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Research Article

A Concession Equilibrium Solution Method without Weighted Aggregation Operators for Multiattribute Group Decision-Making Problems

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This paper introduces a concession equilibrium solution without weighted aggregation operators to multiattribute group decision-making problems (in short MGDMPs). It is of practical significance for all decision-makers to find an optimal solution to MGDMPs or to sort out all candidate solutions to MGDMPs. It is proved that under certain conditions the optimal concession equilibrium solution does exist, and on this important result the optimal concession equilibrium solution is obtained by solving a single objective optimization problem. Moreover, the optimal concession equilibrium solution is equivalent to the robust optimal solution with the group weight aggregation under the worst weight condition. Finally, it is proved that the concession equilibrium solution is equivalent to a complete order, i.e. all candidate alternatives can be sorted by concession equilibrium solution. By defining the triangular fuzzy numbers of target concession value, the optimal concession equilibrium solution or the order of the alternative solutions can be obtained in the range of objective concession ambiguity. Numerical experiment shows that the solution can balance the evaluations of multiattribute group decision makers. This paper provides a new approach to solving multiattribute group decision-making problems.

1. Introduction

The multiattribute group decision-making problems (MGDMP) exist in many areas such as social network, supplier selection, competitive business environment, economic analysis, strategic planning, medical diagnosis, venture capital, and etc. Because of the conflict between attributes and decision makers, it is very difficult to solve a MGDMP.

Suppose that there are r decision makers, $DM_1, DM_2, \ldots, DM_r$ $(r > 1)$ –a group of experts– and the multiattribute evaluation (cost or benefit) function for DM_i is $f^i : R^n \longrightarrow R^m$, $i = 1, 2, \ldots, r$. Let $\boldsymbol{x}$ be a candidate scheme (solution) and the set of all candidate schemes be $X \subset R^n$. Each decision maker selects a solution or a ranking of the candidate schemes from X by evaluating $\boldsymbol{f}^i(\boldsymbol{x})$. This paper studies multiattribute group decision-making problem as follows:

(MGDMP)

$$\begin{aligned}
&(\mathrm{P}_1) && \min && \boldsymbol{f}^1(\boldsymbol{x}) \\
&(\mathrm{P}_2) && \min && \boldsymbol{f}^2(\boldsymbol{x}) \\
& && \vdots && \\
&(\mathrm{P}_r) && \min && \boldsymbol{f}^r(\boldsymbol{x}) \\
& && \text{s.t.} && \boldsymbol{x} \in X.
\end{aligned} \tag{1}$$

When $r = 1$, (MGDMP) becomes a group decision-making problem (GDMP), which is a single-attribute one. So far, almost all the studies on MGDMPs focus on the weighted aggregation methods and fall within the following four main areas:

(1) The weighted aggregation method used earlier is mainly based on given weights. For example, Choi (1998) [1] and Kim (1999) [2] et al. gave a method involving multiattribute weights and group members' weights to find out the best candidate. Wei (2000) [3] et al. gave the compromise weight method with individual preference. This idea of the aggregating OWA operator weights has been widely studied and promoted (see Xu (2009) [4]; Yue (2012) [5]; Zhou, Chen, and Liu (2012) [6]; Liu, Cai, and Martnez (2013) [7]; Liu, Zhang, and Zhang (2014) [8]; Gao, Li, and Liu (2015) [9]; Dong, Xiao, Zhang, and Wang (2016) [10]).

(2) Some studies focus on weighted aggregation methods that consider the uncertainty of weights, such as weights being an interval or a probability distribution. For example, Merig, Casanovas, and Yang (2014) [11] studied the uncertain generalized probabilistic weighted averaging (UGPWA) operator. Qi, Liang, and Zhang (2015) [12] presented a method of generalized cross-entropy based group decision-making with unknown experts and attribute weights under interval-valued intuitionistic fuzzy environment.

(3) In recent years, some new complex methods using fuzzy theory are applied to the weighted aggregation methods. For example, Sadi-Nezhad and Akhtari (2008) [13] studied a possibilistic programming approach with fuzzy multidimensional analysis preference. Yan and Ma (2015) [14] proposed an approach to uncertain quality function deployment based on fuzzy preference relation and fuzzy majority. More research literatures can be seen in Xu, Chen, Rodrguez, Herrera, and Wang (2016) [15]; Bayrama and Sahin (2016) [16]; Chen and Kuo (2017) [17]; Banaeian, Mobli, Fahimnia, Nielsen, and Omida (2018) [18].

(4) The hesitant fuzzy linguistic term set and the linguistic distribution are becoming popular tools to solve MGDMPs. For example, Thuong, Zhang, Li, and Hong (2018) [19] proposed a quantitative hesitant fuzzy judgment description with an embedded assessing attitude to evaluate financial statement quality (FSQ) to overcome the weighting difficulties, and applied a distance-based method to determine the evaluators' weights and a weighted averaging operator to compute the criteria weights of MGDMPs. Wu, Xu, and Xu (2016) [20] proposed an entropy method that is generalized to a linguistic setting to derive the important weights for the attributes with quite different values, which are considered more important and therefore have higher weights for MGDMPs. Wu and Xu (2018) [21] considered the preferences of the decision-makers using fuzzy preference relations and a novel distance measure over the possibility distribution based hesitant fuzzy elements is defined to compute the various consensus measures. More research literatures can be seen in Wu, Li, Chen, and Dong (2018) [22]; Wu, Dai, Chiclana, Fujita, and Herrera-Viedma (2018) [23]; Li, Rodrguez, Martnez, Dong, and Herrera (2018) [24]; Wu and Xu (2016) [25]; Wu, Jin, and Xu (2018) [26].

All the above literatures on weighted aggregation almost all focus on limited number of candidate schemes (solutions) to MGDMPs. But, the weighted aggregation method is a commonly used method in solving MGDMPs. Its fatal weakness is that different weights lead to different ranking of the candidate schemes (or candidates), and it is impossible to prove which weighted aggregation method is the best. In literatures on MGDMPs, the attributes' weights and the experts' weights should be determined, e.g. in [5] the weights were determined by using all the schemes, then if there are infinite number of candidate schemes, the method will become ineffective. On the other hand, different attributes' weights and experts' weights will lead to different ranking of the final scheme, which would result in an outcome that makes it difficult to determine which ranking is the best. So we propose an s-concession equilibrium solution to MGDMPs to avoid the determination of attributes' weights and experts' weights, and it provides an optimum solution to the situation when there are infinite number of schemes for MGDMPs.

To solve the infinite-number-of-candidate multi-decision-maker decision-making problems, Meng, Hu and Dang (2005) [27] proposed an s-concession equilibrium solution with single attribute mathematical programming model for the coexistence of competitions and cooperation problems. Meng, Hu, Jiang and Zhou (2007) [28] gave an s-concession equilibrium solution with single attribute interactional programming model for the coexistence of competitions and co-operations problems. Xu, Meng, and Shen (2015) [29] introduced an s-concession equilibrium solution and gave a cooperation model based on CVaR measure for a two-stage supply chain with a single-attribute GDMP. Jiang, Meng and Shen (2018) [30] proposed for the first time the target concession value of s-concession equilibrium solution to the single-attribute GDMPs. But, an s-concession equilibrium solution to MGDMPs with the target concession value has not yet seen in published literatures.

Jiang, Meng and Shen (2018) [30] introduced a concept as to the solution to group decision-making problems (GDMPs): s-concession equilibrium solution, which is more adaptive to the situation where the number of candidates is unlimited, and used an example to show how to solve the product ordering and production operation decision-making problem between the retailer and the manufacturer using the s- optimal concession equilibrium solution under the case where the number of alternatives is unlimited. The concept is characterized by that, for each decision maker, each objective attribute gives the corresponding concession value s, and the s-optimal concession equilibrium solution is the minimum concession given. The s-optimal concession equilibrium solution provides a natural criterion for evaluating the merits of the candidates. Obviously, it is different from other existing methods with weighted aggregation operators, because s-optimal concession equilibrium solution is a method that provides a natural criterion for evaluating the merits of the candidates and does not use weighted aggregation operators. According to the definition of s^*-optimal concession equilibrium solution, the s^*-optimal concession equilibrium solution is obviously not dependent on the evaluation function of one DM. On the other hand, for each scheme, the equilibrium value is the same for each decision maker's goal. Therefore, the s-optimal concession equilibrium solution has its individual rationality.

