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Research Article

Optimal Control Strategy for a Discrete Time to the Dynamics of a Population of Diabetics with Highlighting the Impact of Living Environment

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Nowadays, Diabetes is one of the most common diseases, which has a huge and growing socio-economic burden affecting individuals, families, and the whole society. In this paper, we propose an optimal control approach modeling the evolution from pre-diabetes to diabetes with and without complications and the effect of living environment. We show the existence of an optimal control and then use a numerical implicit finite-difference method to monitor the size of population in each compartment.

1. Introduction

Today, all countries of the world suffer from the high number of people with diabetes, which is increasing and expanding at the extreme level. According to the latest statistics from the International Diabetes Federation (IDF) and as reported in the eighth edition of the Atlas Diabetes 2017 [1], diabetes ranks fourth in diseases leading to death which means that it is a serious disease, and all social and age groups suffer from it. Also, IDF says that the number of people with diabetes is more than 425 million people, most of them are 65 years old, which increases the risk of the disease that does not exclude children and adolescents under 20 years. The last estimates of the number of people with diabetes in this category are that more than one million people have diabetes type I or type II. This is required to raise the degree of danger and preparedness to find solutions to it and reduce its expansion because the number of people with diabetes could increase to 629 million in the horizon in the year 2045. Despite these predictions, the number of people with the disease has begun to decline in some high-income countries. At the same time, despite this decline, there are about 352 million other people suffering from the disease for a variety of reasons such as impaired glucose tolerance, and these people are more at risk of diabetes, very large, and there are about 10 million adults and 8 million elderly people over the age of 65 years suffering from diabetes in recent years compared to 2015.

Diabetics are more likely to have a variety of other serious diseases; when it is not treated well all types of diabetes can lead to complications in many parts of the body, leading to an early death. Diabetics have an increased risk of a number of serious life-threatening health problems that have psychological, moral, and behavioral effects, leading to increased costs of medical care as well as serious physical deterioration, due to the increased risk of chronic disease with severe complications in various types such as cardiovascular disease, blindness, kidney failure, and amputation of the lower extremities, and so on. According to IFD statistics, diabetics have a two- to three-fold increased risk of cardiovascular disease, as well in more than a third of them, the incidence of retinopathy among all people with diabetes is the main cause of vision loss of adults at work and furthermore they accuse an increasing incidence of end-stage renal diseases (ESRD) up to 10 times in the disease. Besides, complications need to be treated and intensive care and expensive long-term care which have a material impact on individuals, families, and society as a whole have to be considered. The American Diabetes Association estimates that the annual cost of treating a person with diabetes is more than five times higher than the cost of a person who is not diabetic. Other studies suggest that the treatment of diabetic patients with complications is two to five times higher than the treatment of diabetes without complications, compounding burdens that have exceeded the limits of economic problems by incurring indirect and intangible costs [11]. Thus, to ease the social and economic burdens, several studies must be conducted for controlling the number of people developing from pre-diabetes to diabetes stages with and without complications.

During the last decade, a large number of mathematical models have been developed to simulate, analyse, and understand the dynamics of a population of diabetics; in a related research work, Boutayeb and Chetouani [2] and Derouich et al. [3] proposed a mathematical model for the dynamics of the population of diabetes patients with or without complications using a system of ordinary differential equations. Also, many researches have focused on this topic and other related topics [4, 5, 12–17].

Specialists overlooked the living environment as one of the main causes of the development of complications of diabetes, which plays a key role in the development of complications of diabetes in various types, according to studies carried out by the IDF. Therefore, in our proposed model we wanted to highlight the impact of the living environment on diabetic patients (availability of healthy food, diet, exercise, weight for instance) and its main role in the development of complications, which often ends either with loss of vision or amputation of the toes, feet, and lower legs or Paraplegia and so on. To achieve this objective, we consider a compartment model that describes the dynamic of a population of diabetics that is divided into four classes, i.e: the potential diabetic specialy pre-diabetics by through genetics (P), diabetics without complications (D), diabetics with complications (C), and we add a compartment (E) people who are likely to have diabetes through the effect of living environment or psychological problems.

We notice that most of researchers about diabetes and its complications focused on continuous time models, and described by differential equations. Recently, more and more attention has been paid to discrete time models (see [6, 7] and the references mentioned there). Reasons for adopting discrete models are as follows: First, statistical data are collected separately for moments (day, week, month, or year). Therefore, it is more direct, more accurate, and timely to describe the disease using discrete time models instead of continuous time models. Second, discrete time models can be used to avoid some mathematical complexities such as choosing the function space and regularity of the solution. Third, numerical simulations of continuous time models are obtained by abstraction.

In this paper, in Section 2, we propose a discrete *PDEC* Mathematical that describes the dynamic of a population of diabetics. In Section 3, we present an optimal control problem for the proposed model where we give some result concerning the existence of the optimal control and we characterize the optimal controls using the Pontryagin maximum principle in discrete time. Numerical simulations through MATLAB are given in Section 4. Finally, we conclude the paper in Section 5.

