

Research Article

Finite-Time Synchronization of Complex Dynamical Networks with Nondelayed and Delayed Coupling by Continuous Function Controller

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This paper investigates the finite-time synchronization of complex dynamical networks with nondelayed and delayed coupling. By designing a simple continuous function controller, sufficient criteria for finite-time synchronization of dynamical networks with nondelayed and delayed coupling are obtained. As a special case, the continuous function controller designed in this paper may be the simplest and easy to implement for the finite-time synchronization of dynamical networks without delay. Finally, numerical simulations are given to verify the effectiveness of the conclusions presented in this paper.

1. Introduction

As synchronization behavior of complex systems is a common phenomenon in the field of nature and engineering technology, synchronization control of complex systems has been attracted people to study [1–8]. However, in many fields, people begin to realize that finite-time control of complex systems may be more practical than infinite-time control of complex systems [9–14]. For example, when the brain is stimulated by external signals, the brain's nervous system will respond accordingly, and the response output signals of the central nervous system and the sensory nervous system should be consistent. That is to say, the driving neural network and the response neural network move under different starting conditions. By properly controlling the neural network, the trajectories of the two neural networks will coincide in a finite-time, which is the finite-time control of the neural dynamic network. Since finite-time of

system control mainly depends on the initial state of complex systems, the initial state of complex systems may be random, which is not conducive to solving practical problems. Subsequently, Polyakov proposed a finite-time control method independent of the initial state of complex systems, that is, so-called fixed-time control of complex systems, which compensated for the limitation of the finite-time control method [15, 16]. Therefore, studying the finite-time control problem of complex networks, on the one hand, it expands synchronization control theory of complex networks; on the other hand, the finite-time control method may save the control cost better and may have greater practical value in the stability control application of complex systems.

At present, based on the research of finite-time control in complex networks, the authors have obtained some interesting results, such as finite-time control via optimal method [17], finite-time control via intermittent

feedback [18], finite-time control via linear and nonlinear control method [19], finite-time control with nonlinear dynamics and uncertainties [20], finite-time control with switched discontinuous systems, and so on [21]. However, in these discussions, the designed controller usually contains symbolic functions. As the designed controller contains symbolic function, which is a discontinuous function, a flutter phenomenon may occur during the control process. The occurrence of the flutter phenomenon will obviously affect the finite-time control effect of complex networks. In order to eliminate the flutter phenomenon, some authors have proposed finite-time controller with continuous functions. For example, in [22], the authors effectively synchronize two complex networks by designing continuous function controllers, eliminating the adverse effects caused by symbolic function controllers, that is, the controller is $r_i = -l_i w_i(t) - k_1 w_i^{m_1/m_2}(t) - k_2 w_i^{m_3/m_4}(t)$, where $l_i > 0$, $k_1 > 0$, $k_2 > 0$, m_1, m_2, m_3 , and m_4 are positive odd integers satisfying, $m_1 > m_2, m_3 < m_4$, and w_i means the error. In [23], a simpler continuous function controller for fixed-time synchronization of complex networks with nondelayed coupling was proposed, that is, the controller is $r_i = -k_1 w_i^{m_1/m_2}(t) - k_2 w_i^{m_3/m_4}(t)$. Therefore, designing simple and effective finite-time controllers is still an interesting topic.

In addition, in the network, time-delay is inevitable. For example, in the process of transmission and response, the physical limitations of propagation speed and the existence of network congestion often lead to delay in complex networks. The generation of time-delay may affect the stability of the whole network, such as the oscillation and instability of the system. Therefore, it is undoubtedly of great application value and theoretical significance to study the synchronization control of the time-delay coupled network. Although the finite-time synchronization of time-delay complex networks has been discussed in the existing literatures [24, 25], finite-time controllers designed in their papers were generally complex and contained symbolic functions, which may cause chattering during network synchronization. In general, an important criterion for studying finite-time control of complex systems is that the controller is as simple as possible and easy to implement. As far as we know, finite-time synchronization of dynamical networks with delayed coupling was less considered by using a simple continuous function controller. In this paper, we attempt to design a continuous finite-time controller to realize finite-time control of complex systems with delayed, and we also make the effort to propose the simplest finite-time continuous function controller for complex networks with nondelayed coupling.

