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Research Article

Optimal Control of Rumor Spreading Model with Consideration of Psychological Factors and Time Delay

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Rumors have rapidly increasing influence on the society as well as individual life in the information age. How to control the spread of such rumors effectively has become an urgent problem to be solved. In this paper, we consider an optimal control of rumor spreading model with psychological factors and time delay. Firstly, we introduce a realistic optimal control of rumor spreading model with consideration of Holling-type II functional response and time delay. Secondly, by introducing two control strategies of both promoting scientific knowledge and releasing official information, we formulate an optimal control problem to minimize both the number of ignorant individuals and spreaders and the control cost. Thirdly, we prove the existence and the necessary conditions of optimal control strategies theoretically based on Pontryagin's maximum principle. Our results indicate that the proposed control strategies are effective in reducing the number of spreaders and ignorant individuals and minimizing control cost.

1. Introduction

With the help of network technology, rumors can widely spread in a second and lead to public opinion risks; they may lead to some unpredictable and irreparable consequences. Therefore, the impact and consequences of rumors should not be underestimated; it is a very urgent and meaningful subject to research on how to deal with rumors. Nowadays, mathematical modeling of rumor is one of the most important research areas; it has been studied by many scholars. Among them, the DK model proposed by Daley and Kendall in 1965 [1] is the early classical rumor model. In their model, the crowd was divided into three categories, the people who do not know the rumors, the people who know the rumors and spread rumors, and the people who know the rumor but do not transmit it. Zanette applied complex network theory to study the rumor spreading [2, 3]. Moreno et al. examined the dynamics of rumor spreading on scale-free networks [4]. Zhao et al. provided a good description of the amplitude effect caused by opinion leaders in the rumor spreading process [5]. Several authors considered suspected mechanism [6], incubation mechanism [7], forgetting mechanism [8], and hesitating mechanism [9] to the process of rumor spreading.

Most previous rumor spreading models mainly consider that rumor diffusion process meets the bilinear incidence rate. Rumor propagation is closely related to personal psychological quality. Thus, the bilinear incidence rate in real rumor spreading is not very appropriate. In recent years, some researchers have studied nonlinear incidence rate in rumor propagation process. For example, Huo established a rumor spreading model with consideration of Holling-type II functional response [10]. The Holling-type II functional response was used to describe the excellent explanations of the scientific knowledge. However, one significant drawback to Huo's work was that the rumor spreading model did not consider the time delay in rumor spreading process. As is well known, individuals sometimes may not timely respond to rumors. Time delay is a common and inevitable phenomenon in nature; the development trend of the dynamic system depends on not only the present state, but also the past state. Time delays are often represented as latent period or immune period in epidemic model [11–13] and finite response period in neural network [14–16]. Similarly, time delays also exist in the rumor spreading process. For example, a person who is infected with rumor will not spread the rumor timely; it takes time for thinking and then for spreading rumor; there are little studies including time delay in rumor spreading model. Li et al. established a rumor model with time delay considering forgetting effect [17]. Li et al. proposed a time delay rumor propagation model with a saturated control function in emergencies [18, 19]. Meanwhile, rumor spreading emphasizes its timeliness; there is no doubt that inclusion of time delay in rumor spreading model makes it more realistic by describing the effects of latent period. Thus, considering the time delay when the rumor-infected individuals contact the rumor-susceptible individuals is necessary.

Scientific knowledge and official information have played a great role in the process of the rumor control. Most rumors and suspicions are caused by the lack of scientific knowledge and official information. Promoting scientific knowledge can enhance people's ability to identify rumors indirectly; releasing official information can help people know the truth directly. For example, in March 2011, after the Tohoku earthquake, a buying frenzy for iodized salt erupted in China, people believed that iodized salt can protect people from nuclear radiation, and it has brought inestimable negative effects; however, the situation eased after the government promoting scientific knowledge and releasing official information. Therefore, this means that it is realistic to represent the actual dynamics of rumor spreading model including the effects of scientific knowledge and official information. Zhang et al. considered an 8-state ICSAR rumor propagation model with official rumor refutation [20]. Huo et al. present a way of analysis on interaction between two processes and a frame about the dissemination of scientific knowledge in order to prevent rumor transition [21]. Havakhor et al. argued that the extent of knowledge diffusion in the social media networks depends on the type of mechanism and the relative distribution of these three roles. [22]. Zhu et al. considered government intervention in the process of rumor spreading and proposed a rumor propagation model with government regulation and control [23]. Wang et al. built a knowledge transmission model by considering the self-learning mechanism to describe the dynamics of the knowledge transmission process [24]. Huo and Huang established a dynamic transmission 3D model to explore the influence of scientific knowledge in rumor spreading [25].

Recently, optimal control theory has been widely used to analyze the dynamic equation; an optimal control problem is described by suggesting an objective functional for optimizing the control cost and effect. A number of studies introduce vaccination and treatment on the spread of infectious diseases by using the control theory [26–28]. Due to the high similarity between rumor spreading model and epidemic model, various optimal control models were also proposed in rumor model. Among them, Kandhway formulated an optimal control problem, from the perspective of single campaigner, to maximize the spread of information when the campaign budget is fixed [29]. Dhar J et al. proposed media awareness as a control strategy for reducing the rumor spreading [30]. Huo et al. introduced control signals, such as science education and official media coverage attempt to convert lurkers and spreaders into stiflers [31].

Above all, in this paper, we will investigate the influence of scientific knowledge and official information on rumor spreading with psychological factors and time delay. To do this, we proposed a more realistic optimal control problem of rumor spreading model with consideration of Holling-type II functional response and time delay, in which time delay means a time that is used to think about whether to spread the rumor or not by one's cognition or surrounding influence; two control strategies represent separately promoting scientific knowledge and releasing official information. Secondly, we use optimal control approach to minimize both the number of spreaders and ignorant individuals and the cost of two control strategies. Finally, we proved that the spreading of rumor could be controlled effectively by using the two control strategies together.

The organization of this paper is as follows. In Section 2, to prevent the spread of rumor, we introduce two control variables in rumor spreading model with time delay, and we use the optimal control techniques to find the optimal solution of the dynamics system. In Section 3, we perform some numerical simulations. In Section 4, we obtain the conclusions.

2. The Model and Optimal Control Problem

In this section, we first consider a rumor spreading model with time delay. According to classic rumor model, we supposed rumor spread in a virtual community, where the population is divided into three different types: $x(t)$: ignorant individuals (referred to as 'ignorant' in this paper) who have never heard the rumor and can be easily infected by the rumor; $y(t)$: spreaders who have heard the rumor and passed it to others; and $z(t)$: removal individuals (referred to as 'removal' in this paper) who have heard the rumor but will not spread the rumor to others any more. They satisfy the condition $x(t) + y(t) + z(t) = N$.

