

Research Article

Valuation of Guaranteed Unitized Participating Life Insurance under MEGB2 Distribution

Haitao Zheng ¹, Junzhang Hao ¹, Manying Bai ¹, and Zhengjun Zhang²

¹School of Economics & Management, Beihang University, Beijing, 100083, China

²Department of Statistics, University of Wisconsin, Madison, WI 53706, USA

Correspondence should be addressed to Junzhang Hao; haojz@buaa.edu.cn

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Crisis events have significantly changed the view that extreme events in financial markets have negligible probability. Especially in the life insurance market, the price of guaranteed participating life insurance contract will be affected by a change in asset volatility which leads to the fluctuations in embedded option value. Considering the correlation of different asset prices, MEGB2 (multivariate exponential generalized beta of the second kind) distribution is proposed to price guaranteed participating life insurance contract which can effectively describe the dependence structure of assets under some extreme risks. Assuming the returns of two different assets follow the MEGB2 distribution, a multifactor fair valuation pricing model of insurance contract is split into four components: the basic contract, the annual dividend option, the terminal dividend option, and the surrender option. This paper studies the effect of death rate, minimum guaranteed yield rate, annual dividend ratio, terminal dividend ratio, and surrender on the embedded option values and calculates the single premium of the insurance contract under different influence factors. The Least-Squares Monte Carlo simulation method is used to simulate the pricing model. This article makes a comparison in the sensitivity of the pricing parameters under the MEGB2 distribution and Multivariate Normal distribution asset returns. Finally, an optimal hedging strategy is designed to cover the possible risks of the underlying assets, which can effectively hedge the risks of portfolio.

1. Introduction

Today participating life insurance products are quite popular in insurance market all over the world. In these policies the insured not only gets the guaranteed annual minimum benefit but also receives proceeds from an investment portfolio. Accurate pricing of life insurance participating policies can be traced back to Wilkie [1] through the fair valuation of the embedded options. He initiated the use of modern option pricing theory to study the embedded options in bonuses on participating life-insurance policies. Besides embedding a term life insurance and a minimum guaranteed interest rate, these policies include bonus participation rules and surrender regarding the insurer's benefits. Grosen and Jørgensen [2] utilized a barrier option framework to found the influence of regulatory intervention rules for reducing the insolvency risk of the policies. Concerned with the study of a unit-linked policy with interest rate guarantees, Shen and Xu [3] studied the valuation of unit-linked policies

with or without surrender options. Steffensen [4] provided a general framework about the analysis of surrender and paid-up options and the actuarial approach comparison done by Linnemann [5, 6]. Gerber, Shiu, and Yang [7] calculated the value of equity-linked death benefits in various participating life insurance products. Bacinello [8] supposed the pricing of participating life insurance policies with surrender options using a recursive binomial tree approach. Bernard et al. [9] used techniques which were developed in Carr et al. [10] to achieve the result of the optimal surrender strategy for a variable annuity contract embedded with minimum guaranteed accumulation benefits.

The pricing of participating life insurance products with guarantees has been largely studied under the assumptions of the Black-Scholes option pricing model by Tiong [11], Milevsky and Posner [12], Lin et al. [13], Ng et al. [14], Gatzert and Holzmüller [15], and Fan [16]. However, when the historical prices of the underlying assets are analyzed and used as the basis for setting the pricing assumptions with

respect to the risk model, it is possible to recognize that asset returns rarely meet the standard Brownian motion. Since the last financial crisis, the pricing modeling assumptions which are frequently used in the field of option market and risk management in the financial industry have gradually come into focus. Crisis events such as the collapse of Lehman Brothers have radically changed the view that extreme events in financial markets have negligible probability. Over the last decade, the life insurance industry's situation has deteriorated due to substantial changes in both the economic and the regulatory environment. However, changing market conditions, for instance, a change in asset volatility, may affect these models quite differently. There has been a growing pragmatic and theoretical shift in interest from the modeling of "normal" asset market conditions to the shape and fatness of the tails of the distributions of asset returns which characterize statistical models for extreme events. The Black-Scholes model cannot explicitly account for the negative skewness and the excess kurtosis of asset returns. Typically, in periods when the left skewness of asset prices increases, the Black-Scholes model will overprice out-of-the-money call options and underprice in-the-money call options relative to when there is greater symmetry in the distribution function. Therefore, when we are pricing different forms of investment guarantees typically embedded in participating life insurance products, the brusque variations of the financial market require considering the unpredictable variations of the risk model hypotheses.

The modeling of financial prices is a key assumption to conjecture in the theoretical analysis of participating life insurance products and particularly the dynamics of the value of the referenced financial portfolio or market index; also the embedded options in contracts are very sensitive to the tails of the underlying distribution. It is important to identify the distribution of stock returns over expansion-recession cycles and the occurrence of catastrophic events when the pricing of both the participation and the surrender options are affected by the value of the reference portfolio. Therefore, asymmetries and leptokurtic have to be taken into account. The research has developed models with regime-switching schemes [17] to cope with these stylized facts, or with method of Lévy process, such as Cont [18], Ballotta [19], Coleman et al. [20], Siu et al. [21], Fard and Siu [22], Yu [23], Huang and Yu [24], Lin et al. [25], Orozco-Garcia and Schmeiser [26], and Kélani and Quittard-Pinon [27]. However, Melick and Thomas [28] mentioned that it was more natural to begin with an assumption about the future distribution of the underlying asset, rather than the particular stochastic process by which it evolved and to use option prices to directly recover the parameters of that distribution. Therefore, extreme value theory is a kind of robust framework to analyze the tail behavior of distributions during the financial crisis, which has been applied extensively in various fields especially in the insurance industry, in case of insufficient solvency problem for insurance companies.

The inapplicability of the assumption of financial data following the log normality distribution in option pricing was early proposed by Mandelbrot [29]. The systematic study of extreme value theory for risk management and financial

models was done by Embrechts et al. [30–32] and Mc Neil [33]. Aparicio and Hodges [34] and Corrado [35] studied the option pricing based on generalized Beta functions of the second kind and generalized Lambda distribution respectively. Rocco [36] summarized some new developments of extreme value theory for finance. Yang et al. [37] investigated the applications in modeling multivariate long-tailed data by a generalized beta copula. Markose and Alentorn [38] argued that there was difference in pricing options used Generalized Extreme Value (GEV) distribution and Normal distribution for asset returns. The results presented that how the option price was highly sensitive to the changes in the tail shape, which was distinct to its sensitivity to the variance of the return distribution. Cui and Ma [39] studied the pricing synthetic CDO with MGB2 distribution. Ma et al. [40] evaluated the default risk of bond portfolios with extreme value theory. Zheng et al. [41] made a valuation of guaranteed participating life insurance under GEV distribution.

Existing literatures rarely considered the pricing for participating life insurance products with applications in modeling multivariate shape and heavy tailed asset returns. With fluctuations in the financial markets, does the price of participating life insurance products change sharply under the changing financial market? This will be a problem worth studying. Therefore, the framework of Yang et al. [37] is used in our article; we introduce MEGB2 distribution to fit the distributions of asset returns in order to highlight the characteristic features of heavy tail, the skew, and the dependence structures in asset returns. Then we make a pricing for participating life insurance products embedded the surrender option and analyze the sensitivity of parameters. Finally, we study the differences of pricing of participating life insurance products between MEGB2 joint distribution and Multivariate Normal distribution for asset returns.

The remainder of this paper is structured as follows. In Section 2 we briefly illustrate the MEGB2 asset returns and introduce our valuation framework about four contracts with different embedded options and the single premium of every contract is explicitly written out. We make an empirical study to analyze the influence of different initial parameters on single premium in Section 3. As a comparison, we discuss the parameters sensitivity under the assumption of Multivariate Normal distribution for asset returns. Then, a comparison is made about the difference between the two pricing results under two different multivariate distributions. Moreover, an optimal hedging strategy for this type of insurance product is also designed. Finally our study has been concluded in Section 4. The Appendix is derivation of MEGB2 copula and derivation of assets expected value under MEGB2 distribution.

2. Valuation Model under MEGB2 Asset Returns

2.1. MEGB2 Distribution. We first introduce the definition of MEGB2 distribution.

Set $X = (X_1, \dots, X_n)$ as a real n -dimensional random vector on $(0, \infty]^n$. Each X_i ($i = 1, 2, \dots, n$) given θ follows

a generalized gamma distribution $GG(a_i, b_i\theta^{1/a_i}, p_i)$ with the probability density function (p.d.f.):

$$f_{X_i|\theta}(x_i) = \frac{a_i}{\Gamma(p_i)\theta^{p_i}} \left(\frac{x_i}{b_i}\right)^{a_i p_i} e^{-(x_i/b_i)^{a_i}/\theta}, \quad x > 0 \quad (1)$$

Meanwhile, if parameter θ is a random variable which follows an inverse gamma distribution with parameters $(q, 1)$, i.e., $\theta \sim \text{InvGa}(q, 1)$, then the unconditional p.d.f. of θ is

$$f_{\theta}(\theta) = \frac{1}{\Gamma(q)} \theta^{-q-1} e^{-1/\theta} \quad (2)$$

and then $X=(X_1, \dots, X_n)$ is said to follow multivariate GB2 distribution (MGB2) and the marginal coordinate X_i follows $GB2(a_i, b_i, p_i, q)$ with density:

$$GB2(x; a, b, p, q) = \frac{|a| y^{a p - 1}}{b^{a p} B(p, q) (1 + (y/b)^a)^{p+q}}, \quad (3)$$

$$x > 0$$

Given θ , the conditional cumulative distribution function of X_i is

$$P(X_i \leq x_i | \theta) = \int_0^{x_i} \frac{a_i}{\Gamma(p_i)\theta^{p_i}} \left(\frac{t}{b_i}\right)^{a_i p_i} e^{-(t/b_i)^{a_i}/\theta} dt \quad (4)$$

$$= G_{p_i} \left(\left(\frac{x_i}{b_i}\right)^{a_i} \theta^{-1} \right)$$

where $G_p(z) = (1/\Gamma(p)) \int_0^z t^{p-1} e^{-t} dt, z > 0, \Gamma(p)$ is the Gamma function.

