

Research Article

Forecasting Stock Market Volatility: A Combination Approach

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We find that combining two important predictors, stock market implied volatility and oil volatility, can improve the predictability of stock return volatility. We also document that the stock market implied volatility provides far more significant predictability than the oil volatility and other nonoil macroeconomic and financial variables. The empirical results show the “kitchen sink” combination approach that using two predictors jointly performs better than not only the univariate regression models which use oil volatility or stock market implied volatility separately but also convex combination of the individual forecasts. This improvement of predictability is also remarkable when we consider the business cycle. Furthermore, the robust test based on different lag lengths and different macroinformation shows that our forecasting strategy is efficient.

1. Introduction

Prediction of stock market volatility has many important applications in risk management, asset pricing, market timing decisions, and portfolio selection. Therefore, modeling and forecasting stock market volatility is an important task and a popular research topic in financial markets [1].

The seminal paper on the economic drivers of stock market volatility is by Officer [2] which is followed by Schwert [3]. Officer [2] points towards countercyclical movements of stock market volatility, but the link between volatility and economic activity is not very strong from a statistical perspective. Christiansen et al. [4] provide a comprehensive analysis of volatility predictability in financial markets by economic variables and find that information on economic variables helps in predicting future volatility. Paye [5] shows that some variables in theory (such as national debt spreads and default gains) will affect the volatility of stocks and finds some variables that can predict stock volatility. Engle et al. [6] analyze the effect of inflation and industrial production growth on daily stock return volatility, considering each macroeconomic variable separately. Conrad and Loch [7] investigate the relationship between long-term US stock market risks and the macroeconomic environment using a two-component GARCH-

MIDAS model and show that macroeconomic variables are important determinants of the secular component of stock market volatility. In the recent years, forecasting of stock market volatility has been investigated from different perspectives, for example, Bayesian model averaging (Nonejad [8]), heterogeneous autoregressive models of realized volatility (HAR-RV) [9, 10], simple linear regression [11, 12], and mixed-frequency approach [13].

Although the prediction of stock market volatility has made good progress, there is still room for improvement. Therefore, our main goal is to obtain superior out-of-sample performance for stock market volatility prediction. In detail, firstly, we use the linear regression models which take oil volatility or stock market implied volatility as the predictor separately. Then, we utilize a “kitchen sink” combination approach that uses oil volatility and stock market implied volatility jointly.

One motivation for this paper is mainly that stock market implied volatility indices derived from option prices which reflect market's expectation on the future volatility over the remaining life of the options are generally considered to be a better measure of market's uncertainty. It is well known that the implied volatility index has long been seen as a predictor of volatility, where the GARCH model is used from an in-sample perspective (see [14–21]). In

addition, crude oil is a core input in the modern industry. Crude oil is also one of the most important commodities. Investors' asset reallocations between commodity indices and stocks result in volatility spillovers between crude oil and stock markets (see [22–25]). It is because that stock implied volatility VIX not only contains the historical volatility information but also includes investors' expectation on the future market conditions. Oil price shocks can certainly lead to changes in stock prices by affecting real economic activities (see Kilian [26]). Therefore, we can expect that oil volatility and stock market implied volatility VIX are predictors of stock market volatility.

The other motivation for our paper is that constructing or finding new powerful predictors for stock market volatility is difficult. Therefore, we try to find existing predictors that can offer more efficient information on forecast stock volatilities and then add them into the forecast combination model to generate stock volatility forecasts. This idea is not really new because it has been well demonstrated that combination forecasts can perform even better than individual forecasts (see [27–29]). Furthermore, the forecast combination method has recently received increasing attention in the economic forecasting literature, especially in the stock return (see, for example, [30–35]). Although predictive combination is becoming more popular in stock returns, the predictability of stock return volatility is rarely used. In view of this, we consider the combined approach as an important competitive model in this paper.

We use daily data of the S&P 500 index, the West Texas Intermediate (WTI), and the Brent crude oil price where the time is from January 1990 to December 2018 and the monthly implied volatility VIX from January 1990 to December 2018. The sum of the monthly average daily returns is built to achieve monthly volatility. The in-sample results show that there is a very significant predictability from stock market implied volatility VIX to stock market volatility.