This paper is organized as follows: Section 2 gives the model and preliminaries. Section 3 puts forward some finite-time synchronization criteria for networks. Section 4

gives an illustrative example. The conclusions are given in Section 5.

2. Model and Preliminaries

Consider the following dynamical network with delay coupling:

$$\dot{\mu}_i(t) = g(\mu_i(t)) + \sum_{j=1}^N b_{ij} \mu_j(t) + \sum_{j=1}^N c_{ij} \mu_j(t - \tau), \quad (1)$$

where $\mu_i(t) = (\mu_{i1}(t), \mu_{i2}(t), \dots, \mu_{in}(t))^T \in R^n$ is the state vector, $i = 1, 2, \dots, N$, N denotes natural number, and $g(\cdot)$ is the activation function. $B = (b_{ij})_{n \times n}$ and $C = (c_{ij})_{n \times n}$ denote connection weight matrix. τ means time-delay.

The controlled dynamic network with delay coupling is represented as follows:

$$\dot{\vartheta}_i(t) = g(\vartheta_i(t)) + \sum_{j=1}^N b_{ij} \vartheta_j(t) + \sum_{j=1}^N c_{ij} \vartheta_j(t - \tau) + r_i, \quad (2)$$

where $\vartheta_i(t) = (\vartheta_{i1}(t), \vartheta_{i2}(t), \dots, \vartheta_{in}(t))^T \in R^n$ is the state vector and $i = 1, 2, \dots, N$, r_i means the controller.

Assumed the errors are expressed as $w_i = \vartheta_i - \mu_i$, then

$$\dot{w}_i(t) = g(\vartheta_i(t)) - g(\mu_i(t)) + \sum_{j=1}^N b_{ij} w_j(t) + \sum_{j=1}^N c_{ij} w_j(t - \tau) + r_i. \quad (3)$$

Assumption 1. Assume the function $g(\cdot)$ satisfies the following condition:

$$|g(\vartheta_i(t)) - g(\mu_i(t))| \leq \gamma |\vartheta_i(t) - \mu_i(t)|, \quad \gamma \in R^+. \quad (4)$$

Lemma 1 (see [22]). *For any vectors $\mu, \vartheta \in R^m$ and positive definite matrix $\Omega \in R^{m \times m}$, the following matrix inequality holds:*

$$2\mu^T \vartheta \leq \mu^T \Omega \mu + \vartheta^T \Omega^{-1} \vartheta. \quad (5)$$

Lemma 2 (see [24]). *Consider the dynamical system as follows:*

$$\begin{aligned} \dot{\mu} &= g(\mu(t)), \\ g(0) &= 0, \\ \mu &\in R^n, \\ \mu(0) &= \mu_0. \end{aligned} \quad (6)$$

Suppose a continuous and positive definite $V(\mu)$ satisfies

$$\begin{aligned} \dot{v}(\mu) &\leq l_1 v(\mu) - l_2 v^\xi(\mu), \\ v^{1-\xi}(\mu) &< \frac{l_2}{l_1}, \end{aligned} \tag{7}$$

$$l_1 > 0, l_2 > 0, 0 < \xi < 1.$$

Then, the origin of the dynamical system (6) is finite-time stable, and

$$T_\mu(\mu_0) \leq \frac{1}{l_1(\xi-1)} \ln\left(1 - \frac{l_1}{l_2} v^{1-\xi}(\mu_0)\right). \tag{8}$$

Lemma 3 (see [26]). For $\Theta_i \geq 0, i = 1, 2, \dots, n, 0 < \lambda \leq 1, \kappa > 1$, then

$$\begin{aligned} \sum_{i=1}^n \Theta_i^\lambda &\geq \left(\sum_{i=1}^n \Theta_i\right)^\lambda, \\ \sum_{i=1}^n \Theta_i^\kappa &\geq n^{1-\kappa} \left(\sum_{i=1}^n \Theta_i\right)^\kappa. \end{aligned} \tag{9}$$

3. Main Results

Based on the preparation of the second section, the section will discuss the finite-time synchronization of two complex networks.