In this paper, based on the idea of concession equilibrium to GDMPs (Jiang, 2018) [30], the optimal concession equilibrium solution to MGDMPs without weighted aggregation operators is defined. Our innovation includes (1) a new $\boldsymbol{s}^*$-optimal concession equilibrium solution is proposed, where $\boldsymbol{s}^*$ is a vector, while the concept – s-optimal concession solution defined in [30] cannot solve (MGDMP); (2) the $\boldsymbol{s}^*$-optimal concession equilibrium solution is a robust solution in all the weighted aggression sets; (3) a new triangular-fuzzy-concession ranking method is proposed based on the $\boldsymbol{s}^*$-optimal concession equilibrium solution, and the rankings in the numerical experiments show stability under different concession values.

2. $\boldsymbol{s}^*$-Optimal Concession Equilibrium Solution to MGDMP

In this section, $\boldsymbol{s}^*$-optimal concession equilibrium solution to MGDMPs and its property are discussed. Let $\boldsymbol{\epsilon}^i = (\epsilon_1^i, \epsilon_2^i, \ldots, \epsilon_m^i)^T \in R^m$ $(i = 1, 2, \ldots, r)$ be given, where $\boldsymbol{\epsilon}^i$ is called the target concession value of DM_i $(i = 1, 2, \ldots, r)$. Let $\boldsymbol{\varepsilon} = (\boldsymbol{\epsilon}^1, \boldsymbol{\epsilon}^2, \ldots, \boldsymbol{\epsilon}^r)$.

Definition 1. Let $\boldsymbol{x}^* \in X$, $\boldsymbol{s} \in R^m$, and $\boldsymbol{\epsilon}^i \geq 0$ $(i = 1, 2, \ldots, r)$. If there is

$$\boldsymbol{f}^i(\boldsymbol{x}^*) - \boldsymbol{s} - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r, \tag{2}$$

i.e.,

$$\begin{aligned} &f_j^i(\boldsymbol{x}^*) - s_j - \epsilon_j^i \leq f_j^i(\boldsymbol{x}), \\ &\qquad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r, \ j = 1, 2, \ldots, m, \end{aligned} \tag{3}$$

then $\boldsymbol{x}^*$ is called $\boldsymbol{s}$-concession equilibrium solution to (MGDMPs) at the value $\boldsymbol{\varepsilon}$. The $|\boldsymbol{s}| = \sum_{j=1}^m s_j$ is called an equilibrium value of (MGDMP) to $\boldsymbol{x}^*$. $\boldsymbol{s}$ is called an equilibrium point of (MGDMP) to $\boldsymbol{x}^*$. The set of all equilibrium values $|\boldsymbol{s}|$ of all $\boldsymbol{s}$-concession equilibrium solutions $\boldsymbol{x}^* \in X$ to (MGDMP) is denoted as S. If $\boldsymbol{x}^*$ is the $\boldsymbol{s}^*$-concession equilibrium solution to (MGDMP) and $|\boldsymbol{s}^*|$ is the minimum of the set S, then $\boldsymbol{x}^*$ is called the $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}$. $|\boldsymbol{s}^*|$ is called the optimal equilibrium value, and obviously the optimal equilibrium value is unique. $\boldsymbol{s}^*$ is called the optimal equilibrium point, and the equilibrium point $\boldsymbol{s}^*$ of $\boldsymbol{x}^*$ is not always unique.

This differs from the s^*-optimal concession equilibrium solution to single attribute group decision-making problem (Jiang, 2018) [30]. Furthermore, to solve the infinite-number-of-candidate MGDMPs and avoid the determination of the attributes' weights and the experts' weights, the s-concession equilibrium solution and s∗-concession equilibrium solution are introduced.

Obviously, we have the following property.

Property 2. Let $\boldsymbol{x}^*$ be $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$.

(1) Let $\boldsymbol{v} \in R^m$. If

$$\boldsymbol{f}^i(\boldsymbol{x}^*) - \boldsymbol{v} - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r, \tag{4}$$

then $|\boldsymbol{s}^*| \leq |\boldsymbol{v}|$.

(2) Then $\boldsymbol{x}^*$ be $\boldsymbol{0}$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}(\boldsymbol{s}^*)$, where $\boldsymbol{\varepsilon}(\boldsymbol{s}^*) = (\boldsymbol{s}^* + \boldsymbol{\epsilon}^1, \boldsymbol{s}^* + \boldsymbol{\epsilon}^2, \ldots, \boldsymbol{s}^* + \boldsymbol{\epsilon}^r)$.

(3) If X has only a finite number of solutions, then the $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) exists.

Property 2 indicates that, with the given $\boldsymbol{\epsilon}^i$, $\boldsymbol{s}^*$ is the minimum concession value among all the candidates, so its corresponding $\boldsymbol{s}^*$-optimal concession equilibrium is the best solution in all equilibrium values.

When the different target concession values, i.e., different $\boldsymbol{\epsilon}$, are given, different $\boldsymbol{s}^*$-optimal concession equilibrium solutions are obtained, as shown in the following example.

Example 3. Consider the following GDMP.

(MGDMP)

$$\begin{aligned} &(\mathrm{P}_1) \ \min \quad f^1(x, y) = \left((x-2)^2 + y^2, x^2 + (y-2)^2\right) \\ &(\mathrm{P}_2) \ \min \quad f^2(x, y) = \left(x^2 + (y-2)^2, (x-2)^2 + y^2\right) \\ &\qquad\quad \text{s.t.} \quad (x, y) \in X = R^1 \times R^1. \end{aligned} \tag{5}$$

If $\boldsymbol{\varepsilon}_1 = ((0,0)^T, (0,0)^T)$, we have

$$\begin{aligned} &f_1^1(1,1) - 2 - 0 \leq f_1^1(x, y), \\ &f_2^1(1,1) - 2 - 0 \leq f_2^1(x, y), \\ &\qquad\qquad (x, y) \in R^1 \times R^1, \\ &f_1^2(1,1) - 2 - 0 \leq f_1^2(x, y), \\ &f_2^2(1,1) - 2 - 0 \leq f_2^2(x, y), \\ &\qquad\qquad (x, y) \in R^1 \times R^1. \end{aligned} \tag{6}$$

$(x, y) = (1, 1)$ is the $(2, 2)^T$-optimal concession equilibrium solution to the problem at the concession value $\boldsymbol{\varepsilon}_1$. This solution gives the minimum equilibrium value of each decision-maker's individual objective.

If $\boldsymbol{\varepsilon}_2 = ((8,0)^T, (0,8)^T)$, we have

$$\begin{aligned} &f_1^1(0,2) - 0 - 8 \leq f_1(x, y), \\ &f_2^1(0,2) - 0 - 0 \leq f_2^1(x, y), \\ &\qquad\qquad (x, y) \in R^1 \times R^1, \\ &f_1^2(0,2) - 0 - 0 \leq f_1(x, y), \\ &f_2^2(0,2) - 0 - 8 \leq f_2^2(x, y), \\ &\qquad\qquad (x, y) \in R^1 \times R^1. \end{aligned} \tag{7}$$

$(0, 2)$ is the $(0, 0)^T$-optimal concession equilibrium solution to the problem at the concession value $\boldsymbol{\varepsilon}_2$.

From the above example, it is understood that when $\boldsymbol{\epsilon}$ is given, an optimal concession equilibrium solution is obtained. And Lemma 4 gives the conclusion that under certain conditions any feasible solution to (MGDMP) is the concession equilibrium solution.

For each $i = 1, 2, \ldots, r$; $j = 1, 2, \ldots, m$, we have the following single objective optimization problem:

$$\begin{aligned} \left(P_j^i\right) \ \min \quad & f_j^i(\boldsymbol{x}) \\ s.t. \quad & \boldsymbol{x} \in X, \end{aligned} \tag{8}$$

Let f_j^{i*} be the optimal target value of (P_j^i). Assumes that $f_j^{i*} > -\infty$, $i = 1, 2, \ldots, r$, $j = 1, 2, \ldots, m$. If some $f_j^{i*} = -\infty$, there is no optimal concession equilibrium solution to (MGDMP).

Lemma 4. *Assume that there is an optimal solution to* (P_j^i) $(i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m,)$. *Then for any given* $\overline{\boldsymbol{x}} \in X$, $\overline{\boldsymbol{x}}$ *is an* $\overline{\boldsymbol{s}}$*-concession equilibrium solution to (MGDMP), where* $\overline{\boldsymbol{s}} = (\overline{s}_1, \overline{s}_2, \ldots, \overline{s}_m)^T$ *and* $\overline{s}_j = \max\{f_j^i(\overline{\boldsymbol{x}}) - f_j^{i*} - \epsilon_j^i \mid i = 1, 2, \ldots, r\}$, $j = 1, 2, \ldots, m$.

