Research Article

Sustainable and Optimal “Uniqueness” Contract in Public-Private Partnership Projects of Transportation Infrastructure

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To control the “uniqueness” risk in Public-Private Partnership (PPP) projects of transportation infrastructure, we design a simplified “uniqueness” contract model by incorporating the impact of the initial investment which is based on the Bertrand model. The nonlinear programming method is adopted to derive the optimal “uniqueness” contracts for incumbent private capital, the public, and the social welfare, respectively. The simulation results show that the achievement of the optimal “uniqueness” contract is essentially the result of a compromise between the private capital, the public, and social welfare. The extent to which such a contract reduces the probability of “uniqueness” risk mainly depends on the equilibrium relation between the interests of private capital and the public. The initial investment is not related to the government default when the contract does not take into account the interests of the private capital. Furthermore, the “uniqueness” contracts between private capital and the government are mainly for anticompetitive purpose in the PPP market of transportation infrastructure. Unless the contract terms focus on the improvement of social welfare, entering a “uniqueness” contract will cause social welfare losses.

1. Introduction

Due to the long investment period and large capital expenditures associated with public-private partnership (PPP) projects in transportation infrastructure, participants face complex risky conditions that may hinder the delivery of sustainable projects [1]. The “uniqueness” risk is a special critical risk in PPP projects of transportation infrastructure that adopts a user-paid model. It occurs when there is a new similar project or the government and other investors rebuild the project [2]. The risk of “uniqueness” stems from the conflict of interest between the government and private capital. Although both government and private capital can use the user-paid model to operate transportation infrastructure, the government aims to maximize the total social welfare, while private partners strive to maximize their profits [3]. These two goals are contradictory and cannot be achieved at the same time. For example, a profitable toll road does not necessarily increase the total social welfare; however, a project that maximizes social welfare may suffer losses [4]. Therefore, to increase the output of transportation services, the government is motivated to build new transportation infrastructure near the existing transportation infrastructure operated by other private partners to improve social welfare. But, the impact on the existing PPP projects is usually severe. It will not only reduce the demand for existing transportation infrastructure projects but also cause the project to fail and or become a waste of resources. The results will also discourage other private capitals to participate in PPP projects. “Uniqueness” risk events usually lead to a series of subsequent risks, such as demand risk, income risk, and credit risk [5]. Since the emergence of the PPP model in the twentieth century, there have been many failures in the PPP construction projects of transportation infrastructure due to the outbreak of “uniqueness” risks (the Quanzhou Citong Bridge BOT project was the first
transportation infrastructure project in China to adopt the PPP model, and it was terminated early due to a “uniqueness” risk [6]).

If there is no restriction on the behavior of build new transportation infrastructure by the government, the “uniqueness” risk is wholly borne by the private capital. The government can decide to build a new infrastructure to increase social welfare at any time. However, the private capital will lose the whole market by the competition from the newly built transportation infrastructure. Since the private capital can predict the risks, many projects may fail bids due to the excessive “uniqueness” risk borne by the private capital. To attract the appropriate private capitals to participate in the PPP projects, the government has the motivation to share the “uniqueness” risk with the private capital. To reduce the probability of “uniqueness” risk and reduce the loss caused by the risk to an acceptable level, the private capital participating in the PPP project demands restricting the government from building new competitive transportation infrastructure during the concession period in the form of a contract.

In many countries, governments are often restricted in constructing “unnecessary competitive projects” by law to encourage private capitals to participate in the PPP projects. For example, the pertinent “PPP Project Contract Guide” issued by the Ministry of Finance in China states that “In a project that adopts the user-paid model, the project company needs to charge the project users to generate income from the investment. Thus, it must ensure that there are enough users to use the project facilities and pay the fees” [7]. In view of this, the government is usually obliged to prevent unnecessary competition projects, the so-called “uniqueness” clause in PPP contracts. When the “uniqueness” contract is signed, part of the “uniqueness” risk is transferred to the government. If the government defaults and builds a new competitive transportation infrastructure, a compensation for the liquidation should be paid to the private capital providers by the government. At this time, the probability of “uniqueness” risk can be computed as the probability of default by the government.

Essentially, an exclusive contract per se, the “uniqueness” contract sets up barriers to entry to prevent new competitors from entering the existing market. There are a number of studies on exclusive transactions. Most of them focused on the challenge that unless the project is sufficiently efficient, the current seller will not be able to pay the buyer enough to accept the exclusive contract [8–12]. The consequences of exclusive contracts are usually very complicated. Whether an exclusive contract is relevant, anticompetitive, or enhancing efficiency and social welfare depends on the characteristics of the market. In the PPP relationship, both parties have the motivation to sign a “uniqueness” contract. The private capital is concerned about the extent to which the “uniqueness” contract it signs with the government can lock the two parties in a “uniqueness” relationship. The government expects to increase the output of transportation services and improve social welfare by signing a “uniqueness” contract, in addition to enhancing the government’s reputation and attracting more private capitals to participate in PPP projects in the region.

To control the “uniqueness” risk in the PPP of transportation infrastructure, we adopt the nonlinear programming method to construct four models and simulate the optimal “uniqueness” contract. The nonlinear programming method has two advantages. One is to help identify whether the optimal contract solution exists and the number of solutions. The other is that we can find out an optimal numerical solution by the nonlinear programming method which can provide direct support for the decision-making of both the government and the private capital. Although a clear optimal numerical solution of a “uniqueness” contract may be difficult to achieve in the current institutional environment, it sets goals for the government and private capital. The government is encouraged to improve the institutional environment to reach a contract close to the optimal solution with the private capital and enable it to be effectively implemented.

In this paper, we first find that the effect of “uniqueness” risk depends on the contract objective. The optimal contract of incumbent private capital and the optimal contract of the public can significantly reduce the probability of government default, while the optimal contract of social welfare is weakly related to government default. Among them, the optimal contract of incumbent private capital reduces the risk probability and increases the private capital return the most significant, followed by the optimal contract of the public. The optimal contract after weighting can take into account the objectives of incumbent private capital, the public, and social welfare and can achieve the “uniqueness” risk control objective.

Furthermore, we show that the barrier effect of initial investment depends on whether the contract takes into account the interests of the incumbent private capital. Under the optimal “uniqueness” contract of incumbent private capital, with the increase of the initial investment cost, the “uniqueness” risk probability shows a unilateral downward trend, showing a barrier effect. Under the optimal “uniqueness” contract of the public, the “uniqueness” risk probability does not change with the initial investment cost. Under the optimal “uniqueness” contract of social welfare, as the initial investment cost increases, the “uniqueness” risk probability presents a temporary increase in the initial stage. This shows that when the “uniqueness” contract clause fully considers the benefits of the incumbent private capital, the “uniqueness” risk has a negative co-relationship with the initial investment. To be specific, as the initial investment cost increases, the “uniqueness” risk probability appears downtrend. The more the “uniqueness” contract conforms to the interests of private capital, the stronger the correlation.