We also take the out-of-sample R_{oos}^2 to evaluate the forecasting of out-of-sample performance as in the literature ([4, 5, 36, 37]). We utilize the rolling estimation and recursive estimation to produce a one-step-ahead prediction of stock market volatility from Jan. 1998 to Dec. 2018. Being of our interest, the forecasting performance of the “kitchen sink” combining the predictive regression model of oil volatility and VIX is stronger than the univariate of VIX, and it beats the univariate of oil volatility significantly during various sample periods.

The economic cyclical periods may have an impact on market predictability. To test the robustness of the predictability, we try to link market predictability to the business cyclical periods. During recession periods, R_{oos}^2 values for VIX are larger than R_{oos}^2 values for the oil volatility. The “kitchen sink” model for combining oil volatility WTI and VIX yields higher R_{oos}^2 than the regression model of VIX individually. During expansion periods, R_{oos}^2 values for all of the regressions are smaller than that during the recession periods. In addition, from a comparative perspective, the out-of-sample performance for most of the regressions is basically consistent with that during the recession periods.

To explore the predictability of our models, we use different lag lengths of stock market return realized volatility and different macroinformation to carry out the robustness analysis. Out-of-sample results show that different lag orders have little effect on out-of-sample forecasting performance for stock return volatility. In addition, the out-of-sample performance of the “kitchen sink” model by combining macroeconomic and financial variables and VIX is stronger than the univariate of macroeconomic and financial variables or VIX. Hence, our out-of-sample forecasting results over different lag lengths and different macroinformation are found to be robust.

The remainder of this paper is organized as follows: Section 2 provides our research data and summary statistics. Section 3 presents the predictive regression methodology. We report the in-sample and out-of-sample results in Section 4 and Section 5, respectively. Section 6 gives the extension analysis by linking to the business cycle. Section 7 investigates the robustness test by containing different lag lengths and different macroinformation. Finally, we conclude the paper.

2. Data and Descriptive Statistics

In this paper, we try to use stock implied volatility together with crude oil volatility to accurately predict the volatility of the S&P 500 index. We utilize the daily data for the S&P 500 index, spanning from January 1990 to December 2018. Simultaneously, we use the end-of-month closing price spanning from January 1990 to December 2018 from the implied volatility indices VIX. These data are extracted from the Thomson Reuters Database (<https://www.thomsonreuters.com/en.html>). In addition, we also utilize the prices of Brent crude oil and West Texas Intermediate (WTI) crude oil, which can be freely downloaded from the Energy Information Administration website [1].

Following the literature (e.g., [4, 5, 12, 13]), the volatility of S&P 500 for a month t can be defined as follows:

$$RV_t = \sum_{j=1}^m r_{t,j}^2, \quad j = 1, 2, \dots, m, \quad (1)$$

where m stands for the number of trading days per month and $r_{t,j}$ stands for the j -th daily return in the t -th month. The summary statistics of the basic statistics of the crude oil volatility and the volatility of S&P 500 are reported in Table 1.

Following the literature (e.g., [4, 5, 12]), we also take the natural logarithm as $V_t = \log(RV_t)$, to reduce the impact of leptokurtic on the realized volatility in (1).

3. Methodology

3.1. Forecasting Models. For stock volatility forecasting, a standard benchmark is the following autoregressive model (AR):

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \varepsilon_{t+1}, \quad (2)$$

TABLE 1: Summary statistics.

	Mean	Median	S. E.	Max.	Min.	Skew.	Kurt.
S&P	0.003	0.011	0.013	19.876	0.001	0.008	0.008
BRT	18.235	0.005	0.017	0.020	7.555	0.0001	0.0007
WTI	0.001	10.420	0.057	0.237	0.255	59.890	6.899
VIX	8.218	7.108	1.682	66.802	97.190	75.067	7.374

Note. This table reports the basic statistics of the major regression variables, which is shown in the first column of the table. S&P, BRT, and WTI denote the realized volatility of the S&P500 stock index, Brent oil price, and WTI oil price, respectively. VIX denotes the implied volatility for the S&P500 stock index. The mean value, standard deviation, median, coefficients of kurtosis and skewness, range, and the number of observations are shown in this table.

where $V_t = \log(RV_t)$ and the error term $\varepsilon_{t+1} \sim N(0, 1)$, and it is assumed to be independent and identical.