Proof. For any $\boldsymbol{x} \in X$, we have

$$f_j^{i*} \leq f_j^i(\boldsymbol{x}), \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m. \tag{9}$$

So, we have

$$\begin{aligned} f_j^i(\overline{\boldsymbol{x}}) - \left[f_j^i(\overline{\boldsymbol{x}}) - f_j^{i*} - \epsilon_j^i\right] - \epsilon_j^i \leq f_j^i(\boldsymbol{x}), & \\ i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m. & \end{aligned} \tag{10}$$

So,

$$\boldsymbol{f}^i(\overline{\boldsymbol{x}}) - \overline{\boldsymbol{s}} - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad i = 1, 2, \ldots, r. \tag{11}$$

Therefore, by Definition 1, the conclusion of the theorem is true. □

Define the following optimization problem:

$$\begin{aligned} (S) \ \min \quad & |\boldsymbol{s}| = s_1 + s_2 + \cdots + s_m \\ \text{s.t.} \quad & f_j^i(\boldsymbol{x}) - s_j - \epsilon_j^i \leq f_j^{i*}, \\ & \qquad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m, \\ & \boldsymbol{s} \in R^m,\ \boldsymbol{x} \in X. \end{aligned} \tag{12}$$

Theorem 5. *Assume that there is an optimal solution to* (P_j^i) $(i = 1, 2, \ldots, r)$. *Then* $\boldsymbol{x}^*$ *is* $\boldsymbol{s}^*$*-optimal concession equilibrium solution to (MGDMP) at the value* $\boldsymbol{\varepsilon}$ *if and only if* $(\boldsymbol{x}^*, \boldsymbol{s}^*)$ *is an optimal solution to (S).*

Proof. First, assume that if $(\boldsymbol{x}^*, \boldsymbol{s}^*)$ is an optimal solution to (S), then for any $\boldsymbol{x} \in X$, we have

$$\begin{aligned} f_j^i(\boldsymbol{x}^*) - s_j^* - \epsilon_j^i \leq f_j^{i*} \leq f_j^i(\boldsymbol{x}), & \\ i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m. & \end{aligned} \tag{13}$$

So, by Definition 1, we have that $\boldsymbol{x}^*$ is $\boldsymbol{s}^*$-concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. Let $\overline{\boldsymbol{x}}$ be $\overline{\boldsymbol{s}}$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. By Property 2, we have $|\boldsymbol{s}^*| \geq |\overline{\boldsymbol{s}}|$ and

$$\boldsymbol{f}^i(\overline{\boldsymbol{x}}) - \overline{\boldsymbol{s}} - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X,\ i = 1, 2, \ldots, r. \tag{14}$$

Therefore, $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{s}})$ is a feasible solution to (S). So, $|\overline{\boldsymbol{s}}| \geq |\boldsymbol{s}^*|$. That is $|\overline{\boldsymbol{s}}| = |\boldsymbol{s}^*|$ and $\boldsymbol{x}^*$ is $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. □

Now, assume that $\boldsymbol{x}^*$ is an $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. Then by Definition 1 it is known that $(\boldsymbol{x}^*, \boldsymbol{s}^*)$ is a feasible solution to (S). Let $(\overline{\boldsymbol{x}}, \overline{\boldsymbol{s}})$ be an optimal solution to (S). By the above proof, $\overline{\boldsymbol{x}}$ is an $\overline{\boldsymbol{s}}$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. Therefore, $|\overline{\boldsymbol{s}}| = |\boldsymbol{s}^*|$. So, $\boldsymbol{x}^* \equiv_s \overline{\boldsymbol{x}}$ and $(\boldsymbol{x}^*, \boldsymbol{s}^*)$ is an optimal solution to (S).

Theorem 5 points out that if $\boldsymbol{x}$ is $\boldsymbol{s}$-concession equilibrium solution, then

$$s_j + \epsilon_j^i \geq 0, \quad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m. \tag{15}$$

Furthermore, Theorem 5 gives that if there exists $\boldsymbol{s}^*$-optimal concession equilibrium solution, the optimal solution for (P_j^i) must exist. Then we have the following.

Theorem 6. *Assume that* X *is a compact set and* $f_j^i(\boldsymbol{x})$ $(i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m)$ *is continuous on* X. *Then the* $\boldsymbol{s}^*$*-optimal concession equilibrium solution to (MGDMP) exists.*

Proof. By the assumption, there is an optimal solution to each (P_j^i). By Lemma 4, we have $S \neq \emptyset$. We prove that S is close. Assume that a sequence $\{\boldsymbol{s}_k\} \subset S$ converges to $\boldsymbol{s}^*$. For $k = 1, 2, \ldots$, let $\boldsymbol{x}_k^* \in X$ be an $\boldsymbol{s}_k$- concession equilibrium solution to (MGDMP). Because X is compact, the sequence $\{\boldsymbol{x}_k^*\}$ has a convergent subsequence. Without loss of generality, let $\boldsymbol{x}_k^* \longrightarrow \boldsymbol{x}^* \in X$. By Definition 1, we have

$$\begin{aligned} \boldsymbol{f}^i(\boldsymbol{x}_k^*) - \boldsymbol{s}_k - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), & \\ \forall \boldsymbol{x} \in X,\ i = 1, 2, \ldots, r,\ k = 1, 2, \ldots. & \end{aligned} \tag{16}$$

Let $k \longrightarrow +\infty$, and then we have

$$\boldsymbol{f}^i(\boldsymbol{x}^*) - \boldsymbol{s}^* - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X,\ i = 1, 2, \ldots, r. \tag{17}$$

So, $\boldsymbol{x}^*$ is the $\boldsymbol{s}^*$-concession equilibrium solution. Therefore S is close. By Lemma 4, there is a minimum $\boldsymbol{s}^*$ in S. Given a sufficiently large $A > 0$, define the problem:

$$\begin{aligned} (S)_A \ \min \quad & |\boldsymbol{s}| = s_1 + s_2 + \cdots + s_m \\ \text{s.t.} \quad & f_j^i(\boldsymbol{x}) - s_j - \epsilon_j^i \leq f_j^{i*}, \\ & \qquad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m, \\ & A \geq s_j \geq s_j^*,\ \boldsymbol{x} \in X,\ j = 1, 2, \ldots, m. \end{aligned} \tag{18}$$

It is obvious that the problem $(S)_A$ is equivalent to the problem (S), and the feasible set of the problem $(S)_A$ is compact too. Therefore, there exists the optimal solution $(\boldsymbol{x}_A^*, \boldsymbol{s}_A^*)$ to $(S)_A$, then $(\boldsymbol{x}_A^*, \boldsymbol{s}_A^*)$ is also the optimal solution to the problem (S). By Theorem 5, the conclusion is true. □

To solve (S), (P_j^i) must be solved first, which is quite difficult. Therefore, we have the following Theorem 7, where solving a single objective programming problem $(\overline{\mathrm{S}})$ obtains the $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP).

$$
\begin{aligned}
(\overline{\mathrm{S}})\ \min\quad & |s| + \sum_{i=1}^{r}\sum_{j=1}^{m} f_j^i\left(\boldsymbol{x}_j^i\right) \\
\text{s.t.}\quad & f_j^i(\boldsymbol{x}) - s_j - \epsilon_j^i \le f_j^i\left(\boldsymbol{x}_j^i\right), \\
& \qquad i = 1,2,\ldots,r;\quad j = 1,2,\ldots,m\,, \\
& \boldsymbol{s} \in R^m,\ \boldsymbol{x}, \boldsymbol{x}_j^i \\
& \quad \in X,\quad i = 1,2,\ldots,r\,;\quad j = 1,2,\ldots,m\,,
\end{aligned}
\tag{19}
$$

where $\boldsymbol{s}, \boldsymbol{x}, \boldsymbol{x}_j^i$, $i = 1,2,\ldots,r$; $j = 1,2,\ldots,m$, are variable of $(\overline{\mathrm{S}})$. We have better results as follows.