Theorem 1. If Assumption 1 holds, two complex dynamical networks (1) and (2) can be synchronized by the following simpler finite-time controller:

$$r_i = -\frac{k w_i^T(t-\tau) w_i(t-\tau)}{\|w_i(t)\|^2} w_i(t) - h_i w_i^\xi(t), \tag{10}$$

and

$$T \leq \frac{1}{l(\xi-1)} \ln\left(1 - \frac{2^{(1-\xi)/2} l}{\lambda_{\min}(H \otimes I)} v^{(1-\xi)/2}(w(0))\right), \tag{11}$$

where $k = (N/2)\lambda_{\max}(\Omega^{-1}), l = \gamma + \lambda_{\max}(B \otimes I) + (N\delta^2/2)\lambda_{\max}(\Omega \otimes I), H = \text{diag}\{h_1, \dots, h_N\}, h_i > 0, \xi = n_1/n_2, n_1$ and n_2 are positive odd integers, $n_1 < n_2$, and Ω is the positive definite matrix.

Proof. Consider the following function:

$$v(t) = \frac{1}{2} w^T(t) w(t), \tag{12}$$

where $w = (w_1^T, w_2^T, \dots, w_n^T)^T$.

So,

$$\begin{aligned} \dot{v}(t) &= \sum_{i=1}^N w_i^T(t) \left(g(\vartheta_i(t)) - g(\mu_i(t)) + \sum_{j=1}^N b_{ij} w_j(t) + \sum_{j=1}^N c_{ij} w_j(t-\tau) + r_i \right) \\ &\leq \sum_{i=1}^N \gamma w_i^T(t) w_i(t) + \sum_{i=1}^N w_i^T(t) \sum_{j=1}^N b_{ij} w_j(t) + \sum_{i=1}^N w_i^T(t) \sum_{j=1}^N c_{ij} w_j(t-\tau) - \sum_{i=1}^N w_i^T(t) \frac{k w_i^T(t-\tau) w_i(t-\tau)}{\|w_i(t)\|^2} w_i(t) \\ &\quad - \lambda_{\min}(H \otimes I) w^T(t) w^\xi(t). \end{aligned} \tag{13}$$

In equation (13), obviously,

$$\begin{aligned} w^T(t) w^\xi(t) &= \sum_{i=1}^N \sum_{j=1}^n (w_{ij}^2(t))^{(1+\xi)/2} \geq \sum_{i=1}^N \left(\sum_{j=1}^n w_{ij}^2(t) \right)^{(1+\xi)/2} \geq (w^T(t) w(t))^{(1+\xi)/2} = (2)^{(1+\xi)/2} (v(t))^{(1+\xi)/2} \\ \sum_{i=1}^N w_i^T(t) \sum_{j=1}^N c_{ij} w_j(t-\tau) &\leq \frac{N\delta^2}{2} \sum_{i=1}^N [w_i(t)]^T \Omega w_i(t) + \frac{N}{2} \sum_{j=1}^N (w_j(t-\tau))^T \Omega^{-1} w_j(t-\tau), \end{aligned} \tag{14}$$

where $|c_{ij}| \leq \delta \in R^+$.

