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Research Article

Multiplicity Solutions for Integral Boundary Value Problem of Fractional Differential Systems

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This paper deals with the existence and multiplicity of solutions for the integral boundary value problem of fractional differential systems: $\begin{cases} D_{0+}^{\alpha_1}u_1(t) = f_1(t,u_1(t),u_2(t)), \\ D_{0+}^{\alpha_2}u_2(t) = f_2(t,u_1(t),u_2(t)), \\ u_1(0) = 0,\ D_{0+}^{\beta_1}u_1(0) = 0,\ D_{0+}^{\gamma_1}u_1(1) = \int_0^1 D_{0+}^{\gamma_1}u_1(\eta)\mathrm{d}A_1(\eta), \\ u_2(0) = 0,\ D_{0+}^{\beta_2}u_2(0) = 0,\ D_{0+}^{\gamma_2}u_2(1) = \int_0^1 D_{0+}^{\gamma_2}u_2(\eta)\mathrm{d}A_2(\eta), \end{cases}$ where $f_i\colon [0,1]\times[0,\infty)\times[0,\infty)\longrightarrow[0,\infty)$ is continuous and $\alpha_i - 2 < \beta_i \le 2, \alpha_i - \gamma_i \ge 1, 2 < \alpha_i \le 3, \gamma_i \ge 1\,(i = 1,2)$. D_{0+}^{α} is the standard Riemann–Liouville's fractional derivative of order α. Our result is based on an extension of the Krasnosel'skiĭ's fixed-point theorem due to Radu Precup and Jorge Rodriguez-Lopez in 2019. The main results are explained by the help of an example in the end of the article.

1. Introduction

With the deepening of people's understanding of mathematics, the knowledge of mathematics is more and more closely related to the way of production and life of human beings. In recent years, fractional calculus is very active in the field of applied mathematics. It can be applied not only in biochemistry, mathematical physics equation, physical science experiment, and other academic fields but also in precision production [1–3].

In many recent papers are researched the fractional differential equations with the existence of the solutions [4–43]. For example, Zhang and Zhong [38] used the fixed-point theorem on cones to find the existence result of at least two positive solutions which are considered the nonlinear fractional differential equations of nonlocal boundary value problems as follows:

$$\begin{cases} D_{0+}^{\alpha}u(t) + f(t,u(t)) = 0, \quad 0 < t \le 1, \\ D_{0+}^{\beta}u(1) = \sum_{i=1}^{\infty}\xi_i D_{0+}^{\beta}u(\eta_i), \quad u(0) = 0, D_{0+}^{\beta}u(0) = 0, \end{cases} \tag{1}$$

where $2 < \alpha \le 3, 1 \le \beta \le 2, \alpha - \beta \ge 1, 0 < \xi_i, \eta_i < 1$ with $\sum_{i=1}^{\infty}\xi_i\eta_i^{\alpha-\beta-1} < 1$.

In [22], the authors obtained the uniqueness results of positive solution by the contraction map principle and obtained some existence results of positive solution through the fixed-point index theory, which is as follows:

$$\begin{cases} D_{0+}^{\alpha}u(t) + f(t,u(t)) = 0, \quad 0 < t \le 1, \\ \beta u(\eta) = u(1), \quad u(0) = 0, \end{cases} \tag{2}$$

where $1 < \alpha \le 2$, $0 < \beta\eta^{\alpha-1} < 1, 0 < \eta < 1$, D_{0+}^{α} is the standard Riemann–Liouville differentiation, and the function f is continuous on $[0,1]\times[0,\infty)$.

However, in recent years, many scholars have begun to use the fixed-point index to study the existence and multiplicity of operator equation and operator systems [44–51]. For example, in [46], the authors use the fixed-point index in cones to study the existence, localization, and multiplicity of positive solutions to operator systems of the following form:

$$\begin{cases} L_i(u_i) = F_i(u_1, u_2), \\ u_i \in D(L_i), \\ i = 1, 2, \end{cases} \quad (3)$$

where for each i, L_i: $D(L_i) \subset X \longrightarrow Y$ is invertible, F_i: $Y \times Y \longrightarrow Y$, and $X, Y \subset C[0,1]$. It should be noted that each component of the fixed-point operator systems may have the same or different behaviors.

In particular, only few papers studied the existence and multiplicity of solutions to specific differential systems [52–58]. Therefore, in this paper, we will apply an extension of the Krasnosel'skiĭ's fixed-point theorem to investigate the existence and multiplicity of solutions for a class of fractional differential systems. More precisely, the following fractional differential systems are studied:

$$\begin{cases} D_{0+}^{\alpha_1} u_1(t) = f_1(t, u_1(t), u_2(t)), \\ D_{0+}^{\alpha_2} u_2(t) = f_2(t, u_1(t), u_2(t)), \\ u_1(0) = 0, \quad D_{0+}^{\beta_1} u_1(0) = 0, D_{0+}^{\gamma_1} u_1(1) = \int_0^1 D_{0+}^{\gamma_1} u_1(\eta) dA_1(\eta), \\ u_2(0) = 0, \quad D_{0+}^{\beta_2} u_2(0) = 0, D_{0+}^{\gamma_2} u_2(1) = \int_0^1 D_{0+}^{\gamma_2} u_2(\eta) dA_2(\eta), \end{cases} \quad (4)$$

where $\alpha_i - 2 < \beta_i \le 2, \alpha_i - \gamma_i \ge 1, 2 < \alpha_i \le 3, \gamma_i \ge 1$, $f_i \in C([0,1] \times [0,\infty)^2, [0,\infty))$, and A_i are nondecreasing on $[0,1]$, left continuous at $t = 1$.

