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Research Article

On $(\in, \in \vee q_k)$-Fuzzy Hyperideals in Ordered LA-Semihypergroups

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The concept of $(\in, \in \vee q_k)$-fuzzy hyperideal of an ordered LA-semihypergroup is introduced by the ordered fuzzy points, and related properties are investigated. We study the relations between different $(\in, \in \vee q_k)$-fuzzy hyperideal of an ordered LA-semihypergroup. Furthermore, we study some results on homomorphisms of $(\in, \in \vee q_k)$-fuzzy hyperideals.

1. Introduction

Hyperstructures represent a natural extension of classical algebraic structures and they were introduced by the French mathematician Marty [1]. Since then, several authors continued their researches in this direction. Nowadays hyperstructures are widely studied from theoretical point of view and for their applications in many subjects of pure and applied mathematics. In a classical algebraic structure, the composition of two elements is an element, while, in an algebraic hyperstructure, the composition of two elements is a set. Some basic definitions and theorems about hyperstructures can be found in [2, 3]. The concept of a semihypergroup is a generalization of the concept of a semigroup. Many authors studied different aspects of semihypergroups, for instance, Bonansinga and Corsini [4], Corsini [5] Davvaz [6], Davvaz et al. [7], Davvaz and Poursalavati [8], Gutan [9], Hasankhani [10], Hila et al. [11], and Onipchuk [12]. Recently, Hila and Dine [13] introduced the notion of LA-semihypergroups as a generalization of semigroups, semihypergroups, and LA-semihypergroups. Yaqoob, Corsini, and Yousafzai [14] extended the work of Hila and Dine and characterized intraregular left almost semihypergroups by their hyperideals using pure left identities. Other results on LA-semihypergroups can be found in [15, 16].

The concept of ordered semihypergroup was studied by Heidari and Davvaz in [17], where they used a binary relation "$\leq$" on semihypergroup $(H, \circ)$ such that the binary relation is a partial order and the structure $(H, \circ, \leq)$ is known as ordered semihypergroup. There are several authors who study the ordering of hyperstructures, for instance, Bakhshi and Borzooei [18], Chvalina [19], Hoskova [20], Kondo and Lekkoksung [21], and Novak [22]. The ordering in LA-semihypergroups was introduced by Yaqoob and Gulistan [23].

In 1965, the concept of fuzzy sets was first proposed by Zadeh [24], which has a wide range of applications in various fields such as computer engineering, artificial intelligence, control engineering, operation research, management science, robotics, and many more. Many papers on fuzzy sets have been published, showing the importance and their applications to set theory, group theory, real analysis, measure theory, and topology etc. There are several authors who applied the concept of fuzzy sets to algebraic hyperstructures. Ameri and others studied fuzzy sets in hypervector spaces [25], Γ-hyperrings [26], and polygroups [27]. Corsini et al. [28] studied semisipmle semihypergroups in terms of hyperideals and fuzzy hyperideals. Cristea [29] introduced hyperstructures and fuzzy sets endowed with two membership functions. In [30, 31], Davvaz introduced the concept of fuzzy hyperideals in a semihypergroup and Hv-semigroup. Recently in [32], Davvaz and Leoreanu-Fotea studied the structure of fuzzy Γ-hyperideals in Γ-semihypergroups. Also see [33, 34]. Fuzzy ordered hyperstructures have been considered by some researchers, for instance, Pibaljommee et al. [35, 36], Tang et al. [37–40], Tipachot and Pibaljommee [41].

Murali [42] defined the concept of belongingness of a fuzzy point to a fuzzy subset under a natural equivalence on a fuzzy subset. In [43], the idea of quasi-coincidence of a fuzzy point with a fuzzy set is defined. The concept of a (α, β)-fuzzy subgroup was first considered by Bhakat and Das in [44, 45] by using the "belongs to" relation $(\in)$ and "quasi coincident with" relation (q) between a fuzzy point and a fuzzy subgroup, where $\alpha, \beta \in \{\in, q, \in \vee q, \in \wedge q\}$ and $\alpha \neq \in \wedge q$. Jun et al. [46] gave the concept of $(\in, \in \vee q)$-fuzzy ordered semigroups. Moreover, $(\in, \in \vee q_k)$-fuzzy ideals, $(\in, \in \vee q_k)$ -fuzzy quasi-ideals, and $(\in, \in \vee q_k)$-fuzzy bi-ideals of a semigroup are defined in [47]. Azizpour and Talebi characterized semihypergroups by $(\in, \in \vee q_k)$-fuzzy interior hyperideals [48, 49]. Huang [50] provided characterizations of semihyperrings by their $(\in_\gamma, \in_\gamma, \vee q_\delta)$ -fuzzy hyperideals. Shabir and Mahmood characterized semihypergroups by $(\in, \in \vee q_k)$-fuzzy hyperideals [51] and also by $(\in_\gamma, \in_\gamma, \vee q_\delta)$-fuzzy hyperideals [52].

Motivated by the work of Shabir et al. [47, 51], we applied the concept of $(\in, \in \vee q_k)$-fuzzy sets to LA-semihypergroups. In this paper, we introduce and study several types of $(\in, \in \vee q_k)$-fuzzy hyperideals in an ordered LA-semihypergroup. Further, we study some results on homomorphisms of $(\in, \in \vee q_k)$-fuzzy hyperideals.

2. Preliminaries and Basic Definitions

In this section, we provide some basic definitions needed for our further work.

Let H be a nonempty set. Then the map $\circ : H \times H \longrightarrow \mathscr{P}^*(H)$ is called hyperoperation or join operation on the set H, where $\mathscr{P}^*(H) = \mathscr{P}(H)\backslash\{\emptyset\}$ denotes the set of all nonempty subsets of H. A hypergroupoid is a set H together with a (binary) hyperoperation. For any nonempty subsets A, B of H, we denote

$$A \circ B = \bigcup_{a \in A, b \in B} a \circ b \tag{1}$$

Instead of $\{a\} \circ A$ and $B \circ \{a\}$, we write $a \circ A$ and $B \circ a$, respectively.

Recently, in [13, 14] authors introduced the notion of LA-semihypergroups as a generalization of semigroups, semihypergroups, and LA-semihypergroups. A hypergroupoid $(H, \circ)$ is called an LA-semihypergroup if for every $x, y, z \in H$, we have $(x \circ y) \circ z = (z \circ y) \circ x$. The law $(x \circ y) \circ z = (z \circ y) \circ x$ is called a left invertive law. An element $e \in H$ is called a left identity (resp., pure left identity) if for all $x \in H$, $x \in e \circ x$ (resp., $x = e \circ x$). In an LA-semihypergroup, the medial law $(x \circ y) \circ (z \circ w) = (x \circ z) \circ (y \circ w)$ holds for all $x, y, z, w \in H$. An LA-semihypergroup may or may not contain a left identity and pure left identity. In an LA-semihypergroup H with pure left identity, the paramedial law $(x \circ y) \circ (z \circ w) = (w \circ z) \circ (y \circ x)$ holds for all $x, y, z, w \in H$. If an LA-semihypergroup contains a pure left identity, then by using medial law, we get $x \circ (y \circ z) = y \circ (x \circ z)$ for all $x, y, z \in H$.

Definition 1 (see [23]). Let H be a nonempty set and $\leq$ be an ordered relation on H. The triplet $(H, \circ, \leq)$ is called an ordered LA-semihypergroup if the following conditions are satisfied.

(1) $(H, \circ)$ is an LA-semihypergroup.

(2) $(H, \leq)$ is a partially ordered set.

(3) For every $a, b, c \in H$, $a \leq b$ implies $a \circ c \leq b \circ c$ and $c \circ a \leq \ c \circ b$, where $a \circ c \ \ \leq b \circ c$ means that for $x \in a \circ c$ there exist $y \in b \circ c$ such that $x \leq y$.

Definition 2 (see [23]). If $(H, \circ, \leq)$ is an ordered LA-semihypergroup and $A \subseteq H$, then $(A]$ is the subset of H defined as follows:

$$(A] = \{t \in H : t \leq a, \text{ for some } a \in A\}. \tag{2}$$

If $x \in H$ and A is a nonempty subset of H, then $A_x = \{(y, z) \in H \times H \mid x \leq y \circ z\}$.

Definition 3 (see [23]). A nonempty subset A of an ordered LA-semihypergroup $(H, \circ, \leq)$ is called an LA-subsemihypergroup of H if $(A \circ A] \subseteq (A]$.