Compared with traditional rumor spreading, we introduce the nonlinear incidence rate; we assume that rumor spreads between ignorant and spreaders, and the propagation meets Holling II functional response $\beta x(t)y(t)/(1+\alpha y(t))$, in which β represents the transmission rate and α measures the inhibitory effect of $y(t)$. In the process of rumor spreading, on the one hand, due to the lack of the identification ability, one may be willing to expand the rumor; on the other hand, with the increase of spreaders, the government further intensifies the work of releasing official information; the effect of the blocking action of official information on individual's choice whether to believe and spread the rumor is gradually increased. Thus, the contradictory psychological activity of individuals will lead to a decelerating spreading rate, which is coincident with Holling II functional response.

Time delay is introduced in the system as follows: at time t only a percentage of ignorant, who leave the ignorant class τ time unit ago, enter the spreader class at the rate of $e^{-\mu\tau}(\beta x(t-\tau)y(t-\tau)/(1+\alpha y(t-\tau)))$, a Holling-type II saturated incidence function. It represents the proportion of individuals who acquire infection by contacting spreaders and become spreaders at time $t-\tau$, where $e^{-\mu\tau}$ is the removal rate during thinking time τ; it represents a thinking time that is used to think about whether to spread the rumor or not by one's cognition or surrounding influence. $1/(1+\alpha y(t-\tau))$ measures the individual psychological inhibition effect from the behavioral change of ignorant when their number increases. This incidence rate is more realistic than the usual bilinear incidence rate because it includes the individual psychological factors in the process of rumor spreading.

Finally, we assume there are two ways for spreader to become removal; spreader $y(t)$ contact the removal $z(t)$; because the removal $z(t)$ knows the rumor, spreader $y(t)$ loses motivation to spread rumor with a certain probability η.

Based on the classical D-K model, Cintron [32] proposes a deterministic dynamic model. He thinks that people are enthusiastic about passing on the word as long as it is news; once they meet with others who already know the rumor, it is no longer exciting to spread it. In this paper, we suppose that two spreaders $y(t)$ meet, they spread information to each other. It is possible that one feels that the other person already knows the information, so they continue to spread the rumors with little motivation; only the initiating spreader loses motivation to spread rumor again and becomes a stifler; we denote by the parameter η the spreader's contact rate.

The model is formulated as the following system of delayed differential equations:

$$\begin{aligned}\frac{dx(t)}{dt} &= (1-p)b - \frac{\beta x(t)y(t)}{1+\alpha y(t)} - \mu x(t)\\ \frac{dy(t)}{dt} &= e^{-\mu\tau}\frac{\beta x(t-\tau)y(t-\tau)}{1+\alpha y(t-\tau)}\\ &\quad - \eta y(t)(y(t)+z(t)) - \mu y(t)\\ \frac{dz(t)}{dt} &= pb + \eta y(t)(y(t)+z(t)) - \mu z(t)\end{aligned} \tag{1}$$

where b is the number of newly coming people; p is the immune rate of a newly coming people; and μ is natural removal rate.

Based on the above analysis, we know that scientific knowledge and official information have significant effect on controlling rumor spreading. In this paper, we introduce two control strategies to decrease the number of ignorant and spreaders. Promoting scientific knowledge can help the ignorant identify information when rumor occurs; as a result, they will not believe the rumors. Therefore, we assume that ignorant become the removal directly under the action of u_1. However, scientific knowledge plays a weak role in influencing the spreaders, because scientific knowledge is not aimed at one specific rumor, and spreader chooses to believe the rumor with preconceived notions, and it is difficult to change their thought. In order to study the subject conveniently, we assume that u_1 has no effect on spreader. Similarly, when rumor occurred, the government released official information against the rumor, which can help the spreaders learn the rumor truth; as a result, they will not spread the rumor again. Therefore, we assume that spreaders become the removal directly under the action of u_2. However, official information has no direct effect on ignorant compared with scientific knowledge. In order to study the subject conveniently, we assume that u_2 has no effect on the ignorant.

Hence, the optimal control model becomes

$$\begin{aligned}\frac{dx(t)}{dt} &= (1-p)b - \frac{\beta x(t)y(t)}{1+\alpha y(t)} - (\mu+u_1(t))x(t)\\ \frac{dy(t)}{dt} &= e^{-\mu\tau}\frac{\beta x(t-\tau)y(t-\tau)}{1+\alpha y(t-\tau)}\\ &\quad - \eta y(t)(y(t)+z(t)) - (\mu+u_2(t))y(t)\\ \frac{dz(t)}{dt} &= pb + \eta y(t)(y(t)+z(t)) - \mu z(t)\\ &\quad + u_1(t)x(t) + u_2(t)y(t)\end{aligned} \tag{2}$$

with initial conditions

$$\begin{aligned}x(0) &= x_0,\\ y(0) &= y_0,\\ z(0) &= z_0.\end{aligned} \tag{3}$$

In order to set an optimal control problem, firstly, we make the following notational conventions. Let T be a given constant and define the control set:

$$\begin{aligned}U = \{u = (u_1,u_2) \mid u_i(t)\,\textit{measurable},\ 0 \le u_i(t)\\ \le 1,\ 0 \le t \le T,\ i = 1,2\}.\end{aligned} \tag{4}$$

It indicates an admissible control set; in this optimal problem, we assume a restriction on the control variable such as $0 \le u_i(t) \le 1$, because vaccination of all the susceptible individuals at one time is impossible.

The purpose of optimal control aims to spend the least control cost to reduce negative effect caused by the rumor and maximize social utility. As a result, everybody knows the truth and they all become the removal, and they will not believe and spread the rumor easily. In the model, the physical meaning of the control variable in this problem is that if the low levels of the numbers of spreaders and ignorant individuals are reached, we get more removal individuals. In case of no control, the number of spreaders and ignorant individuals increases while the number of removal individuals decreases. The prefect time for control strategies can bring the number of infected individuals down to a small level, the number of ignorant individuals begins to be built again, and more individuals recover from the infection.

Now, we consider an optimal control problem to minimize the objective (cost) functional given by

$$\begin{aligned}&J(u_1,u_2)\\ &= \int_0^T \left[A_1x(t) + A_2y(t) + \frac{1}{2}B_1u_1^2(t) + \frac{1}{2}B_2u_2^2(t)\right]dt\end{aligned} \tag{5}$$

where A_1, A_2 denote positive weight parameters that balance the size of the term $x(t)$, $y(t)$. B_1, B_2 are positive weight parameters which are associated with the control u_1, u_2, and the square of the control variable reflects the severity of the side effects. $u_1 = 1$ and $u_2 = 1$ represent maximizing the intensity of promoting scientific knowledge and releasing official information separately.

2.1. Existence of an Optimal Control. The existence of the optimal control pair without delay can be obtained by using the result in Fleming and Rishel [33]. On the basis of the literature, the principle with delay given in [34] provides necessary conditions for an optimal control problem. According to these papers, the optimal control problem (2) and (3) has the Lagrangian

$$L(x, y, u_1, u_2) = A_1 x(t) + A_2 y(t) + \frac{1}{2} B_1 u_1^2(t) + \frac{1}{2} B_2 u_2^2(t). \tag{6}$$

Theorem 1. *An optimal control pair $u_1^*(t)$, $u_2^*(t)$exists so that*

$$J(u_1^*(t), u_2^*(t)) = \min_{(u_1, u_2 \in U)} J(u_1(t), u_2(t)). \tag{7}$$

Proof. To prove the existence of an optimal control pair, we must check the following properties:

(I) The set of controls and corresponding state variables is nonempty.