So,

$$\begin{aligned}
\dot{v}(t) &\leq \sum_{i=1}^N \gamma w_i^T(t) w_i(t) + \sum_{i=1}^N w_i^T(t) \sum_{j=1}^N b_{ij} w_j(t) + \frac{N\delta^2}{2} \sum_{i=1}^N [w_i(t)]^T \Omega w_i(t) + \frac{N}{2} \sum_{j=1}^N (w_j(t-\tau))^T \Omega^{-1} w_j(t-\tau) \\
&\quad - \sum_{i=1}^N \frac{k w_i^T(t-\tau) w_i(t-\tau)}{\|w_i(t)\|^2} w_i^T(t) w_i(t) - \lambda_{\min}(H \otimes I) (2)^{(1+\xi)/2} (v(t))^{(1+\xi)/2} \\
&\leq \sum_{i=1}^N \gamma w_i^T(t) w_i(t) + \sum_{i=1}^N w_i^T(t) \sum_{j=1}^N b_{ij} w_j(t) + \frac{N\delta^2}{2} \sum_{i=1}^N [w_i(t)]^T \Omega w_i(t) + \frac{N}{2} \lambda_{\max}(\Omega^{-1}) \sum_{j=1}^N (w_j(t-\tau))^T w_j(t-\tau) \\
&\quad - k \sum_{i=1}^N w_i^T(t-\tau) w_i(t-\tau) - \lambda_{\min}(H \otimes I) (2)^{(1+\xi)/2} (v(t))^{(1+\xi)/2} \\
&\leq \sum_{i=1}^N \gamma w_i^T(t) w_i(t) + \sum_{i=1}^N w_i^T(t) \sum_{j=1}^N b_{ij} w_j(t) + \frac{N\delta^2}{2} \sum_{i=1}^N [w_i(t)]^T \Omega w_i(t) + \left(\frac{N}{2} \lambda_{\max}(\Omega^{-1}) - k\right) \sum_{j=1}^N (w_j(t-\tau))^T w_j(t-\tau) \\
&\quad - \sum_{i=1}^N l w_i^T(t) w_i(t) + \sum_{i=1}^N l w_i^T(t) w_i(t) - \lambda_{\min}(H \otimes I) (2)^{(1+\xi)/2} (v(t))^{(1+\xi)/2}.
\end{aligned} \tag{15}$$

When $k = (N/2)\lambda_{\max}(\Omega^{-1})$, $l = \gamma + \lambda_{\max}(B \otimes I) + (N\delta^2)/2\lambda_{\max}(\Omega \otimes I)$, we have

$$\begin{aligned}
\dot{v}(t) &\leq \sum_{i=1}^N l w_i^T(t) w_i(t) - \lambda_{\min}(H \otimes I) (2)^{(1+\xi)/2} (v(t))^{(1+\xi)/2} \\
&= 2lv(t) - \lambda_{\min}(H \otimes I) (2)^{(1+\xi)/2} (v(t))^{(1+\xi)/2}.
\end{aligned} \tag{16}$$

Then, the origin of the error systems (3) is finite-time stable, and T satisfies

$$T \leq \frac{1}{l(\xi-1)} \ln \left(1 - \frac{2^{(1-\xi)/2} l}{\lambda_{\min}(H \otimes I)} v^{(1-\xi)/2}(w(0)) \right). \tag{17}$$

The proof is completed. \square

Remark 1. At present, few continuous function controllers are designed for finite-time synchronization of delay dynamical networks. Therefore, the continuous function finite-time controller in this paper may be more practical.

Remark 2. From the proof of Theorem 1, when the delay is time-varying, Theorem 1 still holds.

Remark 3. If $c_{ij} = 0$, the coupled dynamical networks (1)-(2) can reduce to complex dynamical networks with the non-delayed coupling dynamical network. In this case, the finite-time controller (10) is not available. In this case, since the network does not contain delay, and the following Corollary 1 can be obtained.

