility constraint of EN. EN determines his or her efforts to maximize his or her own returns.

Formula (37) is the incentive compatibility constraint of AN. AN determines his or her efforts to maximize his or her own returns.

From formula (36) and (37), the optimal efforts are

$$\widetilde{e}_{11}^{*} = r_1\left(\frac{\widetilde{a}_1^2}{\widetilde{D}_1} - \frac{2\widetilde{a}_1^2}{\widetilde{D}_1^2}\right) + \delta\rho\frac{\widetilde{a}_2^2}{\widetilde{D}_2^2} - \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2^2}. \quad (39)$$

$$\widetilde{e}_{21}^{*} = r_1\left(-2\widetilde{a}_1 + \frac{3\widetilde{a}_1^2}{\widetilde{D}_1} + \frac{2\widetilde{a}_1^2\widetilde{\varphi}^2}{\widetilde{D}_1}\right). \quad (40)$$

From the optimal efforts (39) and (40) in the first period, we can get the optimal utility of AN if he or she participates in the project:

$$\widetilde{\Pi}_{AN}^{**} = \widetilde{\Pi}_{AN1}\left(\widetilde{e}_{11}^{*}, \widetilde{e}_{21}^{*}\right) + \delta\widetilde{\Pi}_{AN2}\left(\widetilde{e}_{11}^{*}, \widetilde{e}_{21}^{*}, \widetilde{e}_{12}^{*}, \widetilde{e}_{32}^{*}\right). \quad (41)$$

4.3. Optimal Contract and Contract Characteristics. In Sections 4.1 and 4.2, the optimal decision in the first and second stage is obtained. In this section, we need to find the optimal contract and further analyze the characteristics of the contract.

Substituting (33), (34), (39), and (40) to successful probability of second stage, we get

$$\begin{aligned}\widetilde{p}_1 &= \widetilde{\theta}_1\widetilde{e}_{12}^{*} + \widetilde{\theta}_3\widetilde{e}_{32}^{*} + \rho\left(\widetilde{e}_{11}^{*} + \widetilde{e}_{21}^{*}\right) \\ &= \widetilde{\theta}_1\widetilde{e}_{12}^{*} + \frac{\delta\rho^2}{\widetilde{\theta}_1} + \widetilde{\theta}_3\widetilde{e}_{32}^{*} + \rho\left(-2\widetilde{a}_1 + \frac{4\widetilde{a}_1}{\widetilde{D}_1}\right).\end{aligned} \quad (42)$$

To simplify model, we assume $\widetilde{\theta}_1 = \widetilde{\theta}_2 = \widetilde{\theta}_3 = \widetilde{\theta}, \widetilde{r}_1 = \widetilde{r}_2 = \widetilde{r}$. Then, from (41), we get

$$\begin{aligned}\widetilde{p}_1 &= \delta\rho^2\left(\frac{2\widetilde{a}_2}{\widetilde{D}_2} - \frac{2\widetilde{a}_2\varphi_1^2}{\widetilde{D}_2-}\right) + \rho r\left(-2a_1 + \frac{4a_1}{\widetilde{D}_1}\right) \\ &\quad + \widetilde{\theta}^2\left(-2\widetilde{a}_2 + \frac{4\widetilde{a}_2}{\widetilde{D}_2}\right).\end{aligned} \quad (43)$$

Substituting (33), (34), (39), and (40) to the second stage utility function of VC, there is

$$\begin{aligned}\widetilde{\Pi}_{VC2}^{*} &= 2\widetilde{a}_2 - \frac{2\widetilde{a}_2^2}{\widetilde{D}_2} + \left[\delta\rho^2\left(\frac{\widetilde{a}_2^2}{\widetilde{D}_2} - \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\right)\right. \\ &\quad \left. + \theta^2\left(-2\widetilde{a}_2 + \frac{4\widetilde{a}_2^2}{\widetilde{D}_2}\right) + r\rho\left(-2a_1 + \frac{4a_1^2}{\widetilde{D}_1}\right)\right] \\ &\quad \times\left(-2a_2 + \frac{3\widetilde{a}_2^2}{\widetilde{D}_2} + \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\right) - \frac{1}{2}\widetilde{\theta}_1^2\left(-2\widetilde{a}_2 + \frac{3\widetilde{a}_2^2}{\widetilde{D}_2}\right. \\ &\quad \left. + \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\right) - I_2.\end{aligned} \quad (44)$$

