

Research Article

Multiple Criteria Decision-Making Based on Vector Similarity Measures under the Framework of Dual Hesitant Fuzzy Sets

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Similarity measures have a great importance in the decision-making process. In order to identify the similarity between the options, many experts have established several types of similarity measures on the basis of vectors and distances. The Cosine, Dice, and Jaccard are the vector similarity measures. The present work enclosed the modified Jaccard and Dice similarity measures. Founded on the Dice and Jaccard similarity measures, we offered a multiple criteria decision-making (MCDM) model under the dual hesitant fuzzy sets (DHFSS) situation, in which the appraised values of the alternatives with respect to criteria are articulated by dual hesitant fuzzy elements (DHFES). Since the weights of the criteria have a much influence in making the decisions, therefore decision makers (DMs) allocate the weights to each criteria according to their knowledge. In the present work, we get rid of the doubt to allocate the weights to the criteria by taking an objective function under some constraints and then extended the linear programming (LP) technique to evaluate the weights of the criteria. The Dice and Jaccard weighted similarity measures are practiced amongst the ideal and each alternative to grade all the alternatives to get the best one. Eventually, two practical examples, about investment companies and selection of smart phone accessories are assumed to elaborate the efficiency of the proposed methodology.

1. Introduction

In everyday life, decision-making plays a central role in choosing the best option out of certain choices. Generally, the decision-making [1, 2] process enables the experts or decision makers (DMs) to tackle problems by analyzing alternative selection and choosing the best course to adopt. However, an inconvenience is encountered by the DMs when they deal with the vague and ambiguous information. Zadeh [3] reduced the difficulties of the DMs by introducing the idea of fuzzy sets (FSs). FSs have opened the new horizons to treat the hesitation and vagueness involved in the process of decision-making. Recently, Citil [4] analyzed the fuzzy issues by using the fuzzy Laplace transform. In the FSs' environment, DMs consider only membership values. In [5], Atanassov further extended by adding a nonmembership value in FSs, known as intuitionistic fuzzy sets (IFSs). Later

on, Torra [6] presented a novel extension of FSs, the hesitant fuzzy sets (HFSs), which augmented by adding diverse values to the membership. Researchers divert their attention towards HFSs and ample work is carried out in the MCDM process with the help of HFSs [7–9]. The DMs feel that the above extensions of FSs have inadequacy of data because FSs treat only one membership value; IFSs deal two kinds of information that are membership and nonmembership while HFSs consider the set values in its membership value but ignore the nonmembership value. In order to overcome this deficiency, Zhu et al. [10] defined another extension of FSs, named dual hesitant fuzzy sets (DHFSS) which have the behavior of HFSs as well as IFSs. DHFSSs can take more information into account because DHFSSs have a set of hesitant values as belonging and nonbelonging values. More values are obtained from the decision makers due to which DHFSSs can be regarded as a more comprehensive set which

supports a more flexible approach when the DMs provide their decisions. Xia and Xu [11] presented the idea of dual hesitant fuzzy element (DHFE), which can be considered as the fundamental unit of the DHFSs and thereby becomes the basic and successful instrument used to express the DMs reluctant inclinations in the procedure of decision-making.

In order to develop the theory of DHFSs, Zhu and Xu [12] presented the idea of typical DHFSs (T-DHFSs) and deliberate certain distinct properties of T-DHFSs. Chen and Huang [13] presented the concept of dual hesitant fuzzy probability (DHFP) dependent on some important outcomes, including the characteristic of DHFP, dual hesitant fuzzy contingent probability, and dual hesitant fuzzy complete probability. Recently, MCDM techniques have been established under the DHFSs environment. For example, Ren et al. [14] used DHFS-based VIKOR method for multicriteria group decision-making. Afterwards, the distance and similarity measures [15, 16], the correlation measures [17, 18], and the entropy measures [19] based on DHFSs have been constructed to handle the MCDM problems. Jamil and Rashid [20] developed the weighted geometric Bonferroni and Choquet geometric Bonferroni means based on DHFSs and then applied it in MCDM issue to elect the best alternative. Moreover, due to the specification of DHFSs, DMs are working more and more in different fields of management sciences by using DHFSs [21–23].

The similarity measure denotes the most resemblance amongst the two particles, and it is plausible to give the preferred arrangement according to the significance. Most of the similarity measures are developed on the basis of distances under the dual hesitant fuzzy environment. Beg and Ashraf discussed the various characteristic of similarity measures under the framework of FSs [24]. Measures of vector similarity also play a dominant role in decision-making, such as Ye [25] applied the Cosine similarity measures to pattern recognition and medical diagnosis under IFSS' environment. Intarapaiboon [26] applied two new similarity measures to pattern recognition under IFSS situations. Furthermore, Song and Hu [27] established two similarity measures between hesitant fuzzy linguistic term sets and used it for MCDM problems. Recently, Zang et al. [28] developed the Heronian mean aggregation operators and applied them for multiattribute decision-making (MADM) under the interval-valued dual hesitant fuzzy framework. Zhang et al. [29] introduced a new concept of Cosine similarity measure based on DHFSs and implemented it for the weapon selection problem. Jiang et al. [30] presented a novel similarity measure dependent on distance between IFSS by transforming the isosceles triangles from IFSS and determined the validity and practicality of the proposed similarity measure by employing on different pattern recognition examples. Chen and Barman [31] proposed an adaptive weighted fuzzy interpolative reasoning (AWFIR) method on the basis of representative values (RVs) and similarity measures of interval type-2 polygonal fuzzy sets to handle the flaws of adaptive fuzzy interpolative reasoning (AFIR) method given by Cheng et al. [32]. Moreover, Chen and Barman [33] established a novel adaptive fuzzy interpolative reasoning (AFIR) method on the basis of similarity measures under the polygonal fuzzy

sets' (PFSS) framework to diagnose the diarrheal disease in the specified persons.

The linear programming (LP) [34] technique allows some target function to be minimized or maximized inside the system of giving situational limitations. LP is a computational technique that enables DMs to solve the problems which they face in a decision-making process. It encourages the DMs to deal with the constrained ideal conditions which they need to make the best of their resources. For example, one limitation for a business is the number of employees it can contract. Another could identify the measure of crude material it has access. Wang and Chen [35] presented a new MCDM method on the basis of the linear programming model, new score, and accuracy function of interval-valued intuitionistic fuzzy values (IVIFVs). He et al. [36] presented an input-output LP model to study energy-economic recovery resilience of an economy. Wang and Chen [37] presented LP methodology and the extended TOPSIS method for interval-valued intuitionistic fuzzy numbers for the selection of the best alternative, which deals with two interval values: belonging and nonbelonging. Lanbaran et al. [38] used the interval-valued fuzzy TOPSIS to determine the opportunities of investment in an MCDM problem. Aliyev [39] presented interval LP where the ambiguous location is termed by interval numbers. Recently, Sindhu et al. implemented the LP technique to evaluate the weights of criteria under distinct extensions of FSs [40–42].