Remark 4. The matrix Ω is a positive definite matrix given arbitrarily. To guarantee the optimality of result, the matrix Ω should have minimum eigenvalue. As the convergence time is related to the matrix Ω , the control strength k is also related to the matrix Ω . From their expressions, we can see that the smaller the eigenvalue of the matrix Ω , the shorter the convergence time, but the greater the control strength k , and how to choose the matrix Ω to guarantee the optimality of result will be the topic for further consideration in the future.

When $C = 0$, the complex networks (1)-(2) changed into the following complex dynamical networks with nondelayed coupling:

$$\dot{\mu}_i(t) = g(\mu_i(t)) + \sum_{j=1}^N b_{ij} \mu_j(t), \tag{18}$$

$$\dot{\vartheta}_i(t) = g(\vartheta_i(t)) + \sum_{j=1}^N b_{ij} \vartheta_j(t) + r_i. \tag{19}$$

Corollary 1. *If Assumption 1 holds, two complex dynamical networks with nondelayed coupling (18) and (19) can be synchronized by the simplest finite-time controller:*

$$r_i = -h_i w_i^\xi(t), \tag{20}$$

and the time is bounded as

$$T \leq \frac{1}{l(\xi-1)} \ln \left(1 - \frac{2^{(1-\xi)/2} l}{\lambda_{\min}(H \otimes I)} v^{(1-\xi)/2}(w(0)) \right), \tag{21}$$

where $l = \gamma + \lambda_{\max}(B \otimes I)$, $\xi = n_1/n_2$, and n_1 and n_2 are positive odd integers, $n_1 < n_2$.

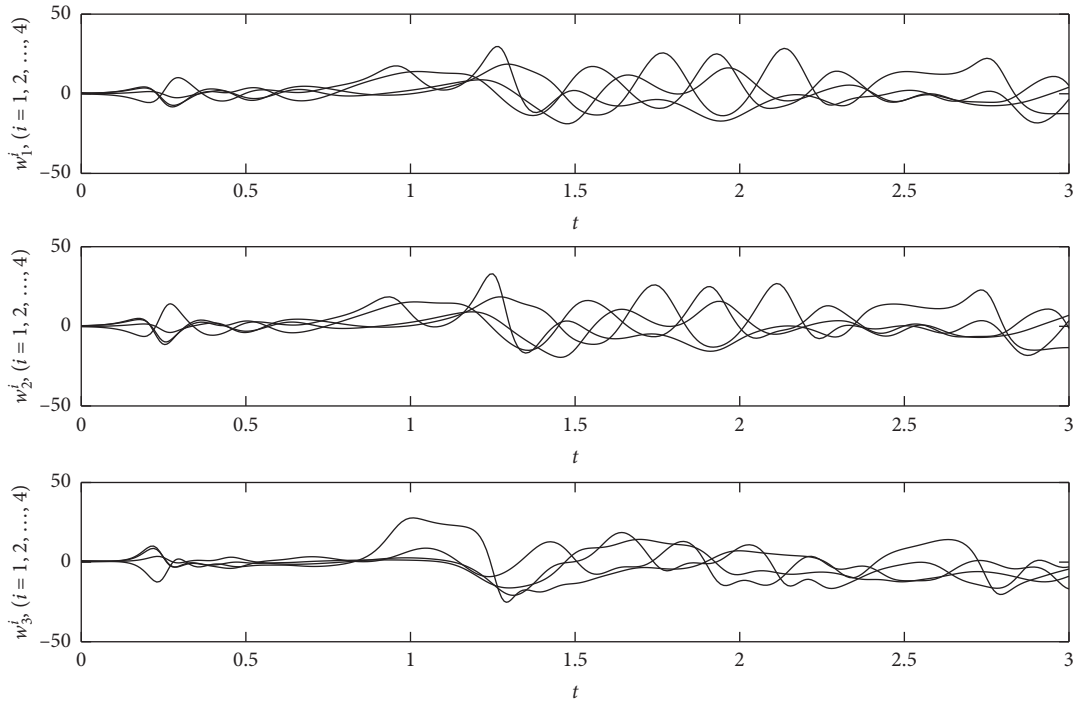


FIGURE 1: Error evolution of two networks.