Theorem 7. *Suppose that $\overline{\boldsymbol{x}}_j^{i*}$ is an optimal solution to (P_j^i) $(i = 1,2,\ldots,r;\ j = 1,2,\ldots,m)$. If $[(\overline{\boldsymbol{x}},\overline{\boldsymbol{s}}), (\overline{\boldsymbol{x}}_1^1, \ldots, \overline{\boldsymbol{x}}_m^1), (\overline{\boldsymbol{x}}_1^2, \ldots, \overline{\boldsymbol{x}}_m^2), \ldots, (\overline{\boldsymbol{x}}_1^r, \ldots, \overline{\boldsymbol{x}}_m^r)]$ is an optimal solution to $(\overline{\mathrm{S}})$, then $\overline{\boldsymbol{x}}$ is $\overline{s}^*$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\epsilon}$, where $\boldsymbol{s}^* = \overline{\boldsymbol{s}} + \widehat{\boldsymbol{s}}$, $\widehat{\boldsymbol{s}} = (\widehat{s}_1, \widehat{s}_2, \ldots, \widehat{s}_m)^T$,*

$$
\widehat{s}_j = \max\left\{ f_j^i\left(\overline{\boldsymbol{x}}_j^i\right) - f_j^i\left(\overline{\boldsymbol{x}}_j^{i*}\right) \mid i = 1,2,\ldots,r \right\}, \qquad j = 1,2,\ldots,m.
\tag{20}
$$

Proof. Let $\overline{\boldsymbol{x}}^*$ be an $\overline{s}^*$-optimal concession equilibrium solution to (MGDMP). By Theorem 5, we have that $(\overline{\boldsymbol{x}}^*, \overline{\boldsymbol{s}}^*)$ is an optimal solution to (S). It is clear that $[(\overline{\boldsymbol{x}}^*, \overline{\boldsymbol{s}}^*), (\overline{\boldsymbol{x}}_1^{1*}, \ldots, \overline{\boldsymbol{x}}_m^{1*}), (\overline{\boldsymbol{x}}_1^{2*}, \ldots, \overline{\boldsymbol{x}}_m^{2*}), \ldots, (\overline{\boldsymbol{x}}_1^{r*}, \ldots, \overline{\boldsymbol{x}}_m^{r*})]$ is a feasible solution to $(\overline{\mathrm{S}})$. Therefore, we have

$$
|\overline{\boldsymbol{s}}| + \sum_{i=1}^{r}\sum_{j=1}^{m} f_j^i\left(\overline{\boldsymbol{x}}_j^i\right) \le \left|\overline{\boldsymbol{s}}^*\right| + \sum_{i=1}^{r}\sum_{j=1}^{m} f^i\left(\overline{\boldsymbol{x}}_j^{i*}\right).
\tag{21}
$$

By (20), (21), and $f_j^i(\overline{\boldsymbol{x}}_j^{i*}) = f_j^{i*}$, we have $|\boldsymbol{s}^*| \le |\overline{\boldsymbol{s}}^*|$. On the other hand, according to the theorem hypothesis, we have

$$
f_j^i(\overline{\boldsymbol{x}}) - \overline{s}_j - \epsilon_j^i \le f_j^i\left(\overline{\boldsymbol{x}}_j^i\right), \qquad i = 1,2,\ldots,r;\ j = 1,2,\ldots,m.
\tag{22}
$$

So,

$$
f_j^i(\overline{\boldsymbol{x}}) - \overline{s}_j - \epsilon_j^i \le f_j^i\left(\overline{\boldsymbol{x}}_j^i\right) - f_j^i\left(\overline{\boldsymbol{x}}_j^{i*}\right) + f_j^i\left(\overline{\boldsymbol{x}}_j^{i*}\right), \qquad i = 1,2,\ldots,r;\ j = 1,2,\ldots,m.
\tag{23}
$$

Here,

$$
f_j^i(\overline{\boldsymbol{x}}) - \epsilon_j^i \le s_j^* + f_j^i\left(\overline{\boldsymbol{x}}_j^{i*}\right), \qquad i = 1,2,\ldots,r;\ j = 1,2,\ldots,m.
\tag{24}
$$

We have that $(\boldsymbol{x}^*, \boldsymbol{s}^*)$ is a feasible solution to (S). Therefore, $|\boldsymbol{s}^*| \ge |\overline{\boldsymbol{s}}^*|$ and $\boldsymbol{x}^*$ is $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. □

Based on Theorem 7, we have the following corollary.

Corollary 8. *Let $X = R^n$ and $f_j^i(\boldsymbol{x})$ $(i = 1,2,\ldots,r,\ j = 1,2,\ldots,m)$ on R^n be a continuous differentiable convex function. Suppose that $\overline{\boldsymbol{x}}_j^{*i}$ is an optimal solution to (P_j^i) $(i = 1,2,\ldots,r,\ j = 1,2,\ldots,m)$. Then $\boldsymbol{x}^*$ is an $\boldsymbol{s}^*$-optimal concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$ if and only if there is an incomplete zero of $(\lambda_j^1, \lambda_j^2, \ldots, \lambda_j^r)$ $(j = 1,2,\ldots,m)$ to satisfy the following KKT-condition:*

$$
\begin{aligned}
& \lambda_j^1 + \lambda_j^2 + \cdots + \lambda_j^r = 1, \quad j = 1,2,\ldots,m, \\
& \sum_{i=1}^{r} \lambda_j^i \nabla f_j^i\left(\boldsymbol{x}^*\right) = 0, \quad j = 1,2,\ldots,m, \\
& \lambda_j^i\left(f_j^i\left(\boldsymbol{x}^*\right) - s_j^* - \epsilon_j^i - f_j^{i*}\right) = 0, \\
& \qquad i = 1,2,\ldots,r,\ j = 1,2,\ldots,m, \\
& f_j^i\left(\boldsymbol{x}^*\right) - s_j^* - \epsilon_j^i \le f_j^{i*}, \\
& \qquad i = 1,2,\ldots,r,\ j = 1,2,\ldots,m, \\
& \lambda_j^i \ge 0, \\
& \qquad i = 1,2,\ldots,r,\ j = 1,2,\ldots,m.
\end{aligned}
\tag{25}
$$

As per Corollary 8, there is $(\lambda_j^1, \lambda_j^2, \ldots, \lambda_j^r)$ $(j = 1,2,\ldots,m)$ which can be seen as the group weighted aggregation of (MGDMP). Next, in the Set (27) consisting of all the aggregations, the $\boldsymbol{s}^*$-optimal concession equilibrium solution is a robust optimal solution, as shown in Theorem 9.

Let DM_i's evaluation be $f_j^i(\boldsymbol{x}) - \epsilon_j^i - f_j^{i*}$ for $\boldsymbol{x}$. By using weights $1 \ge \lambda_j^i \ge 0$, DM_i's evaluation becomes $\lambda_j^i(f_j^i(\boldsymbol{x}) - \epsilon_j^i - f_j^{i*})$ for $\boldsymbol{x}$ and $j = 1,2,\ldots,m$. With the linear weighting method, the evaluation of all decision-makers for $\boldsymbol{x}$ is defined as

$$
F(\boldsymbol{x}, \boldsymbol{\lambda}) := \sum_{i=1}^{r}\sum_{j=1}^{m} \lambda_j^i\left(f_j^i(\boldsymbol{x}) - \epsilon_j^i - f_j^{i*}\right),
\tag{26}
$$

where $\lambda_j^1 + \lambda_j^2 + \cdots + \lambda_j^r = 1$, $\lambda_j^i \ge 0$, $i = 1,2,\ldots,r$; $j = 1,2,\ldots,m$. Let group weight set

$$
\begin{aligned}
\Lambda = \Bigg\{ \boldsymbol{\lambda} = \Big[& \left(\lambda_1^1, \lambda_1^2, \ldots, \lambda_1^r\right)^T, \left(\lambda_2^1, \lambda_2^2, \ldots, \lambda_2^r\right)^T, \ldots, \\
& \left(\lambda_m^1, \lambda_m^2, \ldots, \lambda_m^r\right)^T \Big] \mid \sum_{i=1}^{r} \lambda_j^i = 1,\ \lambda_j^i \ge 0,\ i = 1,2,\ldots, \\
& r;\ j = 1,2,\ldots,m \Bigg\}.
\end{aligned}
\tag{27}
$$

The worst evaluation score of the solution $\boldsymbol{x}$ solves: $\max_{\boldsymbol{\lambda}\in\Lambda} F(\boldsymbol{x}, \boldsymbol{\lambda})$ for each $\boldsymbol{x} \in X$. Let $\boldsymbol{\lambda}(\boldsymbol{x}) = \operatorname{argmax}\{F(\boldsymbol{x}, \boldsymbol{\lambda}) \mid \boldsymbol{\lambda} \in \Lambda\}$. Then, we are to find a minimum score from these worst scores $\min_{\boldsymbol{x}\in X} F(\boldsymbol{x}, \boldsymbol{\lambda}(\boldsymbol{x}))$, i.e.,

$$
(\widetilde{\mathrm{S}})\ \min_{\boldsymbol{x}\in X} \max_{\boldsymbol{\lambda}\in\Lambda} F(\boldsymbol{x}, \boldsymbol{\lambda}).
\tag{28}
$$

We prove the following conclusion.