If $X \sim GB2(a, b, p, q)$, we can get $(X/b)^a/[1 + (X/b)^a] \sim B(p, q)$. From the above relation, the cumulative distribution function of X_i can be derived:

$$F_{X_i}(x_i) = B_{p_i, q} \left(\frac{x_i^{a_i}}{(x_i^{a_i} + b_i^{a_i})} \right) \quad (5)$$

where $B_{p_i, q}$ is a cumulative distribution function of the standard Beta distribution with the parameters (p_i, q) and the function expression is

$$B_{p_i, q}(z) = \frac{1}{B(p_i, q)} \int_0^z t^{p_i-1} (1-t)^{q-1} dt, \quad 0 \leq z \leq 1 \quad (6)$$

Based on the distribution of MGB2, MEGB2 distribution is defined as follows. Just as the relationship between the Lognormal and Normal distribution, define $G=LnX$ as a real n -dimensional random vector on $(-\infty, +\infty)^n$. If X follows MGB2 distribution, then the variable G is said to be distributed as a multivariate exponential generalized beta of the second kind (MEGB2).

Although the MEGB2 distribution is a useful tool to deal with the complex multivariate long-tailed data, there are still some constraints faced by the MEGB2 distribution. Firstly, the MEGB2 distribution requires the univariate marginal distributions belonging to the same family. Secondly, the analytical expression of MEGB2 distribution becomes increasingly

complicated when dimension rises. Thirdly, all margins of the MEGB2 distribution must have a common parameter q , while other parameters may vary across margins. For these reasons, we now introduce the corresponding MEGB2 copula.

Based on the study of Yang et al. [37], we construct the n -dimensional MEGB2 copula which is defined by

$$C(u_1, \dots, u_n) = F_{\mathbf{g}}(F_{g_1}^{-1}(u_1), \dots, F_{g_n}^{-1}(u_n)) \quad (7)$$

$$= \int_0^{\infty} \prod_{i=1}^n F_{G_i|\theta}(F_{G_i}^{-1}(u_i)) dG(\theta)$$

where $(u_1, \dots, u_d) \in [0, 1]^d$. It is easier to acquire the n -dimensional MEGB2 distribution by using the MEGB2 copula function. Details of the derivation are given in Appendix A.

2.2. The Structure of the Contract. For pricing the guaranteed unitized participating life insurance, we start with a basic endowment insurance contract B_1 including an initial guaranteed death benefit. According to the features of different options, three options are included into the basic contract. On the basis of the basic endowment insurance contract, we define a model for a contract B_2 added an annual dividend option to B_1 ; a contract B_3 added a terminal dividend option to B_2 ; a contract B_4 added a surrender option to B_3 . Finally, the fair valuation of guaranteed unitized participating life insurance contract can be obtained.

2.2.1. Basic Contract. Firstly, we consider a basic endowment insurance contract B_1 with periodic premium payments. For the contract B_1 , if they die within the term of the contract, the insured will receive the death benefits from the insurer. Meanwhile, the insured will obtain the expected premium payments in the case of survival until maturity. It is assumed that the death benefits are equal to the survival payments. According to the actuarial mathematics theory for an endowment policy, if the age of the insured is x , single premium U^{P_1} can be calculated as follows:

$$U^{P_1} = C_1 A_{x:\overline{T}|r} \quad (8)$$

$$= C_1 \left(\sum_{t=1}^{T-1} (1+r)^{-t} {}_{t-1}q_x + (1+r)^{-T} {}_{T-1}p_x \right)$$

where C_1 denotes death or survival payments and r is used as the annual compounded interest rate for discounting future benefits and premiums. ${}_{t-1}q_x$ represents the probability that the insured dies between $t-1$ and t^{th} year of contract and ${}_t p_x$ is the probability for an x year old policyholder surviving for the next t years. The accounts will receive the annual premium P_i at each policy time. The single premium can be used to calculate the value of annual dividend option.

2.2.2. Contract with Annual Dividend Option. Next, in addition to the features of the basic contract B_1 , we include an annual dividend option and call this contract B_2 . The single premium is set as U^{P_2} .

The expected value of the benefits to the insured will be increased by annual dividend. Set annual benefits for the insured as C_t ; then $C_t = C_1 + \text{annual dividends}$. Since C_t is the benefit payable at time t ($t=2, \dots, T$), we need to consider the insurance policy reserve of this contract at the end of each policy year t in order to calculate C_t . We denote by ${}_tV$ the insurance policy reserve at time t ($t=1, \dots, T-1$) and ${}_tV$ can be calculated according to the actuarial equivalence principle for the endowment policy:

$$\begin{aligned} {}_tV &= C_t A_{x+t: \overline{T-t}|r} \\ &= C_t \left[\sum_{h=1}^{T-t} (1+r)^{-h} {}_{h-1|}q_{x+t} + (1+r)^{-T} {}_{T-t}P_{x+t} \right] \quad (9) \\ & \quad t = 1, 2, \dots, T-1 \end{aligned}$$

Equation (9) is just a theoretical formula because C_t is unknown among the equation. Hence, ${}_tV$ cannot be calculated directly. In order to price the contract with an annual dividend option, assuming that the added value of insurance policy reserve at the end of the policy year is determined by a certain percentage z_t of the reserve at the beginning of the policy year. Then

$$(\Delta V)_t = {}_tV \cdot z_t \quad t = 1, 2, \dots, T-1 \quad (10)$$

According to Bacinello [8], $(\Delta V)_t$ and $(\Delta C)_t$ exist in the following relationship:

$$(\Delta V)_t + (\Delta P)_t \ddot{a}_{x+t: \overline{T-t}|r} = (\Delta C)_t A_{x+t: \overline{T-t}|r} \quad (11)$$

where $(\Delta P)_t$ is the increase value of the premium in the case of the annual premium.

In the payment form of single premium, (11) should be written as

$$(\Delta V)_t = (\Delta C)_t A_{x+t: \overline{T-t}|r} \quad (12)$$

Then,

$$(\Delta C)_t = \frac{(\Delta V)_t}{A_{x+t: \overline{T-t}|r}} \quad (13)$$

Denoting $a_t = (\Delta C)_t / C_t$ as the ratio of change annual payment, we have

$$a_t = \frac{(\Delta C)_t}{C_t} = \frac{(\Delta V)_t / A_{x+t: \overline{T-t}|r}}{C_t} = \frac{(\Delta V)_t}{{}_tV} = z_t \quad (14)$$

Insurance policy reserves will be invested in the financial market, taken as a special portfolio of assets. First introduce the following notation: A_t denotes the special portfolio and g_t represents the rate of return on the reference portfolio at the t^{th} year of contract. Annual benefits will be allocated to the insured at a certain percentage which is denoted as η . Then insurance policy reserves will be increased $e^{\eta g_t}$. According to the provisions of the participating life insurance, fund growth rate cannot be less than the minimum guaranteed yield rate i . We can obtain some relationships written as follows:

$$(1 + z_t) = \max(1 + i, e^{\eta g_t}) \quad t = 1, 2, \dots, T \quad (15)$$

Combining and solving (9) and (10), we can deduce the expression of annual benefits C_t then,

$$C_t = C_{t-1} (1 + z_{t-1}) = C_1 \prod_{k=1}^{t-1} (1 + z_k) \quad t = 2, 3, \dots, T \quad (16)$$

In order to calculate the annual equilibrium premium of contract B_2 , supposing financial market risks and mortality risks are independent, we assume that the risks of insurance companies are neutral. On the basis of these two assumptions, the specific calculation is implemented in two steps. Firstly, the expected value of the benefit C_t is discounted to the beginning of the insurance period and denote $\pi(C_t)$ as discounted present value. Considering the expected value of the death risk, by $\pi(C_t)$ we can calculate the single premium U^{P_2} .

At time $t=1$, one gets

$$\pi(C_1) = C_1 (1 + r)^{-1} \quad (17)$$

At time $t>1$,

$$\begin{aligned} \pi(C_t) &= E [C_t (1 + r)^{-t}] = C_1 (1 + r)^{-t} \\ &\cdot \left\{ \prod_{k=1}^{t-1} E(1 + z_k) \right. \\ &\quad \left. - \sum_{k=1}^{t-1} \left[\left(1 - \frac{k}{T}\right) E(z_k) \prod_{h=k+1}^{t-1} E(1 + z_h) \right] \right\} \quad (18) \end{aligned}$$

where $z_t = \max(i, e^{\eta g_t} - 1)$ $t = 1, 2, \dots, T$, the minimum guaranteed yield rate is i , and annual dividend ratio is denoted as η .

Then, the expression of U^{P_2} is given by

$$U^{P_2} = \sum_{t=1}^{T-1} \pi(C_t) {}_{t-1|}q_x + \pi(C_T) {}_{T-1}P_x \quad (19)$$

2.2.3. Contract with Terminal Dividend Option. Considering the annual dividend, the actual benefit payable of contract B_3 at time T is given by

$$C'_T = \begin{cases} A_T, & A_T \leq C_T \\ C_T + \beta(A_T - C_T)^+, & A_T > C_T \end{cases} \quad (20)$$

where the value of asset is denoted as A_T at the expired date of contract. When A_T is greater than ${}_TV$, the insured will receive the final dividend which is a certain proportion β of surpluses. The terminal dividend of the insured is expressed as $\beta(A_T - C_T)^+$.

Assume that the different assets obey the MEGB2 joint distribution. Then, combined with the formulation of single premium about B_2 , the single premium of B_3 U^{P_3} can be written as

$$\begin{aligned} U^{P_3} &= \sum_{t=1}^{T-1} \pi(C_t) {}_{t-1|}q_x + \pi(C'_T) {}_{T-1}P_x \\ &= U^{P_2} + [\pi(C'_T) - \pi(C_T)] {}_{T-1}P_x \quad (21) \end{aligned}$$

where $\pi(C'_T) = E[C'_T \cdot (1 + r)^{-T}]$.

The difference $U^{P_3} - U^{P_2} = [\pi(C'_T) - \pi(C_t)]_T P_x$ in (21) is the value of terminal dividend option under the MEGB2 asset returns model.