2. Preliminaries

In this part, we first give the basic definitions, lemmas, and theorems related to fractional calculus.

Definition 1 (see [2]). Define the Riemann–Liouville fractional derivative of order $\alpha > 0$ for function σ as

$$D_{0+}^{\alpha} \sigma(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_0^t \frac{\sigma(s)}{(t-s)^{\alpha+1-n}} \mathrm{d}s, \quad n = [\alpha] + 1, \quad (5)$$

where $\Gamma(\cdot)$ is the Euler gamma function.

Definition 2 (see [2]). Let σ define the Riemann–Liouville fractional integral of order $\alpha > 0$ for σ as

$$I_{0+}^{\alpha} \sigma(t) = \frac{1}{\Gamma(\alpha)} \int_0^t \sigma(s)(t-s)^{\alpha-1} \mathrm{d}s, \quad (6)$$

where $\Gamma(\cdot)$ is the Euler gamma function.

Lemma 1 (see [2]). *Let $n - 1 < \alpha \le n$ and $u \in C(0,1) \cap L^1(0,1)$; then,*

$$I_{0+}^{\alpha} D_{0+}^{\alpha} u(t) = u(t) + c_1 t^{\alpha-1} + c_2 t^{\alpha-2} + \cdots + c_1 t^{\alpha-n}, \quad (7)$$

where $c_i \in \mathbb{R}$, $i = 1, 2, \ldots$.

For convenience, we first consider the following linear fractional differential equation:

$$\begin{cases} D_{0+}^{\alpha_1} u_1(t) + \sigma(t) = 0, \quad 0 < t \le 1, \\ D_{0+}^{\gamma_1} u_1(1) = \int_0^1 D_{0+}^{\gamma_1} u_1(\eta) \mathrm{d}A_1(\eta), \\ u_1(0) = 0, \quad D_{0+}^{\beta_1} u_1(0) = 0, \end{cases} \quad (8)$$

where $\alpha_1 - 2 < \beta_1 \le 2$, $\alpha_1 - \gamma_1 \ge 1$, $2 < \alpha_1 \le 3$, $\gamma_1 \ge 1$, and $A_1(t)$ is nondecreasing on $[0,1]$, left continuous at $t = 1$.

Lemma 2. *Let $1 - \int_0^1 \eta^{\alpha_1 - \gamma_1 - 1} dA_1(\eta) > 0$ and $\sigma \in C[0,1]$; then, boundary value problem (8) has an unique solution $u_1(t) = \int_0^1 G_1(t,s)\sigma(s)ds$, where*

$$G_1(t,s) = \frac{1}{\Gamma(\alpha_1) p_1(0)} \begin{cases} t^{\alpha_1-1} p_1(s)(1-s)^{\alpha_1-\gamma_1-1}, \quad 0 \le t \le s \le 1; \\ t^{\alpha_1-1} p_1(s)(1-s)^{\alpha_1-\gamma_1-1} - p_1(0)(t-s)^{\alpha_1-1}, \quad 0 \le s \le t \le 1, \end{cases}$$
$$p_1(s) = 1 - \int_s^1 \frac{(\eta-s)^{\alpha_1-\gamma_1-1}}{(1-s)^{\alpha_1-\gamma_1-1}} \mathrm{d}A_1(\eta). \quad (9)$$

Proof. It follows from Lemma 1 that $u_1(t) = -I_{0+}^{\alpha_1}\sigma(t) + c_1 t^{\alpha_1-1} + c_2 t^{\alpha_1-2} + c_3 t^{\alpha_1-3}$. With consideration of the boundary value conditions $u_1(0) = 0$, we can get $c_3 = 0$. Consequently,

$$u_1(t) = -I_{0+}^{\alpha_1}\sigma_1(t) + c_1 t^{\alpha_1-1} + c_2 t^{\alpha_1-2}. \tag{10}$$

Notice that $D_{0+}^{\beta_1} t^{\alpha-i} = (\Gamma(\alpha-i+1)/\Gamma(\alpha-i-\beta_1+1))t^{\alpha-i-\beta_1}$ $(i = 1,2)$; we get

$$\begin{aligned} D_{0+}^{\beta_1} u_1(t) &= D_{0+}^{\beta_1}\left(-\int_0^t \frac{1}{\Gamma_1(\alpha_1)}(t-s)^{\alpha_1-1}\sigma(s)\mathrm{d}s\right) \\ &\quad + c_1\frac{\Gamma_1(\alpha_1)}{\Gamma_1(\alpha_1-\beta_1)}t^{\alpha_1-\beta_1-1} \\ &\quad + c_2\frac{\Gamma_1(\alpha_1-1)}{\Gamma_1(\alpha_1-\beta_1-1)}t^{\alpha_1-\beta_1-2}. \end{aligned} \tag{11}$$

