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Research Article

Day-to-Day Traffic Assignment Model considering Information Fusion and Dynamic Route Adjustment Ratio

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A new day-to-day traffic assignment model is proposed to describe travelers' day-to-day behavioral changes with advanced traffic information system. In the model, travelers' perception is updated by a double exponential-smoothing learning process combining experience and traffic information that is explicitly modelled. Route adjustment ratio is dynamically determined by the difference between perceived and expected utilities. Through theoretical analyses, we investigate the existence of its fixed point and the influence factors of uniqueness of the fixed point. An iterative-based algorithm that can solve the fixed point is also given. Numerical experiments are then conducted to investigate effects of several main parameters on its convergence, which provides insights for traffic management. In addition, we compare the system efficiencies under the static route adjustment ratio and dynamic route adjustment ratio and show the application of the model.

1. Introduction

Equilibrium assignment has been providing theoretical basis for the transport planners and operators during the past few decades. While useful, it ignores traffic adjustment process. To analyze it, a stream of research about traffic dynamics has been developed. A variety of day-to-day traffic assignment (DTD) models have been proposed including deterministic process (DP) models, derived from the discrete-time nonlinear dynamic systems theory, introduced for transportation by [1], and stochastic process (SP) models considering random fluctuation of demand and (or) supply, introduced for transportation systems by [2]. Some DP models are established on the assumption that travelers' perception is consistent with the reality and converge to user equilibrium [3, 4] while others incorporate the randomness of perception and converge to stochastic user equilibrium [1, 2]. Cantarella and Cascetta [5] have unified DP and SP models. Apart from these models based on route flow variables, link-based DTD models recently are developed to deal with route overlapping and numeration problem [6].

Nowadays, with the development of information and communication technologies and the popularity of navigation services, it is easy for travelers to access various traffic information. The information might affect travelers' behavior significantly, which arises the interest to study traffic dynamics with traffic information [7–16]. Some of them focus on traffic dynamics with the traffic information obtained from social network [10–12, 15, 16], while others focus on traffic dynamics with the publicly available information provided by advanced traveler information system (ATIS) [7–9, 13, 14]. We focus on the latter.

In the majority of DTD models that aim to capture traffic dynamics with traffic information provided by ATIS, travelers are divided into two categories based on their dependent information resources. Information dependent travelers choose the route according completely to the information, while experience dependent travelers do it according entirely to the perceived cost learned from experience [7–9, 14]. However, Bogers has evidenced that travelers comprehensively use traffic information and experience in the route choice scenario [17].

In addition, these studies generally assume that the proportion of travelers to adjust route every day is fixed [7–9]. Although the fixed value has been changed from 1 to less than 1 to capture the inertia [18], this way is too simple to demonstrate that inertia is controlled by the difference between perceived utilities of alternatives, which is found by Mahmassani and Liu using a set of laboratory experiments [19] and by Wang et al. using stated preference survey data [20].

This study simultaneously deals with the above two problems to establish a more realistic day-to-day traffic assignment model with ATIS and reinvestigates traffic dynamics. In our model, travelers learn from experience to form their initial perception and the perception is then modified by the traffic information provided by ATIS, for example, Liu et al. [13]. However, different from them, we use the single day information to adjust travelers' perceived route cost rather than the difference between information in consecutive two days because it is nearly impossible for travelers to clearly recall traffic information in the previous day. Besides, route adjustment ratio is dynamically determined by the difference between perceived and expected utilities. We also explicitly model traffic information, especially, the predictability of traffic information, for example, Liu et al. [13].

The rest of the paper is organized as follows. Section 2 describes our day-to-day traffic assignment model. Section 3 theoretically analyses the existence of its fixed point and influence factors of uniqueness and stability of the fixed point. An iterative algorithm to solve the fixed point is also given. Section 4 conducts numerical experiments to specifically illustrate how several main parameters affect convergence of the proposed model. The system efficiencies under static route adjustment ratio and dynamic route adjustment ratio are also compared. Furthermore, the proposed model is applied to examine traffic dynamics when there are changes on link capacities. Section 5 concludes the paper and gives some future directions.

2. Day-to-Day Dynamic Traffic Assignment Model

2.1. Travelers' Perceived Travel Time. Experienced travel time and descriptive traffic information provided by traffic agency both affect travelers' perception for route cost. When travelers finish a travel, they can obtain route cost and thus update their perception. The process can be built up through an exponential smooth approach. When the pre-trip descriptive information is provided by ATIS, travelers will further modify their perception, which can also be described through an exponential smooth approach. Hence, a double exponential smoothing approach is constructed to describe travelers' perception updating as follows:

$$C_{r,p}^{n+1} = (1-\delta)\left((1-\lambda)C_{r,p}^{n} + \lambda C_{r,e}^{n}\right) + \delta C_{r,f}^{n+1}, \quad \forall r \in R_w,\ w \in W, \tag{1}$$

where $\lambda\,(0 \le \lambda \le 1)$ is learning rate and describes the extent that travelers put an emphasis on experience and $\delta\,(0 \le \delta \le 1)$ is information fusion rate; the rest of notations are listed in Appendix A.

In the case with $\delta = 0$, equation (1) deteriorates into single exponential smoothing approach and travelers just learn from experience as typically done in the literature.

2.2. Predictive Information. Traffic information is mainly represented in two ways in the DTD model. The first one views traffic information to be fully accurate and without an explicit model. However, due to the uncertainty of traffic supply and demand, there is no fully accurate information. Therefore, the second way aims at modelling imperfect traffic information. Generally, it directly adopts free-flow travel time or the updated value based on historical data. These methods ignore travelers' behavior and traffic information generated lacks predictability. Inspired by Liu et al. [13], we generate predictive traffic information based on travelers' behavior.