Our results indicate that an optimal “uniqueness” contract in practice is a compromise result of multiple objectives. Although social welfare is an important aspect that needs to be taken into account in the PPP projects, maximizing social welfare will harm the construction and
operation of the projects. Project risk will be increased. The interests of the public and incumbent private capital are directly related to the project; they are more concerned about the signing of a “uniqueness” contract. They have more motivation to lobby the government. Therefore, the optimal “uniqueness” contract should simultaneously encourage the incumbent private capital to do a good job in the construction and operation of the project, protect the interests of the public, and improve the total social welfare by striving to make the cake bigger. The “uniqueness” contract programming model after weighting can take into account the objectives of all three stakeholders while improving the welfare of all parties, while effectively controlling the “uniqueness” risks, and enhance project sustainability.

Our work not only makes contributions to the academic literature but also to the policy debate on preventing competitive projects. Policymakers have concerns that the law preventing competitive projects may induce private capitals to set up anticompetitive practices as a protection against potential competitors in the PPP market. We find that the incumbent private capital signs the “uniqueness” contract with the government at the cost of lowering tolls. In the short term, this will increase the benefit of the public and improve the social welfare. But, in the long run, the “uniqueness” contract will prevent more efficient private capitals from entering the PPP market of transportation infrastructure. This will deter the improvement of social welfare. This is very valuable for the government to formulate policies on PPP projects of transportation infrastructure.

2. Literature Review

The “uniqueness” contract is essentially an exclusive contract. Aghion and Bolton [8] established the analysis framework of “one seller and one buyer” and found that current sellers facing the threat of entering the market are willing to sign an exclusive contract with the buyer, even if the contract does not completely prevent low-cost sellers from entering. In addition, when the seller has the information advantage of the possibility of entry, the duration of the exclusive contract can be used as a signal of the true possibility of entry. Rasmussen et al. [9] extended the model of Aghion and Bolton and found that when a monopolist needs multiple buyers to pay its fixed costs, the incumbent monopolist only needs to lock in some customers to effectively prevent potential competitors from entering the market. Bernheim and Whinston [10] found that exclusive transactions may not matter, anticompetitive, or improve efficiency depending on the setting. Bedre-Defolie and Biglaiser [11] studied the market of noncritical buyers and found that whether new entrants are more efficient or consumers’ default amounts are high or low, signing a “uniqueness” contract can effectively prevent potential competitors from entering. This study shows that prohibiting exclusive transactions will improve social welfare, unless incumbents and entrants have similar inefficiency. Kitamura et al. [12] incorporated Nash bargaining into the “one buyer and seller” model and proved that the low efficient existing suppliers can prevent effective suppliers from entering the society through exclusive contracts. Zhu and Berry [13] analyzed the wireless service market under the background of greater unlicensed access to spectrum and found that incumbents will always offer an exclusive contract to customers. The expected social welfare may increase or decrease depending on the demand and technology. Ultsaker [14] studied the downstream market where differentiated buyers compete with each other and proved that the seller can prevent the entry of potential competitors, even when the entry would increase the industry profit.

To verify the theoretical expectations of exclusive dealings, some papers have conducted research using empirical data. Sass [15] analyzed the company-level data of nearly 400 beer distributors in the United States and investigated the impact of economies of scale and restricted promotional activities on exclusive dealings, as well as the impact of exclusive dealings on cost, pricing, and output. The results support the view that exclusive dealings can minimize conflicts between manufacturers and distributors and improve social welfare. Askier [16] assessed the impact of exclusive distribution arrangements on competition in the Chicago beer market in 1994, and found no anticompetitive effects. Ater [17] found exclusive contracts would reduce sales and did not comply with the efficiency principle based on the evidence of the exclusive dealings between burger restaurants and large shopping malls in Israeli market. As a result, shopping malls should use exclusive contracts for anticompetitive purposes. Nurski and Verboven [18] studied whether exclusive contracts between manufacturers and retailers constitute barriers to entry. Through modeling and empirical analysis using data from the European auto market, they found that there is no unilateral motive for maintaining monopoly transactions but collective motives for the entire industry. The widespread use of exclusive contracts in the entire automotive industry will not have anticompetitive effects. However, consumers will benefit from a ban of exclusive dealings, mainly because of increased space availability not because of fierce price competition.

In addition, there is some experimental literature studying exclusive contracts. Consistent with the expectations of theoretical research, these studies found that the consequences of signing exclusive contracts depend on the characteristics of the market environment, including communication, price discrimination [19], sequentiality and confidentiality [20], and the number of buyers [21].

Since the government can only increase the output of transportation services and social welfare by adding new transportation infrastructure in a state of high demand, the “uniqueness” risk event can only occur when the demand for transportation services is high. The high demand refers to the state in which consumers are willing to pay higher than the upper limit set by the government [22]. Due to the high demand, the monopoly nature of the transportation service market will change, and the market will become competitive and the excess return rate will attract potential competitors to enter the market [23]. This provides the government with
an incentive to mobilize new entrants to build and operate a competitive transportation infrastructure.

In the private goods market, monopolists can maximize their profits by determining monopoly prices based on marginal revenue and marginal costs. Transportation services are quasi-public products, and the output of public services under monopoly pricing is suboptimal for social welfare. Therefore, to increase the output of transportation services and improve social welfare, the government will place restrictions on transportation pricing or private capital’s return on capital, so that private capital only can obtain an “acceptable” project profit. This “acceptable” profit should be lower than the monopoly profit. This is the trigger for the “uniqueness” risk. If the rate of return obtained by the private capital by providing public services is too high, it will reduce the social welfare generated by the project and cause government dissatisfaction and public opposition. The direct consequence of government dissatisfaction and public opposition is the occurrence of “uniqueness” risk events. Anecdotal evidence from the previous project supports this inference: when the Hangzhou Bay Bridge project in China has been under construction for less than two years and the Shaoxing Hangzhou Bay Bridge in Shangyu which is only about 50 kilometers apart has been stepping up preparations. One of the reasons may be due to the local government dissatisfaction with the high rate of return on the Hangzhou Bay Bridge. In the case of Xinyuan Minjiang No. 4 Bridge in China, although the local government and private capital agreed on a return rate of 18% in the contract, the government later believed that the agreed return rate was too high. As a result, the local government built the third phase of the Second Ring Road. A large number of vehicles bypassed the toll station of Xinyuan Minjiang No. 4 Bridge. Private capital’s income shrank sharply. At last, the government failed to fulfill its promise to compensate for insufficient private capital income. The project was hopeless to recover the investment cost [24]. In addition, the Shenzhen Wutong channel in China has also attracted dissatisfaction from the government and public opposition because of the high rate of return of private capital [25].