Wang et al. [12] extended the benchmark AR in (2) including the volatility of oil as an additional predictor:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,oil} + \varepsilon_{t+1}, \quad (3)$$

where $V_{t,oil}$ is the log-realized oil volatility in the t -th month and the lag order $p = 6$.

To investigate the predictive ability of the stock implied volatility indices VIX, we also extend the benchmark AR in (2) including a log stock market implied volatility as an additional predictor:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \lambda IV_t + \varepsilon_{t+1}, \quad (4)$$

where IV_t is the stock implied volatility of the t -th month and λ reflects the effect of implied volatility VIX of the t -th month on the $(t+1)$ -th month volatility.

We will utilize the information of each forecast through the following combination of prediction methods. As in [2], we also use a “kitchen sink” model that includes stock implied volatility together with crude oil volatility in a multiple linear regression model:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,oil} + \lambda IV_t + \varepsilon_{t+1}, \quad (5)$$

where IV_t and $V_{t,oil}$ are the stock implied volatility and the volatility of oil in the t -th month, respectively. In this paper, the lag order p is also set to be 6.

Another practical solution is to use a convex combination forecast method (e.g., [28]). First, individual forecasts are obtained in running predictive regressions on each predictor. Then, we can take a convex combination of the individual forecasts as the forecast:

$$\widehat{V}_{t+1}^{MC} = \lambda \widehat{V}_{t+1}^{oil} + (1 - \lambda) \widehat{V}_{t+1}^{IV}, \quad (6)$$

where $0 < \lambda < 1$ is the combining weights and \widehat{V}_{t+1}^{oil} and \widehat{V}_{t+1}^{IV} are obtained in the oil volatility-based predictive regression model (3) and stock implied volatility-based predictive model (4) in the t -th month, respectively. In this paper, we set $\lambda = 0.1$.

3.2. Out-Of-Sample Forecast and Evaluation. For out-of-sample forecast, we use the recursive estimation method and the rolling estimation method to generate a one-step-ahead forecasting volatility. For both recursive and rolling estimation methods, the whole T -observation sample of oil volatility, VIX, and stock market volatility series are divided into two parts, the first M observations are used for the in-sample and the remaining $T-M$ observations are used for the out-of-sample.

Following the literature [38–43], we take a wide spread out-of-sample R^2 statistics (R_{OoS}^2 hereafter) to evaluate the prediction performance of a given model. This statistic tests whether the out-of-sample prediction performance of the given model outperforms the benchmark. The R_{OoS}^2 is computed as

$$R_{OoS}^2 = 1 - \frac{MSPE_{\text{model}}}{MSPE_{\text{bench}}}, \quad (7)$$

where $MSPE_{\text{model}} = (1/T - M) \sum_{t=M+1}^T (V_t - \widehat{V}_t)^2$ and $MSPE_{\text{bench}} = (1/T - M) \sum_{t=M+1}^T (V_t - \bar{V}_t)^2$. $MSPE_{\text{bench}}$ and $MSPE_{\text{model}}$ are the mean-squared predictive errors (MSPE) of the benchmark model and the tested model, respectively. In addition, the p value of the one-sided test can be easily obtained from the standard normal distribution.

As by Christiansen et al. [4], Paye [5], Conrad and Loch [7], Nonejad [8], and Wang et al. [12], we also take the autoregressive model AR (6) in (2) as the benchmark model because stock realized volatility is highly persistent and the autoregressive model AR (6) is a strong benchmark for stock volatility prediction.

4. In-Sample Analysis

Inoue and Kilian [44] showed that it is unreasonable to obtain out-of-sample predictability without good performance for in-sample predictability. And, Table 2 reports the coefficient estimates of predictive regression models in (3), (4), and (5) which are estimated by using the ordinary least squares and t -statistics based on the C-W statistic method [45] which is used to test the null hypothesis of no predictability. We also give increase percentage terms in R^2 for the predictive regression models in (3), (4), and (5), relative to the benchmarks of AR (6) models in Table 2.

The coefficient estimates of α_1 and α_2 in the predictive regression models in (3) and the coefficient estimate of α_1 in the predictive regression models in (4) and (5) are significant and positive. It is obvious that there is volatility persistence for the stylized fact. In the regression models in (3), the estimated coefficient of β is 0.073 and 0.112 for Brent and WTI oil volatility, respectively. And, the coefficient estimate of β in the predictive regression models in (3) is significantly positive with 5% level for WTI oil volatility and at 10% level for Brent oil volatility. The coefficient estimate of λ is 1.647 in the predictive regression models in (4). And, the coefficient estimate of λ in the predictive regression models in (4) is significantly positive at 1% level for stock market implied volatility VIX, which indicates that there is a significant in-sample predictability from stock market implied volatility to stock volatility.

