Research Article

On Face Index of Silicon Carbides

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1.Introduction

Several invariants assigning a matrix, a polynomial, a numeric number, or a sequence (of numbers) to a graph have been defined and studied during the last few decades. One such significant class among such invariants is the class of topological indices (TIs), which assigns a number to the given graph. The value of a TI of a molecular structure is dependent on its shape, size, symmetry, patterns of the bonds, and the contents of heteroatoms in it. Consequently, the notion of the TI provides the quantitative characterization of the molecular structures. Several researchers have studied different aspects and applications of the TIs. For contents related to studies on lower/upper bounds maxima/minima of TIs, see [1–5], studies on chemistry and drugs see [6–10], and other applications, see [11–13]. Another vital contribution of the study of the TIs is their effectiveness in studying the different aspects of new drugs and chemical compounds, which is an immense need of the medical science and industry. For details, we refer [14, 15] to the readers. Consequently, computing and studying the behavior of the values of the TIs of the molecular structures provide significant information and hence is one of the trends in modern research.

Before proceeding further with the study of a specific index, we set the notations used in this paper. The notions of a planar graph, its faces, and an infinite face are well known in the literature. Let G be a graph with vertex set V(G), edge set E(G), and face set F(G). A face f is said to be incident with an edge e, whenever e is among the edges which surrounds f. Moreover, face is said to be incident with a vertex w, whenever w is incident to an edge e which surrounds f. The incidence of w to the face f is represented by w f. The degree of a face f in G is given as d(f) = \sum w f d(w). For the notions and notations not given here, we refer [16] to the readers. Recently, Jamil et al. [17] introduced a novel topological index named as the face index. They showed that the face index can help to predict the boiling points and the energy of selected benzenoid hydrocarbons with the correlation coefficient r > 0.99. For a planar graph G, the face index (FI) can be defined as

\[ FI(G) = \sum_{f \in F(G)} d(f) = \sum_{w \in V(G)} \sum_{e \in E(G)} \sum_{f \in F(G)} \delta(e, f) \]

where \( \delta(e, f) \) is the multiplicity of the edge e in the face f.
Si which we are going to study in this paper, such as applications [20–23]. Here are several silicon carbides well as high temperature and radiation tolerant electronic centers, solar cells, wind turbines, and electric vehicles, as power savings as rectifiers or switches in the system for data a vital role in power industries by setting new principles in the most extensively used wide bandgap materials, performs incredibly uncommon mineral moissanite. SiC being one of chemical stability of SiC [18, 19]. SiC occurs in nature as the strength. Hisisthebaseofextremelyhighmechanicallyand very short bound length and hence a very high bound tetrahedrally oriented molecules of C and Si atoms, with a and Si atoms, typically in biatomic layers. These layers form foremost material that contained the covalent bound of C with the face f.

On the contrary, the Silicon Carbide (SiC) is the first and foremost material that contained the covalent bound of C and Si atoms, typically in biatomic layers. These layers form tetrahedrally oriented molecules of C and Si atoms, with a very short bound length and hence a very high bound strength. This is the base of extremely high mechanically and chemical stability of SiC [18, 19]. SiC occurs in nature as the incredibly uncommon mineral moissanite. SiC being one of the most extensively used wide bandgap materials, performs a vital role in power industries by setting new principles in power savings as rectifiers or switches in the system for data centers, solar cells, wind turbines, and electric vehicles, as well as high temperature and radiation tolerant electronic applications [20–23]. There are several silicon carbides which we are going to study in this paper, such as Si$_2$C$_3$ – I[a, b], Si$_2$C$_3$ – II[a, b], Si$_2$C$_3$ – III[a, b], and SiC$_3$ – III[a, b]. Several papers have been devoted to the study of silicon, carbon-based structures, for details, see [24–26]. The main objective of this paper is to find the analytic formula of the face index of these silicon carbides. Moreover, we also present the graphical analysis of the obtained results. For this, we use the Chemsketch for plotting the figures of silicon carbides, Maple for calculations, and MATLAB for graphical analysis.