Theorem 9. *Supposing that the optimal solution to* (P_j^i) $(i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m)$ *exists, then the problem (S) is equivalent to the problem* $(\widetilde{S})$.

Proof. For a fixed $\boldsymbol{x}$, the problem $\max_{\lambda \in \Lambda} F(\boldsymbol{x}, \boldsymbol{\lambda})$ is a linear programming:

$$\begin{aligned}
\Lambda(\boldsymbol{x})\ \ \max\quad & F(\boldsymbol{x}, \boldsymbol{\lambda}) := \sum_{i=1}^{r} \sum_{j=1}^{m} \lambda_j^i \left(f_j^i(\boldsymbol{x}) - \epsilon_j^i - f_j^{i*} \right) \\
\text{s.t.}\quad & \lambda_j^1 + \lambda_j^2 + \cdots + \lambda_j^r = 1, \\
& \qquad\qquad j = 1, 2, \ldots, m, \\
& \lambda_j^i \geq 0, \\
& \qquad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m.
\end{aligned} \tag{29}$$

The dual problem of $\Lambda(\boldsymbol{x})$ is

$$\begin{aligned}
S(\boldsymbol{x})\ \ \min\quad & |\boldsymbol{s}| = s_1 + s_2 + \cdots + s_m \\
\text{s.t.}\quad & f_j^i(\boldsymbol{x}) - s_j - \epsilon_j^i \leq f_j^{i*}, \\
& \qquad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m, \\
& \boldsymbol{s} \in R^m,\ \boldsymbol{x} \in X.
\end{aligned} \tag{30}$$

Based on the strong duality theorem, there exists the optimal solution to the problem $\Lambda(\boldsymbol{x})$ and the problem $S(\boldsymbol{x})$, and the optimal objective values are equal at their optimal solutions. Let $\boldsymbol{\lambda}^*$ be an optimal solution to $\Lambda(\boldsymbol{x})$ and $\boldsymbol{s}^*$ an optimal solution to $S(\boldsymbol{x})$, then $|\boldsymbol{s}^*| = F(\boldsymbol{x}, \boldsymbol{\lambda}^*) = \max_{\lambda \in \Lambda} F(\boldsymbol{x}, \boldsymbol{\lambda})$. Therefore, the conclusion of theorem is true. □

From the viewpoint of robustness, Theorem 9 means the $\boldsymbol{s}^*$-optimal concession equilibrium solution is the robust solution for the decision-makers under the worst weights in Λ.

The evaluation function $f_j^i(\boldsymbol{x})$ $(i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m)$ of all decision-makers should be consistent as far as possible. f_j^{i*} $(i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m)$ is the ideal goal. Obviously, the closer the solution to the ideal goal, the better. A deviation function is defined by

$$\triangle(\boldsymbol{x}) = \frac{1}{rm} \sqrt{\sum_{i=1}^{r} \sum_{j=1}^{m} \left(f_j^i(\boldsymbol{x}) - f_j^{i*} \right)^2}, \quad x \in X. \tag{31}$$

According to Theorem 5, if $\boldsymbol{x}$ is the s-concession equilibrium solution, then we have

$$\begin{aligned}
& 0 \leq f_j^i(\boldsymbol{x}) - f_j^{i*} \leq s_j + \epsilon_j^i, \\
& \qquad\qquad i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, m.
\end{aligned} \tag{32}$$

That is

$$\begin{aligned}
0 &\leq \frac{1}{rm} \sqrt{\sum_{i=1}^{r} \sum_{j=1}^{m} \left(f_j^i(\boldsymbol{x}) - f_j^{i*} \right)^2} \\
&\leq \frac{1}{rm} \sqrt{\sum_{i=1}^{r} \sum_{j=1}^{m} \left(s_j + \epsilon_j^i \right)^2}, \\
&\leq \frac{1}{m} |\boldsymbol{s}| + \frac{1}{rm} \sum_{i=1}^{r} \sum_{j=1}^{m} \epsilon_j^i,
\end{aligned} \tag{33}$$

for $\boldsymbol{x} \in X$. Let $|\boldsymbol{\varepsilon}| = \sum_{i=1}^{r} \sum_{j=1}^{m} \epsilon_j^i$. $|\boldsymbol{\varepsilon}|$ is the sum of target concession values of all DM_i $(i = 1, 2, \ldots, r)$. So, we have the following conclusion.

Corollary 10. *Assume that there is an optimal solution to* (P_j^i) $(i = 1, 2, \ldots, r,\ j = 1, 2, \ldots, m)$. *If* $\boldsymbol{x}^*$ *is* $\boldsymbol{s}^*$*-optimal concession equilibrium solution to (MGDMP) at the value* $\boldsymbol{\varepsilon}$, *then* $\min_{\boldsymbol{x} \in X} \triangle(\boldsymbol{x}) \leq \triangle(\boldsymbol{x}^*) \leq (1/m)|\boldsymbol{s}^*| + (1/rm)|\boldsymbol{\varepsilon}|$.

Corollary 10 means that the optimal concession equilibrium solution $\boldsymbol{x}^*$ has a minimal upper bound $(1/m)|\boldsymbol{s}^*| + (1/rm)|\boldsymbol{\varepsilon}|$. $\triangle(\boldsymbol{x}^*)$ can be used to evaluate the closeness between the $\boldsymbol{s}^*$-optimal concession equilibrium solution and the ideal objective.

Example 11. Consider problem (MGDMP)

(MGDMP)

$$\begin{aligned}
(\mathrm{P}_1)\ \ \min\quad & f_1(x) = (x, -x) \\
(\mathrm{P}_2)\ \ \min\quad & f_2(x) = (-x, x) \\
\text{s.t.}\quad & x \in X = [-1, 1].
\end{aligned} \tag{34}$$

We know that $f_1^{1*} = f_2^{1*} = f_1^{2*} = f_1^{2*} = -1$. When $\boldsymbol{\varepsilon} = [(0, 0)^T, (0, 0)^T]$, 0 is the $(1, 1)^T$-optimal concession equilibrium solution to the problem at the concession value $\boldsymbol{\varepsilon}$. This solution gives the minimum equilibrium value of each decision-maker's individual objective. Define a weighted function by

$$\begin{aligned}
F(x, \boldsymbol{\lambda}) &= \lambda_1 f_1^1(x) + (1 - \lambda_1) f_1^2(x) + \lambda_2 f_2^1(x) \\
&\quad + (1 - \lambda_2) f_2^2(x) = 2(\lambda_1 - \lambda_2) x.
\end{aligned} \tag{35}$$

By Theorem 9, the optimal solution to $\min_{x \in X} \max_{\lambda_1, \lambda_2 \in [0,1]} F(x, \boldsymbol{\lambda})$ is $x^* = 0$. As a comparison, we are to use the linear weighted method to solve this problem, a very famous method (Kim and Han (1999)) [2] where weighted value $\lambda_1, \lambda_2 \in [0, 1]$. When $\lambda_1 > \lambda_2$, an optimal solution to $\min_{x \in X} F(x, \boldsymbol{\lambda})$ is $x^* = -1$. When $\lambda_1 < \lambda_2$, an optimal solution to $\min_{x \in X} F(x, \lambda)$ is $x^* = 1$. But, when $\lambda_1 = \lambda_2$, no optimal solution to $\min_{x \in X} F(x, \boldsymbol{\lambda})$ exists. On other hand, the deviation function $\triangle(\boldsymbol{x})$ is minimum at $x^* = 0$, but maximum at $x^* = 1$ or $x^* = -1$. It means that the linear weighted method is invalid or bad in this example. Therefore, no matter how the weight is obtained, the linear weighting method may be invalid.

3. Ranking and Fuzzy Target Concession Value of MGDMP

Now, we define the ranking in the set X of $\boldsymbol{s}$-concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$. Deviation of equilibrium value $\boldsymbol{s}$ of $\boldsymbol{s}$-concession equilibrium solution to (MGDMP) at the value $\boldsymbol{\varepsilon}$ is defined as

$$\sigma(\boldsymbol{s}) = \frac{1}{m}\sqrt{\sum_{j=1}^{m}\left(s_j - \frac{1}{m}|\boldsymbol{s}|\right)^2}. \quad (36)$$

$\sigma(\boldsymbol{s})$ represents the difference among attribute values.