2.2.4. Contract with Surrender Option. Finally, we consider the framework on the basis of contract B_3 and include a surrender option named contract B_4 , which the policyholder can exercise annually at time $t=1, 2, \dots, T$ until maturity. When exercising the surrender option at time τ , the policyholder can obtain the insurance policy reserve ${}_{\tau}V$ at time of termination of the contract B_4 . The contract payoff at time of surrender $t = \tau$ can be written out as follows:

$${}_{\tau}V = C_{\tau} A_{x+\tau; \overline{T-\tau}|r} - P_3 \ddot{a}_{x+\tau; \overline{T-\tau}|r}, \quad (22)$$

$$t = 1, 2, \dots, T - 1$$

The insured will select an optimal admissible exercise strategy according to some information at time τ , which includes the assets value, reserve, and risk-free interest rate. For the insured, the optimal surrender exercise time is the moment that can obtain the maximum expected value of cash flow. The expected total discounted payoff of cash flow acquired by the policyholder after exercising the surrender option is given by

$${}_{\tau}U^{P_4} = \sum_{t=1}^{\tau} \pi(C_t) {}_{t-1|}q_x + \pi({}_{\tau}V) P_x \quad (23)$$

The single premium U^{P_4} of contract B_4 with surrender option is denoted by

$$U^{P_4} = E \left[\sup_{\tau \in \Gamma[0, T-1]} ({}_{\tau}U^{P_4}) \right] \quad (24)$$

We denote by $\Gamma[0, T - 1]$ all the values at stopping time from 0 to $T-1$. U^{P_4} minus U^{P_3} equals the value of surrender option.

2.3. Assets Value and Return on Assets Model. Assume that the insured paid the single premium U within the expiration date of the policy. The insurance company will use the single premium as the initial capital of the insured to set up the capital account, and the account funds are invested in the specified portfolio for the return on investment. Suppose that an insurance company invests funds in d risky assets and uses A_t to represent the value of the assets at the end of time t . A_t can be expressed as

$$A_t = A_{t-1} \cdot e^{g_t} = U \cdot \exp \left(\sum_{i=0}^t g_i \right) \quad (25)$$

where $g_t = \sum_{i=1}^d w_{it} g_{it}$ is annual weighted rate of return on investment of the asset in the interval $[t - 1, t)$. w_{it} represents the investment proportions of i assets in the interval $[t - 1, t)$, which satisfied $\sum_{i=1}^d w_{it} = 1$, $0 \leq w_{it} \leq 1$. w_{it} can be determined by mean-variance model of Markowitz. g_{it} is the return rate of the i assets in the interval $[t - 1, t)$ and assume g_{it} obeys the MEGB2 distribution.

Because the annual dividend and terminal dividend are related to the changes of annual payments, we need to investigate the changes of the assets value. The insured have their own capital accounts for their insurance contracts. The accounts will receive the annual premium P at each policy time and form a new initial value of portfolio combined with the existing portfolio. Assume that investment strategy does not change due to the inflow of funds and the annual return on asset g_{it} follows the same distribution, which is $g_{it} = g_i$, $w_{it} = w_i$. Let A_t denote the value of assets at time t before the inflow of annual premium. Then, combined with (25), A_t can be expressed as

$$A_t = P \cdot \exp(tg_t) \quad (26)$$

where $g_t = \sum_{i=1}^d w_i g_i$

Additionally, the expectation of A_t is given by

$$E(A_t) = [E(A_{t-1}) + P] \cdot \left[1 + \int g_t f(g_t) dg_t \right] \quad (27)$$

In order to get the expected value $E(A_t)$ of A_t , the distribution of g_t should to be investigated.

From (4), we can calculate

$$\begin{aligned} F_{G_i|\theta}(g_i) &= P(G_i \leq g_i | \theta) = P(\ln X_i \leq g_i | \theta) \\ &= F_{X_i|\theta}(e^{g_i}) \\ &= \int_0^{e^{g_i}} \frac{a_i}{\Gamma(p_i)} t^{p_i} \left(\frac{t}{b_i}\right)^{a_i p_i} e^{-(e^{g_i}/b_i)^{a_i} \theta^{-1}} dt \quad (28) \\ &= G_{p_i} \left(\left(\frac{e^{g_i}}{b_i}\right)^{a_i} \theta^{-1} \right) \end{aligned}$$

where $G_p(z) = (1/\Gamma(p)) \int_0^z t^{p-1} e^{-t} dt$, $z > 0$.

According to the research findings of Yang et al. [37], the MEGB2 Copula function is constructed as follows:

$$\begin{aligned} C_{p_1, \dots, p_d, q}^{MEGB2}(u_1, \dots, u_d) &= \int_0^{\infty} \prod_{i=1}^d G_{p_i}(e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}) dG(\theta) \quad (29) \\ &= \int_0^{\infty} \frac{1}{\Gamma(q)} \theta^{-q-1} e^{-1/\theta} \prod_{i=1}^d G_{p_i} \left(\frac{B_{p_i, q}^{-1}(u_i)}{1 - B_{p_i, q}^{-1}(u_i)} \theta^{-1} \right) d\theta \end{aligned}$$

The density function of $C_{p_1, \dots, p_d, q}^{MEGB2}$ is

$$\begin{aligned} c_{p_1, \dots, p_d, q}(u_1, \dots, u_d) &= \frac{\Gamma(q)^{d-1} \Gamma(\sum_{i=1}^d p_i + q)}{\prod_{i=1}^d \Gamma(p_i + q)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{(1 + \sum_{i=1}^d x_i)^{\sum_{i=1}^d p_i + q}} \quad (30) \end{aligned}$$

where $x_i = B_{p_i, q}^{-1}(u_i)/(1 - B_{p_i, q}^{-1}(u_i))$.

Then, we derive $E(A_t)$:

$$\begin{aligned}
 E(A_t) &= U \int \int_{\mathbb{R}^d} \cdots \int e^{t \sum_{i=1}^d w_i g_i} \frac{\Gamma(\sum_{i=1}^d p_i + q)}{\Gamma(q) \prod_{i=1}^d \Gamma(p_i)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{(1 + \sum_{i=1}^d x_i)^{\sum_{i=1}^d p_i + q}} \prod_{i=1}^d \frac{e^{p_i (g_i - \delta_i) / \sigma_i}}{|\sigma_i| (1 + e^{(g_i - \delta_i) / \sigma_i})^{p_i + q}} dg_1 dg_2 \cdots dg_d \\
 &= U \int \int_{\mathbb{R}^d} \cdots \int \frac{\Gamma(\sum_{i=1}^d p_i + q)}{\Gamma(q) \prod_{i=1}^d \Gamma(p_i)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{(1 + \sum_{i=1}^d x_i)^{\sum_{i=1}^d p_i + q}} \frac{e^{\sum_{i=1}^d ((t w_i \sigma_i + p_i) g_i - p_i \delta_i / \sigma_i)}}{\prod_{i=1}^d |\sigma_i| (1 + e^{(g_i - \delta_i) / \sigma_i})^{p_i + q}} dg_1 dg_2 \cdots dg_d
 \end{aligned} \tag{31}$$

The derivation procedure is detailed in Appendix B.

3. Data Analysis and Empirical Results

3.1. Model Solution. In order to price the guaranteed unitized participating life insurance, we need to get the expected assets value $E(A_t)$ at the end of each policy year. According to (6), the calculation of $E(A_t)$ relates to the distribution of the return on assets and single premium U . We suppose that the return on assets follows the MEGB2 distribution. Then the parameters of marginal distributions ($g_i \sim EGB2(g_i; a_i, b_i, p_i, q)$) and the parameters (p_1, p_2, q) of copula function should be estimated. The expectation of annual return rate can be simulated after having estimated the corresponding parameters. Actually, $E(A_t)$ is used to calculate the terminal dividend, which existed at contract B_3 and B_4 , whereas other contracts can be calculated only by the annual return rate of the assets value. Hence, when pricing contract B_4 , we use the single premium U^{P_3} of contract B_3 to simulate the expected value of assets in each policy year. Similarly, pricing contract B_3 , we use the single premium U^{P_2} of contract B_2 . The contract B_2 uses the single premium U^{P_1} of contract B_1 to simulate the expected value of assets.

In order to price the contract B_4 with three options mentioned above, we should simulate from B_1 to B_4 . Firstly, according to the actuarial principle, we calculate the single premium of contract B_1 . Secondly, we assume return on assets following the MEGB2 distribution and then calculate the single premium of contract B_2 . Thirdly, based on the single premium of contract B_2 , we calculate the single premium of contract B_3 . Finally, we can calculate the single premium of contract B_4 by the Least-Squares Monte Carlo simulation (LSM) method. The difference of single premium between contracts is the value of embedded option. Specifically, the value of annual dividend is equal to $U^{P_2} - U^{P_1}$; the value of terminal dividend is equal to $U^{P_3} - U^{P_2}$; the value of surrender dividend is equal to $U^{P_4} - U^{P_3}$.

3.2. Parameter Setting and Premium Results. In order to empirical analysis, we need to decide the parameters involved in the previous sections firstly. The mortality used in this study was derived from the experience life table of China Life Insurance (2010-2013). The experience life table includes four groups of mortality for Chinese people. Because the impact of pricing for different population mortality is not the focus of this article, we use the nonpension male mortality as the

initial parameter. Then the initial age of the insured is set as 40 years old. The basic payment of guaranteed unitized participating life insurance C_1 is set as 10000 yuan and the insurance period is 10 years. Considering that the scheduled interest rate of China Life Insurance Policy was failed to 2.5% in 1999 and has maintained this level so far, therefore, the minimum guaranteed yield rate i is set as 2.5%. The risk-free interest rate r is set as the current one-year fixed deposit rate 1.5%. Ratios of annual dividend and final dividend, according to the provisions of the CIRC (China Insurance Regulatory Commission), are set as 75% and 50%, respectively. In this study, we select the CSI 300 stock index and CSI Smallcap 500 index as the two underlying assets in the portfolio and use the logarithmic yield of these two indexes from June 2012 to June 2017 to estimate the parameters of marginal EGB2 distribution and MEGB2 copula function and the parameters of MEGB2 copula are $p_1=1.12$, $p_2=0.057$, and $q=0.028$. The descriptive statistics results of stock index are shown in Table 1. The weights of the two underlying assets in the portfolio are $w_1=0.33$, $w_2=0.67$, respectively, which are calculated by mean-variance model.