In order to show preference strength among the alternatives, the similarity measures have achieved more attention from the DMs from the previous few decades. Many experts have presented a number of similarity measures for MCDM problems to select the most favorable alternative from the various options having identical features under the certain criteria, for example, similarity measures based on distance, Cosine similarity measure, Jaccard similarity measure, and Dice similarity measure. Since DHFSs have adequate information on their formation, therefore, we can deal well with the circumstances that allow both the membership and the non-membership of an element to a particular set having some diverse values. The LP model is simple, user friendly, and responds quickly with the adoptability of Matlab. Also Jaccard and Dice similarity measures are modest and easy to compute. However, it has not been studied under the DHFSs' framework. This motivated us to deal with the problems under the influence of DHFSs' environment. It is noteworthy that the decision-making under DHFSs' environment may acquire more attention and is deserved wider recognition and further research. Thereby, we modified the Jaccard and Dice similarity measures and applied them for the information provided by DMs under DHFSs' environment.

The remaining part of the present work is organized as follows. Section 2 encompasses the basics of DHFSs, the similarity measures, and the LP model. Section 3 comprises the Jaccard and Dice similarity measures with their modified forms. We proposed a MCDM model on the basis of Jaccard and Dice weighted vector similarity measures of DHFSs in Section 4. In Section 5, we utilize MCDM problems to examine the outcomes of the proposed model. A comparative analysis and conclusions are given in Sections 6 and 7, respectively.

2. Preliminaries

A brief review about the fundamentals of IFSSs, HFSs, the Jaccard and Dice vector similarity measures, and the LP model is discussed in the present section.

Definition 1 (see [5]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a discourse set, an intuitionistic fuzzy set (IFS) A on X is represented in terms of two functions $m: X \rightarrow [0, 1]$ and $n: X \rightarrow [0, 1]$ such as

$$A = \{\langle x, m_A(x), n_A(x) \rangle: x \in X\}, \quad (1)$$

with the condition $0 \leq m_A(x) + n_A(x) \leq 1$, for all $x \in X$. Moreover, $\pi(x) = 1 - m_A(x) - n_A(x)$ is called a degree of hesitancy or an intuitionistic index of x in A . For the special case when $\pi(x) = 0$, that is, $m_A(x) + n_A(x) = 1$, then the IFS A becomes a fuzzy set.

Definition 2 (see [6]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a universal set; a hesitant fuzzy set B on X is defined in terms of a function $h_B(x)$ that when applied to X returns a finite subset of $[0, 1]$. For convenience, the HFS can be written mathematically as described by Xia and Xu [11]:

$$H = \{\langle x, h_{H(x)} \rangle: x \in X\}, \quad (2)$$

where $h_{H(x)}$ is the collection of some different values in $[0, 1]$ representing the plausible belonging degrees of the component $x \in X$ to the set H .

Xia and Xu [11] also presented the core element of HFSs called the hesitant fuzzy element (HFE) and simply written as $h = h(x)$. The hesitant fuzzy elements, $h = \{0\}$ and $h = \{1\}$, are called the empty and full HFEs, respectively. For our convenience, HFSs can be calculated with the help of HFEs by using aggregation techniques or some other actions involved in the decision-making methods.

Definition 3 (see [10]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a discourse set; then, a dual hesitant fuzzy set DHFS D on X is defined as follows:

$$D = \{\langle x, h(x), g(x) \rangle: x \in X\}, \quad (3)$$

where $h(x)$ and $g(x)$ are the collections of set values which lie in $[0, 1]$, denoting the plausible MD_g and NMD_g of the element $x \in X$ to the set D , respectively, and satisfying the conditions penned below: $0 \leq \alpha, \beta \leq 1, 0 \leq \alpha^+ + \beta^+ \leq 1$, where

$$\begin{aligned} \alpha &\in h(x), \\ \beta &\in g(x), \\ \alpha^+ \in h^+(x) &= \bigcup_{\alpha \in h(x)} \max\{h(x)\}, \\ \beta^+ \in g^+(x) &= \bigcup_{\beta \in g(x)} \max\{g(x)\}, \\ x &\in X. \end{aligned} \quad (4)$$

For the sake of convenience, the order pair $d(x) = (h(x), g(x))$ is used as DHFE and simply written as $d(h, g)$ in the thesis with a few limitations:

$$\begin{aligned} \alpha &\in h, \\ \beta &\in g, \\ \alpha^+ \in h^+ &= \bigcup_{\alpha \in h} \max\{h\}, \\ \beta^+ \in g^+ &= \bigcup_{\beta \in g} \max\{g\}, \\ 0 \leq \alpha, \beta \leq 1, & 0 \leq \alpha^+ + \beta^+ \leq 1. \end{aligned} \quad (5)$$

Sometimes DMs gave their information in the form of HFSs with distinct cardinalities such as $|h_1| \neq |h_2|$ and $|g_1| \neq |g_2|$ which are the belonging and nonbelonging components of two DHFSs D_1 and D_2 . In order to make the cardinalities to be equal, we can increase or decrease the number of elements by using the definition given by Xu and Zhang [43]. To calculate the distance between two DHFEs $d_1 = (h_1, g_1)$ and $d_2 = (h_2, g_2)$ with unequal lengths, i.e., $|h_1| \neq |h_2|$ and $|g_1| \neq |g_2|$, we will first equalize the cardinalities of d_1 and d_2 .

Definition 4 (see [10]). Let $Y = \{y_1, y_2, \dots, y_n\}$ be a universe set and $\mathcal{D} = \{\{h\}, \{g\}\}$ be the DHFSs on X ; then, the complement of \mathcal{D} denoted by \mathcal{D}^c is defined as follows:

$$\mathcal{D}^c = \bigcup_{\alpha \in h, \beta \in g} \{\{\beta\}, \{\alpha\}\}. \quad (6)$$

Definition 5. Let $S = \{s_1, s_2, \dots, s_n\}$ and $T = \{t_1, t_2, \dots, t_n\}$ be two positive vectors having length n . The Jaccard $J(S, T)$ and Dice $D(S, T)$ similarity measures [44, 45] are defined as follows:

$$\begin{aligned} J(S, T) &= \left(\frac{S \cdot T}{\|S\|^2 + \|T\|^2 - S \cdot T} \right) = \left(\frac{\sum_{k=1}^n s_k \cdot t_k}{\sum_{k=1}^n s_k^2 + \sum_{k=1}^n t_k^2 - \sum_{k=1}^n s_k \cdot t_k} \right), \\ D(S, T) &= \left(\frac{2S \cdot T}{\|S\|^2 + \|T\|^2} \right) = \left(\frac{2 \sum_{k=1}^n s_k \cdot t_k}{\sum_{k=1}^n s_k^2 + \sum_{k=1}^n t_k^2} \right), \end{aligned} \quad (7)$$

where $S \cdot T = \sum_{k=1}^n s_k t_k$ is the inner product of the vectors S and T and $\|S\| = \sqrt{\sum_{k=1}^n s_k^2}$ and $\|T\| = \sqrt{\sum_{k=1}^n t_k^2}$ are the Euclidean norms of S and T . Both Jaccard $J(S, T)$ and Dice $D(S, T)$ similarity measures fulfil the following conditions:

- (1) $J(S, T) = J(T, S)$ and $D(S, T) = D(T, S)$
- (2) $J(S, T) = 1$ and $D(S, T) = 1$ if $S = T$
- (3) $0 \leq J(S, T) \leq 1$ and $0 \leq D(S, T) \leq 1$

Definition 6 (see [34]). The linear programming model is constructed as follows:

$$\begin{aligned}
\text{Maximize: } & S = c_1 t_1 + c_2 t_2 + c_3 t_3 + \dots + c_n t_n \\
& a_{11} t_1 + a_{12} t_2 + a_{13} t_3 + \dots + a_{1n} t_n \leq b_1 \\
& a_{21} t_1 + a_{22} t_2 + a_{23} t_3 + \dots + a_{2n} t_n \leq b_2 \\
\text{Subject to: } & \vdots \\
& a_{m1} t_1 + a_{m2} t_2 + a_{m3} t_3 + \dots + a_{mn} t_n \leq b_m \\
& t_1, t_2, \dots, t_n \geq 0,
\end{aligned} \tag{8}$$

where m and n denote the number of constraints and the number of decision variables, respectively. A solution (t_1, t_2, \dots, t_n) is called feasible point if it fulfils all of the restrictions. The LP model is used to find the optimal solution of the decision variables t_1, t_2, \dots, t_n to maximize the linear function S .

3. Jaccard and Dice Similarity Measures for DHFSs

Let \mathcal{A} and \mathcal{B} be two DHFSs defined on a fixed set $X = \{x_1, x_2, \dots, x_n\}$ represented as $\mathcal{A} = \{\langle x, h_{\mathcal{A}}(x), g_{\mathcal{A}}(x) \rangle : x \in X\}$ and $\mathcal{B} = \{\langle x, h_{\mathcal{B}}(x), g_{\mathcal{B}}(x) \rangle : x \in X\}$, respectively. We can consider any two dual hesitant fuzzy elements $d_1 = (h_1(x), g_1(x)) \in \mathcal{A}$ and $d_2 = (h_2(x), g_2(x)) \in \mathcal{B}$ as two vectors. Then, according to the aforementioned similarity measures in the vector space, we can modify the Jaccard and Dice similarity measures between DHFSs as follows.

Definition 7. Let \mathcal{A} and \mathcal{B} be two DHFSs defined on a fixed set $X = \{x_1, x_2, \dots, x_n\}$; then, the modified Jaccard $J_s(\mathcal{A}, \mathcal{B})$ is defined as follows:

$$\begin{aligned}
J_s(\mathcal{A}, \mathcal{B}) = \frac{1}{2} & \left\{ \frac{\sum(\alpha.\bar{\alpha})}{\sum(\alpha)^2 + \sum(\bar{\alpha})^2 - \sum(\alpha.\bar{\alpha})} \right. \\
& \left. + \frac{\sum(\beta.\bar{\beta})}{\sum(\beta)^2 + \sum(\bar{\beta})^2 - \sum(\beta.\bar{\beta})} \right\}, \tag{9}
\end{aligned}$$

where $\alpha \in h_1, \beta \in g_1 \in \mathcal{A}$ and $\bar{\alpha} \in h_2, \bar{\beta} \in g_2 \in \mathcal{B}$, respectively.

Definition 8. Let \mathcal{A} and \mathcal{B} be two DHFSs defined on a universal set $X = \{x_1, x_2, \dots, x_n\}$; then, the modified Dice $D_s(\mathcal{A}, \mathcal{B})$ is defined as follows:

$$D_s(\mathcal{A}, \mathcal{B}) = \frac{\sum(\alpha.\bar{\alpha})}{\sum(\alpha)^2 + \sum(\bar{\alpha})^2} + \frac{\sum(\beta.\bar{\beta})}{\sum(\beta)^2 + \sum(\bar{\beta})^2}, \tag{10}$$

where $\alpha \in h_1, \beta \in g_1 \in \mathcal{A}$ and $\bar{\alpha} \in h_2, \bar{\beta} \in g_2 \in \mathcal{B}$, respectively.

Since the weights of the criteria has a great worth in making decision. Thus, we can further extend the Jaccard and Dice similarity measures into the Jaccard and Dice weighted similarity measures. Let $w = (w_1, w_2, \dots, w_m)^T$ be a weight vector of the m criteria with $\sum_{j=1}^m w_j = 1$. Then, the Jaccard and Dice similarity measures take the form:

$$J_s^w(\mathcal{A}, \mathcal{B}) = \frac{1}{2} \left\{ \frac{\sum_{j=1}^m w_j(\alpha.\bar{\alpha})}{\sum(\alpha)^2 + \sum(\bar{\alpha})^2 - \sum(\alpha.\bar{\alpha})} + \frac{\sum_{j=1}^m w_j(\beta.\bar{\beta})}{\sum(\beta)^2 + \sum(\bar{\beta})^2 - \sum(\beta.\bar{\beta})} \right\}, \tag{11}$$

$$D_s^w(\mathcal{A}, \mathcal{B}) = \frac{\sum_{j=1}^m w_j(\alpha.\bar{\alpha})}{\sum(\alpha)^2 + \sum(\bar{\alpha})^2} + \frac{\sum_{j=1}^m w_j(\beta.\bar{\beta})}{\sum(\beta)^2 + \sum(\bar{\beta})^2}. \tag{12}$$

Theorem 1. The Jaccard similarity measure $J_s(\mathcal{A}, \mathcal{B})$ between two DHFSs with $d_1 = (h_1, g_1) \in \mathcal{A}$ and $d_2 = (h_2, g_2) \in \mathcal{B}$ satisfies the following properties:

- (1) $J_s(\mathcal{A}, \mathcal{B}) = J_s(\mathcal{B}, \mathcal{A})$
- (2) $J_s(\mathcal{A}, \mathcal{B}) = 1$, if and only if \mathcal{A} is equivalent to \mathcal{B} by definition given in [10]
- (3) $0 \leq J_s(\mathcal{A}, \mathcal{B}) \leq 1$

Proof

(1–2) are obvious.