Definition 12. Let $\boldsymbol{x}^1, \boldsymbol{x}^2 \in X$, and $\boldsymbol{\varepsilon} \geq 0$. Let $\boldsymbol{x}^1$ be an $\boldsymbol{s}_1$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}$ and $\boldsymbol{x}^2$ be an $\boldsymbol{s}_2$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\epsilon}$. If $|\boldsymbol{s}_1| < |\boldsymbol{s}_2|$, we denote $\boldsymbol{x}^1 \prec_s \boldsymbol{x}^2$ to indicate that $\boldsymbol{x}^1$ is superior to $\boldsymbol{x}^2$. If $|\boldsymbol{s}_1| = |\boldsymbol{s}_2|$ and $\sigma(\boldsymbol{s}_1) < \sigma(\boldsymbol{s}_2)$, we denote $\boldsymbol{x}^1 \preceq_s \boldsymbol{x}^2$ to indicate that $\boldsymbol{x}^1$ is better than $\boldsymbol{x}^2$. If $|\boldsymbol{s}_1| = |\boldsymbol{s}_2|$ and $\sigma(\boldsymbol{s}_1) = \sigma(\boldsymbol{s}_2)$, we denote $\boldsymbol{x}^1 \equiv_s \boldsymbol{x}^2$ to indicate that $\boldsymbol{x}^1$ is equivalent to $\boldsymbol{x}^2$.

Obviously, the set X is a serially ordered set about the order $\prec_s, \preceq_s$ or $\equiv_s$.

Theorem 13. *Let $\boldsymbol{x}^1, \boldsymbol{x}^2 \in X$, and $\boldsymbol{\varepsilon} \geq 0$. Let $\boldsymbol{x}^1$ be an $\boldsymbol{s}_1$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}$ and $\boldsymbol{x}^2$ be an $\boldsymbol{s}_2$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}$. If $\boldsymbol{f}^i(\boldsymbol{x}^1) \leq \boldsymbol{f}^i(\boldsymbol{x}^2)$, $i = 1, 2, \ldots, r$, Then $\boldsymbol{x}^1 \prec_s \boldsymbol{x}^2$, $\boldsymbol{x}^1 \preceq_s \boldsymbol{x}^2$, or $\boldsymbol{x}^1 \equiv_s \boldsymbol{x}^2$.*

Proof. According to assumption, we have

$$\boldsymbol{f}^i(\boldsymbol{x}^1) - \boldsymbol{s}_1 - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r, \quad (37)$$

$$\begin{aligned}\boldsymbol{f}^i(\boldsymbol{x}^1) - \boldsymbol{s}_2 - \boldsymbol{\epsilon}^i &\leq \boldsymbol{f}^i(\boldsymbol{x}^2) - \boldsymbol{s}_2 - \boldsymbol{\epsilon}^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \\ &\forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r,\end{aligned} \quad (38)$$

By Definition 1, we have $|\boldsymbol{s}_1| \leq |\boldsymbol{s}_2|$. □

Theorem 13 shows that the concession equilibrium solution must be nondominated for all decision-makers.

Example 14. Consider a MGDMP. Three decision-makers are to rank three candidate solutions through scoring, as shown in Table 1. Let $\boldsymbol{\varepsilon}_1 = ((0,0)^T, (0,0)^T, (0,0)^T)$ and $\boldsymbol{\varepsilon}_2 = ((0,0)^T, (1,1)^T, (2,2)^T)$.

Here, by Definition 12, when $\boldsymbol{\varepsilon}_1 = ((0,0)^T, (0,0)^T, (0,0)^T)$, we have $\boldsymbol{x}^1 \equiv_s \boldsymbol{x}^2 \equiv_s \boldsymbol{x}^3$. But when $\boldsymbol{\varepsilon}_2 = ((0,0)^T, (1,1)^T, (2,2)^T)$, we have $\boldsymbol{x}^1 \prec_s \boldsymbol{x}^2 \prec_s \boldsymbol{x}^3$. Example 14 shows a fact that when the decision-makers do not have the same target concession value $\boldsymbol{\varepsilon}$, the ranking order of $\boldsymbol{s}$-concession equilibrium solutions is not the same.

Theorem 15. *Let $\boldsymbol{\varepsilon} \geq 0$, and let $\boldsymbol{s}' = (s'_1, s'_2, \ldots, s'_m)^T \geq 0$. If $\boldsymbol{x}$ is an $\boldsymbol{s}$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}$, then $\boldsymbol{x}$ is an $\boldsymbol{s} - \boldsymbol{s}'$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}'$, where $\boldsymbol{\epsilon}'^i = (s'_1 + \epsilon_1^i, s'_2 + \epsilon_2^i, \ldots, s'_m + \epsilon_m^i)^T \in R^m$ $(i = 1, 2, \ldots, r)$ and $\boldsymbol{\varepsilon}' = (\boldsymbol{\epsilon}'^1, \boldsymbol{\epsilon}'^2, \ldots, \boldsymbol{\epsilon}'^r)$. In other word, if the same target concession value $\boldsymbol{\varepsilon}$ for each DM is increased, then the ranking orders of X do not change.*

Theorem 16. *Let $\boldsymbol{x}^* \in X$, $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_1 \geq 0$. Let $\boldsymbol{x}^*$ be an $\boldsymbol{s}_1$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}_1$ and an $\boldsymbol{s}_2$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}_2$. If $\boldsymbol{\varepsilon}_1 \leq \boldsymbol{\varepsilon}_2$, then $|\boldsymbol{s}_1| \geq |\boldsymbol{s}_2|$.*

Proof. According to assumption, we have

$$\boldsymbol{f}^i(\boldsymbol{x}^*) - \boldsymbol{s}_1 - \boldsymbol{\epsilon}_1^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r. \quad (39)$$

Then,

$$\begin{aligned}\boldsymbol{f}^i(\boldsymbol{x}^*) - \boldsymbol{s}_1 - \boldsymbol{\epsilon}_2^i &\leq \boldsymbol{f}^i(\boldsymbol{x}^*) - \boldsymbol{s}_1 + \left(\boldsymbol{\epsilon}_2^i - \boldsymbol{\epsilon}_1^i\right) - \boldsymbol{\epsilon}_2^i \\ &\leq \boldsymbol{f}^i(\boldsymbol{x}), \quad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r.\end{aligned} \quad (40)$$

By Property 2 (1), we have $|\boldsymbol{s}_1| \geq |\boldsymbol{s}_2|$.

Theorem 16 means that the bigger the target concession values for all decision-makers, the smaller the equilibrium values. But, it is very difficult to take the target concession value for each decision-maker. The decision-maker can give the approximate range of the target concession value, using fuzzy number. Now, the decision-makers give fuzzy number of the target concession value $\boldsymbol{\epsilon}$ which is defined as follows:

$$\begin{aligned}\boldsymbol{\epsilon}_L^i &= \left(\epsilon_{L1}^i, \epsilon_{L2}^i, \ldots, \epsilon_{Lm}^i\right)^T \in R^m, \quad i = 1, 2, \ldots, r, \\ \boldsymbol{\epsilon}_M^i &= \left(\epsilon_{M1}^i, \epsilon_{M2}^i, \ldots, \epsilon_{Mm}^i\right)^T \in R^m, \quad i = 1, 2, \ldots, r, \\ \boldsymbol{\epsilon}_U^i &= \left(\epsilon_{U1}^i, \epsilon_{U2}^i, \ldots, \epsilon_{Um}^i\right)^T \in R^m, \quad i = 1, 2, \ldots, r,\end{aligned} \quad (41)$$

where $(\boldsymbol{\epsilon}_L^i, \boldsymbol{\epsilon}_M^i, \boldsymbol{\epsilon}_U^i)$ is called a triangular fuzzy target concession value of DM_i $(i = 1, 2, \ldots, r)$. Let $\boldsymbol{\varepsilon}_L = (\boldsymbol{\epsilon}_L^1, \boldsymbol{\epsilon}_L^2, \ldots, \boldsymbol{\epsilon}_L^r)$, $\boldsymbol{\varepsilon}_M = (\boldsymbol{\epsilon}_M^1, \boldsymbol{\epsilon}_M^2, \ldots, \boldsymbol{\epsilon}_M^r)$, and $\boldsymbol{\varepsilon}_U = (\boldsymbol{\epsilon}_U^1, \boldsymbol{\epsilon}_U^2, \ldots, \boldsymbol{\epsilon}_U^r)$. □

Definition 17. Given $\boldsymbol{x}^* \in X, \boldsymbol{\varepsilon}_L, \boldsymbol{\varepsilon}_M, \boldsymbol{\varepsilon}_U$. If there is

$$\begin{aligned}&\boldsymbol{f}^i(\boldsymbol{x}_L^*) - \boldsymbol{s}_L - \boldsymbol{\epsilon}_L^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \\ &\qquad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r, \\ &\text{and } \boldsymbol{f}^i(\boldsymbol{x}_M^*) - \boldsymbol{s}_M - \boldsymbol{\epsilon}_M^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \\ &\qquad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r, \\ &\text{and } \boldsymbol{f}^i(\boldsymbol{x}_U^*) - \boldsymbol{s}_U - \boldsymbol{\epsilon}_U^i \leq \boldsymbol{f}^i(\boldsymbol{x}), \\ &\qquad \forall \boldsymbol{x} \in X, \ i = 1, 2, \ldots, r,\end{aligned} \quad (42)$$

then $\boldsymbol{x}_L^*$ is called $\boldsymbol{s}_L$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}_L$, $\boldsymbol{x}_M^*$ is called $\boldsymbol{s}_M$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}_M$, and $\boldsymbol{x}_U^*$ is called $\boldsymbol{s}_U$-concession equilibrium solution to (MGDMP) at the target concession value $\boldsymbol{\varepsilon}_U$.