(II) The control set U is convex and closed.

(III) The right-hand side of the state system is bounded by a linear function in the state and control variables.

(IV) The integrand of the objective functional is convex on U.

(V) There exist constants $c_1, c_2 > 0$ and $\rho > 1$ such that the integrand of the objective functional $L(x, y, u_1, u_2)$ satisfies

$$L(x, y, u_1, u_2) \geq c_2 + c_1 \left(|u_1|^2 + |u_2|^2\right). \tag{8}$$

According to Theorem 9.2.1 in Lukes [35], the existence of solution of system (2) is given with bounded coefficients, which gives condition (I). By the above definition, the control set is convex and closed, which gives condition (II). By definition, since the state system is linear in u_1, u_2, the right side of system (3) satisfies condition (III). Using the boundedness of the solution, the integrand in the objective functional (5) is convex on U (condition (IV) is proved). Next, let $c_2 = \min(A_1 x + A_2 y)$, $c_1 = \inf(B_1(t), B_2(t))$, and $\rho = 2$; we can have

$$L(x, y, u_1, u_2) = A_1 x(t) + A_2 y(t) + \frac{1}{2} B_1 u_1^2(t) + \frac{1}{2} B_2 u_2^2(t) \geq c_2 + c_1 \left(|u_1|^2 + |u_2|^2\right) \tag{9}$$

which satisfies condition (V).

We conclude that there exists an optimal control.

This completes the proof. □

2.2. Characterization of the Optimal Control. Next, let us derive a necessary condition for the optimal control strategies by using the Pontryagin's maximum principle [34].

Theorem 2. *There exist adjoint variables λ_1, λ_2, λ_3 that satisfy*

$$\begin{aligned}
\frac{d\lambda_1}{dt} &= -A_1 + \lambda_1(t)\left(\frac{\beta y^*}{1 + \alpha y^*} + (\mu + u_1^*)\right) \\
&\quad - \lambda_3(t) u_1^* \\
&\quad - \chi_{[0, t_f - \tau]} \lambda_2(t + \tau) e^{-\mu\tau} \frac{\beta y^*}{1 + \alpha y^*} \\
\frac{d\lambda_2}{dt} &= -A_2 + \lambda_1(t) \frac{\beta x^*}{(1 + \alpha y^*)^2} \\
&\quad - \lambda_2(t)\left(\eta(2y^* + z^*) + (\mu + u_2^*)\right) \\
&\quad - \lambda_3(t)\left(\eta(2y^* + z^*) + u_2^*\right) \\
&\quad - \chi_{[0, t_f - \tau]} \lambda_2(t + \tau) e^{-\mu\tau} \frac{\beta x^*}{(1 + \alpha y^*)^2} \\
\frac{d\lambda_3}{dt} &= \lambda_2(t) \eta y^* - \lambda_3(t)(\eta y^* - \mu)
\end{aligned} \tag{10}$$

with boundary conditions

$$\lambda_i(T) = 0 \quad (i = 1, 2, 3). \tag{11}$$

Furthermore, the optimal control variables are given as follows:

$$u_1^*(t) = \max\left(\min\left(\frac{(\lambda_1(t) - \lambda_3(t)) x^*}{B_1}, u_1^{\max}\right), 0\right) \tag{12}$$

$$u_2^*(t) = \max\left(\min\left(\frac{(\lambda_2(t) - \lambda_3(t)) y^*}{B_2}, u_2^{\max}\right), 0\right). \tag{13}$$

Proof. To find the optimal control function for the optimal control problem, define the corresponding Hamiltonian as

$$\begin{aligned}
H(t) &= L(x, y, u_1, u_2) + \lambda_1\left((1 - p) b - \frac{\beta x(t) y(t)}{1 + \alpha y(t)}\right. \\
&\quad \left. - (\mu + u_1(t)) x(t)\right) + \lambda_2\left(e^{-\mu\tau} \frac{\beta x(t - \tau) y(t - \tau)}{1 + \alpha y(t - \tau)}\right. \\
&\quad \left. - \eta y(t)(y(t) + z(t)) - (\mu + u_2(t)) y(t)\right) \\
&\quad + \lambda_3\left(pb + \eta y(t)(y(t) + z(t)) - \mu z(t)\right. \\
&\quad \left. + u_1(t) x(t) + u_2(t) y(t)\right).
\end{aligned} \tag{14}$$

By differentiating the Hamiltonian above with respect to respective states, we obtain the adjoint system as follows:

$$\frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial x} - \chi_{[0, t_f - \tau]} \frac{\partial H}{\partial x_\tau}(t + \tau), \quad \lambda_1(t_f) = 0$$

$$\frac{d\lambda_2}{dt} = -\frac{\partial H}{\partial y} - \chi_{[0, t_f - \tau]} \frac{\partial H}{\partial y_\tau}(t + \tau), \quad \lambda_2(t_f) = 0$$

$$\frac{d\lambda_3}{dt} = -\frac{\partial H}{\partial z} - \chi_{[0,t_f-\tau]}\frac{\partial H}{\partial z_\tau}(t+\tau), \quad \lambda_3(t_f) = 0 \tag{15}$$

and by the optimality conditions we find that

$$\begin{aligned} \frac{\partial H}{\partial u_1} &= B_1 u^*(t) - \lambda_1(t)x^* + \lambda_3(t)x^* = 0 \\ &\qquad \text{at } u_1 = u_1^*(t) \\ \frac{\partial H}{\partial u_2} &= B_2 u^*(t) - \lambda_2(t)y^* + \lambda_3(t)y^* = 0, \\ &\qquad \text{at } u_2 = u_2^*(t) \end{aligned} \tag{16}$$

which implies

$$\begin{aligned} u_1^*(t) &= \frac{(\lambda_1(t) - \lambda_3(t))x^*}{B_1}, \\ u_2^*(t) &= \frac{(\lambda_2(t) - \lambda_3(t))y^*}{B_2}. \end{aligned} \tag{17}$$

Using the property of control set, we obtain

$$\begin{aligned} u_1^*(t) &= 0 \quad \text{if } \frac{(\lambda_1(t) - \lambda_3(t))x^*}{B_1} \le 0 \\ u_1^*(t) &= \frac{(\lambda_1(t) - \lambda_3(t))x^*}{B_1} \\ &\qquad \text{if } \frac{(\lambda_1(t) - \lambda_3(t))x^*}{B_1} < u_1^{\max} \\ u_1^*(t) &= u_1^{\max} \quad \text{if } \frac{(\lambda_1(t) - \lambda_3(t))x^*}{B_1} \ge u_1^{\max} \\ u_2^*(t) &= 0 \quad \text{if } \frac{(\lambda_2(t) - \lambda_3(t))y^*}{B_2} \le 0 \\ u_2^*(t) &= \frac{(\lambda_2(t) - \lambda_3(t))y^*}{B_2} \\ &\qquad \text{if } \frac{(\lambda_2(t) - \lambda_3(t))y^*}{B_2} < u_2^{\max} \\ u_2^*(t) &= u_2^{\max} \quad \text{if } \frac{(\lambda_2(t) - \lambda_3(t))y^*}{B_2} \ge u_2^{\max} \end{aligned} \tag{18}$$

so the optimal control pair is characterized as (12) and (13).

