Research Article

The Dynamic Rent-Seeking Games with Policymaker Cost and Competition Intensity

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In this paper, a dynamic rent-seeking game incorporating policymaker cost and competition intensity is considered. On the basis of the political environment and rent-seekers with incomplete information set, the locally asymptotic stability of Nash equilibrium is proved. The competition intensity and policymaker cost could enlarge the stability region of Nash equilibrium. The higher the competition intensity is, the more the opponent's expenditure reduces the player's success probability, which is beneficial to the maintenance of Nash equilibrium. The higher the policymaker cost is, the less easily both players succeed and the more stable the rent-seeking market is. As the competition parameter decreases or the expenditure parameter increases, there will be chaos in a rent-seeking market. Chaos control is in order to stabilize the equilibrium of the rent-seeking game.

1. Introduction

The rent-seeking phenomenon was observed by Tullock in welfare loss [1]. His research could be roughly divided into two directions: first, the welfare cost of rent-seeking behavior and, second, the economic analysis of rent-seeking political market. Krueger [2] first proposed that the government's restrictions produced the value of rent. The import permissions of India and Turkey were used to verify large losses caused by rent-seeking. Recently, rent-seeking theory has developed rapidly. In early studies, most scholars considered the rent-seekers with risk neutrality. According to rent-seeking activity of expected return, they adjusted their own expenditure cost for rent-seeking. This view was opposed by Buchanan et al. [3] in the rent-seeking game model. They systematically analyzed the social costs of rent-seeking. Their research focused on three issues. The first was the expenditure to obtain and maintain a monopoly for rent-seekers. The second was the expenditure to create rent for government

officials. The last was the resource allocation distortions and social welfare losses caused by a monopoly which is produced by rent-seeking. In the lottery model of rent-seeking, Tullock pointed out that rent-seekers' equilibrium expenditure may be more than, less than, and equal to the expected rent value return.

To make the game more accurate, costs for supplying rents were considered [4]. Increasing policymaker costs made lobbying less effective. In addition, scholars found that political competition among rent-seekers also influenced the rent-seeking outcome [5, 6]. Godwin et al. in [7] checked policymaker cost and competition intensity into the standard rent-seeking game. The lobbying process was discussed more specifically. They concluded that lobbying expenditures, rent-seeking profits, and rent dissipation depended on the intensity of competition and the value of the rent relative to policymaker costs. They considered comparative statics of expenditures and profits with respect to competition. Both Tullock's standard rent-seeking model and Godwin's rent-seeking model of

describing the political policy process could be summarized as static game models of complete information [7]. In the models, equilibrium was used to forecast the rent-seeking behavior.

This kind of models requires players with complete information on game structure, complete rationality behavior, and best reaction capability. Under this strong assumption, players could reach Nash equilibrium through a strategy interaction. This result is based on the assumption that each firm knows the demand curve of the market and what his opponents decide to do, while, in reality, we often observe that the equilibrium is achieved through repeated interactions. Players may not be completely rational, and they have limited knowledge of demand function. Their profit maximization problem is solved by “rule of thumb” or market experiment. How to solve the rationality assumption in a game? If players are not completely rational, could the equilibrium be maintained? Traditional game theory combined with nonlinear dynamics attracts researchers’ concerns.

To make the research more realistic, scholars put forward kinds of rationality expectations, which have formed the cornerstone of this field, such as bounded rationality, adaptive expectation, naïve rationality, and local monopolistic approach (LMA). This kind of research could be applied to motive the competition and changes in the biological systems over time. Costa et al. [8] verified the existence of numerous similarities between the samples of the Leiden herbarium which emphasize the importance and the usefulness of the herbarium in question for the processes of typification. Ferrauto et al. [9] charted and assessed the state of fragmentation and disturbance of areas of apiarian interest, caused by the presence of urban infrastructure and road networks. The supply chain quality management and decisions were considered in competition [10, 11].