Definition 4 (see [23]). A nonempty subset A of an ordered LA-semihypergroup $(H, \circ, \leq)$ is called a right (resp., left) hyperideal of H if

(1) $A \circ H \subseteq A$ (resp., $H \circ A \subseteq A$),

(2) for every $a \in H, b \in A$ and $a \leq b$ implies $a \in A$.

If A is both right hyperideal and left hyperideal of H, then A is called a hyperideal (or two sided hyperideal) of H.

Definition 5 (see [23]). A nonempty subset A of an ordered LA-semihypergroup $(H, \circ, \leq)$ is called an interior hyperideal of H if

(1) $(H \circ A) \circ H \subseteq A$,

(2) for every $a \in H, b \in A$ and $a \leq b$ implies $a \in A$.

Definition 6 (see [23]). A nonempty subset A of an ordered LA-semihypergroup $(H, \circ, \leq)$ is called a generalized bi-hyperideal of H if

(1) $(A \circ H) \circ A \subseteq A$,

(2) for every $a \in H, b \in A$ and $a \leq b$ implies $a \in A$.

If A is both an LA-subsemihypergroup and a generalized bi-hyperideal of H, then A is called a bi-hyperideal of H.

Definition 7. Let $(H, \circ, \leq)$ be an ordered LA-semihypergroup, and $a \in H$. Then a is said to be regular element of H if there exists an element $x \in H$ such that $a \leq (a \circ x) \circ a$, or equivalently $a \leq (a \circ H) \circ a$. If every element of H is regular then H is said to be a regular ordered LA-semihypergroup.

Now, we give some fuzzy logic concepts.

A fuzzy subset f of a universe X is a function from X into the unit closed interval $[0, 1]$, i.e., $f : X \longrightarrow [0, 1]$ (see [24]). A fuzzy subset f in a universe X of the form

$$f(y) = \begin{cases} t \in (0, 1] & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases} \tag{3}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t. For a fuzzy point x_t and a fuzzy set f in a set

X, Pu and Liu [43] introduced the symbol $x_t\alpha f$, where $\alpha \in \{\in, q, \in\vee q, \in\wedge q\}$, which means that

(i) $x_t \in f$ if $f(x) \geq t$,
(ii) $x_t q f$ if $f(x) + t > 1$,
(iii) $x_t \in\vee q f$ if $x_t \in f$ or $x_t q f$,
(iv) $x_t \in\wedge q f$ if $x_t \in f$ and $x_t q f$,
(v) $x_t \overline{\alpha} f$ if $x_t \alpha f$ does not hold.

For any two fuzzy subsets f and g of H, $f \leq g$ means that, for all $x \in H$, $f(x) \leq g(x)$. The symbols $f \wedge g$ and $f \vee g$ will mean the following fuzzy subsets:

$$\begin{aligned} f \wedge g : H &\longrightarrow [0,1] \mid x \longmapsto (f \wedge g)(x) \\ &= f(x) \wedge g(x) = \min\{f(x), g(x)\} \\ f \vee g : H &\longrightarrow [0,1] \mid x \longmapsto (f \vee g)(x) \\ &= f(x) \vee g(x) = \max\{f(x), g(x)\} \end{aligned} \tag{4}$$

for all $x \in H$.

The product of any fuzzy subsets f and g of H is defined by

$$\begin{aligned} f * g : H &\longrightarrow [0,1] \mid x \longmapsto (f * g)(x) \\ &= \begin{cases} \bigvee_{(y,z)\in A_x} \{f(y) \wedge g(z)\}, & \text{if } A_x \neq \emptyset \\ 0 & \text{if } A_x = \emptyset. \end{cases} \end{aligned} \tag{5}$$

For $\emptyset \neq A \subseteq H$, the characteristic function f_A of A is a fuzzy subset of H, defined by

$$f_A = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{if } x \notin A. \end{cases} \tag{6}$$

For any fuzzy subset f of H and for any $t \in (0,1]$, the set $U(f;t) = \{x \in H : f(x) \geq t\}$ is called a level subset of f.

Definition 8 (see [53]). Let $(H, \circ, \leq)$ be an ordered LA-semihypergroup. A fuzzy subset $f : H \longrightarrow [0,1]$ is called fuzzy LA-subsemihypergroup of H if the following assertion are satisfied.

(i) $\bigwedge_{z\leq a\circ b} f(z) \geq \min\{f(a), f(b)\}$,
(ii) if $a \leq b$ implies $f(a) \geq f(b)$, for every $a, b \in H$.

Definition 9 (see [53]). Let $(H, \circ, \leq)$ be an ordered LA-semihypergroup. A fuzzy subset $f : H \longrightarrow [0,1]$ is called a fuzzy right (resp., left) hyperideal of H if

(1) $\bigwedge_{z\leq a\circ b} f(z) \geq f(a)$ (resp., $\bigwedge_{z\leq a\circ b} f(z) \geq f(b)$),
(2) $a \leq b$ implies $f(a) \geq f(b)$, for every $a, b \in H$.

If f is both fuzzy right hyperideal and fuzzy left hyperideal of H, then f is called a fuzzy hyperideal of H.

Definition 10 (see [54]). Let $(H, \circ, \leq)$ be an ordered LA-semihypergroup. A fuzzy subset $f : H \longrightarrow [0,1]$ is called a fuzzy bi-hyperideal of H if

(1) $\bigwedge_{z\leq (a\circ b)\circ c} f(z) \geq \min\{f(a), f(c)\}$,
(2) $a \leq b$ implies $f(a) \geq f(b)$, for every $a, b \in H$.

Definition 11 (see [54]). Let $(H, \circ, \leq)$ be an ordered LA-semihypergroup. A fuzzy subset $f : H \longrightarrow [0,1]$ is called a fuzzy interior hyperideal of H if

(1) $\bigwedge_{z\leq (a\circ b)\circ c} f(z) \geq f(b)$,
(2) $a \leq b$ implies $f(a) \geq f(b)$, for every $a, b \in H$.

3. $(\in, \in\vee q_k)$-Fuzzy Hyperideals

In what follows, let H denote an ordered LA-semihypergroup, $k \in [0,1)$ and $t, r \in (0,1]$ unless otherwise is specified. Note that for a fuzzy point x_t and a fuzzy subset f, $x_t \in f$ means that $f(x) \geq t$, while $x_t q_k f$ means that $f(x) + t + k > 1$. Notice that $(\in, \in\vee q_k)$-fuzzy hyperideals are particular types of (α, β)-fuzzy hyperideals.

Definition 12. A fuzzy subset f of H is called an $(\in, \in\vee q_k)$-fuzzy LA-subsemihypergroup of H if for all $x, y, z \in H$ and $t \in (0,1]$, the following conditions hold:

(i) $x \leq y$, $y_t \in f \Longrightarrow x_t \in\vee q_k f$,
(ii) $x_t, y_u \in f \Longrightarrow z_{\min\{t,u\}} \in\vee q_k f$, for each $z \in x \circ y$.

Definition 13. A fuzzy subset f of H is called an $(\in, \in\vee q_k)$-fuzzy left hyperideal of H if for all $x, y, z \in H$ and $t \in (0,1]$, the following conditions hold.

(i) $x \leq y$, $y_t \in f \Longrightarrow x_t \in\vee q_k f$,
(ii) $y_t \in f \Longrightarrow z_t \in\vee q_k f$, for each $z \in x \circ y$.

Definition 14. A fuzzy subset f of H is called an $(\in, \in\vee q_k)$-fuzzy right hyperideal of H if for all $x, y, z \in H$ and $t \in (0,1]$, the following conditions hold.

(i) $x \leq y$, $y_t \in f \Longrightarrow x_t \in\vee q_k f$,
(ii) $x_t \in f \Longrightarrow z_t \in\vee q_k f$, for each $z \in x \circ y$.

A fuzzy subset f of H is called an $(\in, \in\vee q_k)$-fuzzy hyperideal of H if it is a left hyperideal and a right hyperideal of H.