Since $\alpha_1 - 2 < \beta_1$ and $D_{0+}^{\beta_1}u_1(0) = 0$, we conclude that $c_2 = 0$. Therefore, (10) reduces to

$$u_1(t) = -I_{0+}^{\alpha_1}\sigma_1(t) + c_1 t^{\alpha_1-1}. \tag{12}$$

Taking into account that $D_{0+}^{\gamma_1}u_1(1) = \int_0^1 D_{0+}^{\gamma_1}u_1(\eta)\mathrm{d}A_1(\eta)$ and $D_{0+}^{\gamma_1}I_{0+}^{\alpha_1}\sigma_1(t) = I_{0+}^{\alpha_1-\gamma_1}\sigma_1(t)$, we have

$$\begin{aligned} D_{0+}^{\gamma_1}u_1(1) &= -\frac{1}{\Gamma(\alpha_1-\gamma_1)}\int_0^1 (1-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s + c_1\frac{\Gamma(\alpha_1)}{\Gamma(\alpha_1-\gamma_1)} \\ &= \int_0^1 D_{0+}^{\gamma_1}u_1(\eta)\mathrm{d}A_1(\eta) \\ &= \int_0^1\left[-\frac{1}{\Gamma(\alpha_1-\gamma_1)}\int_0^\eta (\eta-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s\right. \\ &\quad \left. + c_1\frac{\Gamma(\alpha_1)}{\Gamma(\alpha_1-\gamma_1)}\eta^{\alpha_1-\gamma_1-1}\right]\mathrm{d}A_1(\eta) \\ &= -\frac{1}{\Gamma(\alpha_1-\gamma_1)}\int_0^1\int_0^\eta (\eta-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s\mathrm{d}A_1(\eta) \\ &\quad + c_1\frac{\Gamma(\alpha_1)}{\Gamma(\alpha_1-\gamma_1)}\int_0^1 \eta^{\alpha_1-\gamma_1-1}\mathrm{d}A_1(\eta). \end{aligned} \tag{13}$$

This yields

$$c_1 = \frac{\int_0^1 (1-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s - \int_0^1\int_0^\eta (\eta-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s\mathrm{d}A_1(\eta)}{\Gamma(\alpha_1)\left(1-\int_0^1 \eta^{\alpha_1-\gamma_1-1}\mathrm{d}A_1(\eta)\right)}. \tag{14}$$

Taking the above equality into (12), we have

$$\begin{aligned} u_1(t) &= -I_{0+}^{\alpha_1}\sigma(t) + c_1 t^{\alpha_1-1} \\ &= -\frac{1}{\Gamma(\alpha_1)}\int_0^t (t-s)^{\alpha_1-1}\sigma(s)\mathrm{d}s + \frac{t^{\alpha_1-1}}{\Gamma(\alpha_1)p(0)} \\ &\quad \cdot\left[\int_0^1 (1-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s\right. \\ &\quad \left. -\int_0^1\int_0^\eta (\eta-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s\mathrm{d}A_1(\eta)\right] \\ &= -\frac{1}{\Gamma(\alpha_1)}\int_0^t (t-s)^{\alpha_1-1}\sigma(s)\mathrm{d}s + \frac{t^{\alpha_1-1}}{\Gamma(\alpha_1)p(0)} \\ &\quad \cdot\left[\int_0^1 (1-s)^{\alpha_1-\gamma_1-1}\sigma(s)\mathrm{d}s\right. \\ &\quad \left. \cdot\int_0^1\int_s^1 (\eta-s)^{\alpha_1-\gamma_1-1}\mathrm{d}A_1(\eta)\sigma(s)\mathrm{d}s\right] \\ &= -\frac{1}{\Gamma(\alpha_1)}\int_0^t (t-s)^{\alpha_1-1}\sigma(s)\mathrm{d}s + \frac{t^{\alpha_1-1}}{\Gamma(\alpha_1)p(0)} \\ &\quad \cdot\int_0^1 (1-s)^{\alpha_1-\gamma_1-1}p(s)\sigma(s)\mathrm{d}s \\ &= \int_0^1 G_1(t,s)\sigma(s)\mathrm{d}s, \end{aligned} \tag{15}$$

where $G_1(t,s)$ and $p(s)$ are given in Lemma 2. □

The following proposition of Green's function $G_1(t,s)$ will be used throughout the paper.

Lemma 3. *The function* $p_1(s) > 0, s \in [0,1]$, *and* p_1 *is nondecreasing on* $[0,1]$.