2. A Mathematical Model

We consider a discrete mathematical model *PDEC* that describes the dynamics of a population of diabetics; we highlight the impact of the living environment, such as unhealthy food and health habits on diabetics without complications. We divide the population denoted by N into four compartments.

2.1. Description of the Model. The potential diabetic (P) refers to people who are likely to have diabetes through genetics, P is increasing by Λ_1 (denotes the incidence of pre-diabetes) and decreasing by the amount μ (natural mortality), β_1 (the rate of patients who develop diabetes without complications), and β_3 (the probability of developing diabetes at stage of complications).

$$P_{k+1} = \Lambda_1 + (1 - \mu - \beta_1 - \beta_3)P_k. \tag{1}$$

Compartment (D) is the number of diabetics without complications $D = D_k$ is increasing by amount $\beta_1 P_k$ and by the amount γE_k (patients who become diabetics without complications because of the effect of the living environment), and decreasing by μD_k (natural mortality) and $\alpha(D_k E_k/N)$ (the rate of patients who become diabetic with complication because of the bad contact with the living environment) and decreasing by $\beta_2 C_k$ (the probability of a diabetic person developing a complication).

$$D_{k+1} = \beta_1 P_k + \gamma E_k - \alpha \frac{D_k E_k}{N} + (1 - \mu - \beta_2)D_k. \tag{2}$$

Compartment (E) refers to people who are likely to have diabetes through the effect of living environment or psychological problems $E = E_k$ that describes the dynamics of a population of diabetics; we highlight the impact of the living environment, leading to an increase in the number of people diabetics to diabetics without complications and to diabetics with complications as people who live in the Middle East, Europe, or the USA, so in this compartment, it is increasing by Λ_2 (denote the incidence of effect environment) decreasing by γE_k (the rate of probability developing diabetes) and also decreased by μE_k (natural mortality)

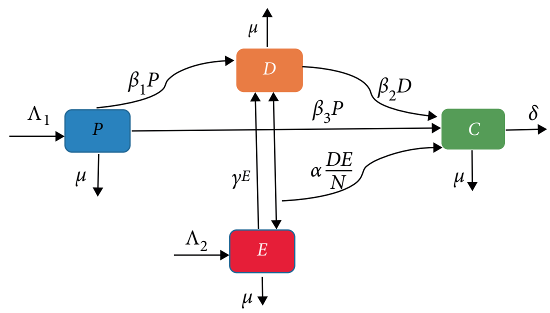
$$E_{k+1} = \Lambda_2 + (1 - \mu - \gamma)E_k. \tag{3}$$

Compartment (C) is the number of diabetics with complications $C = C_k$ is increasing by $\beta_3 P_k$ and increasing $\beta_2 D_k$ and also increasing by $\alpha(D_k E_k/N)$ (the rate of patients who become diabetics with complication because of the bad contact of the living environment) and decreasing by δC_k (mortality rate due to complications) and also decreasing by μC_k (natural mortality)

$$C_{k+1} = \beta_3 P_k + \beta_2 D_k + \alpha \frac{D_k E_k}{N} + (1 - \mu - \delta)C_k. \tag{4}$$

The following diagram proves the flow directions of diabetics among the compartments. These directions are going to be represented by directed arrows in figure of compartement.

2.2. Model Equations. Through the addition of the rates at which diabetics enter the compartment and also by subtracting the rates at which people vacate the compartment, we obtain an equation of difference for the rate at which the patients of each compartment change over discrete time. Hence, we present the diabetic model by the following system of difference equations:



The model is presented

$$\begin{aligned} P_{k+1} &= \Lambda_1 + (1-\mu-\beta_1-\beta_3)P_k, \\ D_{k+1} &= \beta_1 P_k + \gamma E_k - \alpha \frac{D_k E_k}{N} + (1-\mu-\beta_2)D_k, \\ E_{k+1} &= \Lambda_2 + (1-\mu-\gamma)E_k, \\ C_{k+1} &= \beta_3 P_k + \beta_2 D_k + \alpha \frac{D_k E_k}{N} + (1-\mu-\delta)C_k. \end{aligned} \tag{5}$$

And T: final time with $P_0.D_0.E_0, C_0 \geq 0$.