Definition 15. Consider a set $H = \{a, b, c, d\}$ with the following hyperoperation "$\circ$" and the order "$\leq$":

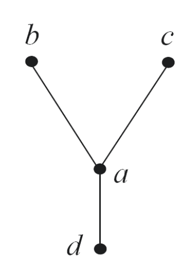
$\circ$	a	b	c	d
a	a	$\{a,d\}$	$\{a,d\}$	d
b	$\{a,d\}$	$\{b,c\}$	$\{b,c\}$	d
c	$\{a,d\}$	b	b	d
d	d	d	d	d

(7)

$$\leq := \{(a,a), (a,b), (a,c), (b,b), (c,c), (d,a), (d,b), (d,c), (d,d)\}.$$

We give the covering relation "$\prec$" and the figure of H as follows:

$$\prec = \{(a,b), (a,c), (d,a)\}.$$



(8)

Then $(H, \circ, \leq)$ is an ordered LA-semihypergroup. Now let f be a fuzzy subset of H such that

$$f : H \longrightarrow [0,1] \mid x \longmapsto f(x) = \begin{cases} 0.6 & \text{if } x = a \\ 0.4 & \text{if } x \in \{b, c\} \\ 0.9 & \text{if } x = d. \end{cases} \quad (9)$$

Let $t = 0.4$ and $k \in [0, 1)$. Then by routine calculations it is clear that f is an $(\in, \in \vee q_k)$-fuzzy hyperideal of H.

Definition 16. A fuzzy subset f of H is called an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H if for all $x, y, z, w \in H$ and $t \in (0, 1]$, the following conditions hold:

(i) $x \leq y$, $y_t \in f \Longrightarrow x_t \in \vee q_k f$,

(ii) $y_t \in f \Longrightarrow w_t \in \vee q_k f$, for each $w \in (x \circ y) \circ z$.

Example 17. Consider a set $H = \{a, b, c, d\}$ with the following hyperoperation "$\circ$" and the order "$\leq$":

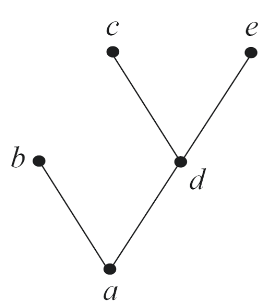
$\circ$	a	b	c	d	e
a	a	a	a	a	a
b	a	b	c	$\{a,d\}$	e
c	a	e	$\{c,e\}$	$\{a,d\}$	e
d	a	$\{a,d\}$	$\{a,d\}$	d	$\{a,d\}$
e	a	c	c	$\{a,d\}$	$\{c,e\}$

(10)

$$\leq := \{(a,a), (a,b), (a,c), (a,d), (a,e), (b,b), (c,c), (d,c), (d,d), (d,e), (e,e)\}.$$

We give the covering relation "$\prec$" and the figure of H as follows:

$$\prec = \{(a,b), (a,d), (d,c), (d,e)\}.$$



(11)

Then $(H, \circ, \leq)$ is an ordered LA-semihypergroup. Now let f be a fuzzy subset of H such that

$$f : H \longrightarrow [0,1] \mid x \longmapsto f(x) = \begin{cases} 0.8 & \text{if } x = a \\ 0.4 & \text{if } x = b \\ 0.5 & \text{if } x \in \{c, e\} \\ 0.7 & \text{if } x = d. \end{cases} \quad (12)$$

Let $t = 0.3$ and $k \in [0, 1)$. Then by routine calculations it is clear that f is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H.

Definition 18. A fuzzy subset f of H is called an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H if for all $x, y, z, w \in H$ and $t \in (0, 1]$, the following conditions hold:

(i) $x \leq y$, $y_t \in f \Longrightarrow x_t \in \vee q_k f$,

(ii) $x_t, y_u \in f \Longrightarrow z_{\min\{t,u\}} \in \vee q_k f$, for each $z \in x \circ y$.

(iii) $x_t, z_u \in f \Longrightarrow w_{\min\{t,u\}} \in \vee q_k f$, for each $w \in (x \circ y) \circ z$.

A fuzzy subset f of H is called an $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal of H if it satisfies (i) and (iii) conditions of Definition 18.

Theorem 19. *Let A be an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H and f a fuzzy subset of H defined by*

$$f(x) = \begin{cases} \geq \dfrac{1-k}{2} & \textit{if } x \in A \\ 0 & \textit{otherwise}. \end{cases} \quad (13)$$

Then

(i) f is a $(q, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H,

(ii) f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H.

Proof. (i) Consider that A is an LA-subsemihypergroup of H. Let $x, y \in H$, $x \leq y$ and $t \in (0, 1]$ be such that $y_t q f$. Then $y \in A$, $f(y) + t > 1$. Since A is an LA-subsemihypergroup of H and $x \leq y \in A$, we have $x \in A$. Thus $f(x) \geq (1-k)/2$. If $t \leq (1-k)/2$, then $f(x) \geq t$ and so $x_t \in f$. If $t > (1-k)/2$, then $f(x)+t+k > (1-k)/2+(1-k)/2+k = 1$ and so $x_t q_k f$. Therefore $x_t \in \vee q_k f$. Let $x, y, z \in H$ and $t, r \in (0, 1]$ be such that $x_t q f$ and $y_r q f$. Then $x, y \in A$, $f(x)+t > 1$ and $f(x)+r > 1$. Since A is an LA-subsemihypergroup of H, we have $x \circ y \subseteq A$. Thus for every $z \in x \circ y$, $f(z) \geq (1-k)/2$. If $\min\{t, r\} \leq (1-k)/2$, then $f(z) \geq \min\{t, r\}$ and so $z_{\min\{t,r\}} \in f$. If $\min\{t, r\} > (1-k)/2$, then $f(z) + \min\{t, r\} + k > (1-k)/2 + (1-k)/2 + k = 1$ and so $z_{\min\{t,r\}} q_k f$. Therefore $z_{\min\{t,r\}} \in \vee q_k f$, for all $z \in x \circ y$.

(ii) Let $x, y \in H$, $x \leq y$ and $t \in (0, 1]$ be such that $y_t \in f$. Then $f(y) \geq t$ and $y \in A$. Since $x \leq y \in A$, we have $x \in A$. Thus $f(x) \geq (1-k)/2$. If $t \leq (1-k)/2$, then $f(x) \geq t$ and so $x_t \in f$. If $t > (1-k)/2$, then $f(x) + t + k > (1-k)/2 + (1-k)/2 + k = 1$ and so $x_t q_k f$. Therefore $x_t \in \vee q_k f$. Let $x, y, z \in H$ and $t, r \in (0, 1]$ be such that $x_t \in f$ and $y_r \in f$. Then $f(x) \geq t > 0$ and $f(y) \geq r > 0$. Thus $f(x) \geq (1-k)/2$ and $f(y) \geq (1-k)/2$, this implies that $x, y \in A$. Since A is an LA-subsemihypergroup of H, we have $x \circ y \subseteq A$. Thus for every $z \in x \circ y$, $f(z) \geq (1-k)/2$. If $\min\{t, r\} \leq (1-k)/2$, then $f(z) \geq \min\{t, r\}$ and so $z_{\min\{t,r\}} \in f$. If $\min\{t, r\} > (1-k)/2$, then $f(z) + \min\{t, r\} + k > (1-k)/2 + (1-k)/2 + k = 1$ and so $z_{\min\{t,r\}} q_k f$. Therefore $z_{\min\{t,r\}} \in \vee q_k f$, for all $z \in x \circ y$.

The cases for left hyperideal, right hyperideal, hyperideal, interior hyperideal and bi-hyperideal can be seen in a similar way. $\square$

Corollary 20. *If a nonempty subset A of H is an LA-subsemihypergroup (resp., left hyperideal, right hyperideal,*

hyperideal, interior hyperideal, and bi-hyperideal) of H, then the characteristic function of A is a $(q, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H.

Corollary 21. *A nonempty subset A of H is an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H if and only if f_A is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi- and hyperideal) of H.*

If we take $k = 0$ in Theorem 19, then we have the following corollary.

Corollary 22. *Let A be an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H and f a fuzzy subset of H defined by*

$$f(x) = \begin{cases} \geq 0.5 & \text{if } x \in A \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Then

(i) f is a $(q, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H,

(ii) f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H.

Theorem 23. *Let f be a fuzzy subset of H. Then, f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H if and only if*

(i) $x \leq y \Longrightarrow f(x) \geq \min\{f(y), (1-k)/2\}$, for all $x, y \in H$;

(ii) $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\}$, for all $x, y, z \in H$.

Proof. Assume that f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H. Let $x, y \in H$ be such that $x \leq y$. If $f(y) = 0$, then $f(x) \geq \min\{f(y), (1-k)/2\}$. Let $f(y) \neq 0$ and assume on the contrary that $f(x) < \min\{f(y), (1-k)/2\}$. Choose $t \in (0, 1]$ such that $f(x) < t \leq \min\{f(y), (1-k)/2\}$. If $f(y) < (1-k)/2$, then $f(x) < t < f(y)$, so $y_t \in f$, but $f(x) + t + k < (1-k)/2 + (1-k)/2 + k = 1$. This implies that $x_t \overline{q_k} f$, therefore $x_t \overline{\in \vee q_k} f$, which is a contradiction. Hence $f(x) \geq \min\{f(y), (1-k)/2\}$, for all $x, y \in H$.