From (44), there is the first-order equation

$$\begin{aligned}\frac{\partial\widetilde{\Pi}_{VC}^{*}}{\partial\widetilde{\varphi}_1} &= \delta\rho^2\left(-\frac{2\widetilde{a}_2^2\widetilde{\varphi}_1}{\widetilde{D}_2}\right)\left(-2\widetilde{a}_2 - \frac{3\widetilde{a}_2^2}{\widetilde{D}_2} + \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\right) \\ &\quad + \frac{2\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\left[\delta\rho^2\left(\frac{\widetilde{a}_2^2}{\widetilde{D}_2} - \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\right) + \widetilde{\theta}^2\left(-2\widetilde{a}_2 + \frac{4\widetilde{a}_2^2}{\widetilde{D}_2}\right)\right. \\ &\quad \left. + r\rho\left(-2\widetilde{a}_1 + \frac{4\widetilde{a}_1^2}{\widetilde{D}_1}\right)\right] - \widetilde{\theta}_1^2\left(-2\widetilde{a}_2 + \frac{3a_2^2}{\widetilde{D}_2} + \frac{a_2^2\varphi_1^2}{\widetilde{D}_2}\right) \\ &\quad \cdot\frac{2\widetilde{a}_2^2\widetilde{\varphi}_1}{\widetilde{D}_2} = 0.\end{aligned} \quad (45)$$

Then we get

$$\begin{aligned}\left(\widetilde{\varphi}_1^{*}\right)^2 &= \frac{2\delta\rho^2\widetilde{a}_2 - 2r\rho a_1 + 4r\rho a_1^2/\widetilde{D}_1}{2\delta\rho^2\widetilde{a}_2^2 + \widetilde{\theta}_1^2\widetilde{a}_2^2}\widetilde{D}_2^{*} \\ &\quad + \frac{-2\delta\rho^2\widetilde{a}_2^2 + \widetilde{\theta}_1^2\widetilde{a}_2^2}{2\delta\rho^2\widetilde{a}_2^2 + \widetilde{\theta}_1^2\widetilde{a}_2^2}.\end{aligned} \quad (46)$$

Similarly, VC's lowest profit is as k_2 times of investment, so in the second stage, the contract is as follows:

$$\begin{aligned}\widetilde{D}_2^{*} &= k_2 I_2. \\ \widetilde{\varphi}_1^{*} &= \sqrt{\frac{2\delta\rho^2\widetilde{a}_2 - 2r\rho a_1 + 4r\rho a_1^2/\widetilde{D}_1}{2\delta\rho^2\widetilde{a}_2^2 + \widetilde{\theta}_1^2\widetilde{a}_2^2}\widetilde{D}_2^{*} + \frac{-2\delta\rho^2\widetilde{a}_2^2 + \widetilde{\theta}_1^2\widetilde{a}_2^2}{2\delta\rho^2\widetilde{a}_2^2 + \widetilde{\theta}_1^2\widetilde{a}_2^2}}.\end{aligned} \quad (47)$$

Substituting (39) and (40) to successful probability of first stage, there is

$$\begin{aligned}\widetilde{p}_1 &= r_1\widetilde{e}_{11}^{*} + r_2\widetilde{e}_{21}^{*} = \frac{\delta\rho r_1}{\widetilde{\theta}_1}\widetilde{e}_{12}^{*} + \rho r\left(-2\widetilde{a}_2 + \frac{4\widetilde{a}_2^2}{\widetilde{D}_2}\right) \\ &= \delta\rho^2 r_1\left(\frac{2\widetilde{a}_2}{\widetilde{D}_2} - \frac{2\widetilde{a}_2\varphi_1^2}{\widetilde{D}_2-}\right) + \rho r\left(-2a_1 + \frac{4a_1}{\widetilde{D}_1}\right).\end{aligned} \quad (48)$$