3. Formulation of the Model

The strategy of control we adopt consists of an awareness program through to correct environment effect in diabetics people without complication. Our main goal is to minimize the number of people evolving from the stage of pre-diabetes to the stages of diabetes with and without complications. In this model, we include three controls u_k, v_k, and w_k, that represent consecutively the awareness program through media and education, treatment, and psychological support with follow-up as measures at time k. So the controlled mathematical system is given by the following system of difference equations:

$$\begin{aligned} P_{k+1} &= \Lambda_1 + (1-\mu-\beta_1-\beta_3)P_k, \\ D_{k+1} &= \beta_1 P_k + \gamma(1-w_k)E_k - \alpha(1-v_k)\frac{D_k E_k}{N} \\ &\quad + (1-\mu-\beta_2)D_k + u_k C_k, \\ E_{k+1} &= \Lambda_2 + (1-\mu)E_k - \gamma(1-w_k)E_k, \\ C_{k+1} &= \beta_3 P_k + \beta_2 D_k + \alpha(1-v_k)\frac{D_k E_k}{N} \\ &\quad + (1-\mu-\delta)C_k - u_k C_k. \end{aligned} \tag{6}$$

4. The Optimal Control Problem

There are three controls $u = (u_0, u_1, \ldots, u_{T-1})$, $v = (v_0, v_1, \ldots, v_{T-1})$, and $w = (w_0, w_1, \ldots, w_{T-1})$. The first control can be interpreted as the proportion to be subjected to treatment. So, we note that uC_k is the proportion of diabetics at stage of complications who moved to diabetics without complications at time step k. The second control can be interpreted as the proportion to be adopted for the awareness program through media and education. So, we note that $v(D_k E_k/N)$ is the proportion of bad contact in living environment in diabetics without complications to complications, who will move to reduce the number of patients, diabetics with complications. The third control wE_k is the rate of probability developing diabetes, who will move to reduce the number of patients becoming diabetics without complications because of the effect of living environment at time step k.

The problem is to minimize the objective functional

$$J(u, v, w) = C_T - D_T + \sum_{k=0}^{T-1} \left[C_k - D_k + \frac{A_k}{2}u_k^2 + \frac{B_k}{2}v_k^2 + \frac{G_k}{2}w_k^2 \right]. \tag{7}$$

where A_k, B_k, and G_k are the cost coefficients. They are selected to weigh the relative importance of u_k, v_k, and w_k at time k,T is the final time.

In other words, we seek the optimal controls u^*,v^*,and w^* such that

$$J(u^*, v^*, w^*) = \min_{u,v,w \in U} J(u, v, w). \tag{8}$$

where U is the set of admissible controls defined by $U = \{(u, v, w)/0 \leq u_{\min} \leq u_k \leq u_{\max} \leq 1, 0 \leq v_{\min} \leq v_k \leq v_{\max} \leq 1 \text{ and } 0 \leq w_{\min} \leq w_k \leq w_{\max} \leq 1, k \in \{0, 1, \ldots, T-1\}\}$.

In order to derive the necessary condition for optimal control, pontryagin's maximum principle, in discrete time, given in was used. This principle converts into a problem of minimizing a Hamiltonian, H_k at time step k defined by

$$\begin{aligned} H_k &= C_k - D_k + \frac{A_k}{2}u_k^2 + \frac{B_k}{2}v_k^2 + \frac{G_k}{2}w_k^2 \\ &\quad + \sum_{i=1}^{4} \lambda_{i,k+1} f_{i,k+1}(P_k, D_k, E_k, C_k), \end{aligned} \tag{9}$$

where $f_{i,k+1}$ is the right side of the difference equation of the i^{th} state variable at time step $k+1$.

5. The Optimal Control: Existence

We first show the existence of solutions of the system, after that we will prove the existence of optimal control [8, 9].

Theorem 1. *Consider the control problem with the system. There are three optimal controls $(u^*, v^*, w^*) \in U^3$ such that*

$$J(u^*, v^*, w^*) = \min_{u,v,w \in U} J(u, v, w). \tag{10}$$

Given the optimal controls (u^*, v^*, w^*) and the solutions P^*, D^*, E^*, and C^* of the corresponding state system (6), there exists adjoint variables $\lambda_{1,k}, \lambda_{2,k}, \lambda_{3,k}$, and $\lambda_{4,k}$ satisfying:

$$\lambda_{1,k} = \frac{\partial H_k}{\partial P_k} = \lambda_{1,k+1}[1-\mu-\beta_1-\beta_3] + \lambda_{2,k+1}\beta_1 + \lambda_{4,k+1}\beta_3, \tag{11}$$

$$\begin{aligned} \lambda_{2,k} = \frac{\partial H_k}{\partial D_k} &= -1 + \lambda_{2,k+1}\left[-\alpha(1-v_k)\frac{E_k}{N} + (1-\mu-\beta_2)\right] \\ &\quad + \lambda_{4,k+1}\left[\alpha(1-v_k)\frac{E_k}{N} + \beta_2\right], \end{aligned} \tag{12}$$

$$\lambda_{3,k} = \frac{\partial H_k}{\partial E_k} = \lambda_{2,k+1}\left[\gamma(1-w_k) - \alpha(1-v_k)\frac{D_k}{N}\right] + \lambda_{3,k+1}[(1-\mu) - \gamma(1-w_k)] + \lambda_{4,k+1}\left(\alpha(1-v_k)\frac{D_k}{N}\right), \quad (13)$$

$$\lambda_{4,k} = \frac{\partial H_k}{\partial C_k} = 1 + \lambda_{2,k+1}u_k + \lambda_{4,k+1}(1-\mu-\gamma-u_k). \quad (14)$$

With the transversality conditions at time T: $\lambda_{1,T} = 0$, $\lambda_{2,T} = -1$, $\lambda_{3,T} = 0$, and $\lambda_{4,T} = 1$.