Travelers' perceived travel time is firstly predicted through an exponential smooth learning process as travelers do, but a different learning parameter is introduced, $\lambda' \neq \lambda$, because traffic agency does not exactly grasp the extent of travelers valuing single day's experience in long perception:

$$C_{r,fp}^{n+1} = (1-\lambda')C_{r,fp}^{n} + \lambda' C_{r,e}^{n}, \quad \forall r \in R_w,\ w \in W. \tag{2}$$

Traffic agency then distributes travelers over traffic network based on perceived route cost. Specifically, a random utility model is adopted here. Through the process, new route travel time is obtained. It is predictive traffic information and provided to travelers. The whole process can be described as follows:

$$\begin{aligned}
P_{r,fp}^{n+1} &= \frac{e^{-\theta' C_{r,fp}^{n+1}}}{\sum_{r\in R_w} e^{-\theta' C_{r,fp}^{n+1}}}, \quad \forall r \in R_w,\ w \in W,\\
y_{r,fp}^{n+1} &= d_w P_{r,fp}^{n+1}, \quad \forall r \in R_w,\ w \in W,\\
x_{l,f}^{n+1} &= \sum_{r\in R_w} \Lambda_{r,l}\, y_{r,fp}^{n+1}, \quad \forall l \in L,\\
C_{r,f}^{n+1} &= \sum_{l\in L} \Lambda_{r,l}\, c\left(x_{l,f}^{n+1}\right), \quad \forall l \in L,
\end{aligned} \tag{3}$$

where θ' reflects perception variation. A higher θ'-value means a smaller perception variation for travelers.

2.3. Route Adjustment Ratio. For each trip, travelers have an expected utility. If the perceived utility of previously chosen route is higher than their expectation, they will continue to travel on the route due to inertia; otherwise, it is possible for them to adjust route. The adjustment ratio has a positive relationship with the difference between perceived utility and expected utility. Based on the work of the literature [21], the adjustment ratio of travelers previously choosing route r is expressed as follows:

$$\chi_{r,p}^{n+1}\left(\nabla C_{r,p}^{n+1}\right)=\frac{\chi_0\left(\nabla C_{r,p}^{n+1}\right)^3}{\left(\nabla C_{r,p}^{n+1}\right)^3+\varpi},\quad \forall r\in R_w,\ w\in W, \tag{4}$$

$$\nabla C_{r,p}^{n+1}=C_{r,p}^{n+1}-E_{w,p}^{n+1},\quad \forall r\in R_w,\ w\in W, \tag{5}$$

$$E_{w,p}^{n+1}=-\frac{\ln\sum_{r\in R_w}e^{-\theta C_{r,p}^{n+1}}}{\theta},\quad w\in W, \tag{6}$$

where $\chi_0\,(0\le\chi_0\le 1)$ is a constant that represents the maximal adjustment ratio; $\varpi\,(\varpi>0)$ is a parameter to adjust the sensitivity of route adjustment behavior for utility difference; θ is similar with θ' and reflects perception variation; and $E_{w,p}^{n}$ is expected utility on day n through OD pair w and expressed as follows:

$$E_{w,p}^{n}=E\left[\max\left\{-C_{r,p}^{n}\right\}\right]. \tag{7}$$

Because travelers differently rely on traffic information, stress single-experience, and so on, their perceived utilities for traveling in a route are not the same. Assuming random residuals of perceived utilities for each route comply the same Gumbel distribution and the random residuals for different routes are independent, equation (6) is derived from equation (7).

2.4. Link Traffic Flow. Route adjustment travelers will rechoose route based on their perceived utilities that negatively depend on perceived travel times. Similarly, there are random residuals for perceived utilities. If random residuals of perceived utilities are the same independent Gumbel distributions, route adjustment travelers will choose route in a logit-based formula:

$$P_{r,p}^{n+1}=\frac{e^{-\theta C_{r,p}^{n+1}}}{\sum_{k\in R_w}e^{-\theta C_{k,p}^{n+1}}},\quad \forall r\in R_w,\ w\in W. \tag{8}$$

The rest will continue to travel on previous routes. Therefore, the total chosen probability of a route is

$$P_{r,e}^{n+1}=\sum_{r\in R_w}\left(\chi_{r,p}^{n+1}P_{r,e}^{n}\right)P_{r,p}^{n+1}+\left(1-\chi_{r,p}^{n+1}\right)P_{r,e}^{n},\quad \forall r\in R_w,\ w\in W. \tag{9}$$

The corresponding route flow and link flow pattern are

$$\begin{aligned} y_{r,e}^{n+1}&=d_wP_{r,e}^{n+1},\quad \forall r\in R_w,\ w\in W,\\ x_{l,e}^{n+1}&=\sum_{r\in R_w}\Lambda_{l,r}y_{r,e}^{n+1},\quad \forall l\in L. \end{aligned} \tag{10}$$

3. Model properties and Solution Algorithm

3.1. Model properties

3.1.1. Existence and State of Fixed Point. Suppose a fixed point is formed at day $\bar{n}$, we have $x_{l,e}^{n}=x_{l,e}^{n+1}=x_{l,e}^{*}$, $C_{r,e}^{n}=C_{r,e}^{n+1}=C_{r,e}^{*}$, $C_{r,p}^{n}=C_{r,p}^{n+1}=C_{r,p}^{*}$, and $C_{r,fp}^{n}=C_{r,fp}^{n+1}=C_{r,fp}^{*}$ for $n\ge\bar{n}$. Furthermore, we can infer $C_{r,fp}^{*}=C_{r,e}^{*}$ and $C_{r,p}^{*}=((\lambda-\lambda\delta)/(\lambda+\delta-\lambda\delta)C_{r,e}^{*})+(\delta/(\lambda+\delta-\lambda\delta)C_{r,f}^{*})$.