Most literature applies this method in the production market; there is almost no article referring to a rent-seeking market. Homogeneous products first attracted much concern. Cournot and Bertrand model was modified and improved by researchers. Nonlinear demand and cost functions increased the complexity of dynamic game [12–15]. Ahmed and Elettrey in [16] introduced a multi-market Cournot model using Puu’s approach [17]. Bischi et al. [18] considered a model with a fixed fraction of firms in two complementary groups: Best Reply and LMA adjustment. The Cournot–Bertrand mixed duopoly game offered a new research possibility. Two firms competing with price and output, respectively, came to some different conclusions [19–21]. Besides the static model of Cournot and Bertrand, the Stackelberg model describes two players with sequential actions. Researchers analyzed chaotic dynamic behaviors in the models [22–24].

The effect of differentiated goods on chaos is also an interesting topic in this field [25–27]. Researchers reached a consensus on the result that bounded rationality with delay enlarged stability region [28–31]. Chaos control also became an important issue that most scholars focused on [16, 32, 33].

In the literature mentioned above, chaos theory is mainly applied in the production of duopoly market. Different demand and cost functions are discussed sufficiently. Firms’ maximization profit problem and classical game model are familiar to researchers. Our study aims at analyzing a dynamic rent-seeking game with policymaker cost and competition intensity. Not only the adjustment parameter of expenditure but also the competition intensity could affect the stability of the rent-seeking market. In our paper, we first point out the deficiency of the model in [7]. Their study could not exclude the possibility of multiequilibrium. We demonstrate that the Nash equilibrium of the rent-seeking game is unique and symmetrical. In addition, rent-seekers’ success probability is not common knowledge of each other. They adjust their expenditure functions according to a gradient rule. Chaos phenomenon is observed in a rent-seeking market.

The paper is arranged as follows. In Section 2, the uniqueness and symmetry of Nash equilibrium are proved. In Section 3, a dynamic rent-seeking game model with policymaker cost and competition intensity is constructed. We discuss the locally asymptotic stability of the Nash equilibrium. In Section 4, some simulation results are presented. In Section 5, we draw some conclusions through theoretical analysis and numerical simulation.

2. A Rent-Seeking Game with Policy Cost and Competition Intensity

Godwin et al. improved Tullock’s lottery model of rent-seeking [7]. They checked a policymaker cost N for supplying a rent value V ($>N$) and a rent-seeking competition intensity $\alpha \in (0, 1]$ into Tullock’s standard rent-seeking model. The greater α is, the more intense the competition of rent-seeking is. It is assumed that the player i ’s rent expenditure is R_i ($i = 1, 2$). The player i ’s success probability function for rent-seeking has been formulated: $P_i = R_i / (R_i + \alpha R_j + N)$, ($i, j = 1, 2, i \neq j$), which means player i may fail in this rent-seeking process. In the static game with complete information and complete rationality of players, player i chooses $R_i \geq 0$ for the fixed R_j . To solve the maximization problem:

$$\max \pi_i(R_i, R_j) = \frac{R_i V}{(R_i + \alpha R_j + N)} - R_i. \quad (1)$$

The optimal first-order condition of formula (1) is

$$\frac{\partial \pi_i}{\partial R_i} = \frac{(\alpha R_j + N)V}{(R_i + \alpha R_j + N)^2} - 1 = 0. \quad (2)$$

Checking formula (2), it is clear that the best response functions $N_1(R_2)$, $N_2(R_1)$ of both players are

$$N_1(R_2) = \begin{cases} \sqrt{(\alpha R_2 + N)V} - \alpha R_2 - N, & 0 \leq R_2 \leq \frac{(V-N)}{\alpha}, \\ 0, & R_2 > \frac{(V-N)}{\alpha}, \end{cases} \quad (3)$$

$$N_2(R_1) = \begin{cases} \sqrt{(\alpha R_1 + N)V} - \alpha R_1 - N, & 0 \leq R_1 \leq \frac{(V-N)}{\alpha}, \\ 0, & R_1 > \frac{(V-N)}{\alpha}. \end{cases} \quad (4)$$

The intersection point of the two response function curves is Nash equilibrium (R_1^e, R_2^e) of the game. In [7], the model could not exclude the possibility of multiequilibrium. We will prove that the Nash equilibrium is unique and symmetrical. Firstly, we need two lemmas as follows.