Furthermore, for $k = 0, 1, 2, \ldots, T-1$, the optimal controls u^*, v^*, and w^* are given by

$$u^* = \min\left(0, \max\left(1, \frac{(\lambda_{4,k+1} - \lambda_{2,k+1})}{A_k}C_k\right)\right), \quad (15)$$

$$v^* = \min\left(0, \max\left(1, \frac{(\lambda_{4,k+1} - \lambda_{2,k+1})}{B_k} \times \frac{\alpha D_k E_k}{N}\right)\right), \quad (16)$$

$$w^* = \min\left(0, \max\left(1, \frac{(\lambda_{2,k+1} - \lambda_{3,k+1})}{G_k} \times \gamma E_k\right)\right). \quad (17)$$

Proof. Since the coefficients of the state equations are bounded and there are a finite number of time steps, $P = (P_0, P_1, \ldots, P_T)$, $D = (D_0, D_1, \ldots, D_T)$, $E = (E_0, E_1, \ldots, E_T)$, and $C = (C_0, C_1, \ldots, C_T)$ are uniformly bounded for all (u, v, w) in the control set U, thus $J(u, v, w)$ is bounded for all $(u, v, w) \in U$. Since $J(u, v, w)$ is bounded, $\inf_{(u,v,w)\in U} J(u, v, w)$ is finite, and there exists a sequence $(u^i, v^i, w^i) \in U$ such that $\lim_{i\to+\infty} J(u^i, v^i, w^i) = \inf_{(u,v,w)\in U} J(u, v, w)$ and corresponding sequences of states P^i, D^i, E^i, and C^i. Since there is a finite number of uniformly bounded sequences, there exist $(u^*, v^*, w^*) \in U$ and P^*, D^*, E^*, and $C^* \in \mathbb{R}^{T+1}$ such that, on a subsequence, $(u^i, v^i, w^i) \mapsto (u^*, v^*, w^*)$, $P^i \mapsto P^*$, $D^i \mapsto D^*$, $E^i \mapsto E^*$, and $C^i \mapsto C^*$. Finally, due to the finite dimensional structure of system (2) and the objective function $J(u, v, w)$, and (u^*, v^*, w^*) is an optimal control with corresponding states P^*, D^*, E^*, and C^*. Therefore $\inf_{(u,v,w)\in U} J(u, v, w)$ is achieved.

The Hamiltonian at time step k is given by

$$\begin{aligned}
H_k = {} & C_k - D_k + \frac{A_k}{2}u_k^2 + \frac{B_k}{2}v_k^2 + \frac{G_k}{2}w_k^2 \\
& + \lambda_{1,k+1}[\Lambda_1 + (1-\mu-\beta_1-\beta_3)P_k] \\
& + \lambda_{2,k+1}\Big[\beta_1 P_k + \gamma(1-w_k)E_k - \alpha(1-v_k)\frac{D_k E_k}{N} \\
& + (1-\mu-\beta_2)D_k + u_k C_k\Big] \\
& + \lambda_{3,k+1}[\Lambda_2 + (1-\mu)E_k - \gamma(1-w_k)E_k] \\
& + \lambda_{4,k+1}\Big[\beta_3 P_k + \beta_2 D_k + \alpha(1-v_k)\frac{D_k E_k}{N} \\
& + (1-\mu-\delta)C_k - u_k C_k\Big].
\end{aligned} \quad (18)$$

For, $k = 0, 1, \ldots, T-1$ the optimal controls u_k, v_k, and w_k can be solved from the optimality condition,

$$\frac{\partial H_k}{\partial u_k} = 0, \quad \frac{\partial H_k}{\partial v_k} = 0, \quad \frac{\partial H_k}{\partial w_k} = 0. \quad (19)$$

That are

$$\begin{aligned}
\frac{\partial H_k}{\partial u_k} &= A_k u_k + (\lambda_{4,k+1} - \lambda_{2,k+1})C_k = 0, \\
\frac{\partial H_k}{\partial v_k} &= B_k v_k + (\lambda_{4,k+1} - \lambda_{2,k+1})\frac{D_k E_k}{N} = 0, \\
\frac{\partial H_k}{\partial w_k} &= G_k w_k + (\lambda_{3,k+1} - \lambda_{2,k+1})\gamma E_k = 0,
\end{aligned} \quad (20)$$

we have

$$\begin{aligned}
u_k &= \frac{(\lambda_{4,k+1} - \lambda_{2,k+1})}{A_k}C_k, \\
v_k &= \frac{(\lambda_{4,k+1} - \lambda_{2,k+1})}{B_k} \times \frac{\alpha D_k E_k}{N}, \\
w_k &= \frac{(\lambda_{2,k+1} - \lambda_{3,k+1})\gamma E_k}{G_k}.
\end{aligned} \quad (21)$$

By the bounds in U of the controls, it is easy to obtain u_k^*, v_k^*, and w_k^* in the form of system.