Suppose on the contrary that there exist $x, y, z \in H$ such that $\bigwedge_{z \in x \circ y} f(z) < \min\{f(x), f(y), (1-k)/2\}$. Then there exists $z \in x \circ y$ such that $f(z) < \min\{f(x), f(y), (1-k)/2\}$. Choose $t \in (0, 1]$ such that $f(z) < t < \min\{f(x), f(y), (1-k)/2\}$. Then $f(x) > t$ and $f(y) > t$ implies that $x_t \in f$ and $y_t \in f$, but $f(z) < t$ and $f(z) + t + k < (1-k)/2 + (1-k)/2 + k = 1$. So $z_t \overline{\in \vee q_k} f$, which is a contradiction. Hence $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\}$.

Conversely, let $y_t \in f$ for $t \in (0, 1]$. Then $f(y) \geq t$. Now $f(x) \geq \min\{f(y), (1-k)/2\} \geq \min\{t, (1-k)/2\}$. If $t > (1-k)/2$, then $f(x) \geq (1-k)/2$ and $f(x) + t + k > (1-k)/2 + (1-k)/2 + k = 1$, it follows that $x_t q_k f$. If $t \leq (1-k)/2$, then $f(x) \geq t$ and so $x_t \in f$. Thus $x_t \in \vee q_k f$.

Now, assume that $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\}$ for all $x, y, z \in H$. Let $x_t \in f$ and $y_r \in f$ for all $t, r \in (0, 1]$. Then $f(x) \geq t$ and $f(y) \geq r$. Thus $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\} \geq \min\{t, r, (1-k)/2\}$. Now if $\min\{t, r\} > (1-k)/2$, then $\bigwedge_{z \in x \circ y} f(z) \geq (1-k)/2$. So for every $z \in x \circ y$, $f(z) + \min\{t, r\} + k > (1-k)/2 + (1-k)/2 + k = 1$, which implies that $z_{\min\{t,r\}} q_k f$. If $\min\{t, r\} < (1-k)/2$, then $\bigwedge_{z \in x \circ y} f(z) \geq \min\{t, r\}$. So for every $z \in x \circ y$, $z_{\min\{t,r\}} \in f$. Thus $z_{\min\{t,r\}} q_k f$. Therefore f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H. □

Theorem 24. *Let f be a fuzzy subset of H. Then, f is an $(\in, \in \vee q_k)$-fuzzy left (resp., right) hyperideal of H if and only if*

(i) $x \leq y \Longrightarrow f(x) \geq \min\{f(y), (1-k)/2\}$, for all $x, y \in H$.

(ii) $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(y), (1-k)/2\}$ (resp., $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(x), (1-k)/2\}$), for all $x, y, z \in H$.

Proof. The proof is similar to the proof of Theorem 23. □

Theorem 25. *Let f be a fuzzy subset of H. Then, f is an $(\in, \in \vee q_k)$ -fuzzy interior hyperideal of H if and only if*

(i) $x \leq y \Longrightarrow f(x) \geq \min\{f(y), (1-k)/2\}$, for all $x, y \in H$.

(ii) $\bigwedge_{w \in (x \circ y) \circ z} f(t) \geq \min\{f(y), (1-k)/2\}$, for all $x, y, z, w \in H$.

Proof. The proof is similar to the proof of the Theorem 23. □

Theorem 26. *Let f be a fuzzy subset of H. Then, f is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H if and only if*

(i) $x \leq y \Longrightarrow f(x) \geq \min\{f(y), (1-k)/2\}$, for all $x, y \in H$.

(ii) $\bigwedge_{z \in x \circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\}$, for all $x, y, z \in H$.

(ii) $\bigwedge_{w \in (x \circ y) \circ z} f(t) \geq \min\{f(x), f(z), (1-k)/2\}$, for all $x, y, z, t \in H$.

Proof. The proof is similar to the proof of Theorem 23. □

A fuzzy subset f of H is called an $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal of H if it satisfies (i) and (iii) conditions of Theorem 26.

Theorem 27. *A fuzzy subset f of an ordered LA-semihypergroup H is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H if and only if $U(f; t) (\neq 0)$ is an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, and bi-hyperideal) of H for all $t \in (0, (1-k)/2]$.*

Proof. Assume that f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H. Let $x, y, z \in H$ be such that $x \leq y \in U(f; t)$ where $t \in (0, (1-k)/2]$. Then $f(y) \geq t$ and by Theorem 23, $f(x) \geq \min\{f(y), (1-k)/2\} \geq \min\{t, (1-k)/2\} = t$. It follows that $x \in U(f; t)$. Let $x, y \in U(f; t)$ where $t \in (0, (1-k)/2]$. Then $f(x) \geq t$ and $f(y) \geq t$. It follows from Theorem 23 that $\bigwedge_{z\in x\circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\} \geq \min\{t, t, (1-k)/2\} = t$. Thus for every $z \in x \circ y$, $f(z) \geq t$ and so $z \in U(f;t)$, that is $x \circ y \subseteq U(f;t)$. Hence $U(f;t)$ is an LA-subsemihypergroup of H.

Conversely, assume that $U(f;t)(\neq 0)$ is an LA-subsemihypergroup of H for all $t \in (0,(1-k)/2]$. If there exist $x, y \in H$ with $x \leq y$ such that $f(x) < \min\{f(y), (1-k)/2\}$. Then $f(x) < t < \min\{f(y), (1-k)/2\}$ for some $t \in (0,(1-k)/2]$. Then, $y \in U(f;t)$ but $x \notin U(f;t)$, a contradiction. Thus $f(x) \geq \min\{f(y), (1-k)/2\}$ for all $x, y \in H$ with $x \leq y$. Now, suppose that there exist $x, y \in H$ such that $\bigwedge_{z\in x\circ y} f(z) < \min\{f(x), f(y), (1-k)/2\}$. Thus there exist $z \in x \circ y$ such that $f(z) < \min\{f(x), f(y), (1-k)/2\}$. Choose $t \in (0,(1-k)/2]$ such that $f(z) < t < \min\{f(x), f(y), (1-k)/2\}$. Then $x, y \in U(f;t)$ but $z \notin U(f;t)$ i.e., $x \circ y \nsubseteq U(f;t)$, which is a contradiction. Hence $\bigwedge_{z\in x\circ y} f(z) \geq \min\{f(x), f(y), (1-k)/2\}$ and so f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H. □

Corollary 28. *A fuzzy subset f of an ordered LA-semihypergroup H is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal) of H if and only if $U(f;t)(\neq 0)$ is an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal) of H for all $t \in (0, 0.5]$.*

Example 29. Consider a set $H = \{a, b, c, d\}$ with the following hyperoperation "$\circ$" and the order "$\leq$":

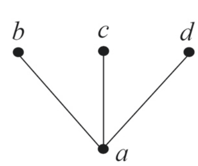
$\circ$	a	b	c	d
a	a	a	a	a
b	a	b	c	d
c	a	d	$\{c,d\}$	$\{c,d\}$
d	a	c	$\{c,d\}$	$\{c,d\}$

(15)

$$\leq := \{(a,a), (a,b), (a,c), (a,d), (b,b), (c,c), (d,d)\}.$$

We give the covering relation "$\prec$" and the figure of H as follows:

$$\prec = \{(a,b), (a,c), (a,d)\}.$$



(16)

Then $(H, \circ, \leq)$ is an ordered LA-semihypergroup. Here, $\{a\}$, $\{b\}$, $\{a,b\}$, $\{a,c,d\}$, $\{b,c,d\}$ and $\{c,d\}$ are LA-subsemihypergroups of H. Now let f be a fuzzy subset of H such that

$$f : H \longrightarrow [0,1] \mid x \longmapsto f(x) = \begin{cases} 0.8 & \text{if } x = a \\ 0.2 & \text{if } x = b \\ 0.6 & \text{if } x = c \\ 0.7 & \text{if } x = d \end{cases} \tag{17}$$

and

$$U(f;t) = \begin{cases} H & \text{if } 0 < t \leq 0.2 \\ \{a,c,d\} & \text{if } 0.2 < t \leq 0.6 \\ \{a\} & \text{if } 0.6 < t \leq 0.8 \\ \emptyset & \text{if } 0.8 < t \leq 1. \end{cases} \tag{18}$$

Then by Theorem 27, f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H for $k \in [0,1)$.