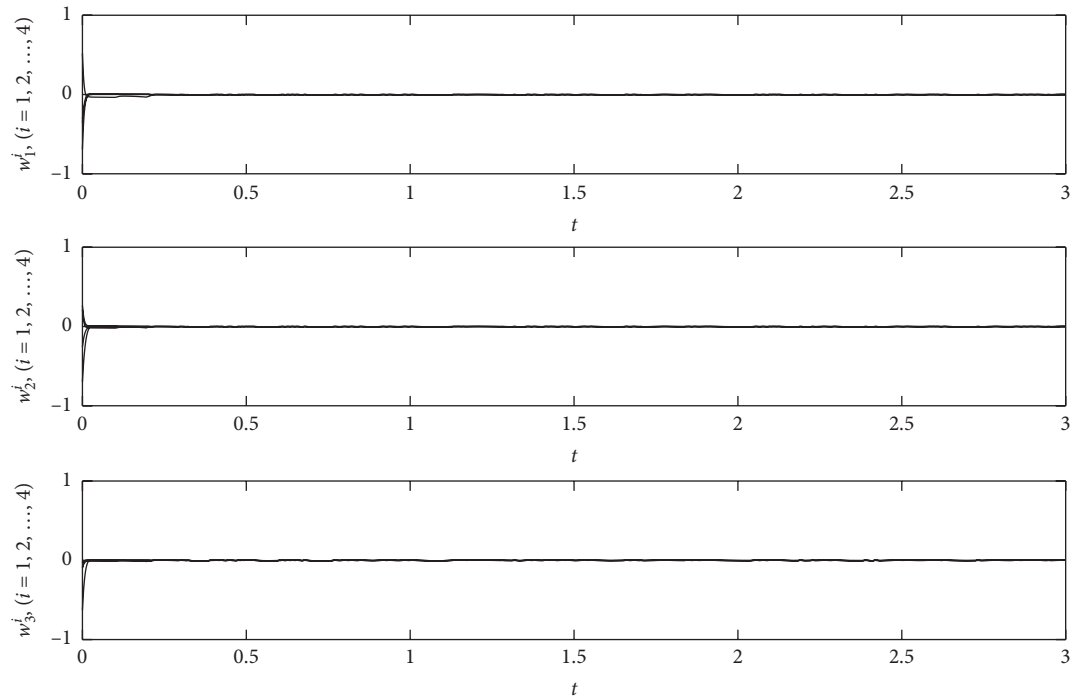


FIGURE 2: Synchronization evolution of two networks.

The proof of Corollary 1 is similar to that of Theorem 1, so we omit their reasoning process here.

Remark 5. Compared with the existing literature [15–18] on finite-time synchronization of complex networks without delay, the controller (20) designed in this paper

may be the simplest one. Generally speaking, simple controllers may have better application in practical engineering.

Remark 6. From (11) and (21), synchronization time of complex networks is the same, that is to say, the delay term

of complex networks does not affect the synchronization time of complex networks.

Remark 7. In [9, 10, 15, 24, 25], the authors proposed continuous function controllers without symbolic functions, but the synchronization of delay networks was not discussed in their paper.

4. Numerical Simulation Example

Considering the following chaos system [27],

$$\dot{\vartheta}_i = \begin{pmatrix} -10 & 10 & 0 \\ 28 & -1 & 0 \\ 0 & 0 & -8/3 \end{pmatrix} \begin{pmatrix} \vartheta_{i1} \\ \vartheta_{i2} \\ \vartheta_{i3} \end{pmatrix} + \begin{pmatrix} 0 \\ -\vartheta_{i1}\vartheta_{i3} \\ \vartheta_{i1}\vartheta_{i2} \end{pmatrix}. \quad (22)$$