Proof. After direct computation, we will get

$$\begin{aligned}
p_1'(s) &= \int_s^1 \frac{(\alpha_1-\gamma_1-1)(\eta-s)^{\alpha_1-\gamma_1-2}(1-s)^{\alpha_1-\gamma_1-1}}{(1-s)^{2(\alpha_1-\gamma_1-1)}}\mathrm{d}A_1(\eta) \\
&\quad - \int_s^1 \frac{(\eta-s)^{\alpha_1-\gamma_1-1}(1-s)^{\alpha_1-\gamma_1-2}(\alpha_1-\gamma_1-1)}{(1-s)^{2(\alpha_1-\gamma_1-1)}}\mathrm{d}A_1(\eta) \\
&= (\alpha_1-\gamma_1-1)\int_s^1 \frac{(\eta-s)^{\alpha_1-\gamma_1-2}-(\eta-s)^{\alpha_1-\gamma_1-1}(1-s)^{-1}}{(1-s)^{\alpha_1-\gamma_1-1}}\mathrm{d}A_1(\eta) \\
&= (\alpha_1-\gamma_1-1)\int_s^1 \frac{(\eta-s)^{\alpha_1-\gamma_1-2}(1-(\eta-s/1-s))}{(1-s)^{\alpha_1-\gamma_1-1}}dA_1(\eta) \\
&= (\alpha_1-\gamma_1-1)\int_s^1 \frac{(1-\eta/1-s)(\eta-s)^{\alpha_1-\gamma_1-2}}{(1-s)^{\alpha_1-\gamma_1-1}}\mathrm{d}A_1(\eta) \\
&\geq 0, s\in[0,1].
\end{aligned} \tag{16}$$

Then, we conclude that $p_1(s)$ is a nondecreasing function on $[0,1]$, which implies that $p_1(s)\geq p_1(0)=1-\int_0^1\eta^{\alpha_1-\gamma_1-1}\mathrm{d}A_1(\eta)>0$, $s\in[0,1]$. This proves the content of lemma. □

Lemma 4. *The function $G_1(t,s)$ has the following properties:*

(i) $G_1(t,s)>0$, $(\partial/\partial t)G_1(t,s)>0$, $0<t$, $s<1$

(ii) $\max_{t\in[0,1]}G_1(t,s)=G_1(1,s)$, $0\leq s\leq 1$

(iii) $G_1(t,s)\geq t^{\alpha_1-1}G_1(1,s)$, $0\leq t$, $s\leq 1$

Proof. According to Lemma 2, we have learned that Green's function G_1 is divided into two cases, and next, we will prove three properties of G_1, respectively.

(i) When $0\leq t\leq s\leq 1$,

$$G_1(t,s)=\frac{t^{\alpha_1-1}}{\Gamma(\alpha_1)p_1(0)}(1-s)^{\alpha_1-\gamma_1-1}p_1(s)>0, \tag{17}$$

then by a direct calculation, it is easy to get

$$\begin{aligned}
\frac{\partial}{\partial t}G_1(t,s) &= \frac{p_1(s)(1-s)^{\alpha_1-\gamma_1-1}}{\Gamma(\alpha_1)p_1(0)}(\alpha_1-1)t^{\alpha_1-2} \\
&\geq 0.
\end{aligned} \tag{18}$$

When $0\leq s\leq t\leq 1$,

$$\begin{aligned}
G_1(t,s) &= \frac{1}{p_1(0)\Gamma(\alpha_1)}\left[t^{\alpha_1-1}p_1(s)(1-s)^{\alpha_1-\gamma_1-1}-p_1(0)(t-s)^{\alpha_1-1}\right] \\
&\geq \frac{1}{p_1(0)\Gamma(\alpha_1)}\left[t^{\alpha_1-1}p_1(0)(1-s)^{\alpha_1-\gamma_1-1}-p_1(0)(t-s)^{\alpha_1-1}\right] \\
&= \frac{1}{p_1(0)\Gamma(\alpha_1)}t^{\alpha_1-1}\left[p_1(0)(1-s)^{\alpha_1-\gamma_1-1}-p_1(0)\left(1-\frac{s}{t}\right)^{\alpha_1-1}\right] \\
&= \frac{t^{\alpha_1-1}}{\Gamma(\alpha_1)}\left[(1-s)^{\alpha_1-\gamma_1-1}-\left(1-\frac{s}{t}\right)^{\alpha_1-1}\right] \\
&\geq 0, \\
\frac{\partial}{\partial t}G_1(t,s) &= \frac{1}{p_1(0)\Gamma(\alpha_1)}\left[(\alpha_1-1)t^{\alpha_1-2}p_1(s)(1-s)^{\alpha_1-\gamma_1-1}-(\alpha_1-1)(t-s)^{\alpha_1-2}p_1(0)\right] \\
&= \frac{\alpha_1-1}{p_1(0)\Gamma(\alpha_1)}\left[t^{\alpha_1-2}p_1(s)(1-s)^{\alpha_1-\gamma_1-1}-p_1(0)(t-s)^{\alpha_1-2}\right] \\
&= \frac{\alpha_1-1}{p_1(0)\Gamma(\alpha_1)}t^{\alpha_1-2}\left[p_1(s)(1-s)^{\alpha_1-\gamma_1-1}-p_1(0)\left(1-\frac{s}{t}\right)^{\alpha_1-2}\right] \\
&\geq \frac{\alpha_1-1}{\Gamma(\alpha_1)}t^{\alpha_1-2}\left[(1-s)^{\alpha_1-\gamma_1-1}-\left(1-\frac{s}{t}\right)^{\alpha_1-2}\right] \\
&\geq 0.
\end{aligned} \tag{19}$$