Then, the fixed-point of our model can be written as follows:

$$\begin{aligned}
C_{r,e}^{*}&=\sum_{l\in L}\Lambda_{r,l}c\left(x_{l,e}^{*}\right),\quad \forall l\in L,\\
P_{r,fp}^{*}&=\frac{e^{-\theta' C_{r,e}^{*}}}{\sum_{r\in R_w}e^{-\theta' C_{r,e}^{*}}},\quad \forall r\in R_w,\ w\in W,\\
y_{r,f}^{*}&=d_wP_{r,fp}^{*},\quad \forall r\in R_w,\ w\in W,\\
x_{l,f}^{*}&=\sum_{r\in R_w}\Lambda_{l,r}y_{r,f}^{*},\quad \forall l\in L,\\
C_{r,f}^{*}&=\sum_{l\in L}\Lambda_{l,r}c\left(x_{l,f}^{*}\right),\quad \forall l\in L,\\
P_{r,p}^{*}&=\frac{e^{-\theta C_{r,p}^{*}}}{\sum_{r\in R_w}e^{-\theta C_{r,p}^{*}}},\quad \forall r\in R_w,\ w\in W,\\
E_{w,p}^{*}&=-\frac{\ln\sum_{r\in R_w}e^{-\theta C_{r,p}^{*}}}{\theta},\quad w\in W,\\
\nabla C_{r,p}^{*}&=C_{r,p}^{*}-E_{w,p}^{*},\quad \forall r\in R_w,\ w\in W,\\
\chi_{r,p}^{*}\left(\nabla C_{r,p}^{*}\right)&=\frac{\chi_0\left(\nabla C_{r,p}^{*}\right)^3}{\left(\nabla C_{r,p}^{*}\right)^3+\varpi},\quad \forall r\in R_w,\ w\in W,\\
P_{r,e}^{*}&=\begin{cases}\dfrac{mP_{r,p}^{*}}{\left(\chi_{r,p}^{*}\right)} & \left|R_w\right|>1,\\ P_{r,p}^{*} & \left|R_w\right|=1,\end{cases}\quad \forall r\in R_w,\ w\in W,
\end{aligned} \tag{11}$$

where $m=1/\sum_{r}^{|R_w|}P_{r,p}^{*}/\chi_{r,p}^{*}$, $\forall r\in R_w$, $w\in W$,

$$\begin{aligned} y_{r,e}^{*}&=d_wP_{r,e}^{*},\quad \forall r\in R_w,\ w\in W,\\ x_{r,e}^{*}&=\sum_{r\in R_w}\Lambda_{l,r}y_{r,e}^{*},\quad \forall l\in L. \end{aligned} \tag{12}$$

The feasible link flow set of our model is closed, bounded, convex, and nonempty [22]. According to Brouwer's fixed-point theorem [23], if the self-map about link flow vector X^* is continuous, there is at least a fixed point.

Assuming that $c(x_{l,e}^{*})$ is continuous with $x_{l,e}^{*}$, $C_{r,f}^{*}$ is a continuous function with X^*. Furthermore, $C_{r,p}^{*}$ is continuous with X^*. With the route choice function and route adjustment ratio function, it can be proved that route choice vector $P_{r,e}^{*}$ is continuous with perceived travel time vector C_{p}^{*}. Therefore, the self-map of X^* are continuous and there is at least a fixed point.

In addition, we can find that the parameters λ, δ, θ, and θ' affect the state of the fixed point. However, χ_0 does not affect the state of fixed point because it can be eliminated in the $P_{r,e}^{*}$. As well, λ' does not affect the state of fixed point.

3.1.2. Uniqueness of Fixed Point. $P_{r,e}^{n+1}$ relies on travelers' perceived travel times which are determined by traffic information and experience. Traffic information depends on perceived travel times predicted by traffic agency. Therefore, we can rewrite our model in an abstract way: $(P_{r,e}^{n+1}, C_{r,p}^{n+1}, C_{r,fp}^{n+1}) = \Psi(C_{r,p}^{n}, C_{r,fp}^{n}, P_{l,e}^{n})$, where Ψ covers all relations formulated in Section 2. Given c is continuously differentiable, we can infer that Ψ is continuously differentiable. Let J denotes the Jacobian matrix of the function Ψ. If $|I - J_{\Psi}| \neq 0$, $\Phi = P^t - \Psi[P^t]$ is invertible according to Hadamard's global inverse theorem [21]. Because Φ is invertible, it also is injective and Φ^{-1} is invertible and injective. $P^* = \Phi^{-1}(\mathbf{0})$ is exclusive.

Specifically,

$$\mathbf{J}_{\Psi}\left(C_{r,p}^{*}, C_{r,fp}^{*}, P_{r,e}^{*}\right) = \mathbf{m}^{*}\left(\left(\boldsymbol{\chi}^{*}\right)^{-1}\mathbf{J}_{\mathbf{P}}^{*}\left(C_{p},\theta\right)\mathbf{J}_{\mathbf{C}}^{*} - \mathbf{B}\mathbf{J}_{\boldsymbol{\chi}}^{*}\mathbf{J}_{\mathbf{C}}^{*}\right) = \mathbf{m}^{*}\left(\left(\boldsymbol{\chi}^{*}\right)^{-1}\mathbf{J}_{\mathbf{P}}^{*}\left(C_{p},\theta\right) - \mathbf{B}\mathbf{J}_{\boldsymbol{\chi}}^{*}\right)\mathbf{J}_{\mathbf{C}}^{*}, \quad (13)$$

where

$$B = \begin{bmatrix} p_{1,p}^{*}/\left(\chi_{1,p}^{*}\right)^{2} & & & & \\ & p_{2,p}^{*}/\left(\chi_{2,p}^{*}\right)^{2} & & & \\ & & \cdots & & \\ & & & p_{|R_w|-1,p}^{*}/\left(\chi_{|R_w|-1,p}^{*}\right)^{2} & \\ & & & & p_{|R_w|,p}^{*}/\left(\chi_{|R_w|,p}^{*}\right)^{2} \end{bmatrix},$$

$$\mathbf{J}_{\mathbf{C}}^{*} = \frac{\lambda - \lambda\delta}{\lambda + \delta - \lambda\delta}\boldsymbol{\Lambda}^{\mathbf{T}}\mathbf{J}_{\mathbf{c}}^{*}\left(C_e\right)\boldsymbol{\Lambda}\mathbf{Q} + \frac{\delta}{\lambda + \delta - \lambda\delta}\boldsymbol{\Lambda}^{\mathbf{T}}\mathbf{J}_{\mathbf{c}}^{*}\left(C_f\right)\boldsymbol{\Lambda}\mathbf{Q}\mathbf{J}_{\mathbf{p}}^{*}\left(C_e,\theta'\right)\boldsymbol{\Lambda}^{\mathbf{T}}\mathbf{J}_{\mathbf{c}}^{*}\left(C_e\right)\boldsymbol{\Lambda}\mathbf{Q}. \quad (14)$$

Therefore, we can infer that if link performance function is a constant without traffic flow on it, $\mathbf{J}_{\mathbf{c}}^{*} = \mathbf{0}$, and the fixed point of our model is uniqueness; otherwise, λ, δ, θ, θ', and χ_0 all affect the uniqueness of the fixed point except λ'.