Substituting (39), (40), and (48) to first stage utility function of AN, there is

$$\begin{aligned}\widetilde{\Pi}_{AN1}^{*} &= 2a_1 - \frac{2a_1^2}{D_1} \\ &\quad + \left[\delta\rho r\left(\frac{\widetilde{a}_2^2}{\widetilde{D}_2} - \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^2}{\widetilde{D}_2}\right) + r^2\left(-2a_1 + \frac{4a_1^2}{\widetilde{D}_1}\right)\right] \\ &\quad \cdot\left(-2a_1 + \frac{3a_1^2}{\widetilde{D}_1} + \frac{a_1^2\widetilde{\varphi}^2}{\widetilde{D}_1}\right) \\ &\quad - \frac{1}{2}\left[r_1\left(-2a_1 + \frac{3a_1^2}{\widetilde{D}_1} + \frac{a_1^2\varphi^2}{\widetilde{D}_1}\right) + \frac{\delta\rho}{\widetilde{\theta}_1}e_{22}^{*}\right]^2 - I_1.\end{aligned} \quad (49)$$

Calculating derivative of (49), we have

$$\widetilde{\varphi}^2 = 5 + \frac{r_1\delta\rho e_{12}/\widetilde{\theta} - 4a_1 r_1^2}{a_1^2 r_1^2}\widetilde{D}_1. \quad (50)$$

Similarly, assuming that AN's minimum value is times of the investment amount, in the first stage, the contract is as follows:

$$\widetilde{D}_1^* = k_1 I_1,$$
$$\varphi^* = \sqrt{\frac{r_1 \delta \rho e_{12}/\widetilde{\theta}}{a_1^2 r_1^2}\widetilde{D}_1^* - 1}. \tag{51}$$

According to the equations above, total utility of AN is as follows:

$$\begin{aligned}\widetilde{\Pi}_{AN1}^* &= 2a_1 - \frac{2a_1^2}{D_1^*} \\ &+ \left[\delta \rho r\left(\frac{\widetilde{a}_2^2}{\widetilde{D}_2^*} - \frac{\widetilde{a}_2^2\widetilde{\varphi}_1^{2*}}{\widetilde{D}_2^*}\right) + r^2\left(-2a_1 + \frac{4a_1^2}{\widetilde{D}_1^*}\right)\right] \\ &\cdot \left(-2a_1 + \frac{3a_1^2}{\widetilde{D}_1^*} + \frac{a_1^2\widetilde{\varphi}^{2*}}{\widetilde{D}_1^*}\right) \\ &- \frac{1}{2}\left[r_1\left(-2a_1 + \frac{3a_1^2}{\widetilde{D}_1^*} + \frac{a_1^2\varphi^{2*}}{\widetilde{D}_1^*}\right) + \frac{\delta\rho}{\widetilde{\theta}_1}e_{22}^*\right]^2 - I_1.\end{aligned} \tag{52}$$

5. Numerical Simulation

To test and verify the conclusions which we get, this section uses simulation method to study relationships among them.

5.1. Simulation of the Second Stage Investment in Which AN Participates. Let values of parameters $I, a_1, a_2, k_1, k_2, \delta, \rho, \theta, r$ be as in Table 2.