In PPP projects, there are three important stakeholders: private capital, government, and the public. Private capital is the provider of transportation services with the goal to maximize profits. The government is the purchaser of PPP services. During the tender period, the government negotiates with private capital on behalf of the public.

The government has established a principal-agent relationship with the public [26]. As an agent, the government should first satisfy the public interest. The government pursues public interest as its first goal, which maximizes consumer surplus [27]. In addition, as the government is the administrator of the society, the government needs to consider the impact of PPP projects on society. Social welfare should be one of the goals that the government should consider at the same time. The public is the principal in the PPP service procurement relationship, the end-user of transportation services, and one of the most important stakeholders of the project. The public’s interest claims are achieved through the government. When the public’s interest is harmed, the public will press the government to take actions to protect its rights through litigation, lobbying, and other means.

The “uniqueness” contract signed by the government and the incumbent private capital is based on the agreement of toll pricing and liquidated damage. The simplified expression of the “uniqueness” contract is \( \{ P, P_0 \} \). \( P \) is the toll pricing agreed by both parties and \( P_0 \) is the liquidated damage parameter. Liquidated damage is the compensation that the government needs to pay to the incumbent private capital for the construction of a competitive transportation infrastructure for violating the “uniqueness” contract. Let it be \( V \). The liquidated damage \( V \) is a function of \( P \) and \( P_0 \). Suppose that \( V \) is equal to the benefit to consumers that the toll pricing is reduced from \( P \) to new equilibrium price \( P_e \) due to the additional transportation service supply by competitive transportation infrastructure:

\[
V = P_0 \cdot \left( D(P) + D(P_e) \right) \frac{1}{2}.
\]

3. Model

3.1. Model Assumptions

3.1.1. Pricing Constraints and Government Default Probability. Assume that private capital invests in the construction of transportation infrastructure in accordance with the PPP contract and the investment cost is compensated by charging users tolls. The maximum traffic volume in the concession period is 100 units, and the upper limit of the price that the public wishes to pay is 1. Let \( Q \) be the demand and \( P \) be the toll pricing, then the inverse demand function is

\[
P = 1 - \frac{Q}{100}.
\]

Let the incumbent private capital’s service cost per standard traffic volume be \( C_e \), the depreciation cost per standard traffic volume be \( \alpha \), and the operating expense per standard traffic volume be \( \beta \), then \( C = \alpha + \beta \). Suppose that the workload method is acceptable by GAAP (generally accepted accounting principles), and it is used to depreciate the transportation infrastructure assets. The unit depreciation amount \( \alpha \) is a fixed value under the accounting method, and the unit operating expense \( \beta \) is generally considered to be stable. So, the unit cost \( C \) is also a fixed value.

Assume that the unit cost of private capital is \( C = (1/2) \). Private capital faces the entry risk of potential competitors, and the unit cost \( C_e \) of potential competitors is unknown. Assume that the unit cost \( C_e \) of potential competitors follows the average distribution among \( (0, 1) \). If a potential competitor decides to enter the market and construct and operate a new competitive transportation infrastructure, it will provide the public with transportation services of the same quality as the incumbent private capital. Since the incumbent infrastructure and the new competitive infrastructure are usually very close in space, coupled with the
provision of transportation services of the same quality, it can be considered that the transportation services provided by the two are homogeneous. This makes the competition between incumbent private capital and the new entrant comply with the Bertrand model. In the Bertrand model, oligarchs producing homogeneous products will compete with price as the decision variable. Since the products of the two oligarchs are completely replaceable, the oligarch pricing the product lower will win the entire market. The one pricing higher will not get any profit at all. This is highly consistent with the transportation market investigated in this paper (in fact, when there are two highly competitive transportation infrastructures, the one with the lower toll price tends to occupy most of the traffic flow, for example, there are four bridges across the Yangtze River in Nanjing, China; only the Nanjing Yangtze River Bridge is free of charge; this has caused most drivers to choose the free but congested Nanjing Yangtze River Bridge; for other bridges, few vehicles are passing due to tolls). According to the Bertrand model, the incumbent private capital will compete with potential competitors in price at this time. The Bertrand equilibrium price \( P_e \) can be defined as

\[
P_e = \max \left\{ \frac{1}{2} C_e \right\}, \tag{3}
\]

If \( C_e = (1/2) \), the entrant is unprofitable. Therefore, for potential competitors, only when their unit cost \( C_e \leq (1/2) \), they will really decide to enter the market. Therefore, the probability that potential competitors agree to the government’s offer and decide to enter the market and build a competitive transportation infrastructure is 50%. Let it be \( \Phi \):

\[
\Phi = \Pr\left( C_e \leq \frac{1}{2} \right) = \frac{1}{2}. \tag{4}
\]

If potential competitors do not enter the market, then the incumbent private capital will operate the existing PPP projects at a monopoly pricing. As shown in Figure 1, according to MR = MC, the equilibrium point \( C \) can be obtained, where the monopoly price is \( (3/4) \) and the equilibrium traffic volume is 25. The triangle \( ABC \) becomes the benefit obtained by consumers. The consumer surplus is \( (25/8) \) by calculating the area of this triangle. According to the principal-agent relationship, the government as the principal-agent relationship, the government as the purchaser of PPP services is the direct agent of the public. Therefore, the direct benefit obtained by the government from the PPP projects should be measured by the consumer surplus obtained by the public through consuming transportation services, that is, the area of triangle \( ABC \) which is \( (25/8) \).

If a potential competitor enters the market, according to the Bertrand model, the market equilibrium price will be reduced to the unit cost of the incumbent private capital which is \( P_e = C_e = (1/2) \). And, the entrant will get the traffic flow of the entire market. Substituting the equilibrium price \( (1/2) \) into the demand function, the equilibrium point is \( E \). At this time, the marginal revenue is 0 and the equilibrium traffic flow increases from 25 to 50 units. The entrant will realize a profit \( \pi = 50 \times \left( (1/2) - C_e \right) \), and the incumbent private capital is unprofitable. The direct benefit obtained by the government can be obtained by calculating the area of the triangle \( ADE \), which is \( (25/2) \).