Before presenting the results, we include Euler’s formula for planar graphs. Evidently, we may observe that the silicon structures presented in Section 2 are consistent with this formula.

\[
\text{FI}(G) = \sum_{f \in F(G)} d(f) = \sum_{w \sim f \in G} d_w,
\]

where $w \sim f$ represents the incidency of the vertex $w$ with the face $f$.

In this section, we investigate the exact formulas of the face index for Si$_2$C$_3$ – I[a, b], Si$_2$C$_3$ – II[a, b], Si$_2$C$_3$ – III[a, b], and SiC$_3$ – III[a, b]. To find the face indices of the molecular graphs, we partitioned the face set depending on the degrees of each face.

**Theorem 1.** For a finite, connected, and planar graph $G$ with vertex set $V(G)$, edge set $E(G)$, and face set $F(G)$, we have $|V(G)| - |E(G)| + |F(G)| = 2.$

**2. Results**

In this section, we investigate the exact formulas of the face index for Si$_2$C$_3$ – I[a, b] and Graphical Representations. The molecular graphs of silicon carbides Si$_2$C$_3$ – I[a, b] are given in Figure 1. Figure 1(a) consists of unit cell of the silicon carbide Si$_2$C$_3$ – I[a, b] and Si$_2$C$_3$ – I[a, b] for $a = 1$ and $b = 1$. Figure 1(b) is Si$_2$C$_3$ – I[a, b] for $a = 3$ and $b = 4$.

**Theorem 2.** Let $G = \text{Si}_2\text{C}_3 - \text{I}[a,b]$, where $a, b \geq 1$. Then,

\[
\text{FI}(G) = \begin{cases} 
70a - 34, & \text{if } b = 1 \\
90ab - 20a - 30b - 4, & \text{if } a, b \neq 1 \\
18(3b - 1), & \text{if } a = 1.
\end{cases}
\]

**Proof.** We consider the following three cases:

Case-I If $b = 1$, let $f_j$ and $|\{f_j\}|$ denote the face with the property $\sum_{w \sim f_j} d_w = j$ and the number of such faces, respectively. The structure Si$_2$C$_3$ – I[a, 1] contains four types of internal faces $f_{14}$, $f_{16}$, $f_{18}$, and $f_{20}$ and an external face, $f_{\infty}$. So, the face index of Si$_2$C$_3$-I[a, 1] is
Case II. When \( a, b \neq 1 \), then, we notice that the structure \( S_2 C_3 - I[1,a,b] \) contains seven types of faces, which are \( f_{14}, f_{15}, f_{16}, f_{18}, f_{19}, f_{20}, \) and \( f_{21} \) and an external face, \( f_{\infty} \). Moreover, the sum of vertex degrees of external face is \( 20a + 30(b-1) \). In each row of Table 1, the number of internal faces is written.

The face index of \( S_2 C_3 - I[a,b] \) is

\[
\text{Table 1: Numbers of } f_{14}, f_{15}, f_{16}, f_{18}, f_{19}, f_{20}, \text{ and } f_{21} \text{ with given number of rows.}
\]

| Rows | \( |f_{14}| \) | \( |f_{15}| \) | \( |f_{16}| \) | \( |f_{18}| \) | \( |f_{19}| \) | \( |f_{20}| \) | \( |f_{21}| \) |
|------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 2    | \( a+1 \)   | \( 2a+3 \)  | \( a-1 \)   | \( a+1 \)   | 2           | \( a-2 \)   | \( 2a-3 \)  |
| 3    | \( a+3 \)   | \( 2(2a-3) \) | \( a-1 \)   | \( 2a \)    | 4           | \( a-2 \)   | \( 2(2a-3) \) |
| 4    | \( a+5 \)   | \( 3(2a-3) \) | \( a-1 \)   | \( 3a-1 \)  | 6           | \( a-2 \)   | \( 3(2a-3) \) |
|      |             |             |             |             |             |             |             |
|      |             |             |             |             |             |             |             |
|      |             |             |             |             |             |             |             |
| \( B \) | \( a+2b-3 \) | \( b-1 \) | \( 2a-3 \)  | \( a-1 \)   | \( ab-a-b+3 \) | \( 2(b-1) \) | \( a-2 \)   |