3.1.3. Stability of Fixed Point. Our model will converge to a fixed point from its attractor region if 3|R| eigenvalues of the matrix $J_{\Psi}^{*}(C_{r,p}^{n}, C_{r,fp}^{n}, P_{r,e}^{n})$ (Appendix B) are within the unit circle [24]. Hence, we can know that λ, λ', δ, θ, θ', and χ_0 all affect the stability of a fixed point. In Section 3.2, we will conduct sensitive analyses to specifically investigate how these parameters affect the convergence of the model as to provide insights for traffic management.

3.2. Solution Algorithm. The fixed point of our model is identical to the fixed point of the dynamic system established in the following [25]:

$$x_{l,e}^{\kappa+1} = \tau\Psi\left(x_{l,e}^{\kappa}\right) + (1-\tau)x_{l,e}^{\kappa}, \quad (15)$$

where parameter $\tau\,(0 < \tau < 1)$ is iterative step length; the superscript $\kappa\,(\kappa = 0, 1, 2, 3, \cdots)$ represents the iterative time.

Based on it, the fixed point of our model can be numerically computed by executing an iteration-based algorithm [26].

Step 1 Initialization: assign a value for the iterative step length τ. Set $\kappa = 0$.

Step 2. Choose a feasible initial vector of link flow $x_{l,e}^{0} \in \boldsymbol{\Theta}$, where $\boldsymbol{\Theta}$ represents the feasible link flow set which satisfies both the nonnegative constraint of link flow and total demand constraint.

Step 3. Renew the link flow vector using the following iterative formula: $x_{l,e}^{\kappa+1} = \tau\Psi(x_{l,e}^{\kappa}) + (1-\tau)x_{l,e}^{\kappa}$.

Step 4. Check the iterative convergence:$|x_{l,e}^{\kappa+1} - x_{l,e}^{\kappa}| < \xi$, $\forall l \in L$. ξ represents the convergence criterion. If the convergence condition is met, $x_{l,e}^{*} = x_{l,e}^{\kappa+1}$, $\forall l \in L$ and stop; otherwise, set $\kappa = \kappa + 1$ and go to Step 2.

According to the work of Cascetta and Cantarella [5], the algorithm can converge to a fixed point when τ is small enough.

4. Numerical Experiments and Analyses

4.1. Small Network. We firstly use a small network with an OD pair to show the effects of λ' and χ_0 on the convergence of the model. The traffic demand of the OD pair is 500 (pcu·h^{-1}). As shown in Figure 1, the small network has two links, whose free-flow travel time and the capacity are, respectively, (2 min, 300 pcu·h^{-1}) and (2 min, 400 pcu·h^{-1}). Bureau of public roads (BPR) link performance function is used to compute the link travel time as do most of relevant literature studies [7, 8, 21, 25, 27]. The function is established

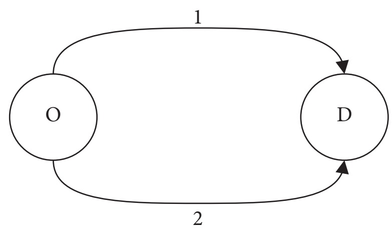

Figure 1: Small network.

based on many field surveys and thus reflects the real relationship between link travel time and link flow:

$$c_l = c_l^0 \left(1.0 + \alpha \left(\frac{x^l}{u_l} \right)^{\beta} \right), \quad \forall l \in L, \tag{16}$$

where α and β are parameters with typical value $\alpha = 0.15$ and $\beta = 4$.

The specific numerical settings are given in Table 1.

When the initial route choice is set as (0.4, 0.6), the fixed point we can get under the above parameters is (0.225, 0.775). Based on it, we can calculate the eigenvalues of $J_{\psi}^{*}(C_{r,p}^{n}, C_{r,fp}^{n}, P_{r,e}^{n})$ under different λ'-values and χ_0-values, which are displayed in Tables 2 and 3, respectively.

We can know from Table 2 that the system can converge to an equilibrium when λ' is 0.4, 0.6, or 0.8 (all eigenvalues are smaller than 1). Figure 2 displays the same result through evolutionary processes of link 1 chosen probabilities under different λ' values. These results indicate that if traffic agency adopts an underestimated learning rate to predict travelers' perceived cost, the system is still able to converge to the equilibrium, but the error should be smaller than a certain value. As well, Figure 2 shows a larger λ' can make the system more quickly converge to the equilibrium.

Table 3 shows that as χ_0 increases, the system starts not to converge to an equilibrium, suggesting that the route adjustment tendency has a negative effect on the stability of the system. We also depict that evolutionary processes of link 1 chosen probabilities under different χ_0 values in Figure 3. It shows the same result. Furthermore, it shows χ_0 has a positive relationship with the convergence speed.

4.2. Nguyen-Dupuis Network. We then conduct numerical experiments on Nguyen-Dupuis network [28] displayed in Figure 4 to further illustrate the other nature of the proposed DTD model and its application. Nguyen-Dupuis network consists of 13 nodes, 19 links, 4 OD pairs, and 25 routes. The characteristics of 19 links are displayed in Table 4. The traffic demand pattern between the four OD pairs is assumed to be $(d_{12}, d_{13}, d_{42}, d_{43}) = (660, 495, 412, 495)(\text{pcu}\cdot\text{h}^{-1})$ [27]. All alternative routes connecting these OD pairs are shown in Table 5. Likewise, we adopt Bureau of Public Roads' (BPR) link performance function as the link travel time function. The specific numerical settings are given in Table 6.

4.2.1. Sensitivity Analysis. We then explore how the other parameters might affect the convergence of the system. Figure 5 displays chosen probabilities of route 1 over time under different parameter values.