Table 1: The score of three decisions and the equilibrium value under different target concession values.

Solution X	score of DM1 $f^1(\boldsymbol{x})$	score of DM2 $f^2(\boldsymbol{x})$	score of DM3 $f^3(\boldsymbol{x})$	$\boldsymbol{\varepsilon}_1$ $\boldsymbol{s}$	$\boldsymbol{\varepsilon}_2$ $\boldsymbol{s}$
$\boldsymbol{x}^1$	(1,1)	(2,2)	(3,3)	$(2,2)^T$	$(0,0)^T$
$\boldsymbol{x}^2$	(2,2)	(3,3)	(1,1)	$(2,2)^T$	$(1,1)^T$
$\boldsymbol{x}^3$	(3,3)	(1,1)	(2,2)	$(2,2)^T$	$(2,2)^T$

Table 2: The scores of 9 candidates and the equilibrium value under triangular fuzzy target concession values.

Solution X	M1 $\boldsymbol{f}^1(\boldsymbol{x})$	DM2 $\boldsymbol{f}^2(\boldsymbol{x})$	DM3 $\boldsymbol{f}^3(\boldsymbol{x})$	DM4 $\boldsymbol{f}^4(\boldsymbol{x})$	DM5 $\boldsymbol{f}^5(\boldsymbol{x})$	$\boldsymbol{\varepsilon}_L$ $\boldsymbol{s}_L$	$\boldsymbol{\varepsilon}_M$ $\boldsymbol{s}_M$	$\boldsymbol{\varepsilon}_U$ $\boldsymbol{s}_U$	$\triangle(\boldsymbol{x})$
$\boldsymbol{x}^1$	(1,5)	(2,2)	(3,3)	(3,4)	(5,7)	$(3,5)^T$	$(1,2)^T$	$(-1,1)^T$	1.2288
$\boldsymbol{x}^2$	(2,3)	(3,3)	(4,2)	(4,3)	(3,6)	$(3,4)^T$	$(2,1)^T$	$(-1,0)^T$	1.1000
$\boldsymbol{x}^3$	(5,5)	(4,2)	(3,2)	(2,3)	(2,2)	$(4,4)^T$	$(3,2)^T$	$(1,1)^T$	1.0198
$\boldsymbol{x}^4$	(4,1)	(2,4)	(5,3)	(6,4)	(6,3)	$(5,2)^T$	$(4,0)^T$	$(1,-1)^T$	1.2961
$\boldsymbol{x}^5$	(5,2)	(5,5)	(3,7)	(4,2)	(4,2)	$(4,5)^T$	$(3,2)^T$	$(1,1)^T$	1.3304
$\boldsymbol{x}^6$	(3,7)	(6,3)	(2,5)	(1,7)	(3,6)	$(4,6)^T$	$(2,4)^T$	$(1,3)^T$	1.5067
$\boldsymbol{x}^7$	(6,6)	(2,3)	(3,3)	(3,5)	(2,5)	$(5,5)^T$	$(4,3)^T$	$(2,2)^T$	1.2884
$\boldsymbol{x}^8$	(7,5)	(3,3)	(4,2)	(7,5)	(5,3)	$(6,4)^T$	$(5,2)^T$	$(3,1)^T$	1.4832
$\boldsymbol{x}^9$	(3,4)	(4,2)	(6,4)	(6,4)	(4,4)	$(5,3)^T$	$(4,1)^T$	$(1,0)^T$	1.3454

By Theorem 16, if $\boldsymbol{\varepsilon}_L \le \boldsymbol{\varepsilon}_M \le \boldsymbol{\varepsilon}_U$, $|\boldsymbol{s}_L| \ge |\boldsymbol{s}_M| \ge |\boldsymbol{s}_U|$. Therefore, $\boldsymbol{s}_L$-optimal concession equilibrium solution, $\boldsymbol{s}_M$-optimal concession equilibrium solution, and $\boldsymbol{s}_U$-optimal concession equilibrium solution to (MGDMP) may not be the same $\boldsymbol{x}$. We have an example as follows.

Example 18. A company is about to produce a new mobile phone. It needs to give consumers a chance to choose their favorite colors and shapes. So 9 colors and shapes are provided as candidates of the new mobile phone. Five decision-makers were randomly selected to score the nine candidates. The smaller the score, the more satisfied the candidate is. The favoritism to colors and shapes of the new mobile phone is described with the score band $0 \sim 7$: $0 \sim 3$ mean like the new mobile phone with 0 meaning very much, 1 like, 2 general, and 3 a little, respectively; 4 means indifferent to the new mobile phone; $5 \sim 7$ means dislike the new mobile phone with 5 a little, 6 dislike, and 7 very much. Hence, (0,0) means like the colors and shapes of the new mobile phone very much. (2,3) means generally like the colors and like a little shapes of the new mobile phone. Then, the triangular fuzzy numbers of target concession value: $\boldsymbol{\varepsilon}_L = [(0,0)^T, (0,0)^T, (0,0)^T, (0,0)^T, (0,0)^T]$, $\boldsymbol{\varepsilon}_M = [(1,2)^T, (2,2)^T, (2,3)^T, (1,2)^T, (2,3)^T]$, and $\boldsymbol{\varepsilon}_U = [(3,3)^T, (3,3)^T, (4,4)^T, (4,3)^T, (4,4)^T]$ describes the degree of preference for color and shape, respectively. Let $\boldsymbol{f}^i(\boldsymbol{x}) = (f_1^i(\boldsymbol{x}), f_2^i(\boldsymbol{x}))$, $i = 1,2,3,4,5$, be multiattribute function of 5 decision makers. $f_1^i(\boldsymbol{x})$ represents color, and $f_2^i(\boldsymbol{x})$ indicates shape in Table 2.

We get the optimal objective value f_j^{i*}: $[(1,1), (2,2), (2,2), (1,2), (2,2)]$. By Lemma 4, we obtain three concession equilibrium value $(\boldsymbol{s}_L, \boldsymbol{s}_M, \boldsymbol{s}_U)$ for $\boldsymbol{s}_L$-concession equilibrium solution, $\boldsymbol{s}_M$- concession equilibrium solution, and $\boldsymbol{s}_U$-concession equilibrium solution to (MGDMP) respectively, as shown in Table 2.

From Table 2, we obtain three orders of the 9 candidates as follows:

$$\begin{aligned}
&\boldsymbol{x}^2 \preceq_{s_L} \boldsymbol{x}^4 \prec_{s_L} \boldsymbol{x}^3 \preceq_{s_L} \boldsymbol{x}^1 \equiv_{s_L} \boldsymbol{x}^9 \prec_{s_L} \boldsymbol{x}^5 \prec_{s_L} \boldsymbol{x}^7 \preceq_{s_L} \boldsymbol{x}^6 \equiv_{s_L} \boldsymbol{x}^8, \\
&\boldsymbol{x}^1 \equiv_{s_M} \boldsymbol{x}^2 \prec_{s_M} \boldsymbol{x}^4 \prec_{s_M} \boldsymbol{x}^3 \equiv_{s_M} \boldsymbol{x}^5 \preceq_{s_M} \boldsymbol{x}^9 \prec_{s_M} \boldsymbol{x}^6 \prec_{s_M} \boldsymbol{x}^7 \preceq_{s_M} \boldsymbol{x}^8, \\
&\boldsymbol{x}^2 \prec_{s_U} \boldsymbol{x}^1 \equiv_{s_U} \boldsymbol{x}^4 \prec_{s_U} \boldsymbol{x}^9 \prec_{s_U} \boldsymbol{x}^3 \equiv_{s_U} \boldsymbol{x}^5 \prec_{s_U} \boldsymbol{x}^7 \preceq_{s_U} \boldsymbol{x}^6 \equiv_{s_U} \boldsymbol{x}^8.
\end{aligned} \tag{43}$$

Under the triangular fuzzy numbers of target concession value, the above three ranking orders of the 9 candidates are different. The optimal concession equilibrium solutions to (MGDMP) are different too. The given target concession value can affect the optimal concession equilibrium solution to (MGDMP). Obviously, the consistency given by this example is very poor. When all the candidates have similar consistency, the orders obtained by $\boldsymbol{s}$-optimal concession equilibrium solution will have better fairness. Of course, it is not difficult to see that if a decision-maker cannot fairly evaluate a program, it directly affects the order of the program.