Therefore, if the government does not sign a “uniqueness” contract with the incumbent private capital, the government’s expected value of the direct benefit can be expressed in terms of expected consumer surplus:

\[
S_{ge} = 1 - \Phi \times \frac{25}{8} + \Phi \times \frac{25}{2} = \frac{125}{16}. \tag{5}
\]

We note that the government is motivated to invite new partners to construct a competitive transportation infrastructure project to obtain more social welfare. By having new competitors, the government’s direct benefit (i.e., consumer surplus) has increased from \( (25/8) \) to \( (25/2) \). The profit of the incumbent private capital has dropped from \( (25/4) \) directly to 0. As shown in Figure 1, the total social welfare will be increased from ADFC to ADE. This is not a Pareto improvement process. The interests of incumbent private capital have been sacrificed. Advocates of PPP believe that the advantage of PPP is that it can achieve a win-win between the government and private capital, which obviously does not occur in this case. Incumbent private capital will withdraw from the PPP relationship as the project is not profitable. The government introducing new partners will also have a negative impact on other private capital to participate in PPP projects in the region in the future. Therefore, to control the “uniqueness” risk, incumbent private capital has the motivation to sign a “uniqueness” contract with the government to prevent potential competitors from entering the market in the future.

Assume that the government is risk-neutral, the calculation of the government’s expected return \( S_{ge} \) indicates the bottom line of the government. Only by ensuring that the government can obtain at least \( S_{ge} = (125/6) \) can it sign a “uniqueness” contract with the incumbent private capital, which stipulates the cost constraints of the “uniqueness” clause.

If the government does not default, the benefit (i.e., consumer surplus) becomes

\[
S = 50 \times \left( 1 - P \right)^2. \tag{6}
\]
If the government defaults, the entrant should at least provide the benefit of $S$ to the public. Assume that the government only considers benefits, it will sign a “uniqueness” contract with the incumbent private capital when the benefits are higher than the expected benefit from the unsigned contract:

$$S \geq S_{gc}. \quad (7)$$

Substituting the formulas $S$ and $S_{gc}$ into the above equation, the constraint of price $P$ can be obtained:

$$P \leq 1 - \frac{1}{4} \sqrt{5 \cdot 2} \approx 0.6047. \quad (8)$$

The government will be attracted to sign the contract only the market equilibrium price after the entry of potential competitors $P_e$ meets the condition $P_e \leq P - P_0$. According to the Bertrand model, $P_e$ can be directly derived as follows:

$$P_e = P - P_0. \quad (9)$$

Note that when the toll price reaches the equilibrium price $P_e$, it must also ensure that the profit of the entrant is non-negative:

$$P_e - C_e \geq 0. \quad (10)$$

The probability of potential competitors entering the market after signing the “uniqueness” contract is available (i.e., the government’s probability of default). Let it be $\varnothing^*$, and the formula is expressed as follows:

$$\varnothing^* = \max\{0, P - P_0\}. \quad (11)$$

### 3.1.2. The Impact of the Initial Investment

Among the transportation infrastructure projects that adopt the PPP mode, some projects do not require an initial investment or require a less initial investment, such as ROT (Renovate-Operate-Transfer) projects, MC (Management Contract) projects, and other PPP projects that do not require construction or do not require large-scale transformation such as part of TOT (Transfer-Operate-Transfer) projects. During the concession period, no matter when private capital withdraws, there will be no fixed cost loss. However, for new construction projects or projects that require large-scale reconstruction and expansion expenditure using DBFO (Design-Build-Finance-Operate) mode, BOT (Build-Operate-Transfer) mode, etc., the private capital must assume the obligation to invest in the construction of the project infrastructure. Therefore, the project will incur huge initial investment costs, and the private capital will bear its risks. This initial investment cost can be gradually compensated by toll revenue during the concession period. Given the annual toll revenue, the higher the initial investment, the longer the compensation period.

If the government decides to build a new competitive transportation infrastructure before the initial investment cost of the incumbent project is fully compensated, the private capital will completely lose the market according to the Bertrand model. The remaining uncompensated part of the initial investment will be directly recognized as a loss. When the incumbent private capital and the government negotiate the terms of the “uniqueness” contract, the contingent losses caused by the uncompensated initial investment cost due to the government’s default must be considered.

Therefore, it is necessary to measure the impact of uncompensated initial investment on profit. Assume that all initial investments cost of the project meets the infrastructure asset recognition and measurement principles specified by GAAP. As the project begins to operate, the fixed value of transportation infrastructure assets will gradually be transferred to the cost of transportation services in the form of depreciation. Correspondingly, with the consumption of infrastructure assets, physical performance will inevitably gradually decrease. In the transportation infrastructure PPP project, to ensure the quality of transportation services and fulfill the promise of infrastructure quality at the end of the concession period, private capital should maintain and repair infrastructure assets on a regular basis. With the gradual consumption of the initial asset value, the maintenance and repair expenses to keep the assets’ physical status will gradually increase. We assume that all the maintenance and repair expenses provided by the incumbent private capital are to restore the infrastructure asset to its original physical status. Assume all maintenance and repair expenses are capitalized into the book value of infrastructure assets and the net book value of transportation infrastructure assets is always in a relatively stable status during the long concession period. Since the initial investment cost will be gradually transferred to the unit cost $C$ in the form of depreciation expenses, the proportion of depreciation of initial investment in the unit cost $C$ decreases as the accumulated traffic volume increases.

Regardless of operating expenses, suppose that the unit cost $C$ consists of two parts: the depreciation of the initial investment cost $(d)$ and the amortization of the maintenance and overhaul expenses $(a)$. Both are a function of the actual traffic volume $Q$. The unit cost $C$ can be expressed as the following formula:

$$C = d + a. \quad (12)$$

Let $d = f(Q)$. Assume that the unit cost $C$ of the first standard traffic only contains the depreciation of the initial investment cost, that is, $f(0) = C$. When the accumulated traffic volume reaches $Q^*$, the value of the initial investment will be all depreciated, that is, $f(Q^*) = 0$. As the maintenance and overhaul expenses will increase with the use of transportation infrastructure, the proportion of the depreciation of the initial investment cost will reduce. Therefore, suppose the first derivative of the depreciation function is less than 0, that is, $f' < 0$. Correspondingly, when the accumulated traffic flow $Q \leq Q^*$, the amortization function of maintenance and overhaul expenses can be expressed as

$$a = C - f(Q). \quad (13)$$

When the accumulated traffic volume $Q > Q^*$, the initial investment cost has been completely depreciated. The unit cost $C$ only includes maintenance and overhaul expenses.
Figure 2 depicts the unit cost $C$ changing with the initial investment cost and maintenance and repair expenses when the actual traffic volume accumulates. Let the initial investment cost of the project be $I$ or the area of $COQ^*$ which can be obtained by integrating $f(Q)$ in $[0, Q^*]$

$$ I = \int_0^{Q^*} f(q) dq. \quad (14) $$

Let the predicted traffic volume be $Q_0$. When the actual accumulated traffic volume $Q \leq Q^*$, the profit (or loss) $\pi$ of the incumbent private capital is

$$ \pi = \int_0^Q [P(Q_0) - C] dQ - \int_Q^{Q^*} f(q) dq. \quad (15) $$

When the actual traffic volume $Q > Q^*$, the initial investment of the incumbent private capital has been fully depreciated. So, the asset-related expenditure only includes maintenance and overhaul expenses. These expenses are highly variable in nature, so the profit (or loss) $\pi$ can be directly expressed as

$$ \pi = \int_0^Q [P(Q_0) - C] dQ. \quad (16) $$

If the actual traffic volume has not been accumulated to fully compensate the initial investment in transportation infrastructure, the incumbent private capital will lose the undepreciated part of the initial investment because of the potential competitor entering the existing market. If the actual traffic volume can fully compensate for the initial investment in transportation infrastructure, the incumbent private capital can withdraw without any loss regardless of whether a potential competitor entering the market.