It can be clearly observed from Figure 5 that the travelers' learning rate, information fusion rate, and perception variations adopted by travelers and traffic agency all affect the convergence of the proposed system, which is consistent with our theoretical analyses.

We further find from Figure 5 that when travelers' learning rate and perception variation parameter are beyond certain values, the system will not be able to converge to an equilibrium. The reason can be, respectively, explained as follows:

(1) The higher travelers' learning rate is, the more travelers put emphasize on single day experience. Travelers' route choice will lack consistency, leading to an unstable system.

(2) A lower perception variation (a higher θ value) means that more travelers choose the same route to travel. As a result, they will frequently adjust the route and the system oscillates.

When δ increases to 0.8, the system starts to converge to an equilibrium. The implication is that appropriate information dependence could help stabilize the system. Furthermore, from the fact that the system with $\delta = 1$ is convergent, we can infer that more dependence on a stable information system might contribute to the system's convergence.

In addition, when θ' is equal to 0.01 (an underestimated value), the traffic system cannot converge to an equilibrium, while the traffic system still is convergent when θ' is equal to 0.4 (an overestimated value). These results indicate that the system can converge to an equilibrium when an inaccurate perception variation is adopted by traffic agency to predict traffic information. As well, when traffic agency cannot obtain the real travelers' perception variation, a relatively larger θ' adopted is more beneficial.

4.2.2. Comparison Analysis of Static Route Adjustment Ratio and Dynamic Route Adjustment Ratio. In our model, travelers decide whether to adjust route based on the difference between perceived and expected utilities, leading to a dynamical adjustment ratio (DR). However, as mentioned above, most relevant studies consider that a fixed proportion of travelers adjust the route every day, called static adjustment ratio (SR). For simulating it, we keep the maximal route adjustment ratio unchanged in the whole evolutionary process. The efficiencies of the systems under the two different adjustment ratios are displayed in Figure 6. It clearly shows that the travelers' total travel times with DR is higher than that with SR, demonstrating the model with SR overestimates the efficiency of the system. This might be because in dynamic route adjustment scenario travelers ignore the small difference among route travel times and keep travelling in routes with slightly high travel times.

It should be noted that, for the network with single route connecting each OD pair, the efficiencies of the systems under the two different adjustment ratios are the same

Table 1: Basic numerical settings on small network.

Parameters	BPR function	Learning rate	Information parameters	Route adjustment ratio	Perception variation parameter
Specification	$\alpha = 2$; $\beta = 4$	$\lambda = 0.8$; $\lambda' = 0.4$	$\delta = 0.1$	$\varpi = 1$; $\chi_0 = 0.8$	$\theta = \theta' = 1$

Table 2: Eigenvalues of $J^*_\Psi(C^n_{r,p}, C^n_{r,fp}, P^n_{r,e})$ under different λ' values.

Eigenvalues	$\lambda\prime$			
	0.2	0.4	0.6	0.8
1	−1.12	−0.96	−0.76	−0.47
2	−0.11	−0.12	−0.13	−0.18
3	0.18	0.18	0.18	0.18
4	0.88	0.74	0.57	0.34
5	<1.00	<1.00	<1.00	<1.00
6	0.80	0.6	0.40	0.20

Table 3: Eigenvalues of $J^*_\Psi(C^n_{r,p}, C^n_{r,fp}, P^n_{r,e})$ under different χ_0 values.

Eigenvalues	χ_0			
	0.6	0.7	0.8	0.9
1	$-0.28 + 0.21i$	−0.63	−0.96	−1.26
2	$-0.28 - 0.21i$	−0.19	−0.12	−0.09
3	$0.18 + 0.00i$	0.18	0.18	0.18
4	$0.74 + 0.00i$	0.74	0.74	0.74
5	$<1.00 + 0.00i$	<1.00	<1.00	<1.00

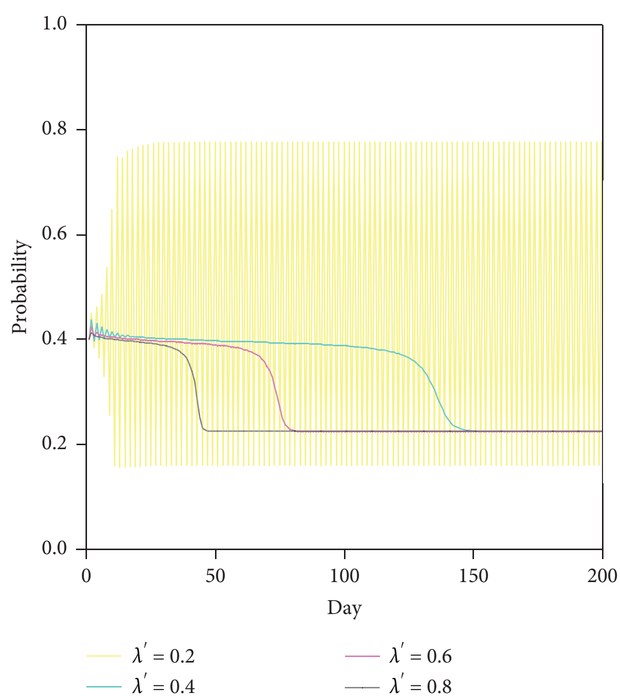

Figure 2: Evolution of link 1 chosen probabilities under different $\lambda\prime$ values.

because no alternative route can be used. Therefore, we can conclude that the model with SR overestimates the efficiency of the system with alternative routes.