(ii) Based on the property (i), it follows that $G_1(t,s)$ is increasing with respect to t. Obviously, we have

$$\max_{t\in[0,1]} G_1(t,s) = G_1(1,s) = \frac{1}{p_1(0)\Gamma(\alpha_1)}\left[p_1(s)(1-s)^{\alpha_1-\gamma_1-1} - p_1(0)(1-s)^{\alpha_1-1}\right]. \tag{20}$$

(iii) For $(t,s)\in[0,1]\times[0,1]$, we discuss two cases.

When $0\le t\le s\le 1$, $G_1(t,s) = (t^{\alpha_1-1}/p_1(0)\Gamma(\alpha_1))(1-s)^{\alpha_1-\gamma_1-1}p_1(s)$, it is easy to get that $G_1(t,s)\ge t^{\alpha_1-1}G_1(1,s)$.

When $0\le s\le t\le 1$,

$$\begin{aligned} G_1(t,s) &= \frac{1}{p_1(0)\Gamma(\alpha_1)}\left[t^{\alpha_1-1}p_1(s)(1-s)^{\alpha_1-\gamma_1-1} - p_1(0)(t-s)^{\alpha_1-1}\right] \\ &= \frac{t^{\alpha_1-1}}{p_1(0)\Gamma(\alpha_1)}\left[p_1(s)(1-s)^{\alpha_1-\gamma_1-1} - p_1(0)\left(1-\frac{s}{t}\right)^{\alpha_1-1}\right] \\ &\ge t^{\alpha_1-1}G_1(1,s). \end{aligned} \tag{21}$$

This yields the desired result. □

The main proof of this research uses the following theorem in [46].

Theorem 1 (see [46]). *Let $(X,\|\cdot\|)$ be a Banat space, $K_1,K_2\subset X$ two cones, and $K=K_1\times K_2$ the corresponding cone of $X^2=X\times X$. Let $p_i,q_i>0$ with $p_i\ne q_i$, $U_{p_i}=\{u\in K_i\colon \|u\|<p_i\}$. Assume that $N\colon \overline{W_1\times W_2}\longrightarrow K$, $T=(T_1,T_2)$ is a compact map (where $W_i=U_{p_i}\cup U_{q_i}$ for $i=1,2$) and there exist $\varphi_i\in K_i\backslash\{0\}, i=1,2$, such that for each $i\in\{1,2\}$, the following condition is satisfied in $\overline{W_1\times W_2}$:*

(i) $\lambda x_i\ne T_i x$ for $\|x_i\|=p_i$ and $\lambda\ge 1$

(ii) $x_i\ne T_i x+\mu\varphi_i$ for $\|x_i\|=q_i$ and $\mu\ge 0$

Then,

(1) T has at least one fixed point in K such that $x_i\in U_{p_i}\backslash\overline{U_{q_i}}$ for $i=1,2$ if $p_i>q_i$ for $i=1,2$

(2) T has at least two fixed points located in $(U_{p_1}\backslash\overline{U_{q_1}})\times U_{p_2}$ and $(U_{p_1}\backslash\overline{U_{q_1}})\times(U_{q_2}\backslash\overline{U_{p_2}})$ if $q_1<p_1$ and $q_2>p_2$

(3) T has at least two fixed points located in $U_{p_1}\times(U_{p_2}\overline{U_{q_2}})$ and $(U_{q_1}\backslash\overline{U_{p_1}})\times(U_{p_2}\backslash\overline{U_{q_2}})$ if $q_1>p_1$ and $q_2<p_2$

(4) T has at least four fixed points located in $U_{p_1}\times U_{p_2}, U_{p_1}\times(U_{q_2}\backslash\overline{U_{p_2}})$, $(U_{q_1}\backslash\overline{U_{p_1}})\times U_{p_2}$, $(U_{q_2}\backslash\overline{U_{p_1}})\times(U_{q_2}\backslash\overline{U_{p_2}})$ if $p_i<q_i$ for $i=1,2$

3. Main Results

Let $X=C[0,1]$, $\|x\|_\infty=\max_{t\in[0,1]}|x(t)|$, $K=\{x\in C[0,1]\colon x(t)\ge 0\}$, and $P_i=\{x\in C[0,1]\colon x(t)\ge t^{\alpha_i-1}\|x\|_\infty\}(i=1,2)$. Then, X becomes a real Banach space with the norm $\|\cdot\|_\infty$ and K, and P_i are cones on X. Also the product space $X\times X$ is a Banach space endowed with norm $\|(x,y)\|=\max\{\|x\|_\infty,\|y\|_\infty\}$ and $P_1\times P_2$ is a cone in $X\times X$.

For convenience, we present some basic conditions as follows which we will be used later:

(H1) $f_i\in C([0,1]\times[0,+\infty)\times[0,+\infty), [0,+\infty))$ $(i=1,2)$.