The optimality system consists of the state system coupled with the adjoint system with the initial and transversality conditions together with the characterization of the optimal control. The optimal control pair and the state are found by solving the following optimality system, which consists of the state system (2), the adjoint system (10), boundary conditions (3) and (11), and the characterization of the optimal control pair (u_1^*, u_2^*) (12) and (13).

$$\begin{aligned} \frac{dx}{dt} &= (1-p)b - \frac{\beta x(t)y(t)}{1+\alpha y(t)} - (\mu + u_1(t))x(t) \\ \frac{dy}{dt} &= e^{-\mu\tau}\frac{\beta x(t-\tau)y(t-\tau)}{1+\alpha y(t-\tau)} - \eta y(t)(y(t)+z(t)) \\ &\quad - (\mu + u_2(t))y(t) \\ \frac{dz}{dt} &= pb + \eta y(t)(y(t)+z(t)) - \mu z(t) + u_1(t)x(t) \\ &\quad + u_2(t)y(t) \\ \frac{d\lambda_1}{dt} &= -A_1 + \lambda_1\left(\mu + u_1 + \frac{\beta y^*(t)}{1+\alpha y^*(t)}\right) - \lambda_3 u_1 \\ &\quad - \frac{\lambda_2(t+\tau)e^{-\mu\tau}y^*(t-\tau)}{1+y^*(t-\tau)} \\ \frac{d\lambda_2}{dt} &= -A_2 + \lambda_1\left(\frac{\beta x^*(t)}{1+\alpha y^*(t)} - \frac{\alpha\beta x^*(t)y^*(t)}{(1+\alpha y^*(t))^2}\right) \\ &\quad + (\lambda_2 - \lambda_3)(y(t)\eta + (y(t)+z(t))\eta + \mu + u_2) \\ &\quad + \lambda_2\left(\frac{e^{-\mu\tau}\beta x^*(t-\tau)}{1+\alpha y^*(t-\tau)}\right. \\ &\quad \left. - \frac{e^{-\mu\tau}\alpha\beta x^*(t-\tau)y^*(t-\tau)}{(1+\alpha y^*(t-\tau))^2}\right) \\ \frac{d\lambda_3}{dt} &= -\lambda_2\eta y^*(t) - \lambda_3(\eta y^*(t) - \mu) \\ u_1^*(t) &= \max\left(\min\left(\frac{(\lambda_1(t) - \lambda_3(t))x^*}{B_1}\right), u_1^{\max}\right), 0) \\ u_2^*(t) &= \max\left(\min\left(\frac{(\lambda_2(t) - \lambda_3(t))y^*}{B_2}\right), u_2^{\max}\right), 0). \end{aligned} \tag{19}$$

This completes the proof. □

3. Numerical Simulations

To illustrate the theoretical analysis, in this section, we solve numerically the optimality system (2) and we present the results found. For the convenience purpose, we suppose that the number of people in the community will not increase over a short time, so we assume that $p = 1$, $b = 0$. Meanwhile, the number of people will not fall over short time; that is $\mu = 0$.

We note that the optimality system is a two-point boundary value problem, with separated boundary conditions at times t=0 and t=t_f, according to [36], this involves use of an appropriate algorithm.

Let there exist a step size and integers $(n, m) \in \mathbb{N}^2$ with $\tau = mh$ and $t_f = nh$. For reasons of programming, we consider m knots to left of 0 and right of t_f, and we obtain the following partition:

$$\Delta = \left(t_{-m} = -\tau < \cdots < t_{-1} < t_0 = 0 < t_1 < \cdots < t_n = t_f < \cdots < t_{n+m}\right). \tag{20}$$

Then, we have $t_i = t_0 + ih (-m \leq i \leq n + m)$. Next we define the state and adjoint variables $x(t), y(t), z(t), \lambda_1(t), \lambda_2(t), \lambda_3(t)$, and the controls $u_1(t)$, $u_2(t)$ in terms of nodal points $x_i, y_i, z_i, \lambda_1^i, \lambda_2^i, \lambda_3^i$ and u_1^i, u_2^i.

Algorithm

Step 1

for i=−m, ⋯ , 0,do

$$\begin{aligned} x_i &= x_0, \\ y_i &= y_0, \\ z_i &= z_0, \\ u_1^i &= 0, \\ u_2^i &= 0. \end{aligned} \tag{21}$$

end for

for i = n, ⋯ , n + m,do

$$\begin{aligned} \lambda_1^i &= 0, \\ \lambda_2^i &= 0, \\ \lambda_3^i &= 0, \end{aligned} \tag{22}$$

end for

Step 2

$$\begin{aligned}
x_{i+1} &= x_i + h\left[(1-p)b - \frac{\beta x_i y_i}{1+\alpha y_i} - \left(\mu + u_1^i\right)x_i\right] \\
y_{i+1} &= y_i + h\left[e^{-\mu\tau}\frac{\beta x_{i-m} y_{i-m}}{1+\alpha y_{i-m}} - \eta y_i\left(y_i + z_i\right)\right. \\
&\quad \left. - \left(\mu + u_2^i\right) y_i\right] \\
z_{i+1} &= z_i + h\left[pb + \eta y_i\left(y_i + z_i\right) - \mu z_i + u_1^i x_i + u_2^i y_i\right] \\
\lambda_1^{n-i-1} &= \lambda_1^{n-i} + h\left[-A_1 + \lambda_1^{n-i}\left(\frac{\beta y_i}{1+\alpha y_i} + \left(\mu + u_1^i\right)\right)\right. \\
&\quad \left. - \lambda_3^{n-i}u_1^i - \chi_{[0,t_f-\tau]}\left(t_{n-i}\right)\lambda_2^{n-i+m}e^{-\mu\tau}\frac{\beta y_i}{1+\alpha y_i}\right] \\
\lambda_2^{n-i-1} &= \lambda_2^{n-i} + h\left[-A_2 + \lambda_1^{n-i}\frac{\beta x_i}{\left(1+\alpha y_i\right)^2}\right. \\
&\quad - \lambda_2^{n-i}(t)\left(\eta\left(2y_i + z_i\right) + \left(\mu + u_2^i\right)\right) \\
&\quad - \lambda_3^{n-i}\left(\eta\left(2y_i + z_i\right) + u_2^i\right) \\
&\quad \left. - \chi_{[0,t_f-\tau]}\left(t_{n-i}\right)\lambda_2^{n-i+m}e^{-\mu\tau}\frac{\beta x_i}{\left(1+\alpha y_i\right)^2}\right] \\
\lambda_3^{n-i-1} &= \lambda_3^{n-i} + h\left[\lambda_2^{n-i}\eta y_i - \lambda_3^{n-i}\left(\eta y_i - \mu\right)\right] \\
\Theta_1^{i+1} &= \frac{\left(\lambda_1^{n-i} - \lambda_3^{n-i}\right)x_{i+1}}{B_1} \\
\Theta_2^{i+1} &= \frac{\left(\lambda_2^{n-i} - \lambda_3^{n-i}\right)y_{i+1}}{B_2} \\
u_1^i &= \max\left(\min\left(\Theta_1^{i+1}, u_1^{\max}\right), 0\right) \\
u_2^i &= \max\left(\min\left(\Theta_2^{i+1}, u_2^{\max}\right), 0\right)
\end{aligned} \tag{23}$$