Lemma 1. *The Nash equilibrium of complete information static rent-seeking game with policymaker cost and competition intensity exists.*

You may refer to Godwin's proof process in [7].

Lemma 2. *The Nash equilibrium (R_1^e, R_2^e) satisfies $R_i^e \in [0, (V-N)/\alpha]$, $i = 1, 2$.*

Proof. We use reduction to absurdity. It is assumed that $R_1^e > (V-N)/\alpha$.

Check formula (4); we know $R_2^e = 0$.

When $R_2^e = 0$, $R_1^e = \sqrt{(\alpha R_2^e + N)V} - \alpha R_2^e - N = \sqrt{VN} - N < (V-N)/\alpha$ from formula (3). There is a contradiction. Therefore, $R_1^e \in [0, (V-N)/\alpha]$. Similarly, $R_2^e \in [0, (V-N)/\alpha]$ QED.

From the two lemmas above, we could prove that the static game of complete information has only one symmetric Nash equilibrium. $\square$

Theorem 1. *If (R_1, R_2) is the Nash equilibrium of the game model, $R_1 = R_2 = R_e$ must be satisfied, where $R_e = \alpha V - 2N(1+\alpha) + \sqrt{\alpha^2 V^2 + 4(1+\alpha)NV} / 2(1+\alpha)^2$.*

Proof. We assume that (R_1, R_2) is the Nash equilibrium; if $R_1 \neq R_2$ from Lemma 2, we know, $R_i \in [0, (V-N)/\alpha]$, $i = 1, 2$. Checking formulas (3) and (4), we know

$$R_1 + \alpha R_2 - \sqrt{(\alpha R_2 + N)V} = R_2 + \alpha R_1 - \sqrt{(\alpha R_1 + N)V}, \quad (5)$$

that is,

$$(1-\alpha)(R_1 - R_2) + \sqrt{(\alpha R_1 + N)V} = \sqrt{(\alpha R_2 + N)V}. \quad (6)$$

Square the ends of the above equation:

$$(1-\alpha)^2(R_1 - R_2)^2 + 2(1-\alpha)(R_1 - R_2)\sqrt{(\alpha R_1 + N)V} + (\alpha R_1 + N)V = (\alpha R_2 + N)V, \quad (7)$$

that is,

$$(1-\alpha)^2(R_1 - R_2)^2 + 2(1-\alpha)(R_1 - R_2)\sqrt{(\alpha R_1 + N)V} + \alpha V(R_1 - R_2) = 0. \quad (8)$$

Both ends of the equation are divided by $R_1 - R_2 (\neq 0)$:

$$(1-\alpha)^2(R_1 - R_2) + 2(1-\alpha)\sqrt{(\alpha R_1 + N)V} + \alpha V = 0. \quad (9)$$

Checking formula (4), $\sqrt{(\alpha R_1 + N)V} = R_2 + \alpha R_1 + N$, we have the following equation:

$$(1-\alpha)(1+\alpha)(R_1 + R_2) + 2N(1-\alpha) + \alpha V = 0. \quad (10)$$

But the left end is positive and there is a contradiction. So, $R_1 = R_2$

We prove that the Nash equilibrium (R_1, R_2) which satisfies $R_1 = R_2$ is unique. In formula (3) or (4), let $R_1 = R_2 = R_e$; we get the quadratic equation about R_e :

$$(1+\alpha)^2 R_e^2 + (2N(1+\alpha) - \alpha V)R_e + N(N-V) = 0. \quad (11)$$

The solution of the equation is

$$\frac{(\alpha V - 2N(1+\alpha) \pm \sqrt{\alpha^2 V^2 + 4(1+\alpha)NV})}{2(1+\alpha)^2}. \quad (12)$$

Because the expenditure for rent-seeking is nonnegative, the minus in front of the radical sign of the molecule in the above equation is removed. We get the rent-seekers' equilibrium expenditure:

$$R_e = \frac{(\alpha V - 2N(1+\alpha) + \sqrt{\alpha^2 V^2 + 4(1+\alpha)NV})}{2(1+\alpha)^2}. \text{ QED.} \quad (13)$$