Therefore, when the accumulated traffic volume is $Q$ and the government chooses to entrust the construction of a new competitive transportation project (i.e., potential competitors choose to enter the market), the profit function of the incumbent private capital is segmented. Taking into account that it takes time to construct a new transportation infrastructure, there will be a certain buffer period before the new infrastructure is put into operation, during which the incumbent private capital can still receive toll income. Let the accumulated traffic flow occurring during the buffer period be $Q'$ and the predicted traffic volume regardless of potential entry be $Q_0$, where $Q_0$ should meet the restriction condition $Q + Q' \leq Q_0$. Then, the profit function of the incumbent private capital can be expressed as

$$ \pi = \begin{cases} \int_0^{Q+Q'} [P(Q_0) - C] dQ - \int_{Q+Q'}^{Q^*} f(q) dq, & Q + Q' \leq Q^*, \\ \int_0^{Q+Q'} [P(Q_0) - C] dQ, & Q + Q' > Q^*. \end{cases} \quad (17) $$

3.1.3. The Expected Time of Government Default. For a “uniqueness” contract signed by the government and incumbent private capital, if the cost information is known by both incumbent private capital and potential competitors, the incumbent private capital will not sign the validity period of the “uniqueness” contract with the government. The “uniqueness” contract should be valid throughout the project period. The probability of potential competitors entering the market is the same at any time as private capital.

Assuming that the traffic volume is evenly distributed on the time axis and the probability of potential competitors entering the market during each period is known to be $\varnothing^*$, it can be derived that the probability of the potential competitor entering the market at time $t$ obeys the exponential distribution with the parameter $Q^*$ during the concession period, denoted as $T \sim E(\varnothing^*)$. The probability density function is written as

$$ f(t) = \{ \begin{array}{ll} \varnothing^* e^{-\varnothing^* t}, & t > 0, \\ 0, & t \leq 0. \end{array} \quad (18) $$

From the probability expectation formula, the expected time $T$ at which the government will incorporate a potential competitor is $(1/\varnothing^*)$

$$ E(T) = \frac{1}{\varnothing^*}. \quad (19) $$

3.2. Model Specifications. Assuming that the concession period of the PPP project is $N$, the predicted traffic volume of each period is $(D(P)/N)$ and the traffic volume that has passed at the time $(1/\varnothing^*)$ is $(D(P)/N\varnothing^*)$. Assuming that the construction period of a new transportation infrastructure is $n$, the new project is completed and operated by the entrant at the time $((1/\varnothing^*) + n)$. The cumulated traffic volume will be $((1/\varnothing^*) + n) \ast (D(P)/N)$. Let $((1/\varnothing^*) + n) \ast (D(P)/N)$ be $Q$. Let the expected return of the incumbent private capital be $Z$. It can be expressed in the form of the following segmented function:

$$ Z = \begin{cases} (P - C) \ast Q + V \ast \left(1 - \frac{Q}{D(P)}\right) - \int_Q^{Q^*} f(q) dq, & Q \leq Q^*, \\ (P - C) \ast Q + V \ast \left(1 - \frac{Q}{D(P)}\right), & Q > Q^*. \end{cases} \quad (20) $$

Let the government’s direct benefit be $S$, that is,
Let the total social welfare generated by the project is $M$. It can be expressed as

\begin{equation}
S = \frac{(1 - P) \cdot Q}{2} \cdot D(P_e) \cdot \left( \frac{1}{2} \cdot (1 - \frac{Q}{D(P)}) \right),
\end{equation}

(21)

\begin{equation}
M = \begin{cases} 
\frac{(P - 2C + 1) \cdot Q}{2} + \frac{P_e - 2C_e + 1}{2} \cdot D(P_e) \cdot (1 - (Q/D(P))) & Q \leq Q^*, \\
\frac{(P - 2C + 1) \cdot Q}{2} + \frac{P_e - 2C_e + 1}{2} \cdot D(P_e) \cdot (1 - (Q/D(P))) & Q > Q^*. 
\end{cases}
\end{equation}

(22)

In the social welfare objective function $M$, $C_e$ is the conditional probability expectation of the unit cost of the new competitor, taking $C_e = 0.25$. To get the optimal “uniqueness” contract, three nonlinear programming models are specified for incumbent private capital, government, and social welfare.

3.2.1. Optimal “Uniqueness” Contract Model for Maximizing the Profit of Incumbent Private Capital. To ensure the intuitiveness and comprehensibility of the solutions, some additional settings for the parameters need to be specified. First, let $N$ be the same as the upper limit of the project operation period prescribed by law. For example, the Regulation on the Administration of Toll Highway issued by China’s Ministry of Transport in 2018 stipulates that “The operating period of toll highway projects is determined in accordance with the principle of recovering investment and reasonable returns, and generally shall not exceed 30 years” [28]. We make $N$ to be 30, i.e., $N = 30$. Second, let the construction period of the project $n$ be 3 for generality. Therefore, the expected time of the government default should be less than the 27th year. Once the contract is successfully fulfilled to the 27th year, the incumbent private capital will lock in all revenue. Therefore, the constraints $0 \leq P_0 \leq P - (1/27)$ can be set. Finally, let the depreciation function $f$ of project assets be a linear function, and its expression is

\begin{equation}
f(q) = C - \frac{C}{Q^2} \cdot q = \frac{1}{2} \left( 1 - \frac{q}{Q} \right).
\end{equation}

(23)

Substituting the above formula into the integral function of the undepreciated initial investment cost of incumbent private capital, we get:

\begin{equation}
\int_Q^{Q^*} f(q)dq = \int_Q^{Q^*} \frac{1}{2} \left( 1 - \frac{q}{Q} \right) dq = \frac{1}{4} \cdot \left(Q^* - 2Q + \frac{Q^3}{Q^2}\right).
\end{equation}