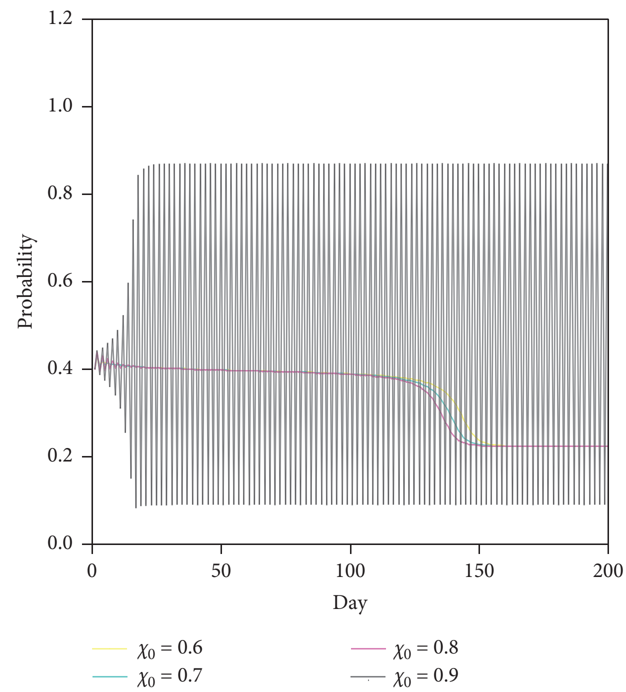

Figure 3: Evolution of link 1 chosen probabilities under different χ_0 values.

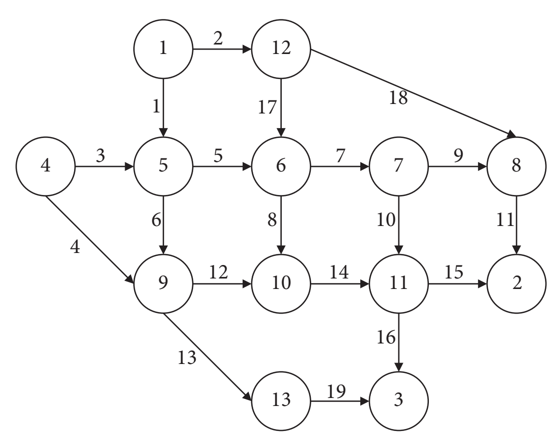

Figure 4: Nguyen-Dupuis network.

4.2.3. Day-to-Day Dynamic after the Change of Road Capacity. The proposed model can be further used to analyze traffic dynamics when there are changes in road capacity, such as, drops in link capacity caused by road maintenance. The scenario can be assumed as at a day, such as 50th day, segment of link 7 is closed due to maintenance and its capacity drops into 200 (pcu·h^{-1}); after road maintenance is finished, its capacity returns into the original value at the 70th day. Corresponding evolutionary process of the system is displayed in Figure 7.

Table 4: Network characteristics.

Link no.	c_l^0 (min)	u_l (pcu·h^{-1})
1	7	300
2	9	200
3	9	200
4	12	200
5	3	350
6	9	400
7	5	500
8	13	250
9	5	250
10	9	300
11	9	500
12	10	550
13	9	200
14	6	400
15	9	300
16	8	300
17	7	200
18	14	300
19	11	200

Table 5: Routes.

OD pair	(1, 2)	(1, 3)	(4, 2)	(4, 3)
Route no. and links sequence	1 (2-18-11)	9 (1-6-13-19)	15 (4-12-14-15)	20 (4-13-19)
	2 (1-5-7-9-11)	10 (1-6-12-14-16)	16 (3-5-7-9-11)	21 (4-12-14-16)
	3 (1-5-7-10-15)	11 (1-5-8-14-16)	17 (3-6-12-14-15)	22 (3-6-13-19)
	4 (1-5-8-14-15)	12 (1-5-7-10-16)	18 (3-5-8-14-15)	23 (3-6-12-14-16)
	5 (1-6-12-14-15)	13 (2-17-8-14-16)	19 (3-5-7-10-15)	24 (3-5-8-14-16)
	6 (2-17-8-14-15)	14 (2-17-7-10-16)		25 (3-5-7-10-16)
	7 (2-17-7-10-15)			
	8 (2-17-7-9-11)			

Table 6: Basic numerical settings on Nguyen-Dupuis network.

Parameters	BPR function	Learning rate	Information parameters	Route adjustment ratio	Perception variation parameter
Specification	$\alpha = 0.15$; $\beta = 4$	$\lambda = \lambda' = 0.4$	$\delta = 0.8$	$\varpi = 1$; $\chi_0 = 0.5$	$\theta = \theta' = 0.1$

As shown in Figure 7, when the capacity of link 7 drops, traffic equilibrium immediately breaks down because traffic agency informs travelers of road maintenance through descriptive information. The whole traffic system is affected, such as route 1 and route 24 that do not include link 7, because links and routes in a traffic system interact with each other. During the whole period of maintenance, the system always oscillates and does not achieve an equilibrium. After recovery of the capacity, the traffic system takes 5 days to return to its initial equilibrium. This demonstrates that the change of temporary link capacity does not cause permanent change.

5. Conclusions

This study extends the day-to-day model by simultaneously considering the combined effect of traffic information and experience on travelers' perception and dynamic route adjustment ratio. Then, we theoretically validate the existence of its fixed point. We find that except the learning rate adopted by traffic agency, travelers' learning rate, information fusion rate, perception variation, and maximal route adjustment ratio in the model all affect uniqueness of its fixed point when link performance functions are the function with link flows. An iteration-based algorithm can be used to solve these fixed points. Finally, we conduct numerical experiments on a small network with two links and Nguyen-Dupuis network to investigate effects of several parameters on the convergence of the model. The results show the system can converge to an equilibrium with traffic information predicted by traffic agency using inaccurate parameters. Also, dependence on traffic information provided by a stable information system contributes to the stability of the system. Furthermore, maximal route adjustment rate has a negative effect on the stability of the fixed point and a positive effect on convergence speed. The results provide insights for ATIS design and traffic management.

An experiment is also conducted to analyze the difference between static adjustment ratio and dynamic adjustment ratio.