end for

Step 3

for i=1, ⋯ n, do,write

$$\begin{aligned} \mathrm{x}^*\left(t_i\right) &= x_i, \\ \mathrm{x}^*\left(t_i\right) &= x_i, \\ \mathrm{y}^*\left(t_i\right) &= y_i, \\ \mathrm{z}^*\left(t_i\right) &= z_i, \\ \mathrm{u}_1^*\left(t_i\right) &= x_i, \\ \mathrm{u}_2^*\left(t_i\right) &= x_i, \end{aligned} \tag{24}$$

end for.

Next, let $\beta = 0.2$, $\alpha = 0.1$, $\eta = 0.01$ separately. Furthermore, we choose the following initial values: $x(0) = 8$, $y(0) = 1$, $z(0) = 1$. The weight constant values in the objective functional are $A_1 = 1$, $A_2 = 1$, $B_1 = 20$, $B_2 = 10$. We investigate and compare numerical results in the following three possible strategies for the control system. We also discuss the effect of different delays in control variables in **Section 3.4**.

3.1. Only Promoting Scientific Knowledge Strategy ($u_2 = 0$). We suppose that the government uses only promoting scientific knowledge strategy to control rumor. With this strategy, only the control variable u_1 is used to optimize the objective function J while the control variable u_2 is set to zero.

Next, let $\tau = 0.1$; by calculation, it can satisfy optimality conditions. The trend of the change of $x(t)$, $y(t)$, $z(t)$ both with and without control is shown in **Figures 1, 2**, and **3** separately. **Figure 4** gives the corresponding optimal control variable $u_1(t)$.

Figure 1 shows that, in the presence of a control $u_1(t)$, the number of ignorant $(x(t))$ decreases fast compared to 'without control'; under the effect of scientific knowledge, some ignorant become the removal. **Figure 2** shows that the number of spreaders $(y(t))$ increases slowly with control

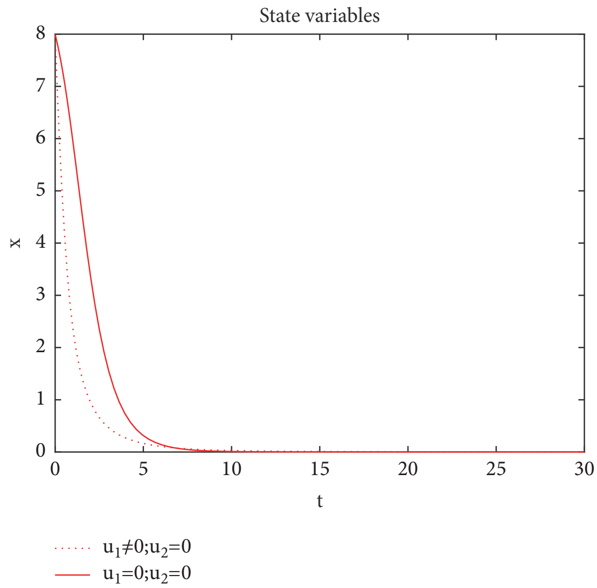

Figure 1: Evolution of number of $x(t)$ with or without control.

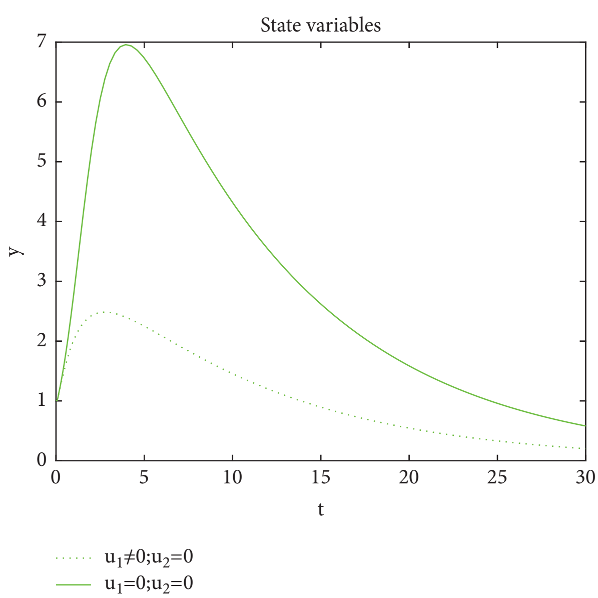

Figure 2: Evolution of number of $y(t)$ with or without control.

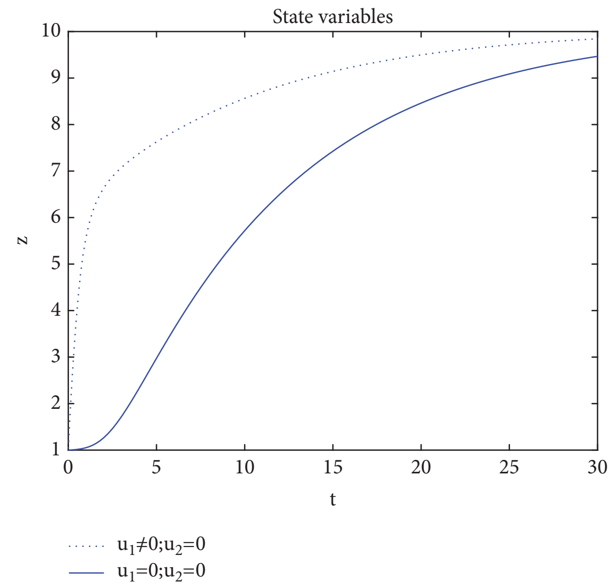

Figure 3: Evolution of number of $z(t)$ with or without control.