(24)

3.2.2. Optimal “Uniqueness” Contract Model for Maximizing the Consumer Surplus. Consumer surplus represents the government’s direct benefit, and the constraints of the function remain unchanged. To get the optimal “uniqueness” contract of maximizing consumer surplus, the nonlinear programming model can be specified as

Based on the above, a nonlinear programming model for the optimal “uniqueness” contract of incumbent private capital can be specified:

\begin{equation}
\begin{align*}
\max Z &= (P - C) \cdot Q + V \cdot \left( 1 - \frac{Q}{D(P)} \right) - \int_Q^{Q^*} f(q)dq, \\
&\quad Q \leq Q^*, \\
&\quad Q > Q^*.
\end{align*}
\end{equation}

(25)

\begin{equation}
\begin{align*}
S.T. 0.5 &\leq P \leq 1 \leq \frac{\sqrt{5}}{2}, \\
0 &\leq P_0 \leq P - \frac{1}{27}, \\
\varnothing^* &= \max \{0, P - P_0\}, \\
\int_Q^{Q^*} f(q)dq &= \frac{1}{4} \cdot \left( Q^* - 2Q + \frac{Q^3}{Q^2} \right), \\
Q^* &= \left( \frac{1}{\varnothing^*} + n \right) \cdot \frac{D(P)}{N}, \\
N &= 30, \\
n &= 3, \\
P_e &= P - P_0, \\
D(P) &= 100 \cdot (1 - P), \\
V &= P_0 \cdot \left( \frac{D(P) + D(P_e)}{2} \right), \\
C &= 0.5.
\end{align*}
\end{equation}

(26)
\[ \text{max } S = \frac{(1 - P) \cdot Q}{2} + \frac{D(P_c) \cdot (1 - P_c)}{2} \left( 1 - \frac{Q}{D(p)} \right) \] 

(27)

\[ \text{max } M = \begin{cases} 
\frac{(P - 2C + 1) \cdot Q}{2} + \frac{[P_c - 2C_c + 1] \cdot D(P_c) \cdot (1 - (Q/D(p)))}{2} - V \cdot \left( 1 - \frac{Q}{D(p)} \right) - \int_{Q}^{Q^*} f(q) dq, & Q \leq Q^*, \\
\frac{(P - 2C + 1) \cdot Q}{2} + \frac{[P_c - 2C_c + 1] \cdot D(P_c) \cdot (1 - (Q/D(p)))}{2} - V \cdot \left( 1 - \frac{Q}{D(P_p)} \right), & Q > Q^*. 
\end{cases} 
\]

(28)

To make the programming function meaningful, we need to ensure that the incumbent private capital can at least obtain the expected benefit of not signing the “uniqueness” contract, which is \( Z_b \). Because there is the initial investment cost of the project in this model, the guaranteed return of incumbent private capital should be the expected profit with the no contract minus the undepreciated initial investment. \( Z_b \) should be a function of \( Q^* \). Therefore, when solving max \( M \) for each \( Q^* \) value, the restriction condition \( Z \geq Z_b \) should be added after the conditions.

4. Model Simulation

4.1. Optimal Contract Simulation of Incumbent Private Capital. From the integral formula of depreciation, we note that \( Q^* \) has a positive correlation with the initial investment cost \( I \). The larger the \( Q^* \), the greater the initial investment cost \( I \). Table 1 lists the simulation results of the optimal contract of the incumbent private capital under the different values of the parameter \( Q^* \).

The explanation of the parameters in the tables is shown below:

- \( P \): optimal price given \( Q^* \)
- \( P_B \): optimal liquidated damage parameter given \( Q^* \)
- \( Q^* \): the accumulated traffic flow where it just happens that the initial investment cost is fully depreciated
- \( Q \): the accumulated traffic flow before the new entry
- \( E(T) \): the time when the government decides to default
- \( \varnothing^* \): the probability of the government default
- \( \text{max } Z \): the expected benefit of the incumbent private capital in the optimal contract of incumbent private capital
- \( \text{max } S \): the expected consumer surplus in the optimal contract of the public
- \( \text{max } M \): the expected social welfare in the optimal contract of social welfare
- \( Z_c \): the least obtained benefit by the incumbent private capital with not signing the “uniqueness” contract

3.2.3. Optimal “Uniqueness” Contract Model for Maximizing the Total Social Welfare. To get the optimal “uniqueness” contract of maximizing total social welfare, the nonlinear programming model can be specified as

\[ Z: \text{the expected benefit of the incumbent private capital in the optimal contract not pursuing maximum profit} \]

\[ S: \text{the expected consumer surplus in the optimal contract not pursuing maximum consumer surplus} \]

\[ M: \text{the expected social welfare in the optimal contract not pursuing for maximum social welfare} \]

Table 1 shows that the optimal solution for toll price is on the upper limit 0.6047. When \( Q^* \leq 14 \), the initial investment cost \( I \) has no effect on the optimal solution of the “uniqueness” contract. The optimal solution is {0.6047, 0.4791}. When \( Q^* \geq 15 \), with the increase of the initial investment cost \( I \) (\( I \) increases with the increase of \( Q^* \)), the profit of the incumbent private capital shows a marginal diminishing trend, and the liquidated damages parameter \( P_B \) and the expected default time \( E(T) \) show a marginal increase trend, although the change is small. The actual liquidated damage parameter \( P_B \) is in the interval (0.4791, 0.5121). The expected time \( E(T) \) of government default remains unchanged when \( Q^* \leq 14 \). When \( Q^* \geq 15 \), \( E(T) \) increases monotonically with \( Q^* \) but not sensitive. In general, the optimal “uniqueness” contract solution and expected default time are not sensitive to the initial investment cost. However, the project profit is sensitive to the initial investment cost. As the initial investment cost increases, the maximum expected profit of the incumbent private capital is rapidly diminishing.

When \( Q^* = 113 \), the profit of incumbent private capital has reached the lower limit. If the initial investment cost increases further, the incumbent private capital will enter the loss range, and the user-pay model will not be feasible. This means that, given the traffic demand, the higher the initial investment cost, the lower the expected return of the incumbent private capital. Therefore, due to demand restrictions, not all transportation infrastructure projects can compensate its costs only by charging tolls. If the project exceeds a certain size, the government needs to subsidize the project to ensure private capital can get enough revenue to compensate for the costs.
The simulation solution indicates that the incumbent private capital transfers part of the benefits in exchange for the signing of the “uniqueness” contract. The price of the contract is lower than the monopoly price. If the “uniqueness” contract is not signed, the probability of “uniqueness” risk is 50%, and the expected time of starting to build a competitive transportation infrastructure is the end of the year 2. The construction will be completed at the end of year 5. If the “uniqueness” contract is signed, the probability of the government default is reduced to between 9.27% and 12.56%. The expected time of the government default is between the end of year 8 and the fourth quarter of the year 11. And, the time when the competitors actually enter the market is approximately between the end of year 11 and the fourth quarter of year 14. Table 1 also shows that the profit of incumbent private capital after signing the “uniqueness” contract is significantly higher than the benefits of not signing the contract.