From (3), let $\alpha \in h_1, \beta \in g_1 \in \mathcal{A}$ and $\bar{\alpha} \in h_2, \bar{\beta} \in g_2 \in \mathcal{B}$, respectively. However, we know that

$$\begin{aligned}
(x - y)^2 & \geq 0 \\
\implies x^2 + y^2 - xy - xy & \geq 0 \\
\implies x^2 + y^2 - xy & \geq xy \\
\implies \sum(\alpha)^2 + \sum(\bar{\alpha})^2 - \sum(\alpha.\bar{\alpha}) - \sum(\alpha.\bar{\alpha}) & \geq 0 \\
\implies \sum(\alpha)^2 + \sum(\bar{\alpha})^2 - \sum(\alpha.\bar{\alpha}) & \geq \sum(\alpha.\bar{\alpha}) \\
\implies \frac{\sum(\alpha.\bar{\alpha})}{\sum(\alpha)^2 + \sum(\bar{\alpha})^2 - \sum(\alpha.\bar{\alpha})} & \leq 1.
\end{aligned} \tag{13}$$

Similarly,

$$\begin{aligned} \sum (\beta)^2 + \sum (\bar{\beta})^2 - \sum (\beta \cdot \bar{\beta}) &\geq \sum (\beta \cdot \bar{\beta}) \\ \implies \frac{\sum (\beta \cdot \bar{\beta})}{\sum (\beta)^2 + \sum (\bar{\beta})^2 - \sum (\beta \cdot \bar{\beta})} &\leq 1. \end{aligned} \tag{14}$$

By adding equations (13) and (14), we obtain

$$\begin{aligned} \frac{\sum (\alpha \cdot \bar{\alpha})}{\sum (\alpha)^2 + \sum (\bar{\alpha})^2 - \sum (\alpha \cdot \bar{\alpha})} + \frac{\sum (\beta \cdot \bar{\beta})}{\sum (\beta)^2 + \sum (\bar{\beta})^2 - \sum (\beta \cdot \bar{\beta})} &\leq 2 \\ \implies \frac{1}{2} \left\{ \frac{\sum (\alpha \cdot \bar{\alpha})}{\sum (\alpha)^2 + \sum (\bar{\alpha})^2 - \sum (\alpha \cdot \bar{\alpha})} + \frac{\sum (\beta \cdot \bar{\beta})}{\sum (\beta)^2 + \sum (\bar{\beta})^2 - \sum (\beta \cdot \bar{\beta})} \right\} &\leq 1, \end{aligned} \tag{15}$$

that is, $0 \leq J_s(\mathcal{A}, \mathcal{B}) \leq 1$.

Lemma 1. Let $X = \{x_1, x_2, \dots, x_n\}$ be a given universe. The modified Jaccard similarity measure $J_s(\mathcal{A}, \mathcal{B})$ satisfies the properties given below:

- (1) $J_s(\mathcal{A}, \mathcal{A}^c) = 0$ if and only if $\mathcal{A} = \{\{1\}, \{0\}\}$
- (2) $J_s(\mathcal{A}, \mathcal{A}^c) = 1$ if and only if $\mathcal{A} = \{\{0.5\}, \{0.5\}\}$
- (3) $J_s(\mathcal{A}, \mathcal{B}) = J_s(\mathcal{A}^c, \mathcal{B}^c)$

Proof

- (1) It is obvious.
- (2) Let $\alpha, \beta \in \mathcal{A}$ and $\alpha_1, \beta_1 \in \mathcal{A}^c$. We obtain

$$\begin{aligned} J_s(\mathcal{A}, \mathcal{A}^c) &= \frac{1}{2} \left\{ \frac{\sum (\alpha \cdot \alpha_1)}{\sum (\alpha)^2 + \sum (\alpha_1)^2 - \sum (\alpha \cdot \alpha_1)} \right. \\ &\quad \left. + \frac{\sum (\beta \cdot \beta_1)}{\sum (\beta)^2 + \sum (\beta_1)^2 - \sum (\beta \cdot \beta_1)} \right\}. \end{aligned} \tag{16}$$

Since $\alpha = \alpha_1$ and $\beta = \beta_1$, therefore,

$$\begin{aligned} J_s(\mathcal{A}, \mathcal{A}^c) &= \frac{1}{2} \left\{ \frac{\sum (\alpha \cdot \alpha)}{\sum (\alpha)^2 + \sum (\alpha)^2 - \sum (\alpha \cdot \alpha)} \right. \\ &\quad \left. + \frac{\sum (\beta \cdot \beta)}{\sum (\beta)^2 + \sum (\beta)^2 - \sum (\beta \cdot \beta)} \right\} = 1. \end{aligned} \tag{17}$$

- (3) Let $\mathcal{A} = \{\{h_1\}, \{g_1\}\}$ and $\mathcal{B} = \{\{h_2\}, \{g_2\}\}$ be two DHFSs and $\mathcal{A}^c = \{\{g_1\}, \{h_1\}\}$ and $\mathcal{B}^c = \{\{g_2\}, \{h_2\}\}$ their complements. Suppose that $\alpha_1, \beta_1 \in \mathcal{A}$, $\alpha_2, \beta_2 \in \mathcal{B}$, $\alpha'_1, \beta'_1 \in \mathcal{A}^c$, and $\alpha'_2, \beta'_2 \in \mathcal{B}^c$, then

$$\begin{aligned} J_s(\mathcal{A}, \mathcal{B}) &= \frac{1}{2} \left\{ \frac{\sum (\alpha_1 \cdot \alpha_2)}{\sum (\alpha_1)^2 + \sum (\alpha_2)^2 - \sum (\alpha_1 \cdot \alpha_2)} \right. \\ &\quad \left. + \frac{\sum (\beta_1 \cdot \beta_2)}{\sum (\beta_1)^2 + \sum (\beta_2)^2 - \sum (\beta_1 \cdot \beta_2)} \right\}. \end{aligned} \tag{18}$$

However, $\alpha_1 = \beta'_1, \alpha_2 = \beta'_2, \beta_1 = \alpha'_1$, and $\beta_2 = \alpha'_2$. Hence,

$$\begin{aligned} J_s(\mathcal{A}, \mathcal{B}) &= \frac{1}{2} \left\{ \frac{\sum (\beta'_1 \cdot \beta'_2)}{\sum (\beta'_1)^2 + \sum (\beta'_2)^2 - \sum (\beta'_1 \cdot \beta'_2)} \right. \\ &\quad \left. + \frac{\sum (\alpha'_1 \cdot \alpha'_2)}{\sum (\alpha'_1)^2 + \sum (\alpha'_2)^2 - \sum (\alpha'_1 \cdot \alpha'_2)} \right\} \\ \implies J_s(\mathcal{A}, \mathcal{B}) &= \frac{1}{2} \left\{ \frac{\sum (\alpha'_1 \cdot \alpha'_2)}{\sum (\alpha'_1)^2 + \sum (\alpha'_2)^2 - \sum (\alpha'_1 \cdot \alpha'_2)} \right. \\ &\quad \left. + \frac{\sum (\beta'_1 \cdot \beta'_2)}{\sum (\beta'_1)^2 + \sum (\beta'_2)^2 - \sum (\beta'_1 \cdot \beta'_2)} \right\} \\ \implies J_s(\mathcal{A}, \mathcal{B}) &= J_s(\mathcal{A}^c, \mathcal{B}^c). \end{aligned} \tag{19}$$

Theorem 2. The Dice similarity measure $D_s(\mathcal{A}, \mathcal{B})$ between two DHFSs with $d_1 = (h_1, g_1) \in \mathcal{A}$ and $d_2 = (h_2, g_2) \in \mathcal{B}$ satisfies the following properties:

- (1) $D_s(\mathcal{A}, \mathcal{B}) = D_s(\mathcal{B}, \mathcal{A})$
- (2) $D_s(\mathcal{A}, \mathcal{B}) = 1$, if and only if \mathcal{A} is equivalent to \mathcal{B} by definition given in [10]
- (3) $0 \leq D_s(\mathcal{A}, \mathcal{B}) \leq 1$

Proof

(1-2) are obvious.