6. Numerical Simulation

Algorithm 1. In this section, we present the results obtained by solving numerically the optimality system.

This system consists of the state system, adjoint system, initial and final time conditions, and control characterization.

So, the optimality system is given by the following:

Step 1. $P_0 = p_0$, $D_0 = d_0$, $E_0 = e_0$, $C_0 = c_0$, $\lambda_{1,T} = 0$, $\lambda_{2,T} = -1$, $\lambda_{3,T} = 0$, $\lambda_{4,T} = 1$ and given $u_{k,0}^*$, $v_{k,0}^*$, and $w_{k,0}^*$

Step 2. For $k = 0; 1; \ldots; T-1$ do:

$$\begin{aligned}
P_{k+1} &= \Lambda_1 + (1-\mu-\beta_1-\beta_3)P_k, \\
D_{k+1} &= \beta_1 P_k + \gamma(1-w_k)E_k - \alpha(1-v_k)\frac{D_k E_k}{N} \\
&\quad + (1-\mu-\beta_2)D_k + u_k C_k, \\
E_{k+1} &= \Lambda_2 + (1-\mu)E_k - \gamma(1-w_k)E_k, \\
C_{k+1} &= \beta_3 P_k + \beta_2 D_k + \alpha(1-v_k)\frac{D_k E_k}{N} + (1-\mu-\delta)C
\end{aligned} \quad (22)$$

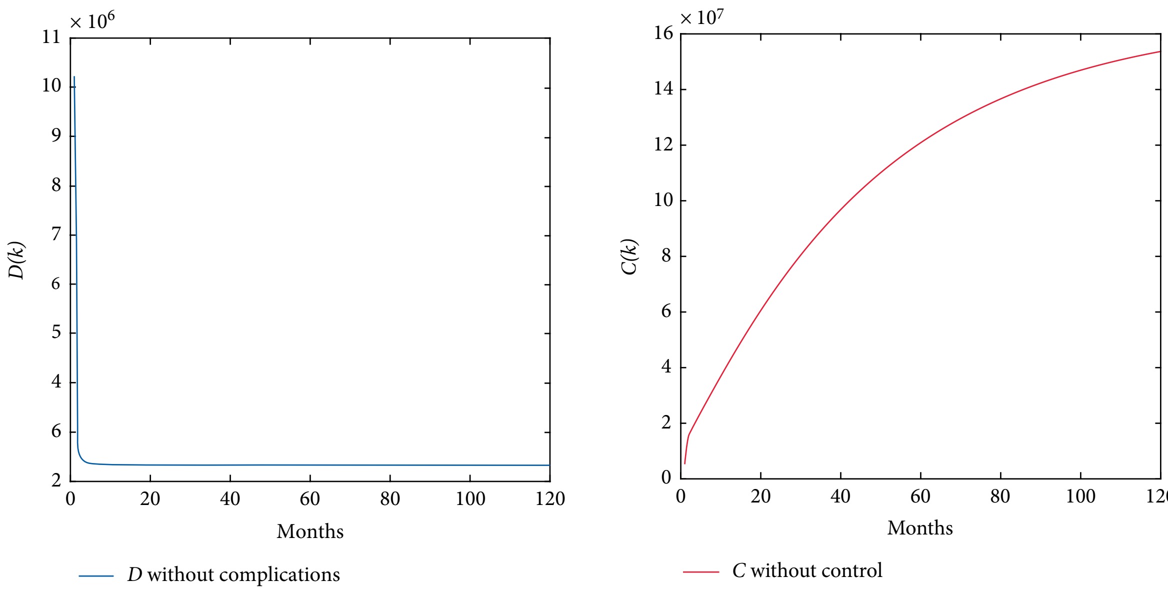


FIGURE 1: The evolution of the number of diabetics with and without complications without controls.

$$
\begin{aligned}
\lambda_{1,T-k} &= \lambda_{1,T-k+1}\left[1-\mu-\beta_1-\beta_3\right] \\
&\quad + \lambda_{2,T-k+1}\beta_1 + \lambda_{4,T-k+1}\beta_3, \\
\lambda_{2,T-k} &= -1 + \lambda_{2,T-k+1}\left[-\alpha\left(1-v_k\right)\frac{E_k}{N} + \left(1-\mu-\beta_2\right)\right] \\
&\quad + \lambda_{4,T-k+1}\left[\alpha\left(1-v_k\right)\frac{E_k}{N} + \beta_2\right], \\
\lambda_{3,T-k} &= \lambda_{2,T-k+1}\left[\gamma\left(1-w_k\right) - \alpha\left(1-v_k\right)\frac{D_k}{N}\right] \\
&\quad + \lambda_{3,T-k+1}\left[\left(1-\mu\right) - \gamma\left(1-w_k\right)\right] \\
&\quad + \lambda_{4,T-k+1}\left(\alpha\left(1-v_k\right)\frac{D_k}{N}\right), \\
\lambda_{4,T-k} &= 1 + \lambda_{2,T-k+1}u_k + \lambda_{4,T-k+1}\left(1-\mu-\delta-u_k\right),
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
u^* &= \min\left(1, \max\left(0, \frac{\left(\lambda_{4,T-k+1}-\lambda_{2,T-k+1}\right)}{A}C_k\right)\right), \\
v^* &= \min\left(1, \max\left(0, \frac{\left(\lambda_{4,T-k+1}-\lambda_{2,T-k+1}\right)}{B} \times \frac{\alpha D_k E_k}{N}\right)\right), \\
w^* &= \min\left(0, \max\left(1, \frac{\gamma\left(\lambda_{2,T-k+1}-\lambda_{3,T-k+1}\right)E_k}{G_k}\right)\right).
\end{aligned}
\tag{24}
$$