Using the parameters we estimated above, the single premium of contract from B_1 to B_4 can be calculated, respectively, as follows:

$$\begin{aligned}
 U^{P_1} &= 8625.97, \\
 U^{P_2} &= 13840.74, \\
 U^{P_3} &= 15369.95, \\
 U^{P_4} &= 15660.3
 \end{aligned} \tag{32}$$

Then, according to the single premium of contract from B_1 to B_4 , the value of embedded option can be calculated. The value of annual dividend equals $U^{P_2} - U^{P_1} = 5214.77$; the value of terminal dividend equals $U^{P_3} - U^{P_2} = 1529.21$; the value of surrender dividend equals $U^{P_4} - U^{P_3} = 290.35$.

As comparison, using the initial parameters we set above, when the return rates of two different indexes follow the Multivariate Normal distribution with parameters $\mathbf{u} = [0.021, 0.032]^T$, $\Sigma = \begin{bmatrix} 0.015 & 0.008 \\ 0.008 & 0.006 \end{bmatrix}$, the single premium of contract from B_1 to B_4 can be calculated, respectively, as follows:

$$\begin{aligned}
 U^{P_1} &= 8625.97, \\
 U^{P_2} &= 13748.05,
 \end{aligned}$$

TABLE 1: The descriptive statistics of stock index.

logarithmic yield of CSI 300				
Min.	Max.	Median	Mean	Deviation
-0.09154	0.06499	0.000422	0.000251	0.015931
logarithmic yield of CSI Smallcap 500				
Min.	Max.	Median	Mean	Deviation
-0.08926	0.06393	0.001878	0.000382	0.018479

TABLE 2: Parameter estimation results of EGB2 distribution.

Stock index	a	b	p	q
CSI 300	0.0005088	-0.0001914	0.01827691	0.0177666
CSI Smallcap 500	0.0016684	0.00181833	0.1486097	0.1641946

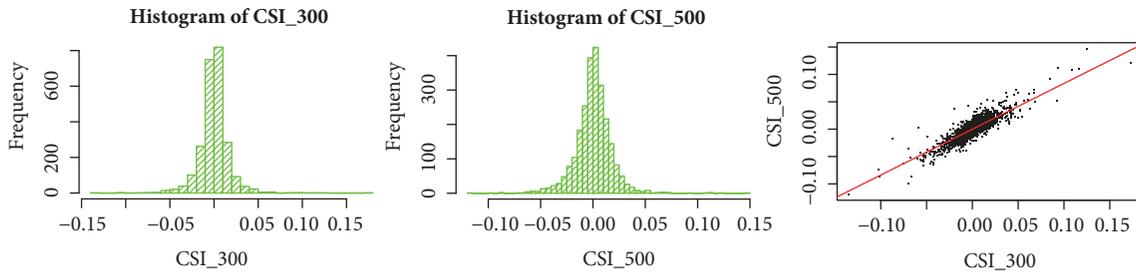


FIGURE 1: Histograms and scatter plot of CSI 300 and CSI Smallcap 500.

$$\begin{aligned}
 U^{P_3} &= 14736.3, \\
 U^{P_4} &= 14876.7
 \end{aligned}
 \tag{33}$$

Then, according to the single premium of contract from B_1 to B_4 , the value of embedded option can be calculated. The value of annual dividend equals $U^{P_2} - U^{P_1} = 5122.08$; the value of terminal dividend equals $U^{P_3} - U^{P_2} = 988.25$; the value of surrender dividend equals $U^{P_4} - U^{P_3} = 140.39$.

3.3. *Parameter Sensitivity Analysis.* Figure 1 shows histograms and scatter plot of the return rates of two indexes and the high correlation can be observed between them. We calculate that their Kendall, Pearson, and Spearman correlation coefficients are 0.65, 0.85, and 0.82, respectively, which illustrates the high degree of correlation between these two indices.

We use the EGB2 distribution as the marginal distribution to fit the data. The parameter estimation results and the fitting effect are shown in Table 2 and Figure 2. Then, we used the MEGB2 copula together with EGB2 marginal to form the bivariate distribution. The fitting results are shown in Figures 3 and 4.

The results in Table 3 illustrate the expected behavior of the premiums with respect to the various parameters.

Then, we compare the values of embedded annual dividend option and surrender option under two different assumptions of the distribution of the return rate. The results are shown in Figures 5 and 6.

We can see that in most cases the values of annual dividend option and surrender option under the MEGB2 distribution of the return rate is higher than the result under the Multivariate Normal distribution. With the increase of age x the difference of annual dividend option under the between MEGB2 distribution and Multivariate Normal distribution is relatively stable, but the difference of surrender option under these two different distributions is gradually reduced. When the age of 60 years is reached, the value of surrender option under assumption of MEGB2 distribution is even higher than it under assumption of Multivariate Normal distribution. The value of annual dividend option and surrender option fluctuates with growing β showed a trend of increase in general. But the annual dividend option value is not very sensitive to β . With the increase of the risk-free interest rate r , under the two assumptions of the distributions, the annual dividend option value shows a downward trend and the difference is relatively stable. But the surrender option value shows a trend of increasing and the gap is narrowing. The value of annual dividend option increases with growing annual dividend ratio η and the gap is very small under both two different distributions of the return rate. However, the surrender option value decreases with growing annual dividend ratio η and the difference increases with the increase of η by weak growth. When other parameters fixed, the value of embedded annual dividend option and surrender option both increase with the growing minimum guaranteed yield rate i and difference remains stable under two distributions of the return rate.

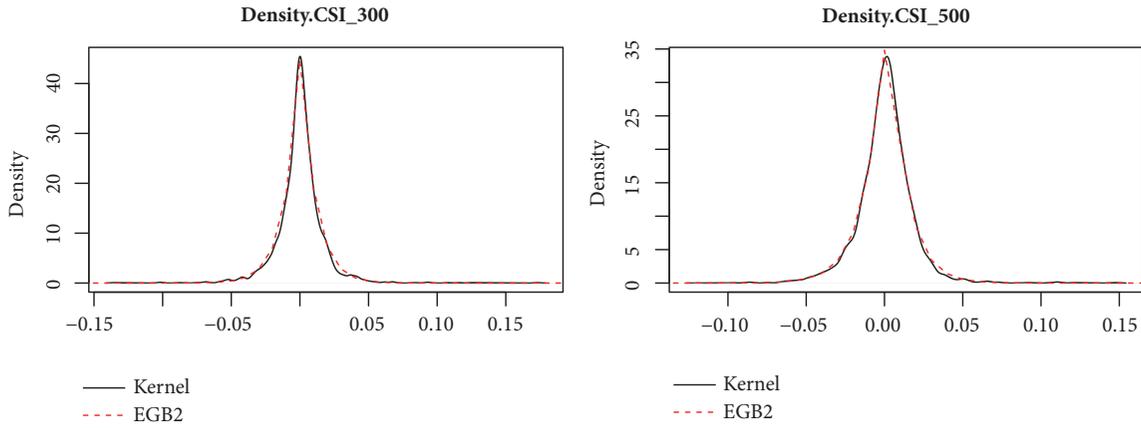


FIGURE 2: Fitting effect of EGB2 distribution as marginal distribution.

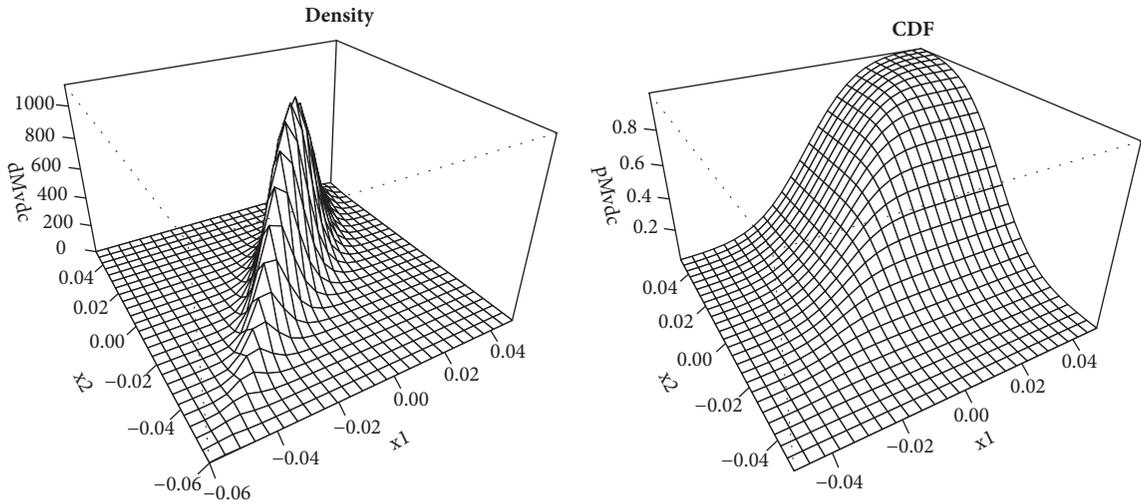


FIGURE 3: The p.d.f and c.d.f of the MEGB2 copula with EGB2 marginals.

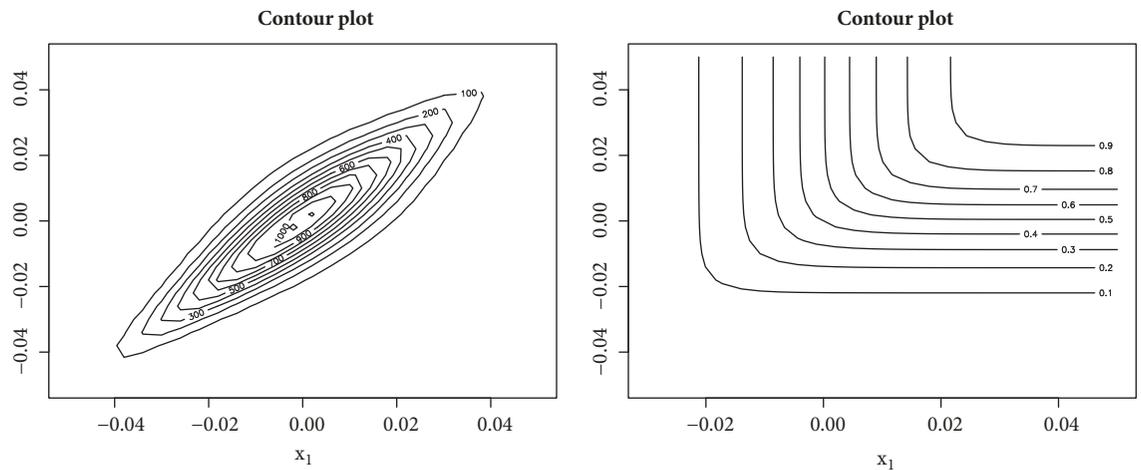


FIGURE 4: Density and cumulative distribution contour plots of distribution having MEGB2 copula.