From (3), let $\alpha \in h_1, \beta \in g_1 \in \mathcal{A}$ and $\bar{\alpha} \in h_2, \bar{\beta} \in g_2 \in \mathcal{B}$, respectively.

However, we know that

$$(x - y)^2 \geq 0$$

$$\implies x^2 + y^2 \geq 2xy$$

$$\sum (\alpha)^2 + \sum (\bar{\alpha})^2 \geq 2 \sum (\alpha \cdot \bar{\alpha}) \tag{20}$$

$$\implies \frac{2 \sum (\alpha \cdot \bar{\alpha})}{\sum (\alpha)^2 + \sum (\bar{\alpha})^2} \leq 1,$$

$$\sum (\beta)^2 + \sum (\bar{\beta})^2 \geq 2 \sum (\beta \cdot \bar{\beta})$$

$$\implies \frac{2 \sum (\beta \cdot \bar{\beta})}{\sum (\beta)^2 + \sum (\bar{\beta})^2} \leq 1. \tag{21}$$

By adding equations (20) and (21), we obtain

$$\frac{2 \sum (\alpha \cdot \bar{\alpha})}{\sum (\alpha)^2 + \sum (\bar{\alpha})^2} + \frac{2 \sum (\beta \cdot \bar{\beta})}{\sum (\beta)^2 + \sum (\bar{\beta})^2} \leq 2 \tag{22}$$

$$\implies \frac{\sum (\alpha \cdot \bar{\alpha})}{\sum (\alpha)^2 + \sum (\bar{\alpha})^2} + \frac{\sum (\beta \cdot \bar{\beta})}{\sum (\beta)^2 + \sum (\bar{\beta})^2} \leq 1,$$

which shows that $0 \leq D_s(d_1, d_2) \leq 1$.

Lemma 2. Let $X = \{x_1, x_2, \dots, x_n\}$ be a given universe. The modified Dice similarity measure $D_s(\mathcal{A}, \mathcal{B})$ satisfies the properties given below:

- (1) $D_s(\mathcal{A}, \mathcal{A}^c) = 0$ if and only if $\mathcal{A} = \{\{1\}, \{0\}\}$
- (2) $D_s(\mathcal{A}, \mathcal{A}^c) = 1$ if and only if $\mathcal{A} = \{\{0.5\}, \{0.5\}\}$
- (3) $D_s(\mathcal{A}, \mathcal{B}) = D_s(\mathcal{A}^c, \mathcal{B}^c)$

Proof

- (1) It is obvious.
- (2) Let $\alpha, \beta \in \mathcal{A}$ and $\alpha_1, \beta_1 \in \mathcal{A}^c$. We obtain

$$D_s(\mathcal{A}, \mathcal{A}^c) = \frac{\sum(\alpha)(\alpha_1)}{\sum\alpha^2 + \sum\alpha_1^2} + \frac{\sum(\beta)(\beta_1)}{\sum\beta^2 + \sum\beta_1^2}. \quad (23)$$

Since $\alpha = \alpha_1$ and $\beta = \beta_1$, therefore,

$$D_s(\mathcal{A}, \mathcal{A}^c) = \frac{\sum(\alpha)(\alpha)}{\sum\alpha^2 + \sum\alpha^2} + \frac{\sum(\beta)(\beta)}{\sum\beta^2 + \sum\beta^2} \quad (24)$$

$$\implies D_s(\mathcal{A}, \mathcal{A}^c) = 1.$$

- (3) Let $\mathcal{A} = \{\{h_1\}, \{g_1\}\}$ and $\mathcal{B} = \{\{h_2\}, \{g_2\}\}$ be two DHFSs and $\mathcal{A}^c = \{\{g_1\}, \{h_1\}\}$ and $\mathcal{B}^c = \{\{g_2\}, \{h_2\}\}$ are their complements. Suppose that $\alpha_1, \beta_1 \in \mathcal{A}$, $\alpha_2, \beta_2 \in \mathcal{B}$, $\alpha'_1, \beta'_1 \in \mathcal{A}^c$, and $\alpha'_2, \beta'_2 \in \mathcal{B}^c$, then

$$D_s(\mathcal{A}, \mathcal{B}) = \frac{\sum(\alpha_1)(\alpha_2)}{\sum\alpha_1^2 + \sum\alpha_2^2} + \frac{\sum(\beta_1)(\beta_2)}{\sum\beta_1^2 + \sum\beta_2^2}. \quad (25)$$

However, $\alpha_1 = \beta'_1, \alpha_2 = \beta'_2, \beta_1 = \alpha'_1$, and $\beta_2 = \alpha'_2$. Hence,

$$\begin{aligned} D_s(\mathcal{A}, \mathcal{B}) &= \frac{\sum(\beta'_1)(\beta'_2)}{\sum(\beta'_1)^2 + \sum(\beta'_2)^2} + \frac{\sum(\alpha'_1)(\alpha'_2)}{\sum(\alpha'_1)^2 + \sum(\alpha'_2)^2} \\ \implies D_s(\mathcal{A}, \mathcal{B}) &= \frac{\sum(\alpha'_1)(\alpha'_2)}{\sum(\alpha'_1)^2 + \sum(\alpha'_2)^2} + \frac{\sum(\beta'_1)(\beta'_2)}{\sum(\beta'_1)^2 + \sum(\beta'_2)^2} \\ \implies D_s(\mathcal{A}, \mathcal{B}) &= D_s(\mathcal{A}^c, \mathcal{B}^c). \end{aligned} \quad (26)$$

We can follow the same way to prove the Jaccard- and Dice-weighted similarity measures:

- (1) $J_s^w(\mathcal{A}, \mathcal{B}) = J_s^w(\mathcal{B}, \mathcal{A})$ and $D_s^w(\mathcal{A}, \mathcal{B}) = D_s^w(\mathcal{B}, \mathcal{A})$
- (2) $J_s^w(\mathcal{A}, \mathcal{B}) = D_s^w(\mathcal{A}, \mathcal{B}) = 1$, if and only if $\mathcal{B} = \mathcal{A}$
- (3) $0 \leq J_s^w(\mathcal{A}, \mathcal{B}) \leq 1$ and $0 \leq D_s^w(\mathcal{A}, \mathcal{B}) \leq 1$