Lemma 30. *The intersection of any family of $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroups (resp., left hyperideals, right hyperideals, interior hyperideals, bi-hyperideals, generalized bi-hyperideals) of H is an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, interior hyperideal, bi-hyperideal, generalized bi-hyperideal) of H.*

Proof. Let $\{f_i\}_{i\in I}$ be a family of $(\in, \in \vee q_k)$-fuzzy left hyperideals of H and $x, y \in H$. Then

$$\bigwedge_{z\in x\circ y} \left\{\underset{i\in I}{\wedge} f_i(z)\right\} = \underset{i\in I}{\wedge} \left\{\bigwedge_{z\in x\circ y} f_i(z)\right\}. \tag{19}$$

Since f_i is a $(\in, \in \vee q_k)$-fuzzy left hyperideals of H. Thus

$$\begin{aligned} \bigwedge_{z\in x\circ y} \left\{\underset{i\in I}{\wedge} f_i(z)\right\} &\geq \underset{i\in I}{\wedge} \left\{f_i(y) \wedge \frac{1-k}{2}\right\} \\ &= \left(\underset{i\in I}{\wedge} f_i(y)\right) \wedge \frac{1-k}{2} \\ &= \left(\underset{i\in I}{\wedge} f_i\right)(y) \wedge \frac{1-k}{2}. \end{aligned} \tag{20}$$

Hence $\wedge_{i\in I} f_i$ is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. The other cases can be seen in a similar way. □

Lemma 31. *The union of any family of $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroups (resp., left hyperideals, right hyperideals, interior hyperideals, bi-hyperideals) of an ordered LA-semihypergroup H is an LA-subsemihypergroup (resp., left hyperideal, right hyperideal, interior hyperideal, bi-hyperideal) of H.*

Proof. The proof is similar to the proof of the Lemma 30. □

Proposition 32. *Let H be an ordered LA-semihypergroup with pure left identity e. Then every $(\in, \in \vee q_k)$-fuzzy right hyperideal of H is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H. For $x, y, z \in H$, we have

$$\begin{aligned} \bigwedge_{z\in x\circ y} f(z) = \bigwedge_{z\in (e\circ x)\circ y} f(z) = \bigwedge_{z\in (y\circ x)\circ e} f(z) \\ \geq \min\left\{\bigwedge_{w\in y\circ x} f(w), \frac{1-k}{2}\right\} \end{aligned}$$

$$\geq \min\left\{f(y), \frac{1-k}{2}, \frac{1-k}{2}\right\}$$
$$= \min\left\{f(y), \frac{1-k}{2}\right\}. \tag{21}$$

Thus f is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. □

Proposition 33. *Let H be an ordered LA-semihypergroup with pure left identity e. Then f is an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H if and only if it is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H. For $x, y, z \in H$, we have

$$\begin{aligned}\bigwedge_{a\in(x\circ y)\circ z} f(a) &\geq \min\left\{\bigwedge_{w\in x\circ y} f(w), \frac{1-k}{2}\right\}\\ &= \min\left\{\bigwedge_{w\in(e\circ x)\circ y} f(w), \frac{1-k}{2}\right\}\\ &= \min\left\{\bigwedge_{w\in(y\circ x)\circ e} f(w), \frac{1-k}{2}\right\}\\ &\geq \min\left\{\bigwedge_{s\in y\circ x} f(s), \frac{1-k}{2}, \frac{1-k}{2}\right\}\\ &\geq \min\left\{f(y), \frac{1-k}{2}, \frac{1-k}{2}, \frac{1-k}{2}\right\}\\ &= \min\left\{f(y), \frac{1-k}{2}\right\},\end{aligned} \tag{22}$$

which implies that f is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal. Conversely, for any x and y in H we have

$$\bigwedge_{a\in x\circ y} f(a) = \bigwedge_{a\in(e\circ x)\circ y} f(a) \geq \min\left\{f(x), \frac{1-k}{2}\right\}. \tag{23}$$

This shows that f is an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H. This completes the proof. □

Proposition 34. *Let f be an $(\in, \in \vee q_k)$-fuzzy left hyperideal of an ordered LA-semihypergroup H with pure left identity e. If f is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H, then it is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. Then for every $z \in H$, there exist $x, y \in H$ such that

$$\bigwedge_{z\in x\circ y} f(z) \geq \min\left\{f(y), \frac{1-k}{2}\right\}. \tag{24}$$

Let e be a pure left identity in H. So,

$$\bigwedge_{z\in x\circ y} f(z) = \bigwedge_{z\in(e\circ x)\circ y} f(z) \geq \min\left\{f(x), \frac{1-k}{2}\right\}. \tag{25}$$

This implies that

$$\bigwedge_{z\in x\circ y} f(z) \geq \min\left\{f(x), f(y), \frac{1-k}{2}\right\}. \tag{26}$$

Thus f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup of H. Now for any $x, y, z \in H$,

$$\begin{aligned}\bigwedge_{a\in(x\circ y)\circ z} f(a) &= \bigwedge_{a\in(x\circ(e\circ y))\circ z} f(a) = \bigwedge_{a\in(e\circ(x\circ y))\circ z} f(a)\\ &\geq \min\left\{\bigwedge_{t\in x\circ y} f(t), \frac{1-k}{2}\right\}\\ &= \min\left\{\bigwedge_{t\in(e\circ x)\circ y} f(t), \frac{1-k}{2}\right\}\\ &\geq \min\left\{f(x), \frac{1-k}{2}, \frac{1-k}{2}\right\}\\ &= \min\left\{f(x), \frac{1-k}{2}\right\}.\end{aligned} \tag{27}$$

Also

$$\begin{aligned}\bigwedge_{a\in(x\circ y)\circ z} f(a) &= \bigwedge_{a\in(z\circ y)\circ x} f(a) = \bigwedge_{a\in(e\circ(z\circ y))\circ x} f(a)\\ &\geq \min\left\{\bigwedge_{t\in z\circ y} f(t), \frac{1-k}{2}\right\}\\ &= \min\left\{\bigwedge_{t\in(e\circ z)\circ y} f(t), \frac{1-k}{2}\right\}\\ &\geq \min\left\{f(z), \frac{1-k}{2}, \frac{1-k}{2}\right\}\\ &= \min\left\{f(z), \frac{1-k}{2}\right\}.\end{aligned} \tag{28}$$

Hence we get

$$\bigwedge_{a\in(x\circ y)\circ z} f(a) \geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}. \tag{29}$$

This shows that f is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H. □

Proposition 35. *Let H be an ordered LA-semihypergroup with pure left identity. If f is an $(\in, \in \vee q_k)$-fuzzy subset of H and g is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H, then $f * g$ is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H.*

Proof. Let $x, y \in H$ such that $x \le y$. Let $(a,b) \in A_y$ then $y \le a \circ b$. Since $x \le y$, so $x \le a \circ b$ implies $(a,b) \in A_x$. Hence $A_y \subseteq A_x$. Now

$$\begin{aligned}(f * g)(y) &= \bigvee_{(a,b)\in A_y} \min\{f(a), g(b)\} \\ &= \bigvee_{(a,b)\in A_y \subseteq A_x} \{f(a), g(b)\} \\ &\le \bigvee_{(a,b)\in A_x} \{f(a), g(b)\} = (f * g)(x).\end{aligned} \tag{30}$$

Thus $(f * g)(x) \ge (f * g)(y)$. Let $x, y \in H$. Then

$$\begin{aligned}&\min\left\{(f * g)(y), \frac{1-k}{2}\right\} \\ &\quad = \min\left\{\left(\bigvee_{(p,q)\in A_y} \min\{f(p), g(q)\}\right), \frac{1-k}{2}\right\} \\ &\quad = \bigvee_{(p,q)\in A_y} \min\left\{f(p), g(q), \frac{1-k}{2}\right\}.\end{aligned} \tag{31}$$

If $y \in p \circ q$, then $x \circ y \subseteq x \circ (p \circ q) = p \circ (x \circ q)$. Now for each $z \in x \circ y$, there exist $a \in x \circ q$ such that $z \in p \circ a$. Since g is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H, $\bigwedge_{a\in x\circ q} g(a) \ge \min\{g(q), (1-k)/2\}$, that is $g(a) \ge \min\{g(q), (1-k)/2\}$. Thus

$$\begin{aligned}&\min\left\{(f * g)(y), \frac{1-k}{2}\right\} \\ &\quad \le \bigvee_{(p,a)\in A_z} \min\{f(p), g(a)\} = (f * g)(z) \\ &\qquad\qquad \text{for every } z \in x \circ y \subseteq p \circ a.\end{aligned} \tag{32}$$

So $\bigwedge_{z\in x\circ y}(f * g)(z) \ge \min\{(f * g)(y), (1-k)/2\}$. Thus $f * g$ is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. □

Definition 36. Let f and g be any two fuzzy subsets of H. We define the product $f *_k g$ by

$$(f *_k g)(x) = \begin{cases} \bigvee_{(y,z)\in A_x} \min\left\{f(y), g(z), \frac{1-k}{2}\right\} & \text{if } A_x \ne \emptyset \\ 0 & \text{if } A_x = \emptyset. \end{cases} \tag{33}$$

Definition 37. The symbols f_k, $f \cap_k g$ and $f \cup_k g$ mean the following fuzzy subsets of H:

$$\begin{aligned} f_k(x) &= \min\left\{f(x), \frac{1-k}{2}\right\}, \quad \forall x \in H, \\ (f \cap_k g)(x) &= \min\left\{f(x), g(x), \frac{1-k}{2}\right\}, \quad \forall x \in H, \\ (f \cup_k g)(x) &= \min\left\{f(x), g(x), \frac{1-k}{2}\right\}, \quad \forall x \in H. \end{aligned} \tag{34}$$

Denote by $\mathcal{F}(H)$ the family of all fuzzy subsets in H.