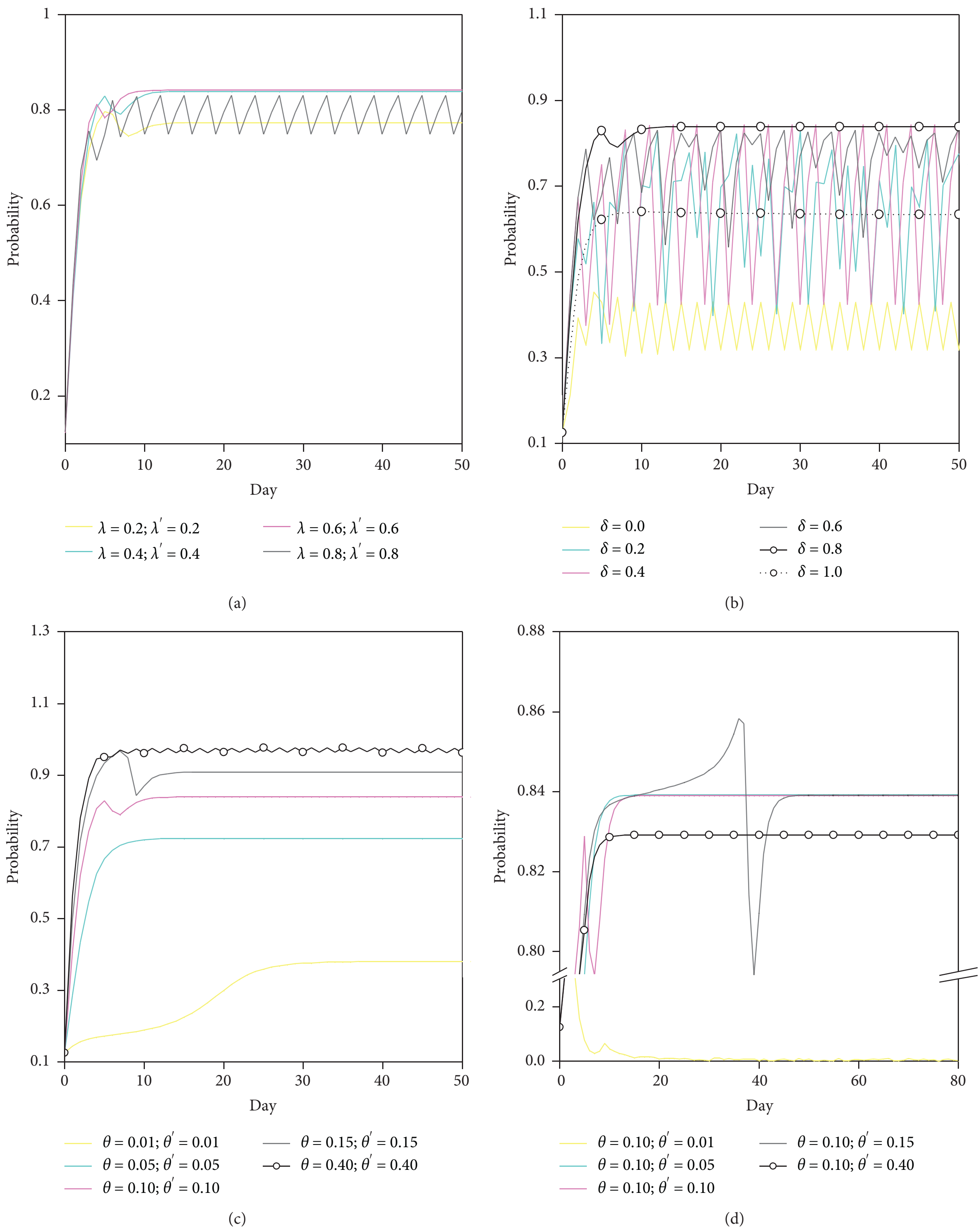

Figure 5: Evolutionary processes under different parameter values: (a)$\lambda = \lambda\prime$ (b)δ, (c)$\theta = \theta\prime$, and (d)$\theta \neq \theta\prime$.

The results show that the model with static route adjustment ratio overestimates the efficiency of the system with alternative routes. Finally, we show the application of our model by analyzing traffic dynamics when there are drops in link capacities.

In the future, various research directions ensuing from this work can be explored. One of them would be to calibrate and validate the model with empirical data. Another line of research would be to analytically derive

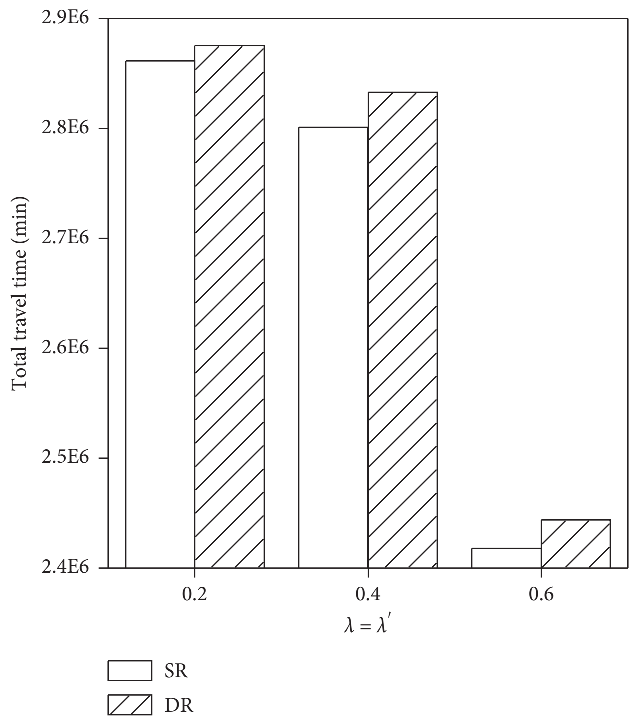

FIGURE 6: System efficiencies under static and dynamic adjustment ratios.

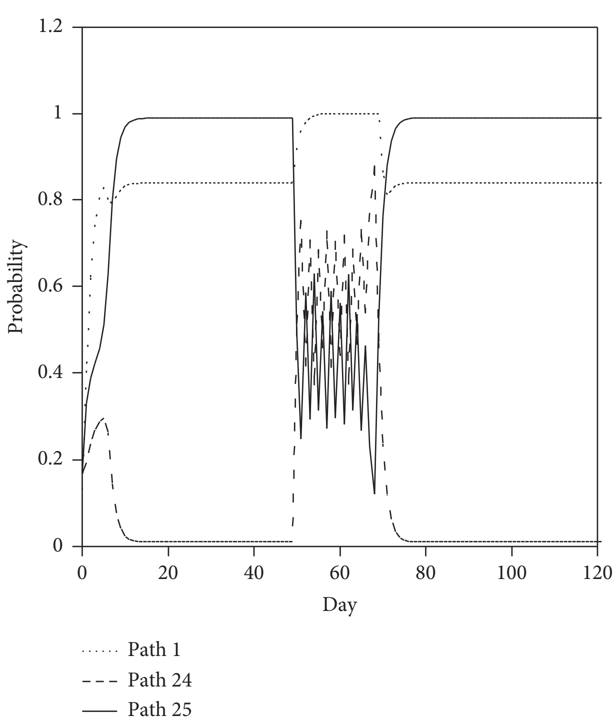

FIGURE 7: Day-to-day dynamics with drop in road capacity.

the sufficient condition that assures the stability of the dynamic system. The third direction would be to design management measures to improve its stability. Last but not least, the model should be further extended into a link-based day-to-day model to avoid numerating all routes among O-D pairs.