(H2) There exist $r_i,\beta_i\in(0,+\infty)$ and $\delta_i\in(0,1)(i=1,2)$ such that

$f_1(t,u_1,u_2)>N_1r_1$ for $(t,u_1,u_2)\in[\delta_1,1]\times[h_1r_1,r_1]\times[0,R_2]$

$f_1(t,u_1,u_2)<M_1\beta_1$ for $(t,u_1,u_2)\in[0,1]\times[0,\beta_1]\times[0,R_2]$

$f_2(t,u_1,u_2)>N_2r_2$ for $(t,u_1,u_2)\in[\delta_2,1]\times[0,R_1]\times[h_2r_2,r_2]$

$f_2(t,u_1,u_2)<M_2\beta_2$ for $(t,u_1,u_2)\in[0,1]\times[0,R_1]\times[0,\beta_2]$

where

$$\begin{aligned} R_i &= \max\{r_i,\beta_i\}, \\ h_i &= \delta_i^{\alpha_i-1}, \\ N_i &= \left(1/h_i\int_{\delta_i}^1 G_i(1,s)\mathrm{d}s\right), \\ M_i &= \left(1/\int_0^1 G_i(1,s)\mathrm{d}s\right), \\ G_i(t,s) &= \frac{1}{p_i(0)\Gamma(\alpha_i)}\begin{cases} t^{\alpha_i-1}(1-s)^{\alpha_i-\gamma_i-1}p_i(s), & 0\le t\le s\le 1, \\ t^{\alpha_i-1}p_i(s)(1-s)^{\alpha_i-\gamma_i-1} - p_i(0)(t-s)^{\alpha_i-1}, & 0\le s\le t\le 1, \end{cases} \\ p_i(s) &= 1-\int_s^1 \frac{(\eta-s)^{\alpha_i-\gamma_i-1}}{(1-s)^{\alpha_i-\gamma_i-1}}\mathrm{d}A_i(\eta). \end{aligned} \tag{22}$$

(H3) $1-\int_0^1 \eta^{\alpha_i-\gamma_i-1}\mathrm{d}A_i(\eta)>0$, $i=1,2$.

Employing Lemma 2 and the condition of (H1), system (4) has the following integral representation:

$$\begin{cases} u_1(t)=\int_0^1 G_1(t,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s, & t\in[0,1],\\ u_2(t)=\int_0^1 G_2(t,s)f_2(s,u_1(s),u_2(s))\mathrm{d}s, & t\in[0,1]. \end{cases} \tag{23}$$

Let us define two operators T_i: $K\times K\longrightarrow X\,(i=1,2)$ as follows:

$$\begin{aligned} T_1(u_1,u_2)(t)&=\int_0^1 G_1(t,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s, \quad t\in[0,1],\\ T_2(u_1,u_2)(t)&==\int_0^1 G_2(t,s)f_2(s,u_1(s),u_2(s))\mathrm{d}s, \quad t\in[0,1]. \end{aligned} \tag{24}$$

Then, we can define an operator T: $K\times K\longrightarrow X\times X$ as follows:

$$T(u_1,u_2)=(T_1(u_1,u_2),T_2(u_1,u_2)), \quad (u_1,u_2)\in K\times K. \tag{25}$$

Lemma 5. *Assume that (H1), (H2), and (H3) hold. Then, T: $K\times K\longrightarrow P_1\times P_2$ is completely continuous.*

Proof. Firstly, we prove that T: $K\times K\longrightarrow P_1\times P_2$. In fact, for $(u_1,u_2)\in K\times K$, by (H1), it is obvious that $T_i(u_1,u_2)(t)\ge 0$ for $i=1,2$ and $t\in[0,1]$. In addition, if $(u_1,u_2)\in K\times K$, then

$$\begin{aligned} T_i(u_1,u_2)(t)&=\int_0^1 G_i(t,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s\\ &\le\int_0^1 G_i(1,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s,\\ &t\in[0,1]. \end{aligned} \tag{26}$$

So,

$$\|T_i(u_1,u_2)\|_\infty\le\int_0^1 G_i(1,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s. \tag{27}$$

On the other hand, for any $(u_1,u_2)\in K\times K$ and any $t\in[0,1]$, it follows from Lemma 4 that

$$\begin{aligned} T_i(u_1,u_2)(t)&=\int_0^1 G_1(t,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s\\ &\ge t^{\alpha_i-1}\int_0^1 G_1(1,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s\\ &\ge t^{\alpha_i-1}\|T_1(u_1,u_2)\|_\infty. \end{aligned} \tag{28}$$

Thus, from the above discussion, we conclude that T: $K\times K\longrightarrow P_1\times P_2$, and then, it obviously shows that T is well defined. The complete continuity of operator T can be given by a standard argument with the help of the Arzela–Arscoli Theorem. We omit the details. □