Theorem 38. *Let H be an ordered LA-semihypergroup. Then the set $(\mathcal{F}(H), *_k, \subseteq)$ is an ordered LA-semihypergroup.*

Proof. Clearly $\mathcal{F}(H)$ is closed. Let f, g and h be in $\mathcal{F}(H)$ and let x be any element of H such that it is not expressible as product of two elements in H. Then we have

$$((f *_k g) *_k h)(x) = 0 = ((h *_k g) *_k f)(x). \tag{35}$$

Let $A_x \ne \emptyset$. Then there exist y and z in H such that $(y, z) \in A_x$. Therefore by using left invertive law, we have

$$\begin{aligned}((f *_k g) *_k h)(x) &= \bigvee_{(y,z)\in A_x} \left\{(f *_k g)(y) \wedge h(z) \wedge \frac{1-k}{2}\right\} \\ &= \bigvee_{(y,z)\in A_x} \left\{\bigvee_{(p,q)\in A_y} \left\{\left\{f(p) \wedge g(q) \wedge \frac{1-k}{2}\right\} \wedge h(z) \wedge \frac{1-k}{2}\right\}\right\} \\ &= \bigvee_{x\le(p\circ q)\circ z} \left\{f(p) \wedge g(q) \wedge h(z) \wedge \frac{1-k}{2} \wedge \frac{1-k}{2}\right\} \\ &= \bigvee_{x\le(z\circ q)\circ p} \left\{h(z) \wedge g(q) \wedge f(p) \wedge \frac{1-k}{2} \wedge \frac{1-k}{2}\right\} \\ &= \bigvee_{(w,p)\in A_x} \left\{\bigvee_{(z,q)\in A_w} \left(h(z) \wedge g(q) \wedge f(p) \wedge \frac{1-k}{2} \wedge \frac{1-k}{2}\right)\right\} \\ &= \bigvee_{(w,p)\in A_x} \left\{(h *_k g)(w) \wedge f(p) \wedge \frac{1-k}{2}\right\} = ((h *_k g) *_k f)(x).\end{aligned} \tag{36}$$

Hence $(\mathcal{F}(H), *_k)$ is an ordered LA-semihypergroup. Assume that $f \subseteq g$ and let $A_x = \emptyset$ for any $x \in H$; then

$$(f *_k h)(x) = 0 = (g *_k h)(x) \Longrightarrow f *_k h \subseteq g *_k h. \tag{37}$$

Similarly we can show that $f *_k h \supseteq g *_k h$. Let $A_x \ne \emptyset$. Then there exist y and z in H such that $(y, z) \in A_x$, therefore

$$\begin{aligned}(f *_k h)(x) &= \bigvee_{(y,z)\in A_x} \left\{f(y) \wedge h(z) \wedge \frac{1-k}{2}\right\} \\ &\le \bigvee_{(y,z)\in A_x} \left\{g(y) \wedge h(z) \wedge \frac{1-k}{2}\right\} \\ &= (g *_k h)(x),\end{aligned} \tag{38}$$

Similarly we can show that $f *_k h \supseteq g *_k h$. It is easy to see that $\mathcal{F}(H)$ is a poset. Thus $(\mathcal{F}(H), *, \subseteq)$ is an ordered LA-semihypergroup. □

Theorem 39. *Let H be an ordered LA-semihypergroup. Then the property*

$$(f *_k g) *_k (h *_k k) = (f *_k h) *_k (g *_k k) \tag{39}$$

holds in $\mathcal{F}(H)$, for all f, g, h and k in $\mathcal{F}(H)$.

Proof. The proof is straightforward. □

Theorem 40. *If an ordered LA-semihypergroup H has a pure left identity, then the following properties hold in $\mathcal{F}(H)$.*
(i) $(f *_k g) *_k (h *_k k) = (k *_k h) *_k (g *_k f)$,
(ii) $f *_k (g *_k h) = g *_k (f *_k h)$,
for all f, g, h and k in $\mathcal{F}(H)$.

Proof. The proof is straightforward. □

Proposition 41. *Let f be an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H and g is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. Then $f *_k g \subseteq f \cap_k g$.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H and g an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. Let $z \in H$ and suppose that there exist $x, y \in H$ such that $z \in x \circ y$. Then

$$\begin{aligned}(f *_k g)(z) &= \bigvee_{(x,y)\in A_z} \min\left\{f(x), g(y), \frac{1-k}{2}\right\} \\ &= \bigvee_{(x,y)\in A_z} \min\left\{f(x), g(y), \frac{1-k}{2}, \frac{1-k}{2}, \frac{1-k}{2}\right\} \\ &\leq \bigvee_{(x,y)\in A_z} \min\left\{\bigwedge_{z\in x\circ y} f(x), \bigwedge_{z\in x\circ y} g(y), \frac{1-k}{2}\right\} \\ &= \min\left\{f(x), g(y), \frac{1-k}{2}\right\} = f \cap_k g.\end{aligned} \tag{40}$$

Let us suppose that there do not exist any $x, y \in H$ such that $z \in x \circ y$. Then, $(f *_k g)(z) = 0 \leq f \cap_k g$. Hence $f *_k g \subseteq f \cap_k g$. □

Lemma 42. *Let f be an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal, generalized bi-hyperideal) of H. Then f_k is a fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal, generalized bi-hyperideal) of H.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H. Then for all $x, y, z \in H$, we have $x \leq y \Longrightarrow f(x) \geq \min\{f(y), (1-k)/2\}$ and $\bigwedge_{z\in x\circ y} f(z) \geq \min\{f(x), (1-k)/2\}$. This implies that

$$\min\left\{f(x), \frac{1-k}{2}\right\} \geq \min\left\{f(y), \frac{1-k}{2}\right\}. \tag{41}$$

So $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$. Also

$$\min\left\{\bigwedge_{z\in x\circ y} f(z), \frac{1-k}{2}\right\} \geq \min\left\{f(x), \frac{1-k}{2}\right\}. \tag{42}$$

So $\bigwedge_{z\in x\circ y} f_k(z) \geq f_k(x)$. Thus f_k is a fuzzy right hyperideal of H. Other cases can be seen in a similar way. □

For an ordered LA-semihypergroups H, the fuzzy subset $\mathcal{H}$ is defined as follows:

$$\mathcal{H} : H \longrightarrow [0,1] \mid x \longmapsto \mathcal{H}(x) := 1. \tag{43}$$

Proposition 43. *For a fuzzy subset f of H, the following conditions are true for all $x, y \in H$.*
(i) If f is a fuzzy LA-subsemihypergroup of H, then $f *_k f \subseteq f_k$ and $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$,
(ii) If f is a fuzzy left hyperideal of H, then $\mathcal{H} *_k f \subseteq f_k$ and $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$,
(iii) If f is a fuzzy right hyperideal of H, then $f *_k \mathcal{H} \subseteq f_k$ and $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$,
(iv) If f is a fuzzy interior hyperideal of H, then $(\mathcal{H} *_k f) *_k \mathcal{H} \subseteq f_k$ and $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$,
(v) If f is a fuzzy generalized bi-hyperideal of H, then $(f *_k \mathcal{H}) *_k f \subseteq f_k$ and $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$,
(vi) If f is a fuzzy bi-hyperideal of H, then $f *_k f \leq f_k$, $(f *_k \mathcal{H}) *_k f \subseteq f_k$ and $x \leq y \Longrightarrow f_k(x) \geq f_k(y)$.