It is well known that the chaos system is bounded, and our analyses show

$$\|g(\vartheta_i) - g(\mu_i)\|_2 \leq 100.5571 \|w_i\|_2. \quad (23)$$

If we use four nodes to describe the coupled complex network, then

$$\dot{\mu}_i(t) = g(\mu_i(t)) + \sum_{j=1}^4 b_{ij}\mu_j(t), \quad (24)$$

$$(b_{ij})_{4 \times 4} = \begin{pmatrix} -4 & 1 & 2 & 1 \\ 1 & -6 & 3 & 2 \\ 2 & 3 & -6 & 1 \\ 1 & 2 & 1 & -4 \end{pmatrix}.$$

Let initial values of the state variable are rand [0, 1] and $\gamma = 115$, $\xi = 0.8$, and $h_i = 200$. Figure 1 shows that the driver network and the response network cannot be synchronized without a controller. Figure 2 shows finite-time synchronization of two networks under the controller. Numerical simulations illustrate the validity of Corollary 1.

5. Conclusion

The paper has studied synchronization of two complex dynamical networks with nondelayed and delayed coupling by the finite-time controller. The designed finite-time controller was continuous function. Especially, the continuous function controller designed might be the simplest for finite-time synchronization of complex dynamical networks without delay. Finally, the numerical simulation verified the validity of the theoretical results. As the results are sufficient conditions in this paper, which implies that there is still room for further improvement, which will be our next research topic.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

- [1] J. Lu, L. Sun, Y. Liu, D. W. C. Ho, and J. Cao, ‘‘Stabilization of Boolean control networks under aperiodic sampled-data control,’’ *SIAM Journal on Control and Optimization*, vol. 56, no. 6, pp. 4385–4404, 2018.
- [2] B. Li, J. Lu, J. Zhong, and Y. Liu, ‘‘Fast-time stability of temporal Boolean networks,’’ *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 8, pp. 2285–2294, 2019.
- [3] Y. Wang, Y. Wei, X. Liu, N. Zhou, and C. Cassandras, ‘‘Optimal persistent monitoring using second-order agents with physical constraints,’’ *IEEE Transactions on Automatic Control*, vol. 64, no. 8, pp. 3239–3252, 2019.
- [4] Y. Wang, Y. Lei, T. Bian, and Z. Guan, ‘‘Distributed control of nonlinear multi-agent systems with unknown and nonidentical control directions via event-triggered communication,’’ *IEEE Transactions on Cybernetics*, 2019.
- [5] H. Shen, S. Huo, J. Cao, and T. Huang, ‘‘Generalized state estimation for markovian coupled networks under round-robin protocol and redundant channels,’’ *IEEE Transactions on Cybernetics*, vol. 49, no. 4, pp. 1292–1301, 2019.
- [6] H. Shen, T. Wang, J. Cao, G. Lu, Y. Song, and T. Huang, ‘‘Nonfragile dissipative synchronization for markovian memristive neural networks: a gain-scheduled control scheme,’’ *IEEE Transactions on Neural Networks and Learning Systems*, vol. 30, no. 6, pp. 1841–1853, 2019.
- [7] Y. Tang, H. Gao, W. Zhang, and J. Kurths, ‘‘Leader-following consensus of a class of stochastic delayed multi-agent systems with partial mixed impulses,’’ *Automatica*, vol. 53, pp. 346–354, 2015.
- [8] W. Zhang, Q.-L. Han, Y. Tang, and Y. Liu, ‘‘Sampled-data control for a class of linear time-varying systems,’’ *Automatica*, vol. 103, pp. 126–134, 2019.
- [9] J. Wang, D. Wang, H. Wu, and T. Huang, ‘‘Finite-time passivity and synchronization of complex dynamical networks with state and derivative coupling,’’ *IEEE Transactions on Cybernetics*, 2019.
- [10] J. Wang, X. Zhang, H. Wu, T. Huang, and Q. Wang, ‘‘Finite-time passivity of adaptive coupled neural networks with undirected and directed topologies,’’ *IEEE Transactions on Cybernetics*, 2018.
- [11] Z. Tang, J. H. Park, Y. Wang, and J. Feng, ‘‘Parameters variation-based synchronization on derivative coupled Lur’e networks,’’ *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 2019.