From Table 1, we observe that the probability of “uniqueness” risk is significantly reduced by signing the “uniqueness” contract. If the “uniqueness” contract is not signed, the probability of “uniqueness” risk is 50%, and the expected time of starting to build a competitive transportation infrastructure is the end of the year 2. The construction will be completed at the end of year 5. If the “uniqueness” contract is signed, the probability of the government default is reduced to between 9.27% and 12.56%. The expected time of the government default is between the end of year 8 and the fourth quarter of the year 11. And, the time when the competitors actually enter the market is approximately between the end of year 11 and the fourth quarter of year 14. Table 1 also shows that the profit of incumbent private capital after signing the “uniqueness” contract is significantly higher than the benefits of not signing the contract.

The simulation solution indicates that the incumbent private capital transfers part of the benefits in exchange for the signing of the “uniqueness” contract. The price of the contract is lower than the monopoly price \( P = 0.75 \). The lower price will cause the incumbent private capital to lose part of the profit and increase the output of transportation services, and the public will benefit from it. In addition, Table 1 reports the change in the social welfare of the project under the optimal “uniqueness” contract of private capital. \( M \) is the social welfare under the “uniqueness” contract, and \( M_0 \) is the social welfare without signing the contract. Due to the negative impact of payment of liquidated damages which is \( V \cdot (1 - (Q/D(P))) \), \( M \) is significantly smaller than \( M_0 \), which means signing the “uniqueness” contract has a negative effect on social welfare.

### 4.2. Optimal Contract Simulation for the Public

Table 2 lists the simulation results of the optimal solution of the “uniqueness” contract with the maximum consumer surplus. The optimal toll price of the maximizing consumer surplus is \( P = 0.5 \), which is the same as the unit cost of incumbent private capital. The optimal contract solution for the public is \([0.5, 0.3753]\), and the consumer surplus is the largest. This contract is not relevant with the initial investment, and the maximum value of consumer surplus is \( \max S = 28.8282 \). The probability of government default under this contract is about 12.47%. The expected time of government default \( E(T) \) is the beginning of year 9, and potential competitors enter the market is about the beginning of the year 12. Under this contract, the incumbent private capital can only compensate for its cost from toll revenue. The liquidated damage is the only source of profit with greater uncertainty, which can only be counted as contingent profit.

Table 2 shows that as a function of the initial investment cost, the expected return of incumbent private capital exhibits the same trend of change as the expected return under the optimal contract of incumbent private capital. When \( Q^* \leq 18 \), the expected return of incumbent private capital \( Z = 16.3282 \), which remains unchanged. When \( Q^* \geq 19 \), the expected return of incumbent private capital presents a marginal diminishing trend as the initial investment cost increases. \( Q^* = 98 \) is the upper limit of the initial investment cost. If it exceeds the threshold, the project will incur losses. The incumbent private capital will find it difficult to maintain the project operation, and the risk of project failure will greatly increase.

Under the optimal “uniqueness” contract of the public, the government can obtain the maximum direct benefit by maximizing consumer surplus. Under this contract, the incumbent private capital compensates its unit cost by operating transportation infrastructure. But, whether there is net profit depends on the probability of government default and the differences between the liquidated damages and the undepreciated part of the initial investment cost, which are highly uncertain. In addition, consistent with the “uniqueness” contract of incumbent private capital, the “uniqueness” contract of the public that seeks to maximize consumer surplus also has a negative impact on social welfare, as shown in Table 2.

### 4.3. Optimal Contract Simulation of Social Welfare

As shown in Table 3, with the goal to maximize social welfare, the “uniqueness” contract tends to limit the toll price to the incumbent private capital’s unit cost, which is \( P = 0.5 \). But,
the liquidated damage is extremely low. The maximum value of the parameter $P_0$ is only about 0.0245. As the initial investment cost increases, $P_0$ decreases monotonically. It shows that this contract fails to effectively reduce the probability of government default, which is about 47.55% to 50%. The expected time $E(T)$ of government default is between the end of year 2 to the beginning of year 3. The window period is very narrow. When $Q^*$ decreases at the beginning and then decreases. When $Q^*$ = 12, $P_0$ drops to the value of 0, which means that, while the price is restricted, the government does not bear any penalty for breach of contract. Such a contract will not be accepted by the private capital. The return of incumbent private capital $Z$, which gradually decreases with the increase of the initial investment cost, reaches the minimum positive value at $Q^* = 10$. Then, the project will cease to attract private capital with initial investment further increasing. This means that if the ultimate objective is to maximize social welfare, the user-pay model has its own limitations. A low expected rate of return is difficult to attract high-quality private capital to participate in the PPP project. Unless the project demand is high, the user-pay model is not feasible for large-scale transportation infrastructure projects.

In addition, under the optimal contract of social welfare, consumer surplus is suboptimal, with a value between 12.5 and 13.54, which is significantly lower than the consumer surplus under the optimal contracts of the public and the incumbent private capital. As for the optimal solution value of the social welfare of the project, as $Q^*$ increases, it increases at the beginning and then decreases. When $Q^*$ = 107, social welfare reaches the lowest positive value. If the initial investment increases further, no positive social welfare will be produced.

The results in Table 3 show that, with the goal to maximize social welfare, signing a "uniqueness" contract can increase social welfare in the interval $Q^* \geq 5$. However, the contract is only feasible for transportation infrastructure PPP projects that do not require an initial investment or low initial investment and cannot effectively motivate incumbent private capital. The expected consumer surplus is also squeezed out by potential competitors. Therefore, both the incumbent private capital and the public have incentives to lobby the government to increase their own benefits.

5. Discussion

The simulation results show that the goal to maximize incumbent private capital profit, consumer surplus, and total social welfare at the same time is not attainable. Although the PPP contract is entered between the government and the incumbent private capital, it involves the interests of all stakeholders. The government not only needs to consider the interests of the public directly but also needs to take into account the social welfares generated by the project. Therefore, the final PPP “uniqueness” contract must be the result of a trade-off between the goals of stakeholders.