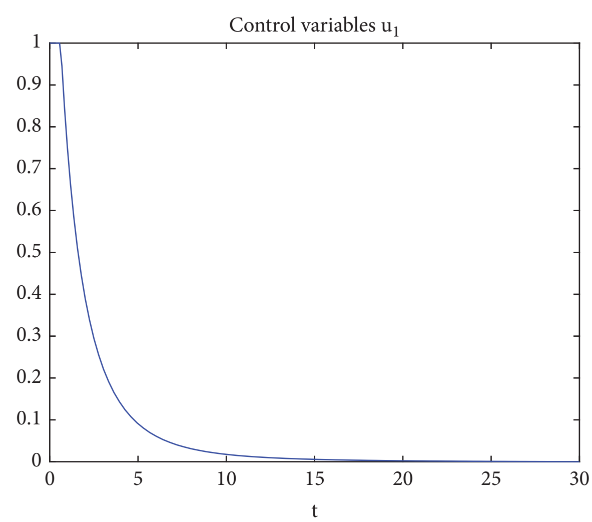

Figure 4: Optimal control function $u_1(t)$ for a controlled system.

compared to ‘without control’ at the beginning and the final size is much smaller. **Figure 3** shows that the number of the removal ($z(t)$) increases fast with control compared to ‘without control’ and the final size is much larger. The dotted line denotes that more ignorant and spreaders become the removal when the control is implemented to the ignorant. **Figure 4** represents the optimal control variable $u_1(t)$. The curve has the maximum value in the beginning because scientific knowledge plays the most important role in the case of the high ignorant level; then it drops off because of the increasing number of spreaders and scientific knowledge has little effect on them.

The comparison shows that promoting scientific knowledge is effective in helping ignorant identify the rumor and become the removal, and it has the maximum utility in the beginning, which is close to realities. Therefore, when the government deals with the rumors, they should promote scientific knowledge to public as fast as possible by phone, TV, and Internet and then prevent the transformation from the ignorant to spreader.

3.2. Only Releasing Official Information Strategy ($u_1 = 0$). We suppose that the government uses only releasing official information strategy to control rumor. With this strategy, only the control variable u_2 is used to optimize the objective function J while the control variable u_1 is set to zero.

Next, let $\tau = 0.1$; by calculation, it can satisfy optimality conditions. The trend of the change of $x(t)$, $y(t)$, $z(t)$ both

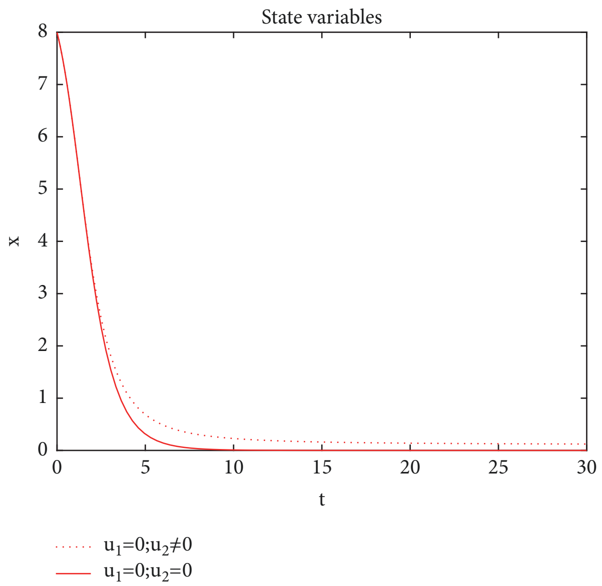

FIGURE 5: Evolution of number of $x(t)$ with or without control.

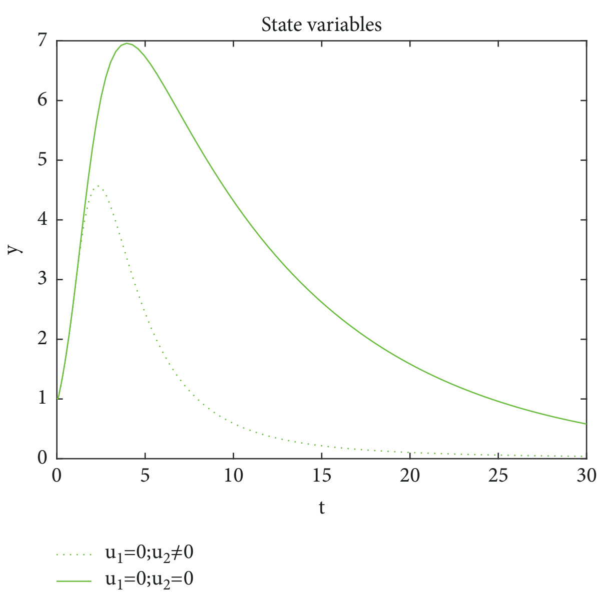

FIGURE 6: Evolution of number of $y(t)$ with or without control.

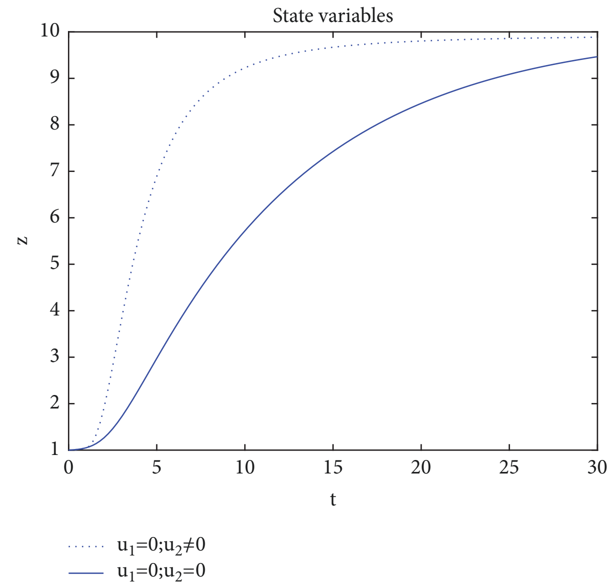

FIGURE 7: Evolution of number of $z(t)$ with or without control.

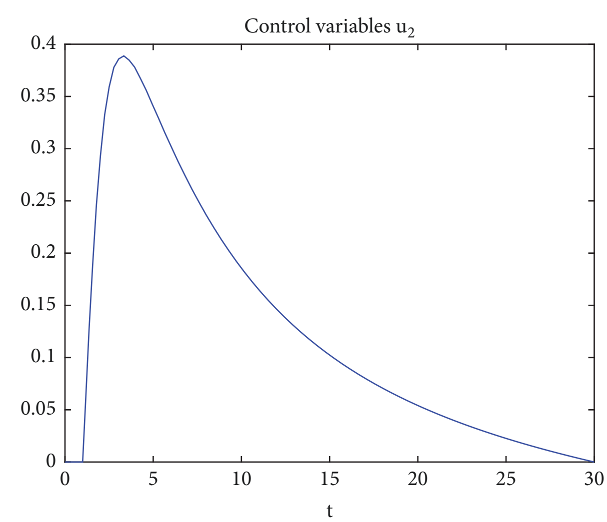

FIGURE 8: Optimal control function $u_2(t)$ for a controlled system.

with and without control is shown in **Figures 5**, **6,** and **7**. **Figure 8** gives the corresponding optimal control variable $u_2(t)$.