TABLE 2: In-sample estimation results.

In-sample estimation results for WTI									
Coefficient	ω	α_1	α_2	α_3	α_4	α_5	α_6	β	λ
	-0.793**	0.470***	0.166***	0.077	-0.095	0.094	0.082	0.112**	—
<i>t</i> -stat	-2.445	5.930	2.744	1.215	-1.261	1.384	1.530	2.308	—
In-sample estimation results for BRT									
Coefficient	ω	α_1	α_2	α_3	α_4	α_5	α_6	β	λ
	-0.846**	0.487***	0.166***	0.077	-0.091	0.095	0.079	0.073*	—
<i>t</i> -stat	-2.579	6.160	2.754	1.224	-1.189	1.374	1.472	1.659	—
In-sample estimation results for VIX									
Coefficient	ω ***	α_1	α_2	α_3	α_4	α_5	α_6	β	λ
	-9.820	0.151*	0.044	0.069	-0.117*	0.057	0.020	—	1.647***
<i>t</i> -stat	-5.953	1.671	0.726	1.213	-1.673	0.864	0.386	—	5.719
In-sample estimation results for WTI + VIX									
Coefficient	ω ***	α_1	α_2	α_3	α_4	α_5	α_6	β	λ
	-9.475	0.131*	0.047	0.061	-0.123*	0.056	0.024	0.085*	1.608***
<i>t</i> -stat	-5.704	1.72	0.775	1.053	-1.676	0.874	0.455	1.820	5.593
In-sample estimation results for BRT + VIX									
Coefficient	ω ***	α_1	α_2	α_3	α_4	α_5	α_6	β	λ
	-9.663	0.134*	0.045	0.059	-0.121*	0.057	0.021	0.065*	1.638***
<i>t</i> -stat	-5.852	1.731	0.757	1.032	-1.668	0.861	0.396	1.771	5.690
Percent increase of R^2									
WTI		BRT		VIX		WTI + VIX		BRT + VIX	
ΔR^2	1.215	ΔR^2	0.500	ΔR^2	9.560	ΔR^2	10.234	ΔR^2	9.954

Note. This table reports the in-sample estimation results for the predictive regression models in (3), (4), and (5) for monthly stock volatility. We report the estimate of the slope coefficients, as well as the corresponding heteroskedasticity-adjusted *t*-statistic, based on the Newey–West method. We also show the percent increase in R^2 of the model of interest relative to that of the benchmark of AR in (2). The asterisks *, **, and *** denote rejections of null hypothesis at 10%, 5%, and 1% significance levels, respectively.

Being of our interest, in the predictive regression models in (5), for combining WTI oil volatility and VIX, the coefficient estimate of β is 0.085 and λ is 1.608 for WTI oil volatility and VIX, respectively. And, the coefficient estimate of β in the predictive regression models in (5) is significantly positive at 10% level for WTI oil volatility, and the coefficient estimate of β in the predictive regression models in (5) is significantly positive at 1% level for VIX.

In the predictive regression models in (5), for combining Brent oil volatility and VIX, the coefficient estimate of β is 0.065 and λ is 1.638 for Brent oil volatility and VIX, respectively. And, the coefficient estimate of β in the predictive regression models in (5) is significantly positive at 10% level for Brent oil volatility, and the coefficient estimate of β in the predictive regression models in (5) is significantly positive at 1% level for VIX.

Comparing the percentage increase in R^2 of the model of interest relative to the AR benchmark (2), we can see that the values of the predictive regression models in (5) for combining oil volatility and VIX are much greater than the percentage increase in R^2 due to predictive regressions models in (3) or (4), which imply that combining oil volatility and VIX can provide more accurate prediction than most popular predictors for stock volatility.

5. Out-Of-Sample Analysis

Table 3 reports the predictive results for the recursive window method, where the values of R^2_{OOS} and the corresponding *p* values of C-W statistics [45] are listed.