We choose the approach to MGDMP based on determining the weights of experts by using projection method in [5] to rank Example 18. In Table 3 the first line in the first column shows the weights of attributes determined by the experts, the other four lines in the first column show the weights of attributes determined randomly, and the second column shows the final ranking of the nine schemes. The ranking is determined via the values of projections of the approach to MGDMP based on determining the weights of experts by using projection method; i.e., the smaller the projection the better the alternative in [5]. From Table 3, it is found that the ranking results rely on the weights of attributes determined by the experts, and different weights lead to different ranking.

Table 3: Approach to MGDMP based on determining the weights of experts by using projection method [5].

Weight vector of attributes of five experts	Rank all alternatives
(0.50,0.50)(0.50,0.50)(0.50,0.50)(0.50,0.50)(0.50,0.50)	$x^4 \prec x^8 \prec x^9 \prec x^3 \prec x^5 \prec x^7 \prec x^1 \prec x^2 \prec x^6$
(0.97,0.03)(0.96,0.04)(0.49,0.51)(0.80,0.20)(0.14,0.86)	$x^4 \prec x^2 \prec x^9 \prec x^8 \prec x^6 \prec x^5 \prec x^7 \prec x^3 \prec x^1$
(0.42,0.58)(0.92,0.08)(0.79,0.21)(0.96,0.04)(0.66,0.34)	$x^8 \prec x^4 \prec x^6 \prec x^1 \prec x^9 \prec x^2 \prec x^5 \prec x^3 \prec x^7$
(0.04,0.96)(0.85,0.15)(0.93,0.07)(0.68,0.32)(0.76,0.24)	$x^1 \prec x^6 \prec x^4 \prec x^8 \prec x^9 \prec x^7 \prec x^3 \prec x^5 \prec x^2$
(0.74,0.26)(0.39,0.61)(0.66,0.34)(0.17,0.83)(0.71,0.29)	$x^2 \prec x^7 \prec x^6 \prec x^3 \prec x^4 \prec x^5 \prec x^9 \prec x^8 \prec x^1$

In particular in the third line, x^8 is ranked as No. 1, while from Table 2 the value of $\triangle(\boldsymbol{x}^8)$ is 1.48 so x^8 should be ranked around No. 8, which is an obvious deviation from the experts'. However, the difference in our ranking is not so big. Using approach to MGDMP based on determining the weights of experts by using projection method in [5], which relies on the weights determined by the experts, might disturb the final ranking of all the schemes and sometime even brings contradictory rankings. However, our proposal avoids the determination of attributes' weights and experts' weights, as the value $(f_1^i(\boldsymbol{x}), f_2^i(\boldsymbol{x}))$ has contained the preference of the attributes and the experts.

The merit of our proposal is that there is no need to determine the attributes' weights and experts' weights and it is easy to determine the triangular fuzzy numbers of target concession value as it is given through attributes' value. Another merit of our proposal is that we may find the optimum solution from the infinite number of candidates, as some methods used to solve MGDMPs do not apply to the situations where there are infinite candidates as shown in [5].

4. Conclusion

The paper defines a new $\boldsymbol{s}^*$- optimal concession equilibrium solution and proves that when there exist optimal solutions to all the subproblems there exists the $\boldsymbol{s}^*$- optimal concession equilibrium solution and that it is equivalent to solving a single objective programming problem. Besides, the paper proves that the $\boldsymbol{s}^*$- optimal concession equilibrium solution is equivalent to the optimal solution with the worst weight using the linear weighted objective method. Finally, we prove that all candidate schemes can be ranked by the concession equilibrium solution. By defining the triangular fuzzy number of target concession value, the ranking order of the schemes or the optimal concession equilibrium solution can be obtained in the range of objective concession ambiguity. The numerical experiments show that the $\boldsymbol{s}^*$- optimal concession equilibrium solution has stable ranking as compared to that by the weighted aggression method and can balance the preferences of different decision-makers about different attributes.

Data Availability

The [DATA TYPE] data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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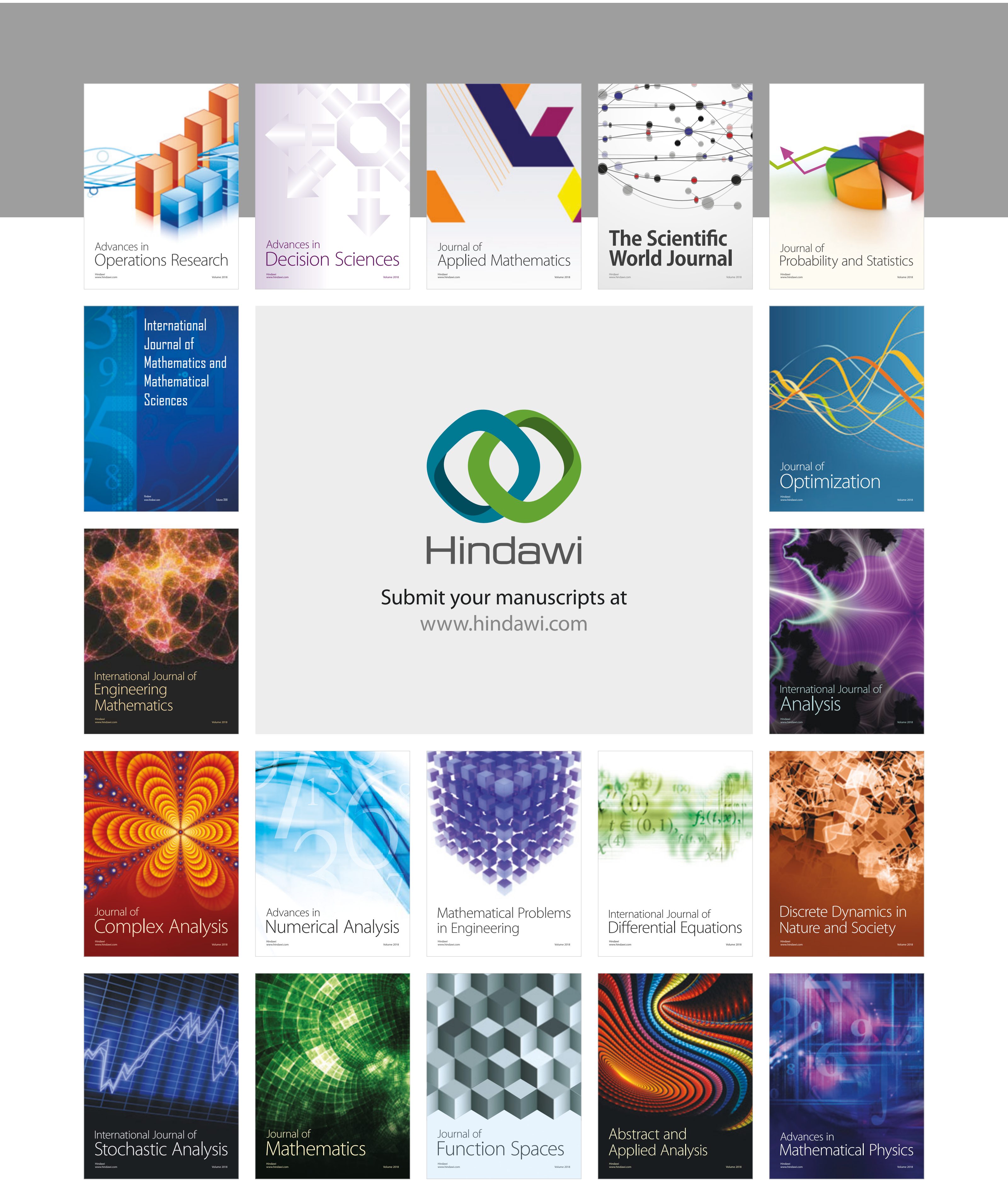
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