4. Multiple Criteria Decision-Making under DHF Environment

The present segment comprises the model for MCDM in which we use the two vector similarity measures based on

DHFSs. For an MCDM problem, under the dual hesitant fuzzy (DHF) environment, let $P = \{P_1, P_2, \dots, P_n\}$ be a discrete set of alternatives and $G = \{G_1, G_2, \dots, G_m\}$ be a discrete set of criteria. If the DMs gave the various values for the alternative $P_i (i = 1, 2, \dots, n)$ under the attribute $G_j (j = 1, 2, \dots, m)$, these values can be considered as a dual hesitant fuzzy element $d_{ij} (i = 1, 2, \dots, n; j = 1, 2, \dots, m)$. Thereby, we can form a dual hesitant fuzzy decision matrix $\mathcal{D} = [\{h_{ij}\}, \{g_{ij}\}]_{n \times m}$. The concept of optimal solution assists the DMs to identify the best alternative from the decision set in the MCDM framework. In spite of the fact that the perfect option does not exist in actual realm, it provides a valuable paradigm to appraise alternatives. Hence, we can find the ideal options P^* from the given information as $P^* = \{\{\max_i h_{i1}, \max_i h_{i2}, \dots, \max_i h_{im}\}, \{\min_i g_{i1}, \min_i g_{i2}, \dots, \min_i g_{im}\}\}$, where $i = 1, 2, \dots, n$. Since the weights of the criteria have excessive importance, therefore a weighting vector of criteria is provided as $w = (w_1, w_2, w_3, \dots, w_m)^T$, where $\sum_{j=1}^m w_j = 1, j = 1, 2, \dots, m$ and $w_j > 0$.

We propose a MCDM model based on the two weighted vector similarity under dual hesitant fuzzy data, which can be formulated as follows:

Step 1: construct a dual hesitant fuzzy decision matrix (DHFM) denoted by $\mathcal{D} = [d_{ij}]_{n \times m}$ according to the given data presented by the DM

Step 2: transform the matrix \mathcal{D} into normalized dual hesitant fuzzy decision matrix NDHFM, $\tilde{\mathcal{D}} = [\tilde{d}_{ij}]_{n \times m}$

Step 3: find the optimal solution P^* from the NDHFM, $\tilde{\mathcal{D}} = [\tilde{d}_{ij}]_{n \times m}$

Step 4: based on Definition 5, using the LP model to find weights of criteria under the given constraints provided by the DM

Step 5: by using equations (11) and (12), calculate the weighted vector similarity measures amongst the alternative $P_i (i = 1, 2, \dots, n)$ and the optimal alternative P^*

5. Practical Examples

Example 1. A particular example is used as a demonstration of the application of the proposed MCDM method in reality-based scenario. For an investment company, who want to invest a sum of money in the best option, there is a particular panel with four possible alternatives to invest the amount: Pakistan microfinance, investment company (P_1), national investment trust limited (P_2), Pak China investment company limited (P_3), power cement limited (P_4), and Pak Kuwait investment company (P_5). In order to make a decision, the investment company follows the following criteria: the risk G_1 , the economic growth G_2 , the environmental impact G_3 , and the interest rate G_4 . The information about the alternatives P_i under the criteria G_j is represented by a dual hesitant fuzzy decision matrix \mathcal{D} in Table 1. Since the weights of the criteria have a great significance in making the decision. DM has an ambiguity to assign the weights to the criteria. DM assumed that the weights of criteria under the constraint conditions are as follows.

TABLE 1: Dual hesitant fuzzy decision matrix \mathcal{D} .

	G_1	G_2	G_3	G_4
P_1	{{0.1, 0.2, 0.3}, {0.6, 0.7}}	{{0.5, 0.6, 0.7}, {0.1, 0.2}}	{{0.3, 0.4, 0.5}, {0.1, 0.2}}	{{0.1, 0.2, 0.3}, {0.4, 0.5}}
P_2	{{0.4, 0.5, 0.6}, {0.1, 0.2}}	{{0.6, 0.7}, {0.1, 0.2}}	{{0.6, 0.7}, {0.1, 0.2}}	{{0.1, 0.2}, {0.3, 0.4}}
P_3	{{0.3, 0.4}, {0.1, 0.2}}	{{0.4, 0.5}, {0.1, 0.2}}	{{0.3, 0.4}, {0.4, 0.5}}	{{0.6, 0.7}, {0.1, 0.2}}
P_4	{{0.7, 0.8}, {0.1, 0.2}}	{{0.2, 0.3, 0.4}, {0.3, 0.4}}	{{0.2, 0.3}, {0.3, 0.4, 0.5}}	{{0.3, 0.4}, {0.2, 0.3}}
P_5	{{0.5, 0.6}, {0.2, 0.3}}	{{0.4, 0.5, 0.6}, {0.1, 0.2, 0.3}}	{{0.1, 0.2, 0.3}, {0.1, 0.2}}	{{0.6, 0.7}, {0.2, 0.3}}

Maximize the objective function $w_1 - 0.2w_2 - 0.3w_3 - w_4$ subject to the constraints $w_j > 0$, where $j = 1, 2, 3, 4$ and $\sum_{j=1}^4 w_j = 1$:

$$\begin{aligned}
 w_1 - w_2 - 0.2w_3 - w_4 &\leq 0.4, \\
 0.2w_1 + w_3 - 0.4w_4 &\leq 0.2, \\
 -0.2w_1 + w_2 + w_4 &\leq 0.1, \\
 0.2 \leq w_1 &\leq 0.4, \\
 0 \leq w_2 &\leq 0.5, \\
 0.2 \leq w_3 &\leq 0.6, \\
 0.1 \leq w_4 &\leq 0.7.
 \end{aligned} \tag{27}$$

We implemented the MCDM approach described in Section 4 to get the preference order of the alternatives to select the best alternative.

Step 1: the information given by the DM is illustrated in Table 1 by a DHFM \mathcal{D}

Step 2: a normalized dual hesitant fuzzy decision matrix, $\tilde{\mathcal{D}}$, is shown in Table 2

Step 3: the optimal solution P^* is obtained from the NDHFM $\tilde{\mathcal{D}}$ as $P^* = \{0.5000, 0.7000, 0.8000, 0.1000, 0.1500, 0.2000\}$

Step 4: we obtained the weights of the criteria by using the LP model on the objective function under the given constraints provided by the DM in equation (27) as $w_1 = 0.2333, w_2 = 0.1000, w_3 = 0.3000, w_4 = 0.3667$

Step 5: based on equations (11) and (12), calculate the weighted vector similarity measures amongst the alternative $P_i (i = 1, 2, \dots, 5)$ and the optimal alternative P^*

From Table 3, we obtain P_3 is the best choice by utilizing the Jaccard and Dice similarity measures. However, we have another ranking order by using Cosine similarity measure presented in Zhang et al. [29], which shows that the alternative P_3 is the best alternative. The preference arrangements may be distinct according to distinct measures because each algorithm emphasizes on different points of interpretation.