\[
\text{Figure 2: FI}(S_2 C_3 - I[a,1]), \text{FI}(S_2 C_3 - I[a,b]), \text{and} \text{FI}(S_2 C_3 - I[1,b]), \text{respectively.}
\]

\[
\text{Table 2: Numbers of } f_{16}, f_{17}, \text{ and } f_{18} \text{ with given number of rows.}
\]

| No. of rows | \( |f_{16}| \) | \( |f_{17}| \) | \( |f_{18}| \) |
|-------------|-------------|-------------|-------------|
| 2           | \( 2a \)    | \( 2a \)    | \( 3a-5 \)  |
| 3           | \( 2a+2 \)  | \( 2a+2 \)  | \( 8a-12 \) |
| 4           | \( 2a+4 \)  | \( 2a+4 \)  | \( 13a-19 \)|
|             |             |             |             |
|             |             |             |             |
|             |             |             |             |
| \( B \)     | \( 2(a+b)-4 \) | \( 2(a+b)-4 \) | \( 5ab-7(a+b)+9 \) |

The rest follows from the definition of the face index.
Case-III When \( a = 1 \), the structure \( Si_2C_3 - I[a, b] \) contains three types of internal faces \( f_{13}, f_{16} \) and \( f_{17} \) and an external face \( f_{\infty} \) with sum of degrees \( 24b - 4 \). Moreover, \( |f_{13}| = b - 1, |f_{16}| = 1, \) and \( |f_{17}| = b - 1 \). So, the face index of \( Si_2C_3 - I[a, b] \) is

\[
FI(G) = \sum_{w \in \text{Face}(G)} d_w
\]

\[
= \sum_{w \in f_{13}} d_w + \sum_{w \in f_{16}} d_w + \sum_{w \in f_{17}} d_w + \sum_{w \in f_{\infty}} d_w
\]

\[
= |f_{13}|(13) + |f_{16}|(16) + |f_{17}|(17) + 24b - 4
\]

\[
= 13(b - 1) + 16(1) + 17(b - 1) + 24b - 4
\]

\[
= 18(3b - 1),
\]

which completes the proof.

Remark 1. It is interesting to observe that \(|V(Si_2C_3 - I[a, b])| = 10ab, \ |E(Si_2C_3 - I[a, b])| = 15ab - 2a - 3b, \) and \(|F(Si_2C_3 - I[a, b])| = 5ab - 2a - 3b + 2 \). Hence, the structure \( Si_2C_3 - I[a, b] \) satisfies the Euler formula.

Now, we present the results obtained for \( Si_2C_3 - I[a, b] \) in a graphical way in Figure 2. Every silicon carbide structure depends on two variables \( a \) and \( b \). We use two kinds of graphs to show the outcomes of face index. One is the 2D graph where the reliance of a face index is drawn against one variable of the structure \( a \) or \( b \) while other is kept fixed. Here, we demonstrate 2D graphs for \( b = 1 \) and change \( a \), and for \( a = 1 \) and change \( b \). The second tool is the 3D graph where reliance of a face index is drawn against both parameters, and we gained a surface that tells about the trends of face index against both parameters \( a \) and \( b \) at the same time.

2.2. Face Index for \( Si_2C_3 - II[a, b] \) and Graphical Representations. The molecular graphs of silicon carbides \( Si_2C_3 - II[a, b] \) are given in Figure 3, which consists of unit cell of silicon carbide \( Si_2C_3 - II[a, b] \) and \( Si_2C_3 - II[a, b] \) for \( a = 4 \) and \( b = 1 \).