In [7], it has given comparative static results on the basis of the Nash equilibrium (R_e, R_e) . However, the rent seekers' complete information and rationality are not always satisfied. Therefore, a dynamic rent-seeking game problem with limited rationality needs to be considered. $\square$

3. A Dynamic Rent-Seeking Game with Gradient Rule Based on Marginal Profit

In this section, we assume that the rent-seekers have no complete information of success probability function and rent value and players have no expectation capability of complete rationality. According to Dixit's assumption of bounded rationality behavior [34], we only assume that player i adjusts his own expenditure for rent-seeking R_i' next

period according to expenditure for rent-seeking R_i and marginal profit $\partial\pi_i/\partial R_i = ((\alpha R_j + N)V/(R_i + \alpha R_j + N)^2) - 1$ in the current period. Thus, the rent seeker's expenditure growth rate is proportional to his marginal profit, that is,

$(R'_i - R_i)/R_i = u_i (\partial\pi_i/\partial R_i)$, $i = 1, 2$, where $u_i > 0$ is the rent-seeker's adjustment parameter of expenditure. Thus, we construct a dynamic system of the rent-seeking game:

$$\begin{cases} R'_1 = R_1 + u_1 R_1 \left(\frac{(\alpha R_2 + N)V}{(R_1 + \alpha R_2 + N)^2} - 1 \right), \\ R'_2 = R_2 + u_2 R_2 \left(\frac{(\alpha R_1 + N)V}{(R_2 + \alpha R_1 + N)^2} - 1 \right). \end{cases} \quad (14)$$

The limit state of the system depends on its fixed points. Thus, we solve the algebraic equations as follows:

$$\begin{cases} u_1 R_1 \left(\frac{(\alpha R_2 + N)V}{(R_1 + \alpha R_2 + N)^2} - 1 \right) = 0, \\ u_2 R_2 \left(\frac{(\alpha R_1 + N)V}{(R_2 + \alpha R_1 + N)^2} - 1 \right) = 0. \end{cases} \quad (15)$$

The solutions of the equations are the four fixed points:

$$\begin{aligned} E_0 &= (0, 0), \\ E_1 &= (0, \sqrt{NV} - N), \\ E_2 &= (\sqrt{NV} - N, 0), \\ E_* &= (R_e, R_e). \end{aligned} \quad (16)$$

E_0, E_1, E_2 are the boundary equilibria and E_* is the Nash equilibrium. R_e is given in Theorem 1.

To forecast the limit state of a dynamic game, the stability of the fixed points should be considered. Thus, we consider the Jacobian matrix of system (14):

$$J = \begin{bmatrix} 1 + u_1 \left(\frac{(\alpha R_2 + N)(\alpha R_2 - R_1 + N)V}{(R_1 + \alpha R_2 + N)^3} - 1 \right) & \frac{\alpha u_1 R_1 (R_1 - \alpha R_2 - N)V}{(R_1 + \alpha R_2 + N)^3} \\ \frac{\alpha u_2 R_2 (R_2 - \alpha R_1 - N)V}{(R_2 + \alpha R_1 + N)^3} & 1 + u_2 \left(\frac{(\alpha R_1 + N)(\alpha R_1 - R_2 + N)V}{(R_2 + \alpha R_1 + N)^3} - 1 \right) \end{bmatrix}. \quad (17)$$

At E_0 , the Jacobian matrix is

$$J(E_0) = \begin{bmatrix} 1 + u_1 \left(\frac{V}{N} - 1 \right) & 0 \\ 0 & 1 + u_2 \left(\frac{V}{N} - 1 \right) \end{bmatrix}. \quad (18)$$

Because $V > N$, the characteristic roots of are $J(E_0)\lambda_1 = 1 + u_1 (V/N - 1) > 1$, $\lambda_2 = 1 + u_2 (V/N - 1) > 1$. Thus, E_0 is unstable.