Example 2. A newly established mobile phone firm wants to launch a smart phone. In order to defeat the global market, we must choose the exceptional fixtures suppliers to fit its supply necessities and technology tactics. The system on chip (SoC) is

the pivot of smart phones which is the main concern of the productive growth. The firm hires the suppliers as alternatives according to their level of effort and market investigation. The alternatives can be represented as $P = \{P_1, P_2, P_3, P_4, P_5\}$ and can be evaluated under the criteria as cost (G_1), technical ability (G_2), product performance (G_3), and financial strength (G_4). Assume that all the criteria are beneficial.

Step 1: the information given by the DM is expressed in Table 4 by a DHFM \mathcal{D} .

Step 2: a normalized dual hesitant fuzzy decision matrix (NDHFM), $\tilde{\mathcal{D}}$, is shown in Table 5.

Step 3: the optimal solution P^* is obtained from the NDHFM $\tilde{\mathcal{D}}$ as $P^* = \{0.7000, 0.7500, 0.8000, 0.1000, 0.1500, 0.2000\}$.

Step 4: same weights are used as calculated for Example 1.

Step 5: based on equations (11) and (12), calculate the weighted vector similarity measures amongst the alternative $P_i (i = 1, 2, \dots, 5)$ and the optimal alternative P^* . Table 6 shows the decision outcomes by applying Jaccard and Dice weighted similarity measures.

Table 6 shows that the two preferences' order of the alternatives are same, and we get P_2 as a best choice by using the Jaccard and Dice similarity measures which coincides with another ranking order obtained by using Cosine similarity measure given in Zhang et al. [29].

The above examples clearly indicate that the proposed decision-making model is simple and effective under dual hesitant fuzzy environments and the true need of new types of models based on the vector similarity measures of DHFSs for dealing with dual hesitant fuzzy MCDM problems. The graphical representation of the preference order of the alternatives achieved by the proposed model and the method used by Zhang et al. [29] are shown in Figures 1 and 2.

6. Comparative Analysis

In order to demonstrate merit and the strength of the proposed modified Jaccard and Dice similarity measures for the selection of investment company and supplier of smart phone accessories, we now apply the already presented Cosine similarity measure by Zhang et al. [29] under the DHFSs situations for the comparison analysis. In Example 1, the results of Table 3 indicate that the alternatives selected by using the Jaccard and Dice similarity measures are the same, that is, P_5 is the best alternative, but on the contrary, P_3 is

TABLE 2: The normalized dual hesitant fuzzy decision matrix $\tilde{\mathcal{D}}$.

	G_1	G_2	G_3	G_4
P_1	{{0.1, 0.2, 0.3}, {0.6, 0.65, 0.7}}	{{0.5, 0.6, 0.7}, {0.1, 0.15, 0.2}}	{{0.3, 0.4, 0.5}, {0.1, 0.15, 0.2}}	{{0.1, 0.2, 0.3}, {0.4, 0.45, 0.5}}
P_2	{{0.4, 0.5, 0.6}, {0.1, 0.15, 0.2}}	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}	{{0.1, 0.15, 0.2}, {0.3, 0.35, 0.4}}
P_3	{{0.3, 0.35, 0.4}, {0.1, 0.15, 0.2}}	{{0.4, 0.45, 0.5}, {0.1, 0.15, 0.2}}	{{0.3, 0.35, 0.4}, {0.4, 0.45, 0.5}}	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}
P_4	{{0.7, 0.75, 0.8}, {0.1, 0.15, 0.2}}	{{0.2, 0.3, 0.4}, {0.3, 0.35, 0.4}}	{{0.2, 0.25, 0.3}, {0.3, 0.4, 0.5}}	{{0.3, 0.35, 0.4}, {0.2, .25, 0.3}}
P_5	{{0.5, 0.55, 0.6}, {0.2, 0.25, 0.3}}	{{0.4, 0.5, 0.6}, {0.1, 0.2, 0.3}}	{{0.1, 0.2, 0.3}, {0.1, 0.15, 0.2}}	{{0.6, 0.65, 0.7}, {0.2, .25, 0.3}}

TABLE 3: Decision results obtained by Jaccard, Dice, and Cosine similarity measures.

	$J_s^w (\mathcal{P}_i, \mathcal{P}^*)$	$D_s^w (\mathcal{P}_i, \mathcal{P}^*)$	$C_s^w (\mathcal{P}_i, \mathcal{P}^*)$ Zhang et al. [29]
P_1	0.3921	0.7353	0.3430
P_2	0.4658	0.8442	0.4122
P_3	0.4920	0.8939	0.4796
P_4	0.4391	0.7976	0.4146
P_5	0.4965	0.8947	0.4661
Preference order	$P_5 > P_3 > P_2 > P_4 > P_1$	$P_5 > P_3 > P_2 > P_4 > P_1$	$P_3 > P_5 > P_4 > P_2 > P_1$

TABLE 4: Dual hesitant fuzzy decision matrix \mathcal{D} .

	G_1	G_2	G_3	G_4
P_1	{{0.5, 0.6}, {0.1, 0.2}}	{{0.7, 0.8}, {0.1, 0.2}}	{{0.6, 0.7}, {0.1, 0.2}}	{{0.3, 0.4}, {0.3, 0.4}}
P_2	{{0.4, 0.5, 0.6}, {0.1, 0.2, 0.3}}	{{0.5, 0.6}, {0.3, 0.4}}	{{0.6, 0.7}, {0.1, 0.2}}	{{0.6, 0.7}, {0.2, 0.3}}
P_3	{{0.1, 0.2, 0.3}, {0.3, 0.4, 0.5}}	{{0.5, 0.6}, {0.2, 0.3}}	{{0.4, 0.5}, {0.1, 0.2}}	{{0.6, 0.7}, {0.1, 0.2}}
P_4	{{0.2, 0.3}, {0.1, 0.2}}	{{0.6, 0.7, 0.8}, {0.1, 0.2}}	{{0.1, 0.2}, {0.3, 0.4}}	{{0.6, 0.7}, {0.2, 0.3}}
P_5	{{0.6, 0.7}, {0.1, 0.2}}	{{0.2, 0.3}, {0.4, 0.5, 0.6}}	{{0.3, 0.4}, {0.1, 0.2}}	{{0.3, 0.4}, {0.4, 0.5}}

TABLE 5: The normalized Dual hesitant fuzzy decision matrix $\tilde{\mathcal{D}}$.