- [12] Z. Tang, J. H. Park, Y. Wang, and J. Feng, "Distributed impulsive quasi-synchronization of Lur'e networks with proportional delay," *IEEE Transactions on Cybernetics*, vol. 49, no. 8, pp. 3105–3115, 2019.
- [13] M. Dai, J. Xia, H. Xia, and H. Shen, "Event-triggered passive synchronization for Markov jump neural networks subject to randomly occurring gain variations," *Neurocomputing*, vol. 331, pp. 403–411, 2019.
- [14] J. Wang, L. Shen, J. Xia, Z. Wang, and X. Chen, "Asynchronous dissipative filtering for nonlinear jumping systems subject to fading channels," *Journal of the Franklin Institute*, vol. 357, no. 1, pp. 589–605, 2020.
- [15] J. Wang, T. Ru, J. Xia, Y. Wei, and Z. Wang, "Finite-time synchronization for complex dynamic networks with semi-Markov switching topologies: an H_{∞} event-triggered control scheme," *Applied Mathematics and Computation*, vol. 356, no. 1, pp. 235–251, 2019.
- [16] A. Polyakov, "Nonlinear feedback design for fixed-time stabilization of linear control systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 8, pp. 2106–2110, 2012.
- [17] G. Mei, X. Wu, D. Ning, and J.-A. Lu, "Finite-time stabilization of complex dynamical networks via optimal control," *Complexity*, vol. 21, no. S1, pp. 417–425, 2016.
- [18] D. Zhang, Y. Shen, and J. Mei, "Finite-time synchronization of multi-layer nonlinear coupled complex networks via intermittent feedback control," *Neurocomputing*, vol. 225, pp. 129–138, 2017.
- [19] B. Zhang and Y. Jia, "Fixed-time consensus protocols for multi-agent systems with linear and nonlinear state measurements," *Nonlinear Dynamics*, vol. 82, no. 4, pp. 1683–1690, 2015.
- [20] Y. Xu, W. Zhou, J. A. Fang, C. Xie, and D. Tong, "Finite-time synchronization of the complex dynamical network with non-derivative and derivative coupling," *Neurocomputing*, vol. 173, pp. 1356–1361, 2016.
- [21] Z. Tang, J. H. Park, and H. Shen, "Finite-time cluster synchronization of Lur'e networks: a nonsmooth approach," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 8, pp. 1213–1224, 2018.
- [22] X. Yang, J. Lam, D. W. C. Ho, and Z. Feng, "Fixed-time synchronization of complex networks with impulsive effects via nonchattering control," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5511–5521, 2017.
- [23] Y. Xu, X. Wu, N. Li, L. Liu, C. Xie, and C. Li, "Fixed-time synchronization of complex networks with a simpler nonchattering controller," *IEEE Transactions on Circuits and Systems II: Express Briefs*, 2019.
- [24] S. Mobayen and J. Ma, "Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay," *Chaos, Solitons & Fractals*, vol. 114, pp. 46–54, 2018.
- [25] R. Wei, J. Cao, and A. Alsaedi, "Finite-time and fixed-time synchronization analysis of inertial memristive neural networks with time-varying delays," *Cognitive Neurodynamics*, vol. 12, no. 1, pp. 121–134, 2018.
- [26] G. Hardy, J. Littlewood, and G. Polya, *Inequalities*, Cambridge University Press, Cambridge, UK, 1952.
- [27] W. Zhou, Y. Xu, H. Lu, and L. Pan, "On dynamics analysis of a new chaotic attractor," *Physics Letters A*, vol. 372, no. 36, pp. 5773–5777, 2008.