TABLE 3: The premiums with respect to the various parameters.

	x	i	r	η	β
U^{P_1}	\nearrow	$=$	\searrow	$=$	$=$
A	\searrow	\nearrow	\searrow	\nearrow	$=$
U^{P_2}	\searrow	\nearrow	\searrow	\nearrow	$=$
T	\searrow	\nearrow	\searrow	\nearrow	\nearrow
U^{P_3}	\searrow	\nearrow	\searrow	\nearrow	\nearrow
S	\searrow	\nearrow	\nearrow	\searrow	\nearrow
U^{P_4}	\searrow	\nearrow	\searrow	\nearrow	\nearrow

Note. U^{P_1} represents the price of basic contract B_1 , annual dividend option is A , $U^{P_2} = U^{P_1} + A$ represents the price of contract B_2 , terminal dividend option is T , $U^{P_3} = U^{P_2} + T$ represents the price of contract B_3 , surrender option is S , and $U^{P_4} = U^{P_3} + S$ represents the price of contract B_4 .

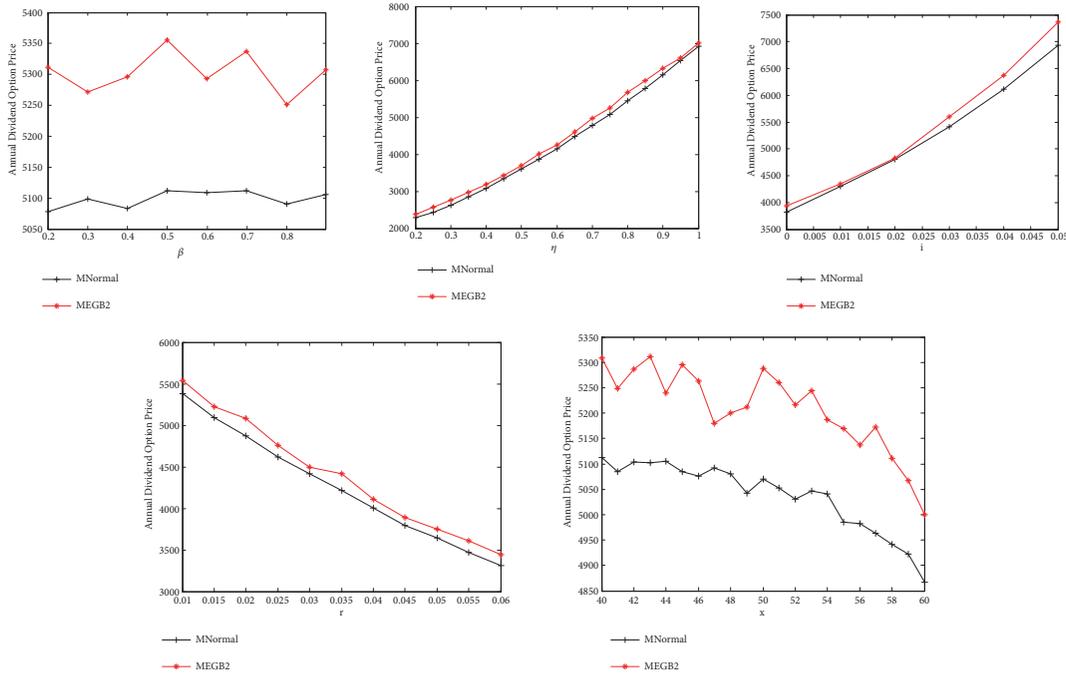


FIGURE 5: Parameter sensitivity analysis of annual dividend option between MEGB2 and Multivariate Normal asset returns.

3.4. *Managing Risks with Hedging Strategy.* In this paper, some put options are embedded in this type of insurance contract. These options may cause huge downside risks for the insurer. Therefore, managing loss risks of the underlying assets and designing a risk hedging strategy are quite necessary. In this section, we study the optimal hedging decision for this kind of guaranteed participating life insurance product when it implements the dynamic hedging strategy. The problem of hedging in insurance markets has been extensively studied, such as Carr and Picron [42], Møller [43], and Vandaele and Vanmaele [44] et al. In this article, following the example of Kélani and Quittard-Pinon [27], we use Delta dynamic optimal hedging strategy. We select riskless asset and the underlying assets of our proposed insurance contract to construct our hedging portfolio. The data period is from June 2012 to June 2017. Transaction costs and short selling constraints are not considered here to simplify the calculation. Then we get the hedging strategies corresponding to 50000 independent simulations.

In this strategy, we use the hedging portfolio to hedge the possible loss of our proposed insurance contracts. In this case, we trade index futures and riskless assets at the end of each t ($t=1, \dots, T-1$) moment. Meanwhile, we define several hedging strategies (δ_t, ζ_t) at every trading time t ($t=1, \dots, T-1$), where δ_t denotes the amount of underlying assets and ζ_t denotes the amount of riskless asset. Let M_t be the value of policy accounts at each policy time t ($t=1, \dots, T-1$) and H_t be the value of the underlying asset in our hedging portfolio at time t ($t=1, \dots, T-1$). Then, we search the optimal (δ_t, ζ_t) to minimize the hedge error ε_t . ε_t can be expressed as

$$\varepsilon_t = M_t - \delta_t H_t - \zeta_t \tag{34}$$

Using a loss function of Least-Squares method, this optimization problem of Delta dynamic hedging strategy with respect to the hedging amounts $(\delta, \zeta) = [(\delta_1, \zeta_1), (\delta_2, \zeta_2), \dots, (\delta_{T-1}, \zeta_{T-1})]$ is defined as

$$\text{Minimize}_{\delta, \zeta} E [\Delta M_t - \delta_t \Delta H_t - \Delta \zeta_t]^2 \tag{35}$$

TABLE 4: The descriptive statistics of P&L.

Return distribution	Mean	Deviation	Median	95%-VaR	99%-VaR
MEGB2	-0.00018	0.0207	-0.0023	0.0366	0.0601
Multivariate Normal	-0.00027	0.0194	-0.0018	0.0345	0.0552

Note. P&L=discounted hedging error/value of the no-hedge portfolio.

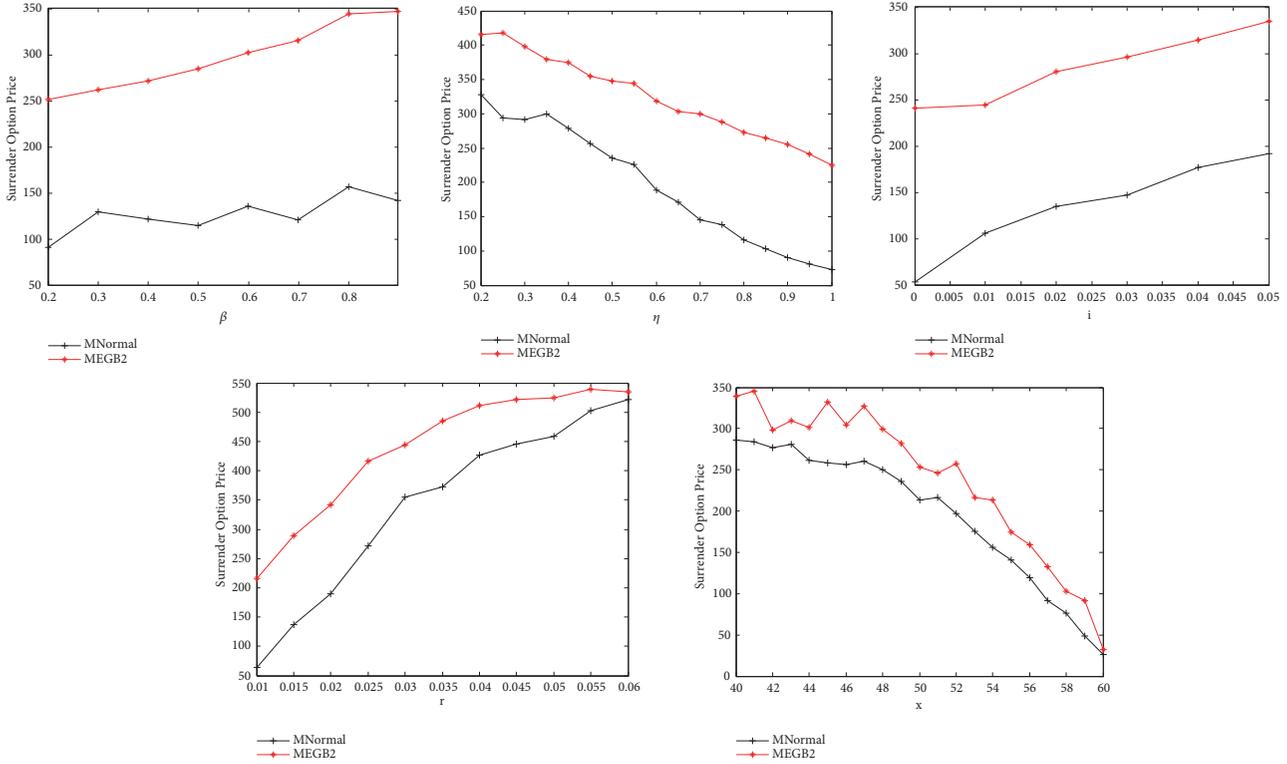


FIGURE 6: Parameter sensitivity analysis of surrender option between MEGB2 and Multivariate Normal asset returns.