Firstly, we can see the predictive performance of Brent and WTI oil volatility, where the predictive regression model is used in (3). The values of R^2_{OOS} suggest that including WTI oil volatility in the regression model can result in a 1.01% decrease in MSPE during the whole out-of-sample period. The R^2_{OOS} value is larger in more recent subperiods. It is obvious that the forecasting accuracy of Brent oil volatility is a little weaker in comparison with WTI oil volatility.

The out-of-sample performance of (4) with stock market implied volatility VIX is stronger than oil volatility. The values of R^2_{OOS} suggest that including VIX in the forecasting model can cause the reduction of MSPE by 12.98% during the whole out-of-sample period. The R^2_{OOS} values are stable during the more recent subperiods. The *p* values based on C-W statistics also show that there exists significant improvement during different periods, which indicates that the predictive ability of VIX is efficient.

Being of our interest, the out-of-sample performance of (5) for the “kitchen sink” combining oil volatility and VIX is stronger than the univariate of oil volatility or VIX. The evidence is that the values of R^2_{OOS} suggest that including VIX and oil volatility in predictive regression can result in a larger improvement of forecasting accuracy during different out-of-sample periods.

The forecasting performance of the convex combination predictive model (6) for combining oil volatility and VIX is a bit weaker in comparison with the “kitchen sink” combining predictive regression model (5). The evidence is that the values of R^2_{OOS} in the convex predictive model (6) is smaller than the “kitchen sink” model (5), even smaller than VIX

TABLE 3: Out-of-sample forecasting results, recursive window.

	WTI	BRT	VIX	WTI + VIX	BRT + VIX	CC (W + V)	CC (B + V)
Out-of-sample forecasting results for 1998–2018							
R^2_{OOS}	1.01	0.12	12.98***	13.29***	13.01***	12.68***	12.58***
p value	0.1323	0.1963	0.0001	0.0001	0.0001	0.0001	0.0001
Out-of-sample forecasting results for 2003–2018							
R^2_{OOS}	1.53*	1.44*	10.41***	11.21***	10.73***	10.38***	10.24***
p value	0.0574	0.0654	0.0003	0.0001	0.0002	0.0003	0.0003
Out-of-sample forecasting results for 2008–2018							
R^2_{OOS}	2.20*	0.63*	12.32***	13.46***	12.72	12.08***	11.92***
p value	0.0522	0.0932	0.0014	0.0007	0.0012	0.0013	0.0014
Out-of-sample forecasting results for 2013–2018							
R^2_{OOS}	3.43*	1.32	18.73***	20.34***	19.69***	18.08***	17.90***
p value	0.0844	0.1243	0.0060	0.0037	0.0048	0.0058	0.0061

Note. This table reports the forecasting results for the predictive regression models in (3), (4), and (5) for monthly stock volatility. The table reports the out-of-sample R^2 , defined in the percent reduction of the mean-squared predictive error (MSPE) of the interest models relative to that of the benchmark of AR (6). The p values of Clark and West [45] (CW) tests for the equivalence of MSPEs between the interest models and the benchmark model are given in the parentheses. The asterisks *, **, and *** indicate rejections of null hypothesis at 10%, 5%, and 1% significance levels, respectively.

over different sample periods. The reason is that the forecasting ability of oil volatility is far weaker than VIX.

Table 4 reports the predictive results for the rolling window method. Overall, the predictive performances based on the rolling window are very similar with the recursive window. From the values of R^2_{OOS} , we can know that the forecasting ability of oil volatility is far weaker than VIX. In addition, the predictive performance of the regression model (5) for the “kitchen sink” combining oil volatility and VIX is stronger than the univariate of oil volatility or VIX. The forecasting performance of the convex combination predictive model (6) for combining oil volatility and VIX is a bit weaker in comparison with the “kitchen sink” combining predictive regression model (5). During different subperiods, both the R^2_{OOS} values and p values of C-W test [45] suggest that there is a very significant improvement of forecasting accuracy for VIX and the “kitchen sink” combining oil volatility and VIX.

6. Business Cycles

The economic cyclical period may have an impact on market predictability. Neely et al. [46] showed that the predictability of the stock market appears with some different results during commercial expansion and recession.