**Theorem 3.** Let \( H = Si_2C_3 - II[a, b] \), where \( a, b \geq 1 \). Then, the face index of \( H \) is given as

\[
FI(H) = \begin{cases} 
18(3a - 2), & \text{if } b = 1, \\
90ab - 30(a + b) - 15, & \text{if } a, b \neq 1, \\
18(3b - 2), & \text{if } a = 1.
\end{cases}
\]

**Proof.** We consider the following three cases.

Case-I If \( b = 1 \), then the graph of \( Si_2C_3 - II[a, 1] \) consists of two faces, an internal face \( f_{15} \) with \( |f_{15}| = 2a - 2 \) and an external face, \( f_{\infty} \) having the sum of vertex degrees \( 24a - 6 \). The rest follows from the definition of the face index.

Case-II When \( a, b \neq 1 \), then, we notice that the structure \( Si_2C_3 - II[a, b] \) contains three types of internal faces, which are \( f_{16}, f_{17}, \) and \( f_{18} \) and an external face, \( f_{\infty} \). Moreover, the sum of vertex degrees of external face is \( 30(a + b) - 45 \). The number of internal faces in each row is given in Table 2.
Case-III When $a = 1$, the structure $Si_2C_3 - II[a, b]$ contains an internal face $f_{15}$ and an external face $f_{\infty}$ with sum of degrees $24b - 6$. Moreover, $|f_{15}| = 2b - 2$.

The rest follows from the definition of the face index.

Remark 2. It is interesting to observe that $|V(Si_2C_3 - II[a, b])| = 10ab$, $|E(Si_2C_3 - II[a, b])| = 15ab - 2a - 3b$, and $|F(Si_2C_3 - II[a, b])| = 5ab - 2a - 3b + 2$. Hence, the structure $Si_2C_3 - II[a, b]$ satisfies the Euler formula.

With the same settings as in Section 2.1, we present the results obtained for $Si_2C_3 - II[a, b]$ in graphical way in Figure 4.

2.3. Face Index for $SiC_3 - III[a, b]$ and Graphical Representations. The molecular graphs of silicon carbides $SiC_3 - III[a, b]$ are given in Figures 5 and 6. Figure 5(a) is for $a = 5$ and $b = 1$, Figure 5(b) is for $a = 5$ and $b = 2$, and Figure 6 consists of unit cell and $SiC_3 - III[a, b]$ for $a = 5$ and $b = 5$.

Theorem 4. Let $K = SiC_3 - III[a, b]$, where $a, b \geq 1$. Then,

$$FI(K) = \begin{cases} 32a - 18; & \text{for } b = 1 \\ 72ab - 30a - 20b - 7; & \text{for } b > 1. \end{cases}$$

Proof. When $b = 1$, then the graph of $SiC_3 - III[a, 1]$ consists of two faces, an internal face $f_{14}$ with $|f_{14}| = a - 1$, and

| Rows | $|f_{13}|$ | $|f_{15}|$ | $|f_{17}|$ | $|f_{18}|$ | $|f_{20}|$ | $|f_{22}|$ | $|f_{24}|$ |
|------|---------|---------|---------|---------|---------|---------|---------|
| 2    | 1       | 3(a-1)  | a-1     | —       | 1       | a-1     | —       |
| 3    | 2       | 5(a-1)  | a-1     | a-1     | 1       | A       | a-1     |
| 4    | 3       | 7(a-1)  | a-1     | 2(a-1)  | 1       | a+1     | 2(a-1)  |
| B    | b-1     | (2b-1)  | a-1     | (b-2)   | a-1     | a+b-3   | (b-2)   |

The rest follows from the definition of the face index.
an external face, \( f_\infty \) having the sum of degrees \( 18a - 4 \). The rest follows from the definition of the face index. Now, when \( b > 1 \), then we notice that the structure \( SiC_3 - III[a, b] \) contains seven types of internal faces which are \( f_{13}, f_{15}, f_{17}, f_{18}, f_{20}, f_{22}, \) and \( f_{24} \) and an external face, \( f_\infty \). Moreover, the sum of vertex degrees of external face is \( 30a + 17b - 30 \).