The optimal holdings are then obtained by solving a least square problem. Then, we use the distribution of profit and loss (P&L) and some standard risk measures to evaluate the hedge effects. When the returns of the underlying assets follow MEGB2 distribution, Figures 7 and 8 report the distribution of P&L and optimal hedging amounts with a yearly rebalancing. Table 4 reports descriptive statistics and common risk measures. The P&L shows that our Delta dynamic hedging strategy is effective. The mean value of P&L is near 0 and its standard deviation is 0.0207. Meanwhile, the value of 95%-VaR is 0.0366 and 99%-VaR is 0.0601. So, the VaR of P&L illustrates that the hedging strategy is not sensitive to the tails of the underlying distributions and the VaR is small. Therefore, both the overall risk and the tail risk are well hedged.

In this paper, we assume that the returns of the underlying assets follow MEGB2 distribution. However, in financial market, the assumption that returns from Normal distribution is more generally applied and have been widely used in pricing of all kinds of life insurance products. So, in order to investigate the robustness of this hedging strategy in a more general condition, we use the same hedging strategy to manage the risk when the returns of the underlying assets follow

a Multivariate Normal distribution. Then, we compared the hedging results of MEGB2 distribution with the results of Multivariate Normal distribution. Figures 9 and 10 report the distribution of P&L and optimal hedging amounts on Multivariate Normal distribution. Table 4 displays the hedging results of the Multivariate Normal distribution. The mean value of P&L is near 0 and its standard deviation is 0.0194. The value of 95%-VaR is 0.0345 and 99%-VaR is 0.0552. The effect of hedging is still good. It means that our dynamic hedging strategy is also useful on the condition of underlying assets returns following an MEGB2 or a Multivariate Normal distribution. So we have reasons to believe that this dynamic hedging strategy is efficient. It can be used in common loss distributions for this proposed insurance contract.

4. Conclusions

According to the characteristics of guaranteed unitized participating life insurance, considering the death rate, surrender, and minimum guaranteed yield rate dividend policy, and assuming that the return rates follow the MEGB2 distribution, we establish the valuation model of guaranteed unitized participating life insurance based on the fair value

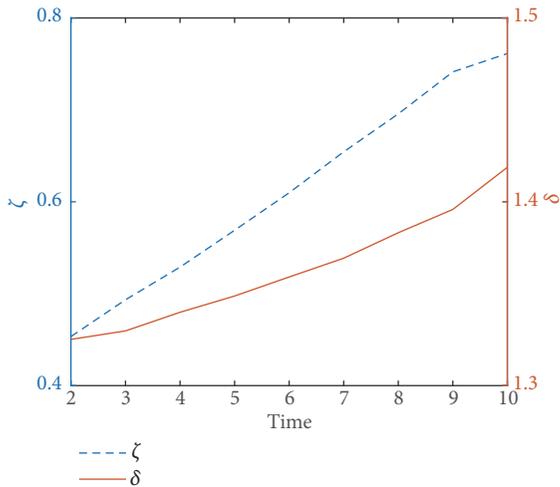


FIGURE 7: Hedging amounts (δ_t, ζ_t) on MEGB2 returns.

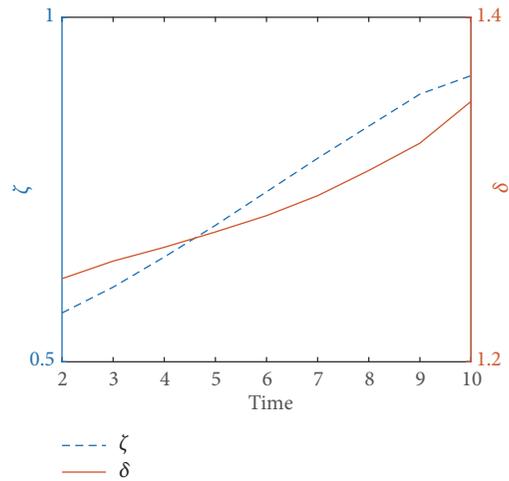


FIGURE 9: Hedging amounts (δ_t, ζ_t) on MNNormal returns.

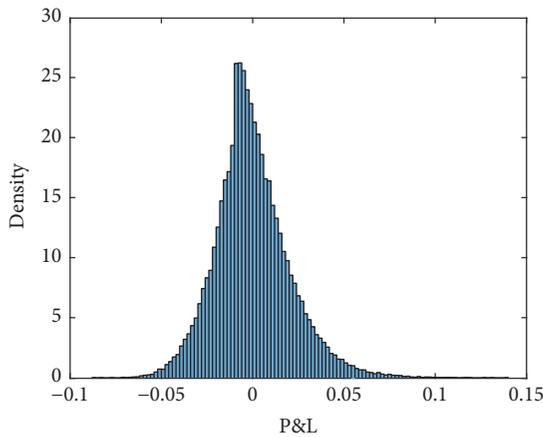


FIGURE 8: Distribution of P&L on MEGB2 returns.

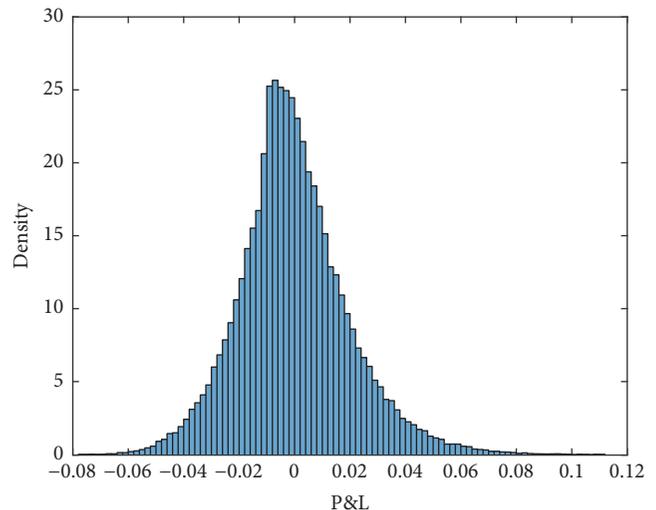


FIGURE 10: Distribution of P&L on MNNormal returns.

measurement. In this article, a multifactor fair valuation pricing model of guaranteed unitized participating life insurance contract is built in four steps. By the valuation model, the guaranteed unitized participating life insurance can be divided into four parts: the basic endowment insurance contract, contract with annual dividend option, contract with terminal dividend option, and contract with surrender option. Finally, we give the single premium of the guaranteed unitized participating life insurance under different influence factors.

From the results of study on pricing of guaranteed unitized participating life insurance embedded surrender options, when the return rates follow the MEGB2 distribution assumption, the value of surrender option is sensitive to the pricing parameters including the age, minimum guaranteed yield rate, the annual dividend ratio, and the risk-free interest rate. Compared with the pricing results that the return rates g_t follow the MEGB2 distribution, it can be seen that when the return rates g_t follow the Multivariate Normal distribution the values of these three embedded options are lower than

the three values with the rate return followed the MEGB2 distribution. Taking into account the distribution of return rates with the characteristics of “asymmetric dependencies,” “non-elliptical,” and “heavy tail” in the capital market, pricing the contracts by using the return rate following the Multivariate Normal distribution with the characteristics of “normal,” “symmetric dependence,” and “light tail” will underestimate the value of embedded options. Therefore, it is important to select an appropriate Multivariate distribution of asset return rates for pricing the model accurately.

Finally, due to insurer facing the infinite risks on falling in underlying asset prices, we designed an optimal dynamic hedging strategy for this insurance contract. By hedging the same underlying stock index futures, we can calculate optimal hedging amounts. The results of P&L illustrate that this kind of dynamic hedging strategy is functional and effective. By comparing the results of underlying assets following Multivariate Normal distribution, our strategy is useful and

can be applied in different loss situations when appropriate pricing of contract is established.

Appendix

A. Derivation of MEGB2 Copula

The probability density function (p.d.f) of the GB2 distribution:

$$GB2(x; a, b, p, q) = \frac{|a| y^{a p - 1}}{b^{a p} B(p, q) (1 + (y/b)^a)^{p+q}}, \quad (A.1)$$

$x > 0$

Set $X = (X_1, \dots, X_n)$ as a real n -dimensional random vector on $(0, \infty]^n$. Each X_i ($i = 1, 2, \dots, n$) given θ follows a generalized gamma distribution $GG(a_i, b_i \theta^{1/a_i}, p_i)$ with the probability density function (p.d.f.)

$$f_{X_i|\theta}(x_i) = \frac{a_i}{\Gamma(p_i) \theta^{p_i}} \left(\frac{x_i}{b_i}\right)^{a_i p_i} e^{-(x_i/b_i)^{a_i}/\theta}, \quad x > 0 \quad (A.2)$$

Meanwhile, suppose that X_1, \dots, X_n are conditionally independent given θ . If parameter θ is a random variable which follows an inverse gamma distribution with parameters $(q, 1)$, i.e., $\theta \sim \text{InvGa}(q, 1)$,

$$f_\theta(\theta) = \frac{1}{\Gamma(q)} \theta^{-q-1} e^{-1/\theta} \quad (A.3)$$

Then the unconditional p.d.f. of \mathbf{X} is

$$f_{\mathbf{X}}(\mathbf{x}) = \frac{\Gamma(\sum_{i=1}^n p_i + q)}{\Gamma(q) \prod_{i=1}^n \Gamma(p_i) x_i} \frac{\prod_{i=1}^n a_i (x_i/b_i)^{a_i p_i}}{[1 + \sum_{i=1}^n (x_i/b_i)^{a_i}]^{q + \sum_{i=1}^n p_i}} \quad (A.4)$$

where $x_i > 0$ with parameters $(p_1, \dots, p_n, q) > 0$. For $i=1, \dots, n$, the marginal coordinate X_i follows $GB2(a_i, b_i, p_i, q)$ with density (A.1).

Given θ , the conditional cumulative distribution function of X_i is

$$P(X_i \leq x_i | \theta) = \int_0^{x_i} \frac{a_i}{\Gamma(p_i) t \theta^{p_i}} \left(\frac{t}{b_i}\right)^{a_i p_i} e^{-(t/b_i)^{a_i}/\theta} dt \quad (A.5)$$

$$= G_{p_i} \left(\left(\frac{x_i}{b_i}\right)^{a_i} \theta^{-1} \right)$$

where $G_p(z) = (1/\Gamma(p)) \int_0^z t^{p-1} e^{-t} dt$, $z > 0$, $\Gamma(p)$ is the Gamma function.