end for

Step 3. For $k = 0; 1; \ldots; T$ write:

$P_k^* = P_k^i$, $D_k^* = D_k^i$, $E_k^* = E_k^i$, and $C_k^* = C_k^i$
$u_k^* = u_k^i$, $v_k^* = v_k^i$, and $w_k^* = w_k^i$

Different simulations can be carried out using various values of parameters. In the present numerical approach, we use the following parameter values taken from [2]:

$P(0) = 6660000$, $D(0) = 10200000$, $E(0) = 10000000$, $C(0) = 5500000$, $n = 100$, $\lambda_1(n) = 0$, $\lambda_2(n) = -1$, $\lambda_3(n) = 0$, and $\lambda_4(n) = 1$.

TABLE 1: Parameter values used in numerical simulation.

Parameter	Value in mth^{-1}
μ	0.02
δ	0.001
β_1	0.2
β_2	0.5
β_3	0.1
α	0.8
γ	0.8
Λ_1	2000000
Λ_2	2000000

Since control and state functions are on different scales, the weight constant value is chosen as follows: $A = 100$, $B = 100$, and $G = 100$ (Figure 1).

After the parameter values (Table 1), we note that diabetics without complications after 120 months decreased from 10.2×10^6 to 2.31×10^6 (Figure 1) This transformation is due to two main things: first is the genetic factors; second, due to the negative impact of the living environment on the patient (nutrition pattern, psychological and moral problems). We note that the number of patients with diabetes complications is increasing. Indeed, we note the number of the transition becomes from 6.66×10^6 to 1.53×10^8 (Figure 1) and, as mentioned above, to come to disease progression for patients with diabetes without complications and also a sudden shift in the potential for people diagnosed with diabetes by means of genetics.

In this formulation, there are initial conditions for the state variables and terminal conditions for the adjoints.

That is, the optimality system is a two-point boundary value problem with separated boundary conditions at time steps $k = 0$ and $k = T$. We solve the optimality system by an iterative method with forward solving of the state system followed by backward solving of the adjoint system. We start with an initial guess for the controls at the first iteration and then

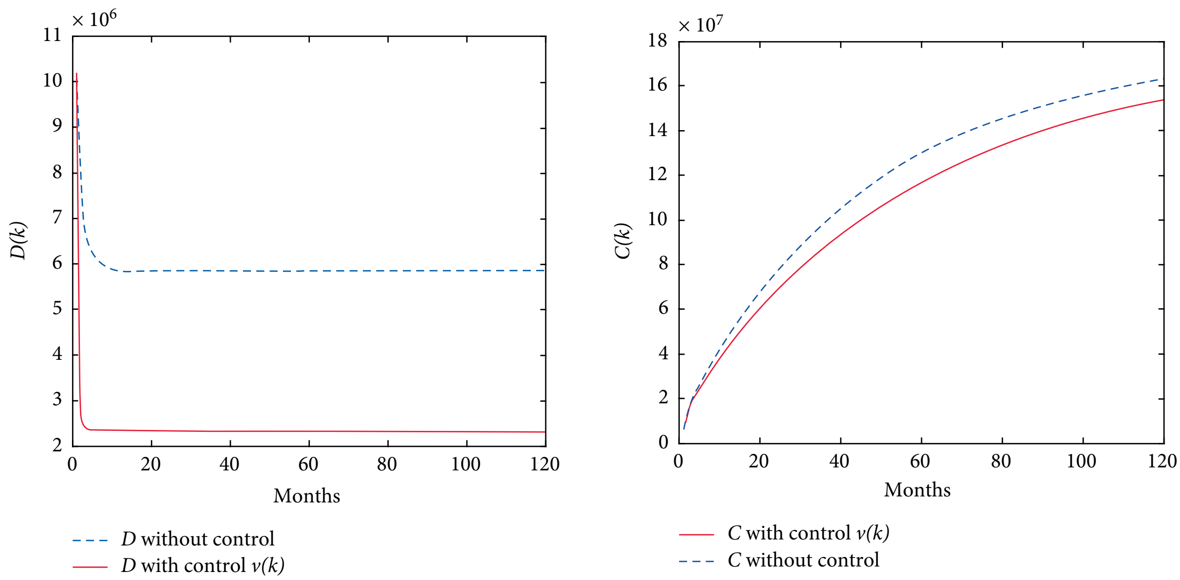


Figure 2: The evolution of the number of diabetics with and without complications with control v_k.