Proof. (i) Let f be a fuzzy LA-subsemihypergroup of H. If $A_x = \{(y,z) \in H \times H : x \in y \circ z\} = \emptyset$, then

$$(f *_k f)(x) = 0 \leq f_k(x). \tag{44}$$

If $A_x \neq \emptyset$, then

$$\begin{aligned}(f *_k f)(x) &= \min\left\{(f * f)(x), \frac{1-k}{2}\right\} \\ &= \min\left\{\bigvee_{x\in y\circ z} \min\{f(y), f(z)\}, \frac{1-k}{2}\right\} \\ &\leq \min\left\{\bigvee_{x\in y\circ z} \bigwedge_{x\in y\circ z} f(x), \frac{1-k}{2}\right\} \\ &\qquad (f \text{ is a fuzzy LA-subsemihypergroup}) \\ &= \min\left\{f(x), \frac{1-k}{2}\right\} \\ &= f_k(x).\end{aligned} \tag{45}$$

Thus $f *_k f \subseteq f_k$. For $x \leq y \Longrightarrow f(x) \geq f(y)$, we have $\min\{f(x), (1-k)/2\} \geq \min\{f(y), (1-k)/2\}$. This implies $f_k(x) \geq f_k(y)$. The proofs of (ii) to (vi) can be seen in a similar way. □

Proposition 44. *Let H be an ordered LA-semihypergroup with pure left identity. Then for any $(\in, \in \vee q_k)$-fuzzy left hyperideal f, which is idempotent in $\mathcal{C}(H)$, the following properties hold:*
(i) f_k is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal;
(ii) f_k is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal.

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H and $f *_k f = f_k$.
(i) We have

$$\begin{aligned}(\mathcal{H} *_k f) *_k \mathcal{H} &\subseteq f_k *_k \mathcal{H} = (f *_k f) *_k \mathcal{H} \\ &= (\mathcal{H} *_k f) *_k f \subseteq f_k *_k f = f *_k f \\ &= f_k.\end{aligned} \tag{46}$$

This implies that f_k is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal H.

(ii) We have

$$\begin{aligned}(f *_k \mathscr{H}) *_k f &= (f *_k \mathscr{H}) *_k f_k \\ &= (f *_k \mathscr{H}) *_k (f *_k f) \\ &= (f *_k f) *_k (\mathscr{H} *_k f) \subseteq f_k *_k f_k \\ &= f_k.\end{aligned} \tag{47}$$

This implies that f_k is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H. □

Theorem 45. *Let H be a regular ordered LA-semihypergroup. Then $(f *_k \mathscr{H}) *_k f = f_k$ for every $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal f of H.*

Proof. Let us suppose that H is a regular ordered LA-semihypergroup and let f be an $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal of H. Let $a \in H$ and so $a \in (a \circ x) \circ a$ for some $x \in H$. Thus we have

$$\begin{aligned}((f *_k \mathscr{H}) *_k f)(a) &= \min\left\{((f^* \mathscr{H}) * f)(a), \frac{1-k}{2}\right\} \\ &= \min\left\{\bigvee_{a \in y \circ z} \min\{(f^* \mathscr{H})(y), f(z)\}, \frac{1-k}{2}\right\} \\ &\geq \min\left\{\min\left\{\bigvee_{t \in a \circ x} (f^* \mathscr{H})(t), f(a)\right\}, \frac{1-k}{2}\right\} \\ &\geq \min\left\{\min\left\{\bigvee_{a \circ x = p \circ q} \min\{f(p), \mathscr{H}(q)\}, f(a)\right\}, \right. \\ &\left. \frac{1-k}{2}\right\} \geq \min\left\{\min\{f(a), 1, f(a)\}, \frac{1-k}{2}\right\} \\ &= \min\left\{f(a), \frac{1-k}{2}\right\} = f_k(a).\end{aligned} \tag{48}$$

Therefore, $f_k \subseteq (f *_k \mathscr{H}) *_k f$. Since $(f *_k \mathscr{H}) *_k f \subseteq f_k$, by Proposition 43(v), we have $(f *_k \mathscr{H}) *_k f = f_k$ for every $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal f of H. □

Proposition 46. *If f is an $(\in, \in \vee q_k)$-fuzzy right hyperideal of a regular ordered LA-semihypergroup H with pure left identity e, then $f_k(x \circ y) = f_k(y \circ x)$ holds for all $x, y \in H$.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy right hyperideal of a regular ordered LA-semihypergroup H with pure left identity e. Let $x, y \in H$. Since H is regular, $x \in (x \circ a) \circ x$ and $y \in (y \circ b) \circ y$ for some $a, b \in H$. Now by using the medial and paramedial laws, we get

$$\begin{aligned}x \circ y &\subseteq ((x \circ a) \circ x) \circ ((y \circ b) \circ y) \\ &= (y \circ x) \circ ((y \circ b) \circ (x \circ a)).\end{aligned} \tag{49}$$

Since f is an $(\in, \in \vee q_k)$-fuzzy right hyperideal, for every $w \in x \circ y \subseteq (y \circ x) \circ ((y \circ b) \circ (x \circ a))$, we have

$$\begin{aligned}&\bigwedge_{w \in x \circ y \subseteq (y \circ x) \circ ((y \circ b) \circ (x \circ a))} f(w) \\ &\quad \geq \min\left\{\bigwedge_{s \in y \circ x} f(s), \frac{1-k}{2}\right\} = \bigwedge_{s \in y \circ x} f_k(s).\end{aligned} \tag{50}$$

Again by using the medial and paramedial laws, we get

$$\begin{aligned}y \circ x &\subseteq ((y \circ b) \circ y) \circ ((x \circ a) \circ x) \\ &= (x \circ y) \circ ((x \circ a) \circ (y \circ b)).\end{aligned} \tag{51}$$

Since f is an $(\in, \in \vee q_k)$-fuzzy right hyperideal, for every $t \in y \circ x \subseteq (x \circ y) \circ ((x \circ a) \circ (y \circ b))$, we have

$$\begin{aligned}&\bigwedge_{t \in y \circ x \subseteq (x \circ y) \circ ((x \circ a) \circ (y \circ b))} f(t) \geq \min\left\{\bigwedge_{p \in x \circ y} f(p), \frac{1-k}{2}\right\} \\ &\quad = \bigwedge_{p \in x \circ y} f_k(p).\end{aligned} \tag{52}$$

This shows that $f_k(x \circ y) = f_k(y \circ x)$ holds for all $x, y \in H$. □

Theorem 47. *Let H be a regular ordered LA-semihypergroup with pure left identity e. Then f is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H if and only if f is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H.*

Proof. Let f be an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. Let $x, y, z \in H$. We have

$$\begin{aligned}\bigwedge_{w \in (x \circ y) \circ z} f(w) &= \bigwedge_{w \in (z \circ y) \circ x} f(w) \geq \min\left\{f(x), \frac{1-k}{2}\right\} \\ &\geq \min\left\{f(x), f(z), \frac{1-k}{2}\right\}.\end{aligned} \tag{53}$$

This shows that f is an $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal of H, and clearly f is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup. Therefore f is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H.

Conversely, assume that f is an $(\in, \in \vee q_k)$-fuzzy bi-hyperideal of H. Let $x, y \in H$. Since H is regular, by the left invertive law and medial law, we have

$$\begin{aligned}x \circ y &\subseteq x \circ ((y \circ z) \circ y) = (y \circ z) \circ (x \circ y) \\ &= ((x \circ y) \circ z) \circ y = ((x \circ y) \circ (e \circ z)) \circ y \\ &= ((x \circ e) \circ (y \circ z)) \circ y = (y \circ ((x \circ e) \circ z)) \circ y.\end{aligned} \tag{54}$$

Thus for every $w \in x \circ y \subseteq (y \circ ((x \circ e) \circ z)) \circ y$, we have

$$\begin{aligned}\bigwedge_{w \in x \circ y} f(w) &\geq \bigwedge_{s \in (y \circ ((x \circ e) \circ z)) \circ y} f(s) \\ &\geq \min\left\{f(y), f(y), \frac{1-k}{2}\right\} \\ &= \min\left\{f(y), \frac{1-k}{2}\right\}.\end{aligned} \tag{55}$$

Hence f is an $(\in, \in \vee q_k)$-fuzzy left hyperideal of H. □

4. Homomorphism

Let $(H_1, \circ_1, \leq_1)$ and $(H_2, \circ_2, \leq_2)$ be two ordered LA-semihypergroups and ψ a mapping from H_1 into H_2. ψ is called isotone if $x, y \in H_1$, $x \leq_1 y$ implies $\psi(x) \leq_2 \psi(y)$. ψ is said to be inverse isotone if $x, y \in H_1$, $\psi(x) \leq_2 \psi(y)$ implies $x \leq_1 y$ [each inverse isotone mapping is 1-1]. ψ is called a homomorphism if it is isotone and satisfies $\psi(x \circ_1 y) = \psi(x) \circ_2 \psi(y)$, for all $x, y \in H_1$. Moreover, ψ is said to be isomorphism if it is onto homomorphism and inverse isotone.