Substitute parameter values of Table 2 to optimal solution $D_2^*, \varphi_1^*, D_1^*, \varphi^*$ of Section 3, so

$$\begin{aligned} D_2^* &= 15, \\ \varphi_1^* &= 0.55, \\ D_1^* &= 12, \\ \varphi^* &= \sqrt{0.39 + 0.40\gamma + 0.45\varphi_2}. \end{aligned} \tag{53}$$

According to the $D_2^*, \varphi_1^*, D_1^*, \varphi^*$ above, two-stage total utility of AN is as follows:

$$\begin{aligned}\Pi_{AN}^* &= \Pi_{AN1}^* + \delta\Pi_{AN2}^* \\ &= 2.517 + 0.0128\gamma + 0.125\varphi_2 \\ &\quad - 0.5\left(-0.826 + 0.063\gamma + 0.126\varphi_2^2\right)^2 \\ &\quad - 0.5\left(0.058\gamma + 0.313\varphi_2\right)^2 - I_1. \end{aligned} \tag{54}$$

5.2. Simulation of Second Stage Investment That AN Does Not Participate. Let values of parameters $I, a_1, a_2, k_1, k_2, \delta, \rho, \theta, r$ be as in Table 3.

Table 2: Parameter value of $I, a_1, a_2, k_1, k_2, \delta, \rho, \theta, r$.

Parameter	I	a_1	a_2	k_1	k_2	δ	ρ	θ	r
Value	10	1	4	1.2	1.5	0.6	0.1	0.267	0.5

Table 3: Parameter value of $I, a_1, a_2, k_1, k_2, \delta, \rho, \theta, r$.

Parameter	I	a_1	a_2	k_1	k_2	δ	ρ	θ	r
Value	10	1	4	1.2	1.5	0.5	0.1	0.45	0.5

Substituting parameter values of Table 3 to optimal solution $\widetilde{D}_2^*, \widetilde{\varphi}_1^*, \widetilde{D}_1^*, \widetilde{\varphi}^*$ of Section 4, we get

$$\begin{aligned} \widetilde{D}_2^* &= 15, \\ \widetilde{\varphi}_1^* &= 0.78, \\ \widetilde{D}_1^* &= 12, \\ \widetilde{\varphi}^* &= 0.57. \end{aligned} \tag{55}$$

In the same way, substituting parameter values of Table 3 to the two-stage total utility of AN, there is

$$\Pi_{AN}^* = 2.25 - I_1. \tag{56}$$

To analyze the exit selection of AN, we compare the value (57), (58) of Π_{AN}^* and $\widetilde{\Pi}_{AN}^*$:

$$\begin{aligned}\Pi_{AN}^* &== 2.517 + 0.0128\gamma + 0.125\varphi_2 \\ &\quad - 0.5\left(-0.826 + 0.063\gamma + 0.126\varphi_2^2\right)^2 \\ &\quad - 0.5\left(0.058\gamma + 0.313\varphi_2\right)^2 - I_1. \end{aligned} \tag{57}$$

$$\widetilde{\Pi}_{AN}^* = 2.25 - I_1. \tag{58}$$

Further, set $\varphi_2 = 0.2$ and assume γ is uniformly distributed in $[0, 1]$. See Table 4.

Substituting these 50 parameter's values into (57), results are shown in Table 5.

According to (58) and Table 5, we have Figure 4.

According to Table 4 and Figure 4, we get Conclusion 6.

Conclusion 6 (numerical conclusion). There is a γ^*; when $\gamma > \gamma^*$, AN chooses to participate in second stage, and when $\gamma < \gamma^*$, AN chooses to exit after the end of first stage.

6. Conclusion

In venture investment projects, AN usually uses common stocks as financial instrument, while VC uses more sophisticated tools such as combination of common stocks and bonds, convertible preferred stocks, and participating convertible preferred stock. As the initial stage during the process of venture investment, it is necessary to improve the awareness of self-protection for AN. In current venture investment projects, AN is motivated to innovate financial instruments, among which convertible preferred stock is the

Table 4: Value of parameter γ.

0.0200	0.0400	0.0600	0.0800	0.1000	0.1200	0.1400	0.1600	0.1800	0.2000
0.2200	0.2400	0.2600	0.2800	0.3000	0.3200	0.3400	0.3600	0.3800	0.4000
0.4200	0.4400	0.4600	0.4800	0.5000	0.5200	0.5400	0.5600	0.5800	0.6000
0.6200	0.6400	0.6600	0.6800	0.7000	0.7200	0.7400	0.7600	0.7800	0.8000
0.8200	0.8400	0.8600	0.8800	0.9000	0.9200	0.9400	0.9600	0.9800	1.0000

Table 5: Second stage utility of AN when AN participates.