Appendix

A. Notations

A traffic network is a directed graph (N, L), where N represents the node set and L corresponds to the link set. The notation used in this paper is

l = link index, $l \in L$

O = origin set, $o \in O, O \subseteq N$

D = destination set, $d \in D, D \subseteq N$

W = origin and destination (OD) pair set

w = OD pair index, $w \in W$

R_w = route set between OD pair w

r = route index, $r \in R_w, w \in W$

$\Lambda_{l,r}$ = link-route index whose value is 1 if link l belongs to route r, zero otherwise

q_w = demand of OD pair w

c_l^0 = free-flow travel time of link l

$x_{l,f}^n$ = flow on link l on day n predicted by traffic agency

$x_{l,e}^n$ = traffic flow on link l on day n

$y_{r,f}^n$ = flow on the route r on day n predicted by traffic agency

$y_{r,e}^n$ = traffic flow on the route r on day n

c = link performance function

$C_{r,e}^n$ = route r travel time on day n

$C_{r,p}^n$ = perceived travel time of route r on day n

$C_{r,fp}^n$ = perceived travel time of route r on day n predicted by traffic agency

$C_{r,f}^n$ = route r travel time on day n predicted by traffic agency

$E_{w,p}^n$ = expected utilities through OD pair w

$\nabla C_{r,e}^{n+1}$ = difference between perceived and expected utilities

$\chi_{r,e}^{n+1}$ = adjustment ratio of travelers on day $n+1$ that choose route r on day n

$P_{r,p}^n$ = probability that route adjustment travelers choose route r

$P_{r,fp}^n$ = r route chosen probability predicted by traffic agency

$P_{r,e}^n$ =route r chosen probability.

B. Jacobian Matrix

The Jacobian matrix is

$$\mathbf{J}_{\psi}^{\mathbf{n}} = \left|\begin{array}{ccc|c} C_p^n & C_{fp}^n & P_e^n & \\ (1-\lambda)(1-\delta)\mathbf{I} & (1-\lambda')\delta\mathbf{J}_{1,2} & (1-\delta)\lambda\mathbf{J}_{2,3} + \lambda'\delta\mathbf{J}_{1,2}\mathbf{J}_{2,3} & \mathbf{C}_p^{n+1} \\ \mathbf{0} & (1-\lambda')\mathbf{I} & \lambda'\mathbf{J}_{2,3} & \mathbf{C}_{fp}^{n} \\ (1-\lambda)(1-\delta)\mathbf{J}_{3,1} & (1-\lambda')\delta\mathbf{J}_{3,1}\mathbf{J}_{1,2} & (\mathbf{P}_p\mathbf{E}\chi + \mathbf{I} - \chi) + \mathbf{J}_{3,1}\mathbf{J}_{1,3} & \mathbf{P}_{\mathbf{e}}^{n+1} \end{array}\right. \tag{B.1}$$

where

$$\begin{aligned} \mathbf{J}_{1,2} &= \Lambda^{\mathrm{T}}\mathbf{J}_{\mathbf{c}}\left(C_f\right)\Lambda\mathbf{Q}\mathbf{J}_{\mathbf{p}}\left(C_{fp}, \theta'\right), \\ \mathbf{J}_{2,3} &= \Lambda^{\mathrm{T}}\mathbf{J}_{\mathbf{c}}\left(C_{fp}\right)\Lambda\mathbf{Q}, \\ \mathbf{J}_{3,1} &= \mathbf{P}_{\mathbf{p}}\mathbf{E}\mathbf{P}_{\mathbf{e}}\mathbf{J}_{\chi} - \mathbf{P}_{\mathbf{e}}\mathbf{J}_{\chi} + \mathrm{Z}\mathbf{J}_{\mathbf{p}}\left(C_p, \theta\right), \end{aligned} \tag{B.2}$$

$$\mathbf{E} = \left|\begin{array}{ccc} \underbrace{\begin{matrix} 1 & \cdots & 1 \\ \cdots & \cdots & \cdots \\ 1 & \cdots & 1 \end{matrix}}_{R_1} & 0 & 0 \\ \cdots & \cdots & \cdots \\ 0 & 0 & \underbrace{\begin{matrix} 1 & \cdots & 1 \\ \cdots & \cdots & \cdots \\ 1 & \cdots & 1 \end{matrix}}_{R_w} \end{array}\right|; \tag{B.3}$$

$$\mathbf{Z} = \left|\begin{array}{ccc} \underbrace{\begin{matrix} \chi_1 P_{e,1} + \cdots + \chi_{R_1} P_{e,R_1} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \chi_1 P_{e,1} + \cdots + \alpha_{R_1} P_{e,R_1} \end{matrix}}_{R_1} & \mathbf{0} & \mathbf{0} \\ \cdots & \cdots & \cdots \\ \mathbf{0} & \mathbf{0} & \underbrace{\begin{matrix} \chi_{e,R_{w-1}+1} P_{e,R_{w-1}+1} + \cdots + \chi_{R_w} P_{e,R_w} & \cdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \cdots & \chi_{R_{w-1}+1} P_{e,R_{w-1}+1} + \cdots + \chi_{R_w} P_{e,R_w} \end{matrix}}_{R_w} \end{array}\right|. \tag{B.4}$$

Data Availability

All data, models, and code generated or used during the study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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