Figure 5 shows that, in the presence of a control $u_2(t)$, the number of ignorant ($x(t)$) decreases slowly compared to 'without control' and the final size is larger. **Figure 6** shows that the number of spreaders ($y(t)$) increases slowly with control compared to 'without control' and the final size is much smaller. Under the effort of official information, some spreaders become the removal. Therefore, we need releasing official information to control the size of spreader. **Figure 7** shows that the number of the removal ($z(t)$) increases fast with control compared to 'without control' and the final size is much larger. The dotted line denotes that more spreaders become the removal when the control is implemented to the spreader. **Figure 8** represents the optimal control variable $u_2(t)$; the curve starts to increase during the first time because of the high spreader level, and then it drops off steadily because of the constant and steady eradication of the rumor.

The comparison shows that releasing official information is effective in making spreader know the truth and become the removal, and it has the maximum utility in the middle time, which is close to realities. Therefore, when the government deals with the rumors, the officials should announce some official information to guide the public opinion and eliminate the popular indignation.

3.3. Combining the Above Two Strategies. We suppose that the government combines both promoting scientific knowledge

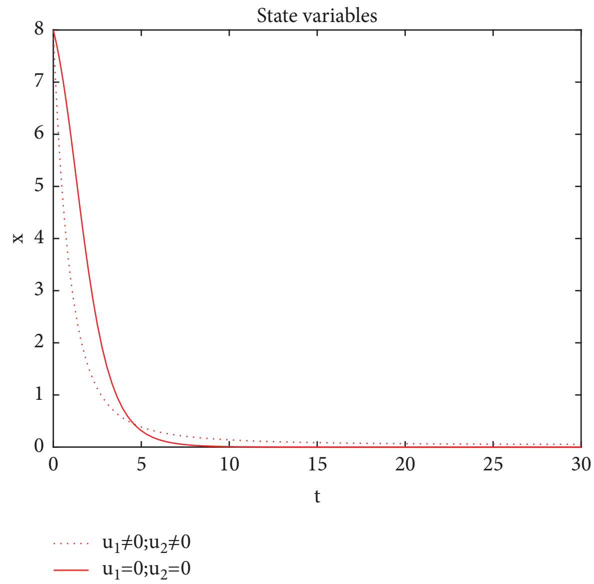

FIGURE 9: Evolution of number of $x(t)$ with or without control.

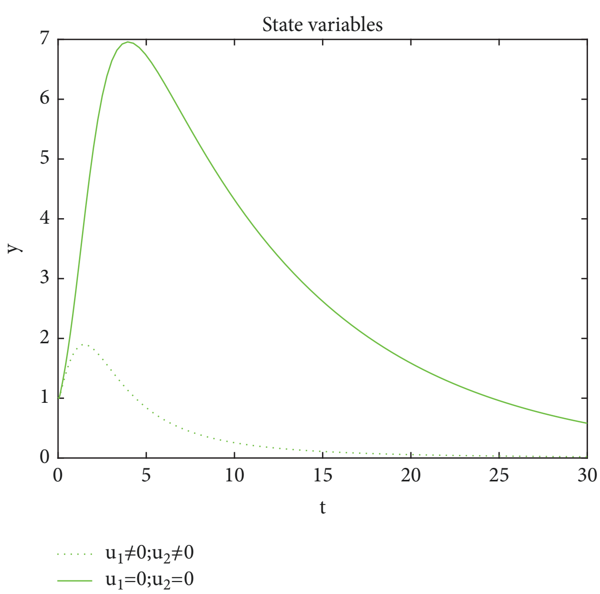

FIGURE 10: Evolution of number of $y(t)$ with or without control.

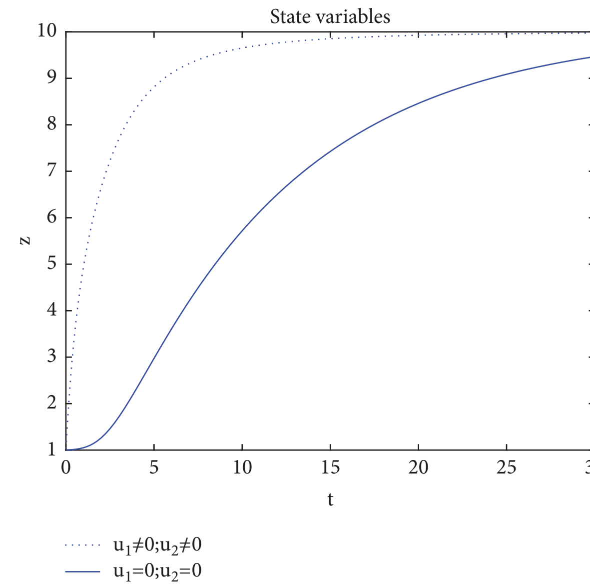

FIGURE 11: Evolution of number of $z(t)$ with or without control.

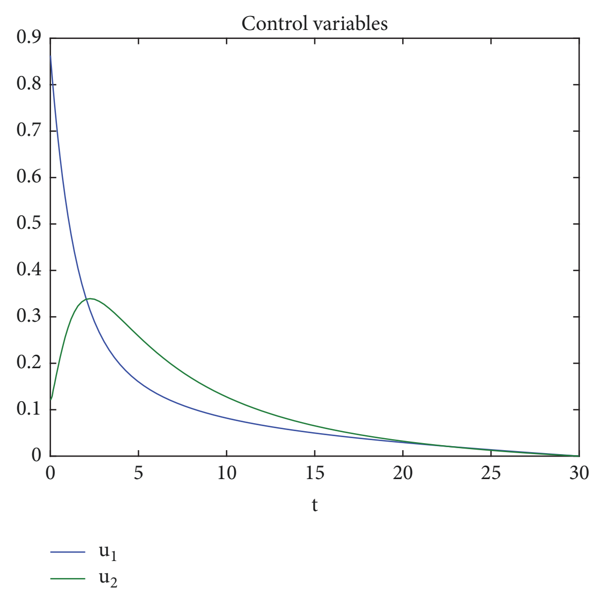

FIGURE 12: Optimal control function $u_1(t), u_2(t)$ for a controlled system.

and releasing official information strategies to control rumor. With this strategy, the two control variables u_1, u_2 are both used to optimize the objective function J.

Next, let $\tau = 0.1$; by calculation, it can satisfy optimality conditions. The trend of the change of $x(t)$, $y(t)$, $z(t)$ both with and without control is shown in **Figures 9**, **10,** and **11**. **Figure 12** gives the corresponding optimal control strategies.

Figure 9 shows that, in the presence of two control variables, the number of ignorant $(x(t))$ decreases fast in the beginning but the final size is larger compared to 'without control'. **Figure 10** shows that the number of spreaders $(y(t))$ increases slowly with control compared to 'without control' and the final size is much smaller. **Figure 11** shows that the number of the removal $(z(t))$ increases fast with control compared to 'without control' and the final size is much larger. **Figure 12** represents the optimal control $u_1(t)$, $u_2(t)$.