γ	0.0200	0.0400	0.0600	0.0800	0.1000
AN's profit	2.2154	2.2166	2.2179	2.2191	2.2204
γ	0.1200	0.1400	0.1600	0.1800	0.2000
AN's profit	2.2216	2.2229	2.2242	2.2254	2.2267
γ	0.2200	0.2400	0.2600	0.2800	0.3000
AN's profit	2.2279	2.2292	2.2304	2.2316	2.2329
γ	0.3200	0.3400	0.3600	0.3800	0.4000
AN's profit	2.2341	2.2354	2.2366	2.2378	2.2391
γ	0.4200	0.4400	0.4600	0.4800	0.5000
AN's profit	2.2403	2.2415	2.2428	2.2440	2.2452
γ	0.5200	0.5400	0.5600	0.5800	0.6000
AN's profit	2.2464	2.2477	2.2489	2.2501	2.2513
γ	0.6200	0.6400	0.6600	0.6800	0.7000
AN's profit	2.2525	2.2538	2.2550	2.2562	2.2574
γ	0.7200	0.7400	0.7600	0.7800	0.8000
AN's profit	2.2586	2.2598	2.2610	2.2622	2.2634
γ	0.8200	0.8400	0.8600	0.8800	0.9000
AN's profit	2.2646	2.2658	2.2670	2.2682	2.2694

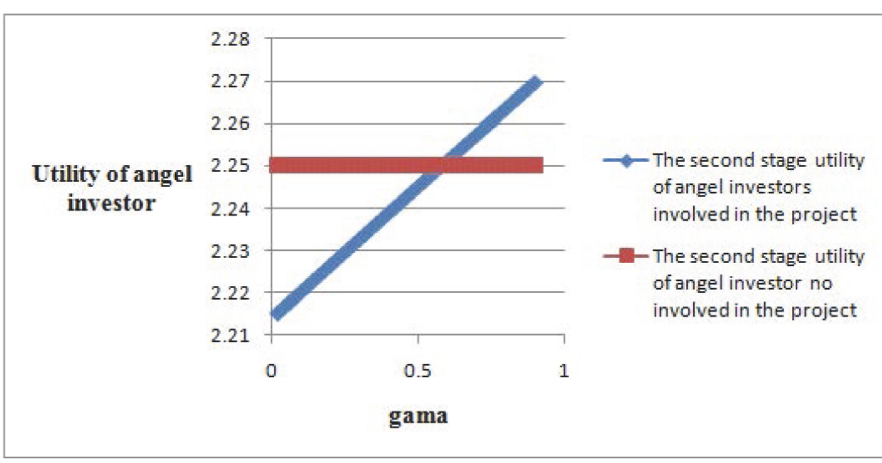

Figure 4: Comparison of AN utility.

most common and simple choice. This article regards convertible preferred stock as an incentive to analyze behaviors of AN.

Capital exit is the main target of investors. Proper exit mechanism enables maximum profit not only for investors but also for ENs. Therefore, capital appreciation and investors' exit are the most essential parts in venture capital operation. As the financier of venture investment, AN usually participates in the beginning stage of projects instead of continuing until the last stage.