At E_1 , the Jacobian matrix is

$$J(E_1) = \begin{bmatrix} 1 + u_1 \left(\frac{V - N - \alpha(\sqrt{NV} - N)}{\alpha\sqrt{NV} + (1 - \alpha)N} \right) & 0 \\ \alpha u_2 (\sqrt{NV} - N) \frac{\sqrt{NV} - 2N}{(NV)^{3/2}} & 1 + u_2 \left(\frac{NV(2N - \sqrt{NV})}{(NV)^{3/2}} - 1 \right) \end{bmatrix}. \quad (19)$$

One of the characteristic roots of $J(E_1)$: is $\lambda_1 = 1 + u_1 ((V - N - \alpha(\sqrt{NV} - N))/(\alpha\sqrt{NV} + (1 - \alpha)N)) > 1$. Thus, the fixed point E_1 is unstable.

At E_2 , the Jacobian matrix is

$$J(E_2) = \begin{bmatrix} 1 + u_1 \left(\frac{NV(2N - \sqrt{NV})}{(NV)^{3/2}} - 1 \right) & \alpha u_1 (\sqrt{NV} - N) \frac{\sqrt{NV} - 2N}{(NV)^{3/2}} \\ 0 & 1 + u_2 \left(\frac{V - N - \alpha(\sqrt{NV} - N)}{\alpha\sqrt{NV} + (1 - \alpha)N} \right) \end{bmatrix}. \quad (20)$$

One of the characteristic roots of $J(E_2)$ is $\lambda_2 = 1 + u_2((V - N - \alpha(\sqrt{NV} - N))/(\alpha\sqrt{NV} + (1 - \alpha)N)) > 1$. Thus, the fixed point E_2 is also unstable.

Finally, at E_* , the Jacobian matrix of the Nash equilibrium is

$$J(E_*) = \begin{bmatrix} 1 - \frac{2R(\alpha R + N)V}{A}u_1 & \alpha VR \frac{(1 - \alpha)R - N}{A}u_1 \\ \alpha VR \frac{(1 - \alpha)R - N}{A}u_2 & 1 - \frac{2R(\alpha R + N)V}{A}u_2 \end{bmatrix}, \quad (21)$$

where $R = R_e$, $A = ((1 + \alpha)R + N)^3$.

The characteristic polynomial of $J(E_*)$ is $f(\lambda) = \lambda^2 - T\lambda + D$, where T is the trace of $J(E_*)$, $T = 2 - (2R(\alpha R + N)V/A)(u_1 + u_2)$, D is the determinant of $J(E_*)$,

$$D = (1 - (2R(\alpha R + N)V/A)u_1)(1 - (2R(\alpha R + N)V/A)u_2) - (\alpha VR((1 - \alpha)R - N/A))^2 u_1 u_2.$$

Because the discriminant is $\Delta = T^2 - 4D = ((2RV(\alpha R + N)/A)u_1 - (2RV(\alpha R + N)/A)u_2)^2 + 4(\alpha VR(1 - \alpha)R - N/A)^2 > 0$, the characteristic roots of $J(E_*)$ are real values. $|\lambda| < 1$ if and only if Jury conditions are established as follows:

$$\begin{cases} (1) 1 + D - T > 0, \\ (2) 1 + D + T > 0, \\ (3) D < 1. \end{cases} \quad (22)$$

Firstly, $1 + D - T = [(2RV(\alpha R + N)/A)^2 - (\alpha VR((1 - \alpha)R - N/A))^2]u_1 u_2 > 0$; thus, the stability of E_* depends on formulas (2) and (3) in Jury's conditions.

That is,

$$\begin{aligned} D < 1 &\iff \left(\frac{1}{2A} \left((2RV(\alpha R + N))^2 - (\alpha VR((1 - \alpha)R - N))^2 u_1 - RV(\alpha R + N) \right) \right) u_2 < RV(\alpha R + N)u_1, \\ 1 + D + T > 0 &\iff \left(\frac{1}{4A} \left((2RV(\alpha R + N))^2 - (\alpha VR((1 - \alpha)R - N))^2 \right) u_1 - RV(\alpha R + N) \right) u_2 > RV(\alpha R + N)u_1 - A. \end{aligned} \quad (23)$$

In summary, we get the following conclusion.