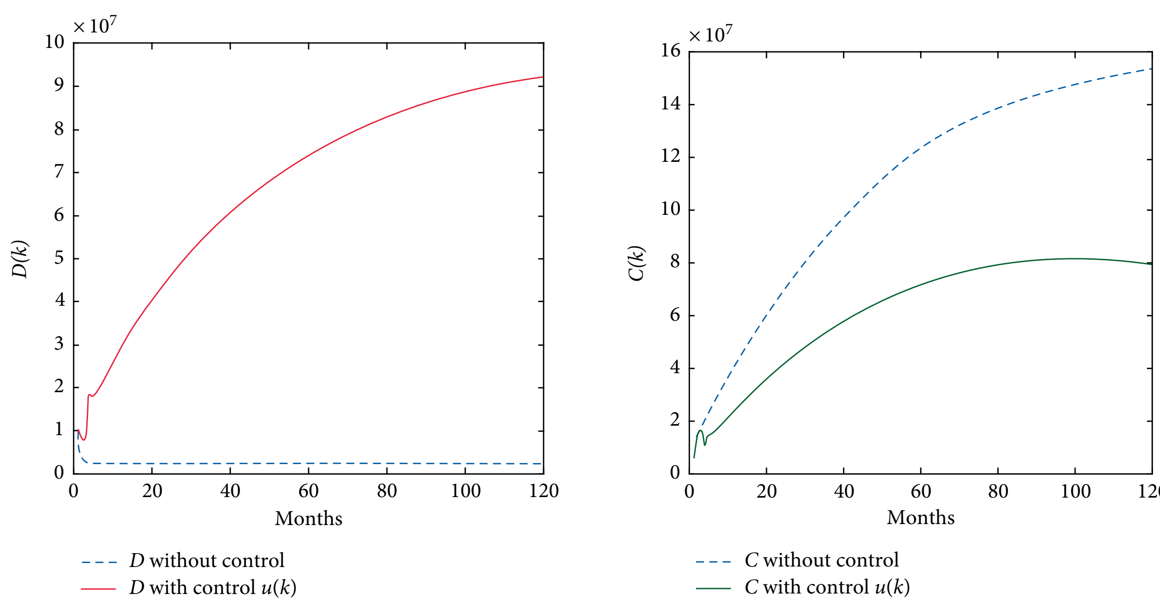


Figure 3: The evolution of the number of diabetics with and without complications with controlu_k.

before the next iteration we update the controls using characterization.

We continue until convergence of successive iterates is achieved.

The proposed control strategy in this work helps to achieve several objectives.

6.1. Objective: Prevention: To Protect Diabetic Patients without Complications from the Negative Impact of the Environment. Awareness programs are to lower the effect of bad contact with living environment, we propose an optimal strategy for this purpose. Hence, we activate the optimal control variable v_k which represents the awareness program for diabetics without complications. Figure 2 compares the evolution of the number of diabetics with complications with and without control v_k in which is the effect of the proposed awareness program through a diet program for diabetics and to keep them as far as possible about problems and family pressures and process.

6.2. Strategy B: Control with Treatment and Psychological. In this strategy, we apply in order to reduce the number of diabetics with complication to diabetics without complications; through Figure 3 note that after applying different Strategic, which is a medicine and psychologist support, the number of diabetics with complication dropped from 1.53×10^8 to 7.96×10^7 the end of the strategic.