Theorem 2. *Assume that (H1), (H2), and (H3) hold. Then, we have*

(i) If $r_1<\beta_1$ and $r_2<\beta_2$, then (23) has at least a positive solution located in $(U_{\beta_1}\backslash\overline{U_{r_1}})\times(U_{\beta_2}\backslash\overline{U_{r_2}})$, where $U_{r_i}=\{u\in P_i\colon \|u\|_\infty<r_i\}$

(ii) If $r_1<\beta_1$ and $r_2>\beta_2$, then (23) has at least two positive solutions located in $(U_{\beta_1}\backslash\overline{U_{r_1}})\times U_{\beta_2}$ and $(U_{\beta_1}\backslash\overline{U_{r_1}})\times(U_{r_2}\backslash\overline{U_{\beta_2}})$

(iii) If $r_1>\beta_1$ and $r_2<\beta_2$, then (23) has at least two positive solutions located in $U_{\beta_1}\times(U_{\beta_2}\backslash\overline{U_{r_2}})$ and $(U_{r_1}\backslash\overline{U_{\beta_1}})\times(U_{\beta_2}\backslash\overline{U_{r_2}})$

(iv) If $r_1>\beta_1$ and $r_2>\beta_2$, then (23) has at least three positive solutions located in $U_{\beta_1}\times U_{\beta_2}$, $U_{\beta_1}\times(U_{r_2}\backslash\overline{U_{\beta_2}})$, $(U_{r_1}\backslash\overline{U_{\beta_1}})\times U_{\beta_2}$, $(U_{r_2}\overline{U_{\beta_1}})\times(U_{r_2}\backslash\,\overline{U_{\beta_2}})$

Proof. It follows from Lemma 5 that the existence of a positive solution of problem (23) is equivalent to the existence of a nontrivial fixed point of T in $P_1\times P_2$. Let $W_i=\{u\in P_i\colon \|u\|_\infty<R_i\}$.

First, note that if $u=(u_1,u_2)\in\overline{W_1\times W_2}$ with $\|u_1\|_\infty=r_1$, then $\|u_2\|_\infty\le R_2$ and by the definition of P_1,

$$r_1t^{\alpha_1-1}\le u_1(t)\le r_1, \quad 0\le u_2(t)\le R_2, t\in[0,1]. \tag{29}$$

In the following, we conclude that for $i\in\{1,2\}$, the following properties hold:

$$\begin{aligned} &\lambda u_i\ne T_iu, \quad \text{for} \quad \|u_i\|_\infty=\beta_i,\lambda\ge 1;\\ &u_i\ne T_iu+\mu t^{\alpha_i-1}, \quad \text{for} \quad \|u_i\|_\infty=r_i,\mu\ge 0, \end{aligned} \tag{30}$$

guaranteeing the validity of Theorem 1.

In fact, if $\|u_1\|_\infty=\beta_1$ and $\lambda u_1=T_1u$ for a $\lambda_1\ge 1$, then by (H2),

$$\begin{aligned} u_1(t)\le\lambda u_1(t)=(T_1u_1)(t)&\le\int_0^1 G_1(1,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s\\ &<M_1\beta_1\int_0^1 G_1(1,s)\mathrm{d}s=\beta_1,\\ &t\in[0,1], \end{aligned} \tag{31}$$

whence, in particular, we conclude $\beta_1<\beta_1$, a contradiction. Now, if $u_1=T_1u+\mu_1t^{\alpha_1-1}$ for $\|u_1\|_\infty=r_1$ and $\mu_1\ge 0$, then by (H2), we obtain

$$\begin{aligned} u_1(t)=(T_1u)(t)+\mu_1t^{\alpha_1-1}&\ge\int_{\delta_1}^1 G_1(t,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s\\ &\ge\delta_1^{\alpha_1-1}\int_{\delta_1}^1 G_1(1,s)f_1(s,u_1(s),u_2(s))\mathrm{d}s\\ &>\delta_1^{\alpha_1-1}\int_{\delta_1}^1 G_1(1,s)N_1r_1\mathrm{d}s=r_1, \end{aligned} \tag{32}$$

for all $t\in[0,1]$. This yields the contradiction $r_1<r_1$. Hence, (30) holds for $i=1$. Similarly, (30) is true for $i=2$. □

Example 1. Consider the following integral boundary value problem of fractional differential systems:

$$\begin{cases} D_{0+}^{11/4}u_1(t) = \sqrt{u_1(t)}\arctan(u_1(t)+1)\left(\dfrac{1}{(u_2(t)+1)^2}+100\right), \\ D_{0+}^{5/2}u_2(t) = \dfrac{1}{2}\sin t + \dfrac{1}{2}\cos u_1(t)u_2(t) + u_2(t) + 3, \\ u_1(0) = 0, \quad D_{0+}^{3/4}u_1(0) = 0, D_{0+}^{5/4}u_1(1) = \displaystyle\int_0^1 D_{0+}^{5/4}u_1(\tau)\mathrm{d}\tau, \\ u_2(0) = 0, \quad D_{0+}^{1/2}u_2(0) = 0, D_{0+}^{3/2}u_2(1) = \dfrac{1}{2}D_{0+}^{3/2}u_2\left(\dfrac{1}{2}\right). \end{cases} \tag{33}$$

Then, (33) has at least two positive solutions (u_1, v_1) and (u_2, v_2) with $(u_1, v_1) \in (U_{100}\backslash\overline{U_{0.72}}) \times U_5$ and $(u_2, v_2) \in (U_{100}\backslash\overline{U_{0.72}}) \times (U_{7100}\backslash\overline{U_5})$.