If $X \sim GB2(a, b, p, q)$, we can get $(X/b)^a / [1 + (X/b)^a] \sim B(p, q)$. From the above relation, the cumulative distribution function of X_i can be derived:

$$F_{X_i}(x_i) = B_{p_i, q} \left(\frac{x_i^{a_i}}{(x_i^{a_i} + b_i^{a_i})} \right) \quad (A.6)$$

where $B_{p_i, q}$ is a cumulative distribution function of the standard Beta distribution with the parameters (p_i, q) and the function expression is

$$B_{p_i, q}(z) = \frac{1}{B(p_i, q)} \int_0^z t^{p_i-1} (1-t)^{q-1} dt, \quad (A.7)$$

$0 \leq z \leq 1$

The joint conditional cumulative distribution function of \mathbf{X} given θ is

$$F_{\mathbf{X}|\theta}(x_1, \dots, x_n) = \prod_{i=1}^n G_{p_i} \left(\left(\frac{x_i}{b_i}\right)^{a_i} \theta^{-1} \right) \quad (A.8)$$

Then, the unconditional cumulative distribution function of \mathbf{X} is

$$F_{\mathbf{X}}(x_1, \dots, x_n) = E_\theta [F_{\mathbf{X}|\theta}(x_1, \dots, x_n)]$$

$$= \int_0^\infty \prod_{i=1}^n G_{p_i} \left(\left(\frac{x_i}{b_i}\right)^{a_i} \theta^{-1} \right) f_\theta(\theta) d\theta \quad (A.9)$$

Additionally, let $S_n = \sum_{i=1}^n p_i$, the p.d.f of \mathbf{X} is

$$f_{\mathbf{X}}(x_1, \dots, x_n) = \int_0^\infty f_\theta(\theta) \prod_{i=1}^n f_{X_i|\theta}(x_i | \theta) d\theta$$

$$= \left(\prod_{i=1}^n \frac{a_i}{\Gamma(p_i) x_i} \left(\frac{x_i}{b_i}\right)^{a_i p_i} \right) \frac{1}{\Gamma(q)}$$

$$\cdot \int_0^\infty \theta^{-q-1-S_n} e^{\sum_{i=1}^n ((x_i/b_i)^{a_i} + 1)/\theta} d\theta$$

$$= \left(\prod_{i=1}^n \frac{a_i}{\Gamma(p_i) x_i} \left(\frac{x_i}{b_i}\right)^{a_i p_i} \right) \frac{1}{\Gamma(q)}$$

$$\cdot \frac{\Gamma(S_n + q)}{(\sum_{i=1}^n (x_i/b_i)^{a_i} + 1)^{S_n + q}} = \frac{\Gamma(S_n + q)}{\Gamma(q) \prod_{i=1}^n \Gamma(p_i) x_i}$$

$$\cdot \frac{\prod_{i=1}^n a_i (x_i/b_i)^{a_i p_i}}{(\sum_{i=1}^n (x_i/b_i)^{a_i} + 1)^{S_n + q}} \quad (A.10)$$

The n -dimensional MEGB2 copula is defined by

$$C(u_1, \dots, u_n) = F_{\mathbf{g}}(F_{g_1}^{-1}(u_1), \dots, F_{g_n}^{-1}(u_n))$$

$$= \int_0^\infty \prod_{i=1}^n F_{G_i|\theta}(F_{G_i}^{-1}(u_i)) dG(\theta)$$

$$= \int_0^\infty \prod_{i=1}^n G_{p_i}(e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}) dG(\theta) \quad (A.11)$$

$$= \int_0^\infty \frac{1}{\Gamma(q)} \theta^{-q-1} e^{-1/\theta} \prod_{i=1}^n G_{p_i} \left(\frac{B_{p_i, q}^{-1}(u_i)}{1 - B_{p_i, q}^{-1}(u_i)} \right) \cdot \theta^{-1} d\theta$$

where $(u_1, \dots, u_d) \in [0, 1]^d$.

We can get a conclusion that $C_{p_1, \dots, p_d, q}^{MEGB2}(u_1, \dots, u_d) = C_{p_1, \dots, p_d, q}^{MGB2}(u_1, \dots, u_d)$. MEGB2 copula function is controlled by $n + 1$ parameters. When $p_m \neq p_n, (m, n) \in \{1, \dots, d\}$, MEGB2 copula is very flexible in modeling asymmetric dependence structure. The p.d.f. of $C_{p_1, \dots, p_d, q}^{MEGB2}$ is

$$c_{p_1, \dots, p_d, q}(u_1, \dots, u_d) = \frac{\Gamma(q)^{d-1} \Gamma(\sum_{i=1}^d p_i + q)}{\prod_{i=1}^d \Gamma(p_i + q)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{(1 + \sum_{i=1}^d x_i)^{\sum_{i=1}^d p_i + q}} \quad (A.12)$$

where $x_i = B_{p_i, q}^{-1}(u_i)/(1 - B_{p_i, q}^{-1}(u_i))$.

B. Derivation of Assets Expected Value under MEGB2 Distribution

Combined with (25), the value of the assets A_t can be expressed as

$$A_t = P \cdot \exp(tg_t) \quad (B.1)$$

where $g_t = \sum_{i=1}^d w_i g_i$

In order to get the expected value $E(A_t)$ of A_t , the distribution of g_t should to be investigated.

From (4), we can calculate

$$\begin{aligned} F_{G_i|\theta}(g_i) &= P(G_i \leq g_i | \theta) = P(\ln X_i \leq g_i | \theta) \\ &= F_{X_i|\theta}(e^{g_i}) \\ &= \int_0^{e^{g_i}} \frac{a_i}{\Gamma(p_i)} t^{\theta p_i} \left(\frac{t}{b_i}\right)^{a_i p_i} e^{-(e^{g_i}/b_i)^{a_i} \theta^{-1}} dt \quad (B.2) \\ &= G_{p_i} \left(\left(\frac{e^{g_i}}{b_i}\right)^{a_i} \theta^{-1} \right) \end{aligned}$$

where $G_p(z) = (1/\Gamma(p)) \int_0^z t^{p-1} e^{-t} dt, z > 0$.

Then, the conditional p.d.f of g_i is

$$\begin{aligned} f_{G_i|\theta}(g_i | \theta) &= \frac{\partial F_{G_i|\theta}(g_i)}{\partial g_i} = \frac{\partial F_{X_i|\theta}(e^{g_i})}{\partial g_i} \\ &= f_{X_i|\theta}(e^{g_i}) \cdot e^{g_i} \end{aligned} \quad (B.3)$$

Assuming $a_i = 1/\sigma_i, b_i = e^{\delta_i}$, the conditional cumulative distribution function and conditional probability density function of g_i can be translated into

$$F_{G_i|\theta}(g_i) = G_{p_i}(e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}) \quad (B.4)$$

$$\begin{aligned} f_{g_i|\theta}(g_i | \theta) &= f_{X_i|\theta}(e^{g_i}) \cdot e^{g_i} \\ &= \frac{a_i}{\Gamma(p_i)} e^{g_i \theta p_i} \left(\frac{e^{g_i}}{b_i}\right)^{a_i p_i} e^{-(e^{g_i}/b_i)^{a_i} \theta^{-1}} e^{g_i} \\ &= \frac{1}{\Gamma(p_i) \theta^{p_i} \sigma_i} \exp\left(\frac{(g_i - \delta_i) p_i}{\sigma_i} - e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}\right) \end{aligned} \quad (B.5)$$

In addition, combining the relation between GB2 distribution and standard Beta distribution by (5) we can get

$$\begin{aligned} F_{G_i}(g_i) &= P(G_i \leq g_i) = P(\ln X_i \leq g_i) = F_{X_i}(e^{g_i}) \\ &= B_{p_i, q}\left(\frac{e^{g_i a_i}}{e^{g_i a_i} + b_i^{a_i}}\right) \\ &= B_{p_i, q}\left(\frac{e^{g_i/\sigma_i}}{e^{g_i/\sigma_i} + e^{\delta_i/\sigma_i}}\right) \end{aligned} \quad (B.6)$$

Setting $F_{G_i}(g_i) = u_i$, then $F_{G_i}^{-1}(u_i) = B_{p_i, q}^{-1}(u_i) = e^{g_i a_i} / (e^{g_i a_i} + b_i^{a_i})$ and

$$e^{(g_i - \delta_i)/\sigma_i} = \frac{B_{p_i, q}^{-1}(u_i)}{1 - B_{p_i, q}^{-1}(u_i)} \quad (B.7)$$

Combined with (B.4), in the case of given θ , the joint conditional cumulative distribution function of \mathbf{g} is

$$F_{\mathbf{g}|\theta}(g_1, \dots, g_d) = \prod_{i=1}^d G_{p_i}(e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}) \quad (B.8)$$

Furthermore, the unconditional cumulative distribution function of \mathbf{g} is

$$\begin{aligned} F_{\mathbf{g}}(g_1, \dots, g_d) &= E_{\theta} [F_{\mathbf{g}|\theta}(g_1, \dots, g_d)] \\ &= \int_0^{\infty} \prod_{i=1}^d G_{p_i}(e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}) f_{\theta}(\theta) d\theta \end{aligned} \quad (B.9)$$

Then the probability density function of \mathbf{g} is

$$\begin{aligned} f_{\mathbf{g}}(g_1, \dots, g_d) &= \int_0^{\infty} f_{\theta}(\theta) \prod_{i=1}^d f_{g_i|\theta}(g_i | \theta) d\theta \\ &= \int_0^{\infty} \frac{1}{\Gamma(q)} \theta^{-q-1} e^{-1/\theta} \prod_{i=1}^d \frac{1}{\Gamma(p_i) \theta^{p_i} \sigma_i} \exp\left(\frac{(g_i - \delta_i) p_i}{\sigma_i} - e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}\right) d\theta \\ &= \frac{1}{\Gamma(q)} e^{\sum_{i=1}^d (g_i - \delta_i) p_i / \sigma_i} \prod_{i=1}^d \frac{1}{\Gamma(p_i) \sigma_i} \int_0^{\infty} \theta^{-q-1} e^{-1/\theta} \prod_{i=1}^d \frac{1}{\theta^{p_i}} \exp(-e^{(g_i - \delta_i)/\sigma_i} \theta^{-1}) d\theta \end{aligned}$$

$$\begin{aligned}
 &= \frac{1}{\Gamma(q)} e^{\sum_{i=1}^d (g_i - \delta_i) p_i / \sigma_i} \prod_{i=1}^d \frac{1}{\Gamma(p_i) \sigma_i} \int_0^\infty \theta^{-q-1 - \sum_{i=1}^d p_i} \exp\left(-\frac{\sum_{i=1}^d e^{(g_i - \delta_i) / \sigma_i} + 1}{\theta}\right) d\theta \\
 &= \frac{1}{\Gamma(q)} e^{\sum_{i=1}^d (g_i - \delta_i) p_i / \sigma_i} \prod_{i=1}^d \frac{1}{\Gamma(p_i) \sigma_i} \frac{\Gamma(q + \sum_{i=1}^d p_i)}{\left(\sum_{i=1}^d e^{(g_i - \delta_i) / \sigma_i} + 1\right)^{q + \sum_{i=1}^d p_i}}
 \end{aligned} \tag{B.10}$$