Based on previous line of reasoning, this article improves exit mechanism of venture capital and analyzes exit mechanism of AN. This article assumes that the level of effort from the former stage influences the success probability of latter stage project, so the design of two-stage contract will interact with and influence the success probability for two-stage project. This article assumes that the choice of AN to exit in first stage is related to their own profits. If AN decides to exit, the final profits that are the difference between the profits and the cost of effort and investment in first stage are negative. If AN decides to stay, then the final profits that are the difference between the profits and the cost of effort and investment in first stage are positive. In the second stage contract, the more the profits VC gives to AN, the smaller the probability of exiting in the first stage.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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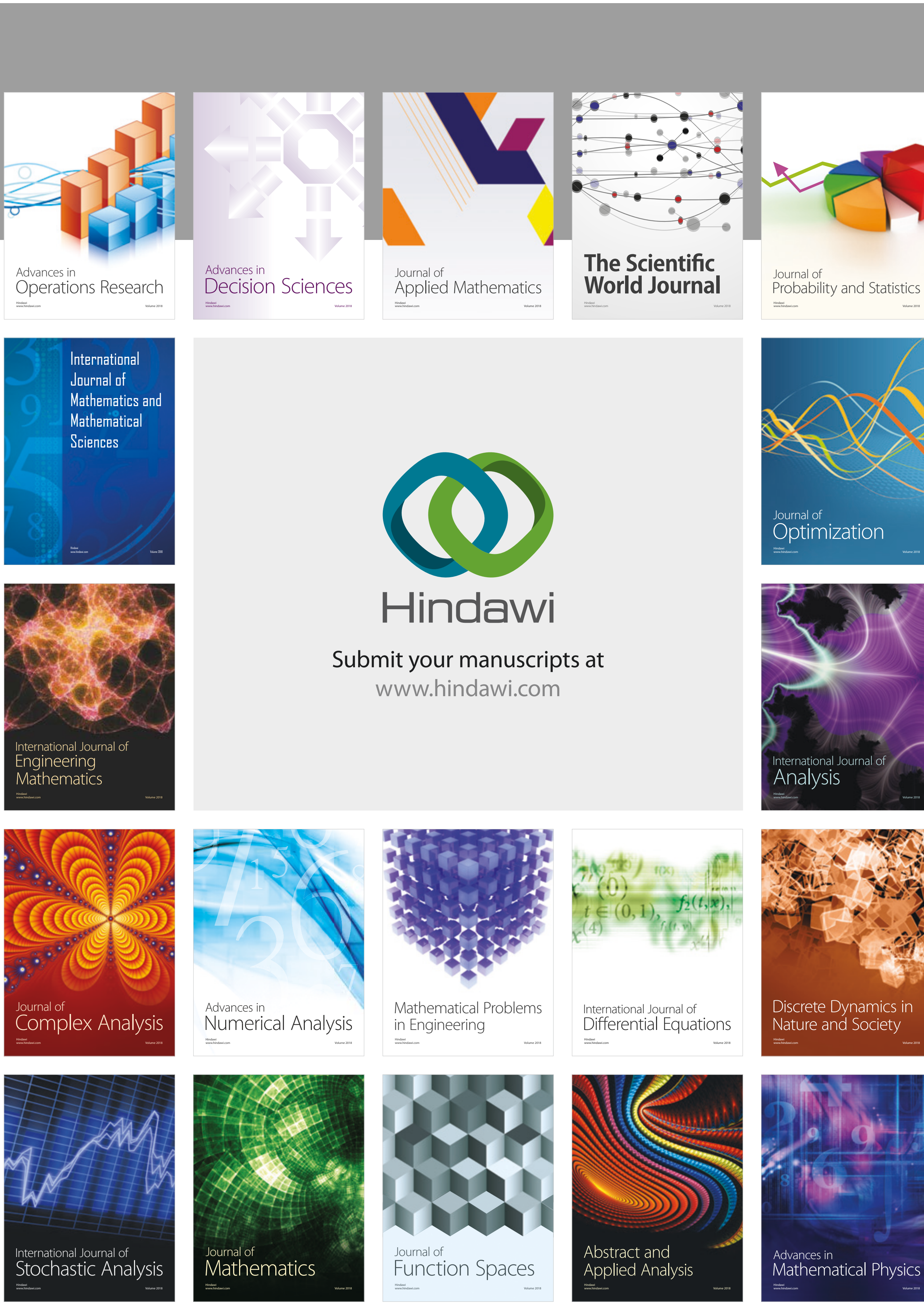
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