To test the predictability of crude oil volatility together with stock implied volatility, we use a business cycle indicator with NBER-date, which is equal to 1 when the economy is in recession. Further, to study the source of predictive ability, we will calculate the out-of-sample R -square R^2_{OOS} during business expansion and recessions periods [47–49], as follows:

$$R^2_{\text{OOS}} = 1 - \frac{\sum_{k=1}^q I^c_{m+k} (V_{m+k} - \hat{V}_{m+k})^2}{\sum_{k=1}^q I^c_{m+k} (V_{m+k} - \bar{V}_{m+k})^2}, \quad (8)$$

$c = \text{REC}, \text{EXP}$,

where I^{REC}_{m+k} (I^{EXP}_{m+k}) is an indicator variable that equals 1 if the month $m+k$ is during a period of recession (expansion). We

will give the performance of out-of-sample forecasting for different business cycles in Table 5.

During recession periods, R^2_{OOS} values for the stock market implied volatility VIX are larger than R^2_{OOS} values for the oil volatility. The “kitchen sink” model for combining oil volatility WTI and VIX yields higher R^2_{OOS} than the regression model of VIX individually. But, the “kitchen sink” model for combining Brent oil volatility and VIX yields almost the same R^2_{OOS} as the regression model of VIX individually. The forecasting performance of the convex combination predictive model (6) for combining oil volatility and VIX is a bit weaker in comparison with the “kitchen sink” combining predictive regression model (5), even for the regression model of VIX individually. During expansion periods, R^2_{OOS} values for all of the regressions are smaller than that during recession periods. In addition, from a comparative perspective, the out-of-sample forecasting performance for most of the regressions is basically consistent with that during the recession periods. Not surprisingly, the “kitchen sink” model for combining oil volatility and VIX yields higher R^2_{OOS} than the other regression models. The results in Table 5 also show that the out-of-sample results in the previous section are robust to alternative business cycles.

7. Robustness Tests

In this section, we will give two robustness analysis that test the predictability of our models, including different lag lengths of stock market return realized volatility and different macroinformation.

7.1. Predictability for Different Lag Lengths. For stock volatility forecasting, it is obvious that different lag orders will have impact on the prediction ability. Hence, we will consider the alternative lag orders: 2, 4, 6, and 8 with the recursive window method where the out-of-sample period is 2003:01–2018:12.

Table 6 reports the out-of-sample predictive results for the recursive window method with the lag orders: 2, 4, 6, and

TABLE 4: Out-of-sample forecasting results, rolling window.

	WTI	BRT	VIX	WTI + VIX	BRT + VIX	CC (W + V)	CC (B + V)
Out-of-sample forecasting results for 1998–2018							
R^2_{oos}	0.56*	0.17	12.81***	12.83***	13.70***	13.12***	13.15***
p value	0.082	0.117	0.0005	0.0001	0.0001	0.0001	0.0001
Out-of-sample forecasting results for 2003–2018							
R^2_{oos}	1.53*	1.43*	12.01***	14.47***	13.22***	12.66***	12.72***
p value	0.057	0.065	0.0001	0.0001	0.0001	0.0001	0.0001
Out-of-sample forecasting results for 2008–2018							
R^2_{oos}	2.52*	1.43	12.10***	13.67***	14.38***	12.89***	12.88***
p value	0.064	0.113	0.0009	0.0001	0.0001	0.0006	0.0007
Out-of-sample forecasting results for 2013–2018							
R^2_{oos}	5.22*	4.32*	21.27***	22.61***	25.50***	21.67***	22.07***
p value	0.067	0.066	0.0026	0.0007	0.0002	0.0020	0.0019

Note. The forecasting results for the predictive regression models in (3), (4), and (5) for monthly stock volatility. The table reports the out-of-sample R^2 , defined in the percent reduction of mean-squared predictive error (MSPE) of the interest models relative to that of the benchmark of AR (6). The p values of Clark and West [45] (CW) tests for the equivalence of MSPEs between the interest models and the benchmark model are given in the parentheses. The asterisks *, **, and *** indicate rejections of null hypothesis at 10%, 5%, and 1% significance levels, respectively.

TABLE 5: Out-of-sample forecasting performance over business cycles.