The MEGB2 copula function is constructed as follows:

$$\begin{aligned}
 C(u_1, \dots, u_d) &= F_{\mathbf{g}}(F_{g_1}^{-1}(u_1), \dots, F_{g_d}^{-1}(u_d)) \\
 &= \int_0^\infty \prod_{i=1}^d F_{G_i|\theta}(F_{G_i}^{-1}(u_i)) dG(\theta) \tag{B.11} \\
 &\quad (u_1, \dots, u_d) \in [0, 1]^d
 \end{aligned}$$

Inputting the cumulative distribution function F_{g_i} of g_i (corresponding to the EGB2 distribution), the cumulative distribution function $F_{g_i|\theta}$ of $g_i|\theta$ (corresponding to the generalized Gamma distribution), and the cumulative distribution function $G(\theta)$ of θ (corresponding to the inverse Gamma distribution) into (B.11), the n -dimensional MEGB2 copula functions can be obtained:

$$\begin{aligned}
 C_{p_1, \dots, p_d, q}^{MEGB2}(u_1, \dots, u_d) &= \int_0^\infty \prod_{i=1}^d G_{p_i}(e^{(g_i - \delta_i) / \sigma_i} \theta^{-1}) dG(\theta) \\
 &= \int_0^\infty \frac{1}{\Gamma(q)} \theta^{-q-1} e^{-1/\theta} \prod_{i=1}^d G_{p_i}\left(\frac{B_{p_i, q}^{-1}(u_i)}{1 - B_{p_i, q}^{-1}(u_i)}\right) \cdot \theta^{-1} d\theta \tag{B.12}
 \end{aligned}$$

As can be seen, $C_{p_1, \dots, p_d, q}^{MEGB2}(u_1, \dots, u_d) = C_{p_1, \dots, p_d, q}^{MGB2}(u_1, \dots, u_d)$, and the MEGB2 copula function is controlled by $d+1$ parameter.

When $p_m \neq p_n, (m, n) \in \{1, \dots, d\}$, the MEGB2 copula function is sufficiently flexible to model the asymmetric dependent structures. The density function of $C_{p_1, \dots, p_d, q}^{MEGB2}$ is

$$\begin{aligned}
 c_{p_1, \dots, p_d, q}(u_1, \dots, u_d) &= \frac{\Gamma(q)^{d-1} \Gamma(\sum_{i=1}^d p_i + q)}{\prod_{i=1}^d \Gamma(p_i + q)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i + q}} \tag{B.13}
 \end{aligned}$$

where $x_i = B_{p_i, q}^{-1}(u_i) / (1 - B_{p_i, q}^{-1}(u_i))$.

We have the p.d.f of the EGB2 distribution:

$$EGB2(z; \delta, \sigma, p, q) = \frac{e^{p(z-\delta)/\sigma}}{|\sigma| B(p, q) (1 + e^{(z-\delta)/\sigma})^{p+q}} \tag{B.14}$$

Then, we derive $E(A_t)$:

$$\begin{aligned}
 E(A_t) &= \int_{-\infty}^\infty U e^{tg_t} c(u_1, \dots, u_d) \cdot f_1(g_1) \cdots f_d(g_d) dg_t \\
 &= U \int \int_{\mathbb{R}^d} \cdots \int e^{t \sum_{i=1}^d w_i g_i} \cdot \frac{\Gamma(q)^{d-1} \Gamma(\sum_{i=1}^d p_i + q)}{\prod_{i=1}^d \Gamma(p_i + q)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i + q}} \tag{B.15} \\
 &\quad \cdot \prod_{i=1}^d \frac{e^{p_i(g_i - \delta_i) / \sigma_i}}{|\sigma_i| B(p_i, q) (1 + e^{(g_i - \delta_i) / \sigma_i})^{p_i + q}} dg_1 dg_2 \cdots dg_d
 \end{aligned}$$

where $x_i = B_{p_i, q}^{-1}(u_i) / (1 - B_{p_i, q}^{-1}(u_i))$, $u_i = F_{g_i}(g_i)$.

Since $\Gamma(p_i + q) B(p_i, q) = \Gamma(p_i) \Gamma(q)$ then,

$$\begin{aligned}
 E(A_t) &= U \int \int_{\mathbb{R}^d} \cdots \int e^{t \sum_{i=1}^d w_i g_i} \cdot \frac{\Gamma(q)^{d-1} \Gamma(\sum_{i=1}^d p_i + q)}{\prod_{i=1}^d (\Gamma(p_i) \cdot \Gamma(q))} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i + q}} \tag{B.16} \\
 &\quad \times \prod_{i=1}^d \frac{e^{p_i(g_i - \delta_i) / \sigma_i}}{|\sigma_i| (1 + e^{(g_i - \delta_i) / \sigma_i})^{p_i + q}} dg_1 dg_2 \cdots dg_d
 \end{aligned}$$

That is,

$$\begin{aligned}
 E(A_t) &= U \int \int_{\mathbb{R}^d} \cdots \int e^{t \sum_{i=1}^d w_i g_i} \frac{\Gamma(\sum_{i=1}^d p_i + q)}{\Gamma(q) \prod_{i=1}^d \Gamma(p_i)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i + q}} \prod_{i=1}^d \frac{e^{p_i(g_i - \delta_i) / \sigma_i}}{|\sigma_i| (1 + e^{(g_i - \delta_i) / \sigma_i})^{p_i + q}} dg_1 dg_2 \cdots dg_d \\
 &= U \int \int_{\mathbb{R}^d} \cdots \int \frac{\Gamma(\sum_{i=1}^d p_i + q)}{\Gamma(q) \prod_{i=1}^d \Gamma(p_i)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i + q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i + q}} \frac{e^{\sum_{i=1}^d ((tw_i + p_i)g_i - p_i \delta_i) / \sigma_i}}{\prod_{i=1}^d |\sigma_i| (1 + e^{(g_i - \delta_i) / \sigma_i})^{p_i + q}} dg_1 dg_2 \cdots dg_d \tag{B.17}
 \end{aligned}$$

In order to price the guaranteed unitized participating life insurance under the MEGB2 distribution, in addition to calculating $E(A_t)$, the analytical expression of $E(1 + z_t)$ is also needed. Then, the theoretical expression of $E(1 + z_t)$ can be derived as follows.

By (15), z_t satisfies $(1 + z_t) = \max(1 + i, 1 + \eta(e^{g_t} - 1))$ $t = 1, 2, \dots, T$; then we have

$$\begin{aligned} & \max(1 + i, 1 + \eta(e^{g_t} - 1)) \\ &= \begin{cases} 1 + \eta(e^{g_t} - 1) & g_t \geq \ln\left(1 + \frac{i}{\eta}\right) \\ 1 + i & g_t < \ln\left(1 + \frac{i}{\eta}\right) \end{cases} \end{aligned} \tag{B.18}$$

Therefore $E(1 + z_t)$ can be expressed as

$$\begin{aligned} E(1 + z_t) &= \int_{\ln(1+i/\eta)}^{\infty} [1 + \eta(e^{g_t} - 1)] f(g_t) dg_t + \int_{-\infty}^{\ln(1+i/\eta)} (1 + i) f(g_t) dg_t = \int_{\ln(1+i/\eta)}^{\infty} [1 + \eta(e^{g_t} - 1)] f(g_t) dg_t \\ &+ (1 + i) \left(1 - \int_{\ln(1+i/\eta)}^{\infty} f(g_t) dg_t\right) = \int_{\ln(1+i/\eta)}^{\infty} [\eta(e^{g_t} - 1) - i] f(g_t) dg_t + 1 + i = 1 + i \\ &+ \int \int_{\mathbb{R}^d} \dots \int \left[\eta\left(e^{\sum_{i=1}^d w_i g_i} - 1\right) - i\right] \frac{\Gamma\left(\sum_{i=1}^d p_i + q\right)}{\Gamma(q) \prod_{i=1}^d \Gamma(p_i)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i+q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i+q}} \prod_{i=1}^d \frac{e^{p_i(g_i - \delta_i)/\sigma_i}}{|1 + e^{(g_i - \delta_i)/\sigma_i}|^{p_i+q}} dg_1 dg_2 \dots dg_d \tag{B.19} \\ &= 1 + i + \int \int_{\mathbb{R}^d} \dots \int \left[\eta\left(e^{\sum_{i=1}^d w_i g_i} - 1\right) - i\right] \\ &\cdot \frac{\Gamma\left(\sum_{i=1}^d p_i + q\right)}{\Gamma(q) \prod_{i=1}^d \Gamma(p_i)} \frac{\prod_{i=1}^d (1 + x_i)^{p_i+q}}{\left(1 + \sum_{i=1}^d x_i\right)^{\sum_{i=1}^d p_i+q}} \frac{e^{\sum_{i=1}^d (p_i(g_i - \delta_i)/\sigma_i)}}{\prod_{i=1}^d |1 + e^{(g_i - \delta_i)/\sigma_i}|^{p_i+q}} dg_1 dg_2 \dots dg_d \end{aligned}$$

Data Availability

In Section 3.2 of the paper, the data of experience life table of China Life Insurance (2010-2013) can be downloaded on the website of <http://xizang.circ.gov.cn/web/site0/tab5168/info4054990.htm>. CSI 300 stock index and CSI Smallcap 500 index (June 2012 to June 2017) can be downloaded from the Wind database. In addition, all data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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