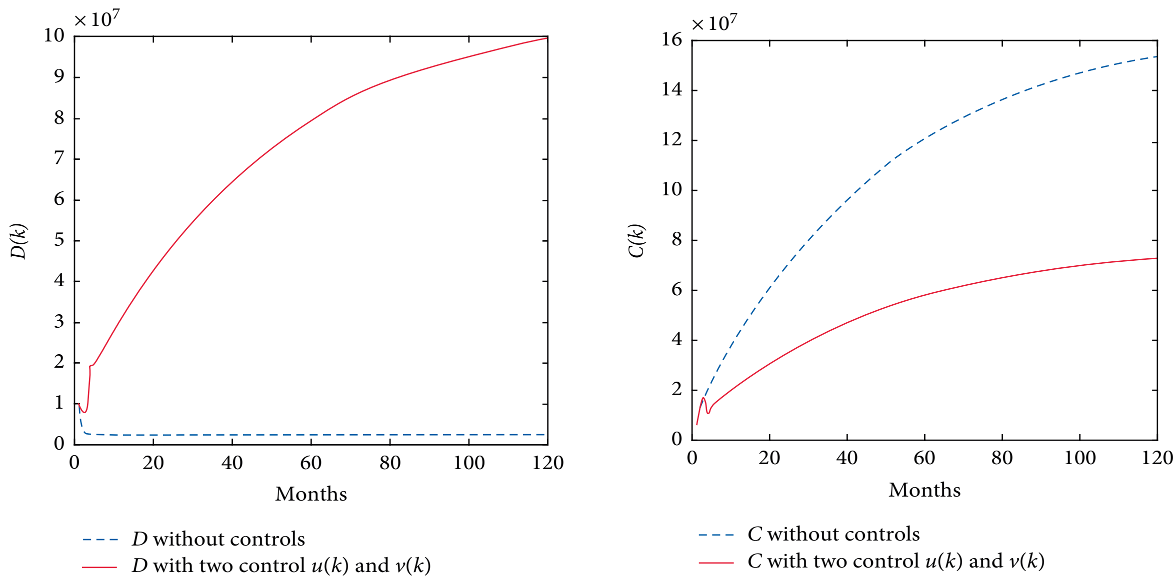


FIGURE 4: The evolution of the number of diabetics with and without complications with two controls u_k and v_k.

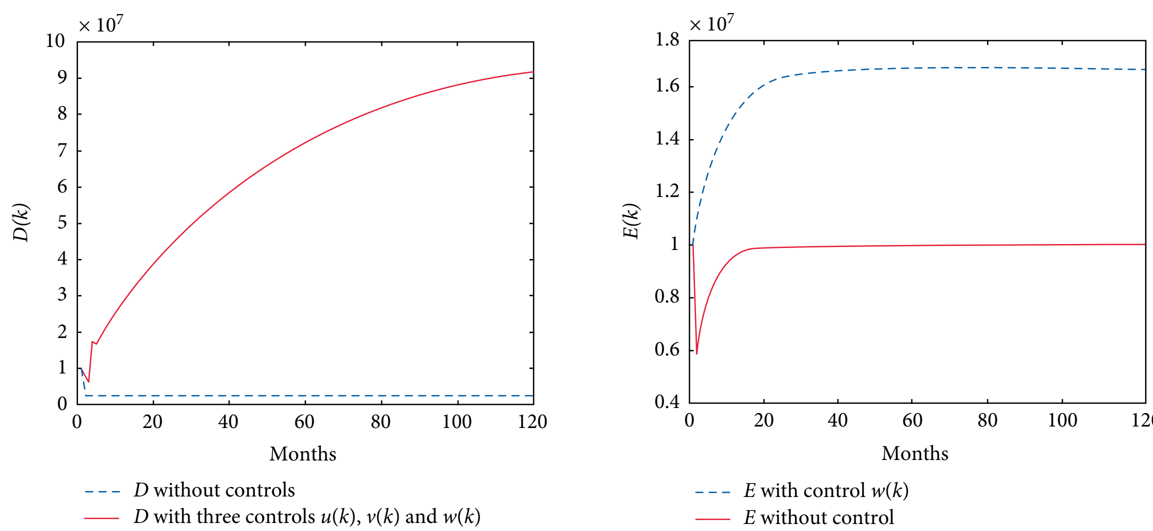


FIGURE 5: The evolution of the number of diabetics without complications with three controls u_k, v_k, and w_k.

The reason for this increase is justified by the fact that the number of diabetics with complications will becomes diabetics without complications. For improving the effectiveness of this strategy, we add the elements of follow-up and psychological support which are represented in the proposed strategy by the optimal control variable u_k (Figure 3). Combining follow-up and psychological support with treatment results in an obvious decrease in the number of diabetics with complications.

6.3. Strategy C: Control with Awareness Program, Treatment, and Psychological Support with Follow-Up. In this strategy, we use two controls optimal u_k and v_k (Figure 4), that we combine the previous two strategies to achieve better results that is represent awareness program through education and media for lowering the effect of bad contact with living environment, and also treatment, and psychological support with follow up. In Figure 4, we observe that the number of diabetics with complications is decreasing from 1.53×10^8 to 7.29×10^7.

6.4. Strategy D: Prevention: Protection E from Diabetes

6.4.1. With Three Controls u_k, v_k and w_k. In this starategy we use three controls u_k, v_k, and w_k (Figure 5), the objectif of the two controls u_k, and v_k as we say above, and the object if of the third control w_k is the aim is to raise awareness campaigns for this target group on the risks of diabetes and its complications with giving recipes for health nutrition.

Remark 1. We can also merge multiple assemblies as (u_k, w_k), (v_k, w_k) and thus get a variety of results.

7. Conclusion

In this paper, we introduced a discrete modeling of populations diabetics with and without complications and effect of living environment in order to minimize the number of diabetics with complications, and lower the effect of bad contact with living environment.

We also introduced three controls which, respectively, represent awareness program through education and media, treatment, and psychological support with follow up.

We applied the results of the control theory and we managed to obtain the characterizations of the optimal controls. The numerical simulation of the obtained results showed the effectiveness of the proposed control strategies.

Data Availability

The disciplinary data used to support the Öndings of this study have been deposited in the Network Repository (http://www.networkrepository.com).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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