	WTI	BRT	VIX	WTI + VIX	BRT + VIX	CC (W + V)	CC (B + V)
Out-of-sample forecasting results for 1998–2018							
Recession period	2.648	0.596	20.781	21.056	21.208	19.854	19.883
Expansion period	0.825	0.116	12.124	13.553	13.112	11.896	11.783
Out-of-sample forecasting results for 2003–2018							
Recession period	0.597	0.572	26.792	26.786	25.937	24.809	24.695
Expansion period	1.572	0.191	8.778	9.659	9.215	8.945	8.803
Out-of-sample forecasting results for 2008–2018							
Recession period	3.151	2.071	12.409	13.352	12.711	12.806	12.468
Expansion period	1.773	0.425	6.482	9.787	9.872	7.531	6.917
Out-of-sample forecasting results for 2013–2018							
Recession period	5.141	0.725	32.515	36.125	35.894	35.003	33.055
Expansion period	1.461	0.555	7.719	11.011	10.874	8.367	7.840

Note. This table reports the out-of-sample performance from statistical perspectives over business cycles based on the recursive window. The forecasting results for the predictive regression models in (3), (4), and (5) for monthly stock volatility are shown. Statistical performance is defined as out-of-sample R -square (R^2_{oos}).

TABLE 6: Out-of-sample forecasting results with different lag lengths.

	WTI	BRT	VIX	WTI + VIX	BRT + VIX	CC (W + V)	CC (B + V)
Out-of-sample forecasting results with lag length 2							
R^2_{oos}	1.41*	1.32*	10.25***	11.18***	10.67***	10.28***	10.18***
p value	0.0765	0.0821	0.0006	0.0001	0.0002	0.0003	0.0003
Out-of-sample forecasting results with lag length 4							
R^2_{oos}	1.48*	1.41*	10.36***	11.20***	10.70***	10.33***	10.20***
p value	0.0574	0.0654	0.0004	0.0001	0.0002	0.0003	0.0003
Out-of-sample forecasting results with lag length 6							
R^2_{oos}	1.53*	1.44*	10.41***	11.21***	10.73***	10.38***	10.24***
p value	0.0574	0.0654	0.0003	0.0001	0.0002	0.0003	0.0003
Out-of-sample forecasting results with lag length 8							
R^2_{oos}	1.50*	1.40*	10.37***	11.17***	10.67***	10.31***	10.21***
p value	0.0786	0.0766	0.0005	0.0003	0.0003	0.0005	0.0005

Note. This table reports the forecasting results for the predictive regression models for monthly stock volatility with the alternative lag orders: 2, 4, 6, and 8 where the out-of-sample period is 2003 : 01–2018 : 12. The table reports the out-of-sample R^2 , defined in the percent reduction of the mean-squared predictive error (MSPE) of the interest models relative to that of the benchmark of AR (p). The p values of Clark and West [45] (CW) tests for the equivalence of MSPEs between the interest models and the benchmark model are given in the parentheses. The asterisks *, **, and *** indicate rejections of null hypothesis at 10%, 5%, and 1% significance levels, respectively.

TABLE 7: Out-of-sample forecasting results with different macroinformation.

Macrovariables	1998–2018		2003–2018		2008–2018	
	Individual	Individual + VIX	Individual	Individual + VIX	Individual	Individual + VIX
cp	0.12	13.18***	0.16	10.78***	0.19	12.88***
dfr	−0.15	13.06***	−0.12**	10.50***	−0.20**	12.68***
dfy	0.45**	13.60***	0.58**	11.02***	0.67**	13.10***
exret	0.11	13.09***	0.18	10.88***	0.22	12.68***
ip	0.67**	13.77***	0.69**	11.10***	0.75**	13.17***
ipvol	1.03**	14.04***	1.12**	11.65***	1.22**	13.89***
npv	0.38*	13.56***	0.45*	11.05***	0.56**	12.97***
ppivol	0.22*	13.24***	0.30*	10.88***	0.36*	12.86***
tms	0.32*	13.35***	0.40**	10.97***	0.45**	12.91***
hs	0.62**	13.67***	0.66**	11.14***	0.72**	13.12***
mkt	0.47**	13.60***	0.50**	11.05***	0.55**	13.06***
VIX	12.98***	—	10.41***	—	12.32***	—

Note. This table reports the forecasting results for the predictive regression models in (9) and (10) for monthly stock volatility. The table reports the out-of-sample R^2 , defined in the percent reduction of the mean-squared predictive error (MSPE) of the interest models relative to that of the benchmark of AR (6). The p values of Clark and West [45] (CW) tests for the equivalence of MSPEs between the interest models and the benchmark model are given in the parentheses. The asterisks *, **, and *** indicate rejections of null hypothesis at 10%, 5%, and 1% significance levels, respectively.