Theorem 2. *The boundary equilibriums E_0, E_1 , and E_2 are unstable. The Nash equilibrium E_* is locally asymptotically stable if parameters V, N, α, u_1, u_2 satisfy*

$$\begin{cases} \left[\frac{1}{4A} \left((2RV(\alpha R + N))^2 - (\alpha VR((1 - \alpha)R - N))^2 u_1 - RV(\alpha R + N) \right) \right] u_2 > RV(\alpha R + N)u_1 - A, \\ \left[\frac{1}{2A} \left((2RV(\alpha R + N))^2 - (\alpha VR((1 - \alpha)R - N))^2 u_1 - RV(\alpha R + N) \right) \right] u_2 < RV(\alpha R + N)u_1, \\ u_1 \geq 0, u_2 \geq 0. \end{cases} \quad (24)$$

Checking Theorem 2, we know the stability region S of E_* about adjustment parameters of expenditure for rent-seeking u_1 and u_2 is surrounded by $u_2(u_1) = ((RV(\alpha R + N)u_1 - A)/(1/4A)((2RV(\alpha R + N))^2 - (\alpha VR((1 - \alpha)R - N))^2)u_1 - RV(\alpha R + N))$, $u_1 \geq 0$, and $u_2 \geq 0$. The intersections of $u_2 = u_2(u_1)$ and u_1, u_2 axis are $(A/RV(\alpha R + N), 0)$ and $(0, A/RV(\alpha R + N))$. The stability region S is as Figure 1 shows.

When the parameters u_1, u_2 are in the stability region S , in the dynamic rent-seeking game, players adjust the expenditure for rent-seeking based on a gradient rule and achieve the Nash equilibrium of complete information static rent-seeking game with complete rationality ultimately. The Nash equilibrium could be used to analyze and forecast the result of the rent-seeking game. However, when the expenditure parameters for rent-seeking are not in the stability

region S , we need to use a numerical simulation method to discuss the complexity of the limit state of the dynamic rent-seeking game with incomplete rationality.

4. Numerical Simulation

In this section, complicated behaviors of the dynamic rent-seeking game model are presented by simulation. A series of bifurcation and chaos with respect to u_2 and α , the largest Lyapunov exponent, the strange attractors, and the sensitivity on initial values are shown in the figures. The sensitivity dependence on initial value is investigated. We set the parameters as follows: $V = 10$, $N = 5$, $\alpha = 0.5$, and $u_1 = 4.1$, so $R^* = 1.78$, $A/RV (\alpha R + N) = 4.304$. Then, draw a bifurcation diagram with respect to u_2 (the adjustment speed of firm 2) when $u_1 = 4.1$ as Figure 2 shows. It is easy to see when $u_2 \in (0, 2.6510)$ that the system is in the Nash equilibrium state. When u_2 exceeds 2.6510, a bifurcation behavior occurs. Period-doubling bifurcation and chaos begin to appear when $u_2 > 2.6510$. A large value of expenditure parameters converges to a more complicated economic behavior.

Figure 3 is the largest Lyapunov exponent with respect to u_2 . A positive value means a chaotic phenomenon. It is evident that L_1, L_2, L_3 , and L_4 in Figure 3 correspond to the first, second, third, and fourth bifurcation point in Figure 2, respectively. When $u_2 \in (0, 2.6510)$, the dynamic system is in the Nash equilibrium state. The first bifurcation occurs when $u_2 \in (2.6510, 4.2370)$; the second bifurcation occurs when $u_2 \in (4.2370, 4.5580)$; the third bifurcation occurs when $u_2 \in (4.5580, 4.6060)$; and chaos occurs when $u_2 \in (4.6060, 4.6190)$. After $u_2 = 4.6190$, the largest Lyapunov exponents are always positive.

The strange attractor is plotted in Figure 4.