	G_1	G_2	G_3	G_4
P_1	{{0.5, 0.55, 0.6}, {0.1, 0.15, 0.2}}	{{0.7, 0.75, 0.8}, {0.7, 0.75, 0.8}}	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}	{{0.3, 0.35, 0.4}, {0.3, 0.35, 0.4}}
P_2	{{0.4, 0.5, 0.6}, {0.1, 0.2, 0.3}}	{{0.5, 0.55, 0.6}, {0.3, 0.35, 0.4}}	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}	{{0.6, 0.65, 0.7}, {0.2, 0.25, 0.3}}
P_3	{{0.1, 0.2, 0.3}, {0.3, 0.4, 0.5}}	{{0.5, 0.55, 0.6}, {0.2, 0.25, 0.3}}	{{0.4, 0.45, 0.5}, {0.1, 0.15, 0.2}}	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}
P_4	{{0.2, 0.25, 0.3}, {0.1, 0.15, 0.2}}	{{0.6, 0.7, 0.8}, {0.1, 0.15, 0.2}}	{{0.1, 0.15, 0.2}, {0.3, 0.35, 0.4}}	{{0.6, 0.65, 0.7}, {0.2, 0.25, 0.3}}
P_5	{{0.6, 0.65, 0.7}, {0.1, 0.15, 0.2}}	{{0.2, 0.25, 0.3}, {0.4, 0.5, 0.6}}	{{0.3, 0.35, 0.4}, {0.1, 0.15, 0.2}}	{{0.3, 0.35, 0.4}, {0.4, 0.45, 0.5}}

TABLE 6: Decision results obtained by Jaccard, Dice, and Cosine similarity measures.

	$J_s^w (\mathcal{P}_i, \mathcal{P}^*)$	$D_s^w (\mathcal{P}_i, \mathcal{P}^*)$	$C_s^w (\mathcal{P}_i, \mathcal{P}^*)$ Zhang et al. [29]
P_1	0.5108	0.9010	0.4432
P_2	0.5439	0.9541	0.4973
P_3	0.4949	0.8943	0.4602
P_4	0.4484	0.8079	0.4103
P_5	0.4534	0.8345	0.4205
Preference order	$P_2 > P_1 > P_3 > P_5 > P_4$	$P_2 > P_1 > P_3 > P_4 > P_5$	$P_2 > P_3 > P_1 > P_5 > P_4$

selected with the help of Cosine similarity measure. In Example 2, Table 6 shows that the alternative P_2 is the best choice obtained by applying the three similarity measures (Jaccard, Dice, and Cosine similarity measures) under the framework of DHFSs. According to the rule of the maximum level of similarity, that is, the similarity increases if the value of the similarity measures approaches to one and become

perfectly similar if the value of the similarity measure become one. From Tables 3 and 6, we can see that value of Jaccard and Dice similarity measures are much greater than Cosine similarity measure. Hence, we can say that our modified Jaccard and Dice similarity measures are more realistic than Cosine similarity measure under the environment of DHFSs.

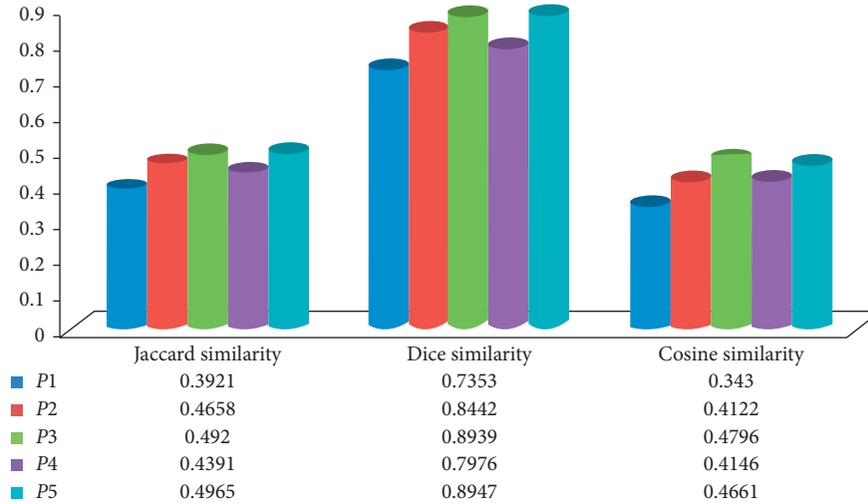


FIGURE 1: Ranking order of alternatives.

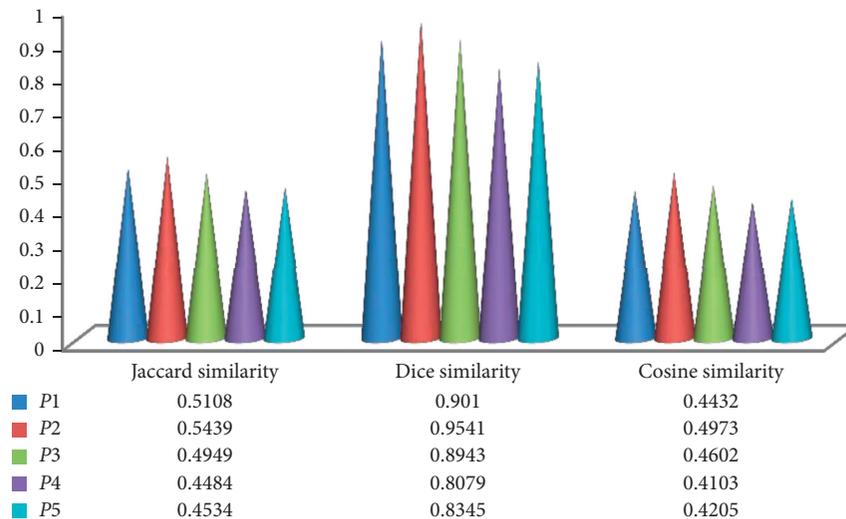


FIGURE 2: Ranking order of alternatives.

7. Conclusions

Jaccard and Dice similarity measures are easy to compute, and LP technique is also user friendly and responds quickly through Matlab. So far, the Jaccard and Dice similarity measures have not been considered under the framework of DHFSs. In the present work, we established the Jaccard and Dice similarity measures between two DHFSs. Most of the DMs assigned the weights to the criteria which are partially or completely known. However, in the present work, we used an objective function which is maximized under the given constraints and then applied the LP technique to find the weights of the criteria. Based on the Jaccard and Dice weighted similarity measures defined in Section 3, we propose an MCDM model to choose the best alternative under the influence of various criteria. Lastly, two practical examples of the constructed model are given to choose the investment company and for the selection of smart phone accessories. Then, we compared the results obtained by using the proposed model and the Cosine

similarity measure presented by Zhang et al. [29]. The decision outcomes illustrate that the Jaccard and Dice similarity measures are more reliable than the Cosine similarity measure because both (Jaccard and Dice) have the same option in the similarity identification and follow the rule of level of maximum similarity. Thus, the numerical examples show that the proposed model in this work is applicable and more effective. In the future research direction, the modified Jaccard and Dice similarity measures will be extended to the picture fuzzy sets, and it can be applied in other related decision-making problem.

Data Availability

No data were used to support the findings of the study.

Conflicts of Interest

The authors declare no conflicts of interest.

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