8. From Table 6, we can see that different lag orders have little effect on out-of-sample forecasting for stock return volatility. Overall, when the lag order is 6, the out-of-sample forecast performance is better. Therefore, a lag order of 6 is a reasonable choice.

7.2. Predictability for Different Macroinformation. Paye [5] provided a series of macroeconomic variables which can influence stock markets in terms of stock market volatility. Is it possible that the forecast improvement which can be obtained by adding macroeconomic variables to the VIX model is sensitive? To examine this question, we also consider the following models to do robustness check.

We can extend the benchmark AR in (2) including macroeconomic variables as an additional predictor:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,\text{mav}} + \varepsilon_{t+1}, \quad (9)$$

where $V_{t,\text{mav}}$ is the nonoil macroeconomic variables in the t -th month.

As in [2], we also use a “kitchen sink” model that includes stock implied volatility together with the nonoil macroeconomic variables in a multiple linear regression model:

$$V_{t+1} = \omega + \sum_{i=0}^{p-1} \alpha_i V_{t-i} + \beta V_{t,\text{mav}} + \lambda IV_t + \varepsilon_{t+1}, \quad (10)$$

where IV_t and $V_{t,\text{mav}}$ are the stock implied volatility and the nonoil macroeconomic variables in the t -th month, respectively, and the lag order p is also set to be 6.

In this section, we use some macroeconomic variables for stock market activity as suggested by Paye [5]. These popular predictor variables are the commercial paper-to-treasury spread (cp), default return spread (dfr), default spread (dfy), expected return (exret), growth in industrial production (ip), industrial production volatility (ipvol), net payout (npv), inflation volatility (ppivol), and term spread

(tms). In addition, some other macroeconomic and financial variables are considered to be powerful predictors of volatility, for example, the US housing starts (hs) and the market factor of Fama–French three factor models (mkt).

Table 7 reports the out-of-sample forecasting results for robustness check with nonoil macroeconomic and financial variables. Firstly, all R^2_{oss} values of the tested macroeconomic and financial variables are positive, except for dfr, and the R^2_{oss} values of ipvol exceeds 1%. From the values of R^2_{oss} , we can know that the forecasting ability of tested macroeconomic and financial variables is far weaker than VIX.

Being of our interest, the out-of-sample performance of (10) for the “kitchen sink” combining macroeconomic and financial variables and VIX is stronger than the univariate of macroeconomic and financial variables or VIX. The evidence is that the values of R^2_{oss} suggest that including VIX and macroeconomic and financial variables in predictive regression can result in a larger improvement of forecasting accuracy during different out-of-sample periods. The p value of the C-W test [45] suggest that there is a very significant improvement of forecasting accuracy for VIX and the “kitchen sink” combining macroeconomic and financial variables and VIX. Overall, the predictive performances are robust to nonoil macroeconomic and financial variables.

8. Conclusions

The goal of this paper is to propose an efficient way to improve the predictability of stock volatility where we seek to use two important predictors, oil volatility and stock market implied volatility. We establish several findings. First, the stock market implied volatility extracts significantly more useful information from the predictors than the oil volatility not only in an in-sample analysis but also in an out-of-sample analysis. Second, the “kitchen sink” combination approach that uses two predictors jointly outperforms not only the univariate regression models that use each predictor’s information separately but also convex combination of the individual forecasts. Our findings

survive other extension analysis, namely, the business cycle. Further analysis demonstrates that the “kitchen sink” combination of oil volatility and stock market implied volatility contributes to improve the predictability of stock volatility over the business cycle. Finally, we test the robustness of forecast ability of different lag lengths of stock market return realized volatility and different macroinformation. The results show that the predictability is robust to different lag lengths and different macroinformation.

Our findings have some implications for market participants. Firstly, the prediction ability of stock market implied volatility is much better than that of macroeconomic and financial variables. Secondly, the “kitchen sink” combination of stock market implied volatility and macroeconomic or financial variables can improve the out-of-sample forecasting performance. Finally, the predictive power of the “kitchen sink” combination is robust to controlling the lagged volatility.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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