To see this, we will apply Theorem 2 with

$$\begin{aligned} \alpha_1 &= \frac{11}{4}, \\ \gamma_1 &= \frac{5}{4}, \\ \alpha_2 &= \frac{5}{2}, \\ \gamma_2 &= \frac{3}{2}, \\ f_1(t, u_1, u_2) &= \sqrt{u_1}\arctan(u_1+1)\left(\frac{1}{(u_2+1)^2}+100\right), \\ f_2(t, u_1, u_2) &= \frac{1}{2}\sin t + \frac{1}{2}\cos u_1u_2 + \frac{u_2^2}{10} \\ A_1(t) &= t, \\ A_2(t) &= \begin{cases} \dfrac{1}{2}, & t \le \dfrac{1}{2}, \\ 1, & \dfrac{1}{2} < t \le 1. \end{cases} \end{aligned} \tag{34}$$

Clearly,

$$\begin{aligned} 1 - \int_0^1 t^{\alpha_1-\gamma_1-1}\mathrm{d}A_1(t) &= 1 - \int_0^1 t^{1/2}\mathrm{d}t = \frac{1}{3} > 0, \\ 1 - \int_0^1 t^{\alpha_2-\gamma_2-1}\mathrm{d}A_2(t) &= 1 - \frac{1}{2} = \frac{1}{2} > 0. \end{aligned} \tag{35}$$

Thus, (H3) holds.

Take

$$\begin{aligned} G_1(t,s) &= \frac{1}{\Gamma(11/4)}\begin{cases} t^{7/4}(1-s)^{1/2}(1+2s), & 0 \le t \le s \le 1, \\ t^{7/4}(1-s)^{1/2}(1+2s) - (t-s)^{7/4}, & 0 \le s \le t \le 1, \end{cases} \\ G_2(t,s) &= \frac{1}{\Gamma(5/2)}\begin{cases} 2t^{3/2}p_2(s), & 0 \le t \le s \le 1, \\ 2t^{3/2}p_2(s) - (t-s)^{3/2}, & 0 \le s \le t \le 1, \end{cases} \\ p_1(s) &= \frac{1+2s}{3}, \\ p_2(s) &= \begin{cases} \dfrac{1}{2}, & s \le \dfrac{1}{2}, \\ 1, & \dfrac{1}{2} < s \le 1. \end{cases} \end{aligned} \tag{36}$$

Let $\delta_1 = \delta_2 = (1/4)$. By simple computation, we have

$$\begin{aligned} h_1 &\approx 0.0884, \\ h_2 &= 0.125, \\ M_1 &\approx 1.9231, \\ M_2 &\approx 1.2085, \\ N_1 &\approx 24.4377, \\ N_2 &\approx 10.0794. \end{aligned} \tag{37}$$

We choose $r_1 = 0.72$, $\beta_1 = 100$, $r_2 = 7100$, and $\beta_2 = 5$. Then, $R_1 = 100$, $R_2 = 7100$,

$$\begin{aligned} &f_1(t,u_1,u_2) > 25\pi\sqrt{h_1r_1} > N_1r_1, \\ &\qquad (t,u_1,u_2) \in [\delta_1, 1] \times [h_1r_1, r_1] \times [0, R_2], \\ &f_1(t,u_1,u_2) < \frac{101\pi}{2}\sqrt{\beta_1} < M_1\beta_1, \\ &\qquad (t,u_1,u_2) \in [0,1] \times [0,\beta_1] \times [0,R_2], \\ &f_2(t,u_1,u_2) > \frac{3}{2} + \frac{h_2^2r_2^2}{10} > N_2r_2, \\ &\qquad (t,u_1,u_2) \in [\delta_2, 1] \times [0, R_1] \times [h_2r_2, r_2], \\ &f_2(t,u_1,u_2) < 3 + \frac{\beta_2^2}{10} < M_2\beta_2, \\ &\qquad (t,u_1,u_2) \in [0,1] \times [0,R_1] \times [0,\beta_2]. \end{aligned} \tag{38}$$

Consequently, (H2) holds with $r_1 < \beta_1$ and $r_2 > \beta_2$, and our conclusion follows from Theorem 2.

4. Conclusions

In this paper, we investigate the existence and multiplicity of positive solutions for the integral boundary value problem of higher-order fractional differential systems. This result is based on an extension of the Krasnosel'skiĭ's fixed-point theorem due to Radu Precup and Jorge Rodriguez-Lopez in [46]. We rewrite the original fractional differential systems as equivalent fractional integral systems. With the help of properties of Green's function, we obtain some sufficient conditions of existence and multiplicity of positive solutions. Finally, an example is presented to illustrate the effectiveness of the main result. The interesting point is that the integral boundary condition is dependent on the lower-order fractional derivative.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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