In each row of Table 3, the number of internal faces is written.

**Remark 3.** It is interesting to observe that 
\[
|V(SiC_3 - III[a, b])| = 8ab, \quad |E(SiC_3 - III[a, b])| = 12ab - 3a - 2b, \quad \text{ and } \quad |F(SiC_3 - III[a, b])| = 4ab - 3a - 2b + 2.
\]

Hence, the structure \( SiC_3 - III[a, b] \) satisfies the Euler formula.

With the same settings as in Section 2.1, we present the results obtained for \( SiC_3 - III[a, b] \) in a graphical way in Figure 7.

**2.4. Face Index for \( SiC_3 - III[a, b] \) and Graphical Representations.** The molecular graphs of silicon carbides \( SiC_3 - III[a, b] \) are given in Figure 8.

**Theorem 5.** Let \( L = SiC_3 - III[a, b] \), where \( a, b \geq 1 \). Then,
\[
Fi(L) = \begin{cases} 
70a - 32, & \text{for } b = 1, \\
90ab - 20a - 30b - 2, & \text{for } b > 1.
\end{cases}
\]
Hence, the structure in row of Table 4, the number of internal faces is written. Moreover, the values of \( a |_{20} \) consists of four types of faces, three internal faces and an external face, \( f \) and \( f \), and an external face, \( f \). Our results obtained for \( \text{SiC}_3 - \text{III}[a, b] \) contains eight types of internal faces, which are \( f_{14}, f_{15}, f_{17}, f_{18}, f_{21}, f_{22}, f_{23} \), and \( f_{24} \) and an external face, \( f_{20} \). Moreover, the sum of vertex degrees of external face is \( 20a + 24(b - 1) \). In each row of Table 4, the number of internal faces is written. 

**Proof.** When \( b = 1 \), then the graph of \( \text{SiC}_3 - \text{III}[a, 1] \) consists of four types of faces, three internal faces \( f_{14}, f_{20}, \) and \( f_{22} \) and an external face \( f_{20} \) having the sum of degrees \( 20a \). Moreover, the values of \( |f_{14}|, |f_{20}|, \) and \( |f_{22}| \) are \( 2(a - 1), 2, \) and \( (a - 2) \), respectively. The rest follows from the definition of the face index. Now, when \( b > 1 \), then we notice that the structure \( \text{SiC}_3 - \text{III}[a, b] \) contains eight types of internal faces, which are \( f_{14}, f_{15}, f_{17}, f_{18}, f_{21}, f_{22}, f_{23}, \) and \( f_{24} \) and an external face, \( f_{20} \). Moreover, the sum of vertex degrees of external face is \( 20a + 24(b - 1) \). In each row of Table 4, the number of internal faces is written.

**Remark 4.** It is interesting to observe that \( |V(\text{SiC}_3 - \text{III}[a, b])| = 10ab, |E(\text{SiC}_3 - \text{III}[a, b])| = 15ab - 2a - 3b, \) and \( |F(\text{SiC}_3 - \text{III}[a, b])| = 5ab - 32 - 3b + 2 \). Hence, the structure \( \text{SiC}_3 - \text{III}[a, b] \) satisfies the Euler formula.

With the same settings as in Section 2.1, we present the results obtained for \( \text{SiC}_3 - \text{III}[a, b] \) in a graphical way in Figure 9.

### 3. Conclusions

An ongoing direction in mathematical and computational chemistry is the assessment of various properties of molecular structures with the help of numerical graph descriptors. These invariants have also established the feasible applications in QSAR/QSPR studies, which are beneficial for the new molecular designs, drug discoveries, and hazard estimation of chemicals. Hence, the novel index, named as the face index for the molecular graphs of silicon carbides, is presented through the numerical way, and we have also displayed our numeric outcomes in the graphical way. Our consequences could be applicable in assessing and comparing the properties of these molecular structures.

### Data Availability

The data used to support the findings of the study are included within the article.

### Conflicts of Interest

The authors declare no conflicts of interest.

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