The comparison shows that combining two control strategies is effective in controlling rumor; not only can the number of spreaders be reduced to a greater extent, but also the cost of two control strategies can be greatly improved. First, we compare **Figures 9**, **5, and 1**; by promoting scientific knowledge, ignorant enhance their ability to identify rumor, so the number of ignorant $(x(t))$ decreases fast at the beginning; then, by releasing official information, most spreaders knowing the truth become the removal and little become ignorant, so the number of ignorant $(x(t))$ is larger compared to 'without control'. Next, we compare **Figures 10, 2, and 6**; we might find that the number of spreaders $(y(t))$ decreases most fast when using two control strategies than that when using only one control strategy; also the peak of spreaders $(y(t))$ is the smallest. Then, we compare **Figures 12, 8, and 4**; we find that the cost of applying two control strategies is smaller than that of applying only one control strategy. To control the rumor, using two control strategies together can control the rumor faster and has better control effect.

Advocating a two-pronged approach can be more effective in controlling the spreader; it has a realistic significance in responding to rumors. Therefore, when the government deals with the rumors, they should promote scientific knowledge in the beginning to prevent the transformation from the ignorant to the spreader and release official information to promote the transformation from the spreader to the removal.

3.4. The Impact of Time Delay on Control Variables. We simulate how different values of τ affect the control variables. Based on the above example, we fixed other parameters; the control variables $u_1(t)$, $u_2(t)$ for two different values of time delays $\tau = 0.1$, $\tau = 1$ are represented in **Figures 13** and **14**, respectively. It proves that introducing both time delay and control strategies into rumor spreading model can have a profound effect on the elimination of rumor spreading.

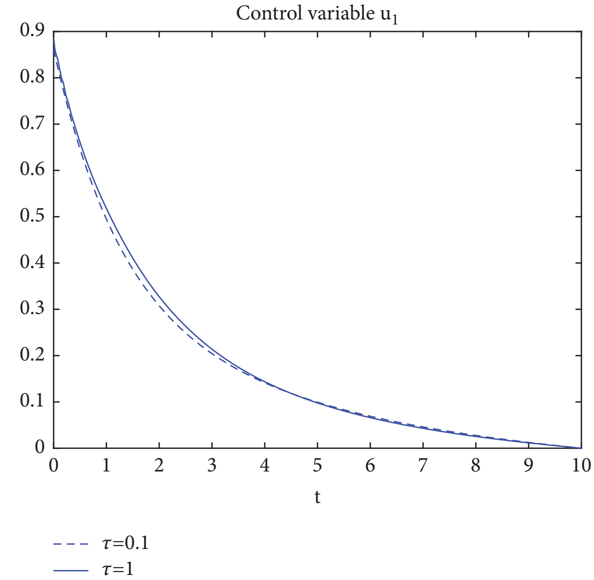

Figure 13: Optimal control function $u_1(t)$.

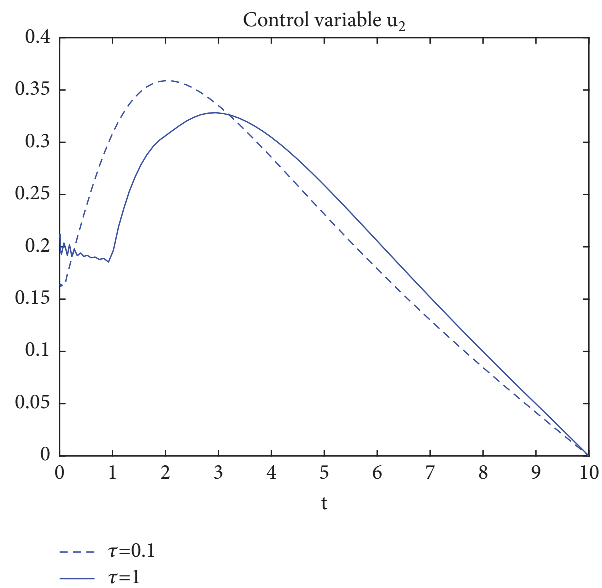

Figure 14: Optimal control function $u_2(t)$.

Figure 13 shows that control variable $u_1(t)$ has the maximum value at the beginning and then decreases in both of the two conditions, but the comparison showed that control variable $u_1(t)$ is smaller at small delay; when time delay is small, the effort of promoting scientific knowledge is not effective enough, and most ignorant will become spreaders because there is no enough time for them to learn scientific knowledge and identify the rumor. **Figure 14** shows that control variable $u_2(t)$ increases at the beginning and then decreases later in both of the two conditions, but the comparison showed that control variable $u_2(t)$ is larger at small delay; because the effort of promoting scientific knowledge is not effective enough, only by releasing official information, most spreaders know the truth and become the removal; control variable $u_2(t)$ decreases over time.

Furthermore, our result shows that the control cost has different values in two situations. When $\tau = 1$, the control cost is $J = 4.399 * 10^{-10}$; when time delay reduces to $\tau = 0.1$, the cost increases to $J = 8.031 * 10^{-10}$. That is to say, with the decreasing of time delay, control cost will continue to increase.

Therefore, the officials' refutation of the rumor must be fast, they should combine promoting scientific knowledge and releasing official information in time to direct the public opinion. If the government misses the moment, it will take a high price to eliminate negative public opinion later.

4. Conclusion

In this paper, we built an optimal control of rumor spreading model with consideration of psychological factors and time delay. Firstly, we considered a more realistic controlled model by including Holling-type II functional response (it can be used to interpret the psychological effect with rumor transmission in emergency) and time delay (a thinking time for the migration from the ignorant to the removal). Secondly, to reduce the number of spreaders, by introducing two control strategies of promoting scientific knowledge and releasing official information, we proposed an optimal control problem by minimizing both the number of spreaders and ignorant individuals and the budgets of two control strategies. Thirdly, by using Pontryagin's maximum principle, we proved the existence and uniqueness of an optimal control strategy and derived a necessary condition for optimal control strategies. Finally, we gave some numerical examples, which indicate that the proposed control strategies are effective in reducing the number of ignorant individuals and spreaders and minimizing the control cost.

No matter which strategy the government chooses, they are all helpful in reducing the spread of rumors. However, when used together, not only can the number of spreaders be reduced to a greater extent, but also the cost of two control strategies can be greatly improved. It has a realistic significance in responding to rumors. On the one hand, in the beginning, the government should promote scientific knowledge to public by phone, TV, and Internet and then prevent the transformation from the ignorant to the spreader. On the other hand, under the circumstances of the prevalence of rumor, the officials should announce some official information to guide the public opinion and eliminate the popular indignation, thus promoting the transformation from the spreader to the removal.

The results also showed that the government could not overlook the role of time delay when they deal with the rumors; if the government misses the moment, it will take a high price to eliminate negative public opinion later. Therefore, the government response must be as fast as possible to seize the initiative and win the support of network public opinion.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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