Definition 48. Let $(H_1, \circ_1, \leq_1)$ and $(H_2, \circ_2, \leq_2)$ be two ordered LA-semihypergroups and ψ a mapping from H_1 into H_2. Let f_1 and f_2 be fuzzy subsets of H_1 and H_2, respectively. Then the image $\psi(f_1)$ of f_1 is defined by

$$\psi(f_1): H_2 \longrightarrow [0,1] \mid y_2 \longrightarrow \begin{cases} \bigvee_{y_1 \in \psi^{-1}(y_2)} f_1(y_1) & \text{if } \psi^{-1}(y_2) \neq \emptyset \\ 0 & \text{otherwise.} \end{cases} \quad (56)$$

And the inverse image $\psi^{-1}(f_2)$ of f_2 is defined by

$$\psi^{-1}(f_2): H_1 \longrightarrow [0,1] \mid y_1 \longrightarrow f_2(\psi(y_1)). \quad (57)$$

Theorem 49. *Let $(H_1, \circ_1, \leq_1)$ and $(H_2, \circ_2, \leq_2)$ be two ordered LA-semihypergroups and ψ a mapping from H_1 onto H_2. Let f and g be $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal) of H_1 and H_2, respectively. Then*

(i) $\psi(f)$ is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal) of H_2, provided ψ is inverse isotone.

(ii) $\psi^{-1}(g)$ is an $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroup (resp., left hyperideal, right hyperideal, hyperideal, interior hyperideal, bi-hyperideal) of H_1.

(iii) The mapping $f \longrightarrow \psi(f)$ defines a one-to-one correspondence between the set of all $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroups (resp., left hyperideals, right hyperideals, hyperideals, interior hyperideals, bi-hyperideals) of H_1 and the set of all $(\in, \in \vee q_k)$-fuzzy LA-subsemihypergroups (resp., left hyperideals, right hyperideals, hyperideals, interior hyperideals, bi-hyperideals) of H_2, provided ψ is inverse isotone.

Proof. The proof is straightforward. □

Theorem 50. *Let $\psi : (H_1, \circ_1, \leq_1) \longrightarrow (H_2, \circ_2, \leq_2)$ be a surjective homomorphism from an ordered LA-semihypergroup H_1 to an ordered LA-semihypergroup H_2. If H_1 contains a pure left identity e, then*

(i) the image of $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H_1 is an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H_2, provided ψ is inverse isotone.

(ii) the preimage of an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H_2 is an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H_1.

Proof. (i) Let f be an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H_1 and let $x_2, y_2, z_2 \in H_2$. Then there exist $x_1, y_1, z_1 \in H_1$ such that $\psi(x_1) = x_2$, $\psi(y_1) = y_2$ and $\psi(z_1) = z_2$. Now, we have

$$\begin{aligned} \bigwedge_{z_2 \in x_2 \circ_2 y_2} \psi(f)(z_2) &= \bigwedge_{z_2 \in x_2 \circ_2 y_2} \left\{ \bigvee_{t \in \psi^{-1}(z_2)} f(t) \right\} \\ &\geq \bigwedge_{z_2 \in x_2 \circ_2 y_2} f(z_1) \\ &= \bigwedge_{\psi(z_1) \in \psi(x_1) \circ_2 \psi(y_1)} f(z_1) \\ &= \bigwedge_{\psi(z_1) \in \psi(x_1 \circ_1 y_1)} f(z_1) \\ &= \bigwedge_{z_1 \in x_1 \circ_1 y_1} f(z_1) \\ &= \bigwedge_{z_1 \in (e \circ_1 x_1) \circ_1 y_1} f(z_1) \\ &\geq \min\left\{ f(x_1), \frac{1-k}{2} \right\} \\ &\geq \min\left\{ \bigvee_{x_1 \in \psi^{-1}(x_2)} f(x_1), \frac{1-k}{2} \right\} \\ &\geq \min\left\{ \psi(f)(x_2), \frac{1-k}{2} \right\}. \end{aligned} \quad (58)$$

Let $x_2 \leq_2 y_2$. Since ψ is inverse isotone, there exist unique $x_1, y_1 \in H_1$ such that $\psi(x_1) = x_2$, $\psi(y_1) = y_2$ and $x_1 \leq_1 y_1$. Thus we have

$$\begin{aligned} \psi(f)(x_2) &= \bigvee_{t \in \psi^{-1}(x_2)} f(t) = f(x_1) \\ &\geq \min\left\{ f(y_1), \frac{1-k}{2} \right\} \\ &= \min\left\{ \bigvee_{t \in \psi^{-1}(y_2)} f(t), \frac{1-k}{2} \right\} \\ &= \min\left\{ \psi(f)(y_2), \frac{1-k}{2} \right\}. \end{aligned} \quad (59)$$

This shows that the image of an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H_1 is an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H_2. The proof of (ii) can be seen in a similar way. □

In the following example we show that in an ordered LA-semihypergroup H_1 without pure left identity, the image of an $(\in, \in \vee q_k)$-fuzzy subset f under ψ can be or cannot be an $(\in, \in \vee q_k)$ -fuzzy right hyperideal of H_2.

Example 51. Let $H_1 = \{x, y, z, w\}$ and $H_2 = \{a, b, c\}$ be two ordered LA-semihypergroups defined by the following hyperoperations "$\circ_1, \circ_2$" and the orders "$\leq_1, \leq_2$":

$\circ_1$	x	y	z	w
x	x	x	x	x
y	x	$\{y,w\}$	$\{x,z\}$	$\{y,w\}$
z	x	$\{x,z\}$	z	$\{x,z\}$
w	x	y	$\{x,z\}$	y

$$\leq_1 := \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c),(d,d)\} \quad (60)$$

$\circ_2$	a	b	c
a	b	b	c
b	$\{a,b\}$	$\{a,b\}$	c
c	c	c	c

$$\leq_2 := \{(a,a),(a,b),(a,c),(a,d),(b,b),(c,c),(d,d)\}.$$

We define a homomorphism $\psi : (H_1, \circ_1, \leq_1) \longrightarrow (H_2, \circ_2, \leq_2)$ by

$$\psi : H_1 \longrightarrow H_2 \mid r \longrightarrow \psi(r) := \begin{cases} c & \text{if } r \in \{x, z\} \\ b & \text{if } r = y \\ a & \text{if } r = w. \end{cases} \quad (61)$$

We take a fuzzy subset f of H_1 as $f(x) = 0.1$, $f(y) = 0.6$, $f(z) = 0.7$ and $f(w) = 0.5$. By routine calculation one can see that the image of f under ψ is an $(\in, \in \vee q_k)$-fuzzy right hyperideal of H_2 for $k \in [0,1)$, but f is not an $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H_1.

Theorem 52. *Let $\psi : (H_1, \circ_1, \leq_1) \longrightarrow (H_2, \circ_2, \leq_2)$ be a homomorphism from an ordered LA-semihypergroup H_1 to an ordered LA-semihypergroup H_2. If H_1 contains a pure left identity e, then the preimage of every $(\in, \in \vee q_k)$-fuzzy interior hyperideal of H_2 is an $(\in, \in \vee q_k)$-fuzzy generalized bi-hyperideal of H_1, provided ψ is inverse isotone.*

Proof. The proof is similar to the proof of Theorem 50(i). □

5. Conclusions

Fuzzification of algebraic hyperstructures plays an important role in mathematics with wide range of applications in many disciplines such as computer sciences, chemistry, engineering, and medical diagnosis. In this paper, we have introduced the concept of $(\in, \in \vee q_k)$-fuzzy hyperideals in ordered LA-semihypergroup and investigated their related properties. We hope that this work would offer foundation for further study of the theory on ordered LA-semihypergroup and fuzzy ordered LA-semihypergroup. The obtained results probably can be applied in various fields such as computer sciences, control engineering, coding theory, theoretical physics, and chemistry. In our future research, we will consider the characterization of intraregular ordered LA-semihypergroup in terms of $(\in, \in \vee q_k)$-fuzzy hyperideals.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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