Based on the participation and influence of each stakeholder in contract negotiations, the importance of the individual party’s goal can be ranked. First, the public is the direct beneficiary. The public’s interests should be guaranteed so that the public can obtain benefit from the use of transportation infrastructure. Second, to ensure the feasibility of the PPP project, the government should attract qualified private capital to participate in the project, encourage private capital to improve the quality of transportation services, and guarantee that the PPP contract can be effectively fulfilled during the whole concession period. So, it is necessary for the government to provide the opportunity that the private capital can make profit. Finally, it is essential to take into account other potential stakeholders of the project by making sure that the project can enhance total social welfare.

According to the importance of these goals, we establish a multiobjective programming model to get the optimal “uniqueness” contract by weighting function $S$ of the public, function $Z$ of the incumbent private capital, and function $M$ of the total social welfare. Assuming that $S$, $Z$, and $M$ are assigned a weight of 1.5, 1.5, and 1, respectively, a new programming function $T$ is established (we weigh $S$, $Z$, and $M$ here to discuss the change of the optimal contract solution under multiobjectives; 1.5, 1.5, and 1 are just a special case of them; objectives of profit, consumer surplus, and social welfare for different projects are different):

$$\max T = 1.5S + 1.5Z + M.$$  (29)
When the initial investment cost is small, the optimal solution of the “uniqueness” contract will gradually scale, the impact of the initial investment cost on the optimal contract programming function under different initial investment cost increases. When the initial investment cost increases, it is expected that the incumbent private capital return reduces to the lowest positive value. Therefore, when the traffic demand remains unchanged, Q* = 53 is the critical point for the implementation of the traffic PPP project of the user-pay model. We note that the optimal contract after linear weighting can support the development of large-scale transportation infrastructure projects.

Table 3: Optimal solutions for the “uniqueness” contract of social welfare.

<table>
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<th>Q*</th>
<th>Z</th>
<th>Z0</th>
<th>E(T)</th>
<th>Ø*</th>
<th>S</th>
<th>max M</th>
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Table 4 lists the simulation results of the “uniqueness” contract programming function under different initial investment costs. Among them, each of the optimal solutions is the result of the compromise of all parties involved in the project. The optimal solutions take into account the interests of the social welfares, the public, and the incumbent private capital.

Table 4 shows that, when the projects have different scales, the impact of the initial investment cost on the optimal solution of the “uniqueness” contract will gradually change. When the initial investment cost is small (Q* ≤ 13), the initial investment cost has no significant impact on the optimal solution of the “uniqueness” contract of the project. In this model, the price P can be set on the upper limit which is P = 0.6047 when Q* ≤ 14. When the project scale increases and the initial investment cost further increases (Q* ≥ 15), the price P will gradually decrease to the unit cost, which is P = 0.5. The liquidated damages parameter P0 changes with Q* which first remains unchanged, then increases, and finally gradually decreases. The gap between the pricing P and the liquidated damage parameter P0 has gradually reduced. So, the government default probability Q* gradually decreases with the increase of the initial investment cost. The expected default time E(T) is at the end of year 7. As the initial investment cost increases, it is expected that the default time point can be delayed up to the beginning of the third quarter of the year 10.

As the initial investment cost increases (Q* ≥ 14), the incumbent private capital return Z under the optimal “uniqueness” contract shows a monotonous downward trend, while the social welfare M shows a downward trend after it first increases. When Q* = 53, the social welfare M reduces to the lowest positive value. Therefore, when the traffic demand remains unchanged, Q* = 53 is the critical point for the implementation of the traffic PPP project of the user-pay model. We note that the optimal contract after linear weighting can support the development of large-scale transportation infrastructure projects.

6. Conclusions

We model whether the incumbent private capital and the government have an optimal “uniqueness” contract in the PPP market by considering the impact of initial investment cost. We construct four models and simulate the optimal “uniqueness” contract. The extent to which such a contract reduces the probability of “uniqueness” risk depends on the equilibrium relation between the interests of private capital and the public. We find that the effect of “uniqueness” risk depends on the contract setting. The optimal contract of incumbent private capital and the optimal contract of the public can significantly reduce the probability of government default, while the optimal contract of social welfare is weakly related to government default. Among them, the optimal contract of incumbent private capital reduces the risk probability and increases the private capital return the most significantly, followed by the optimal contract of the public. The optimal contract after weighting can take into account the objectives of incumbent private capital, the public, and social welfare and can achieve the “uniqueness” risk control objective.

Furthermore, we show that the barrier effect of initial investment depends on whether the contract takes into account the interests of the incumbent private capital. Under the optimal “uniqueness” contract of incumbent private capital, with the increase of the initial investment cost, the “uniqueness” risk probability shows a unilateral downward trend, showing a barrier effect. Under the optimal “uniqueness” contract of the public, the probability of “uniqueness” risk does not change with the initial investment cost. Under the optimal “uniqueness” contract of social welfare, as the initial investment cost increases, the probability of “uniqueness” risk increase to the maximum 0.5 and then remains unchanged. These show that, when the clause of “uniqueness” contract fully considers the benefits of the incumbent private capital, the “uniqueness” risk has a negative co-relationship with the initial investment. The
more the “uniqueness” contract conforms to the interests of private capital, the stronger the correlation.

The simulation results show that the achievement of the optimal “uniqueness” contract is essentially the result of a compromise between the private capital, the public, and social welfare. Although the social welfare is an important issue that needs to be taken into account in PPP projects, maximizing the social welfare will do harm to the construction and operation of the projects. The project risk will be increased. The interests of the public and incumbent private capital are directly related to the project; they are more concerned about the signing of “uniqueness” contract. They have more motivations to lobby the government. Therefore, the optimal “uniqueness” contract should simultaneously encourage the incumbent private capital to do a good job in the construction and operation of the project, protect the interests of the public, and improve the total social welfare by striving to make the cake bigger. The optimal solution after weighting proves that signing the “uniqueness” contract can effectively control the “uniqueness” risk and increase transportation service output of incumbent infrastructure but not enhancing total social welfare. Our findings provide a guide for the two parties of the contract to make their own optimal decisions.

Finally, we show that entering a “uniqueness” contract is not good for social welfare. Consistent with Ater [17] and Nurski and Verboven [18], the incumbent private capital and the government sign the “uniqueness” contract mainly for anticompetitive purposes. Unless the clause considers the improvement of social welfare, the signed “uniqueness” contract is associated with social welfare losses. The government has an incentive to sign a “uniqueness” contract to attract more private capital to participate in the construction of the region. The extent to which this effect can offset the loss of social welfare is not discussed in this article, and it is an important question for further research. In the future, current research can also be extended to consider issues like sustainability [29] and corporate social responsibility [30].

### Data Availability

The data used to support the findings of this study are available upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

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### References


### Table 4: Optimal solutions for the “uniqueness” contract after weighting.

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