Keeping other parameters fixed ($V = 10, N = 5$), Figure 5 presents the effect of different α on the stability of the system. Keeping other parameters fixed ($V = 10, \alpha = 0.5$), Figure 6 presents the effect of different N on the stability of the system. The expression of stability region S indicated that the increase of competition intensity or policymaker cost for supplying a rent heightens the stability of the rent-seeking market. The competition intensity α signifies the importance to player i of player j 's lobbying expenditures. The higher α is, the more player j 's expenditure reduces player i 's success probability, which is beneficial to the maintenance of Nash equilibrium. For any $N > 0$, $P_i, P_j < 1$. Therefore, it is possible that neither rent seeker wins the rent. The higher N is, the less easily both players succeed and the more stable the rent-seeking market is. It means that even though the players adjust the expenditure parameter faster, the rent-seeking market is still relatively stable with a bigger N .

Figure 7 is the bifurcation diagram with respect to α . As the competition parameter increases, chaos would be weakened in a rent-seeking market.

The sensitivity on the initial values has been demonstrated as follows. Considering different u_2 , subgraph 1 and subgraph 2 represent the orbits of R_1 with different initial values $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2.001, 1.9)$ from Figures 8–10. In the three figures, $u_2 = 2$, $u_2 = 4$, and $u_2 = 4.8$ represent equilibrium, bifurcation, and chaos in the state of

FIGURE 1: The stability region of E_* with respect to adjustment parameters of expenditure u_1, u_2 .

FIGURE 2: Bifurcation diagram with respect to u_2 .

FIGURE 3: Lyapunov exponent diagram with respect to u_2 .

system, respectively. As t increases, a tiny change $\Delta = 0.001$ of R_1 could cause a dramatic fluctuation only when the system is in chaos ($u_2 = 4.8$) in Figure 10.

FIGURE 4: Strange attractor for $u_1 = 4.1$ and $u_2 = 4.8$. FIGURE 5: Stability region with a variable α . FIGURE 6: Stability region with a variable N .

Subgraph 1 and subgraph 2 represent the orbits of R_2 with different initial values $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2, 1.901)$ from Figures 11–13. In the three figures, $u_2 = 2$, $u_2 = 4$, and $u_2 = 4.8$ represent equilibrium, bifurcation, and chaos in the state of the system, respectively. As t increases, a tiny change $\Delta = 0.001$ of R_2 could cause a dramatic fluctuation only when the system is in chaos ($u_2 = 4.8$) in Figure 13.

Figure 14 shows the map of bifurcation with respect to control factor k . The feedback control method is used to delay and eliminate chaos. The expenditure for rent-seeking finally stabilizes at the equilibrium levels ($R_1^* = R_2^* = 1.7839$) as Figure 15 shows. Figure 16 presents the effects of control factor k on profits. The rent-seekers' profits finally stabilize at the equilibrium levels ($\pi_1^* = \pi_2^* = 0.5401$).

5. Conclusions

Godwin et al. [7] used a comparative static method to analyze an improved rent-seeking model. They checked the

FIGURE 7: Bifurcation diagram with respect to α .

(a)

(b)

FIGURE 8: Orbits of R_1 with $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2.001, 1.9)$ of initial values ($u_2 = 2$).

(a)

(b)

FIGURE 9: Orbits of R_1 with $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2.001, 1.9)$ of initial values ($u_2 = 4$).

(a)

(b)

FIGURE 10: Orbits of R_1 with $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2.001, 1.9)$ of initial values ($u_2 = 4.8$).

FIGURE 11: Orbits of R_2 with $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2, 1.901)$ of initial values $(u_2 = 2)$.

FIGURE 12: Orbits of R_2 with $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2, 1.901)$ of initial values $(u_2 = 4)$.

FIGURE 13: Orbits of R_2 with $(R_{10}, R_{20}) = (2, 1.9)$ and $(R_{10}, R_{20}) = (2, 1.901)$ of initial values $(u_2 = 4.8)$.

FIGURE 14: Bifurcation diagram with respect to control factor k .

FIGURE 15: Effects of control factor k on R_1 , R_2 ($k=0.4$). FIGURE 16: Effects of control factor k on π_1 , π_2 ($k=0.4$).

policymaker cost and the value of rent into Tullock's lottery model of rent-seeking. Their contribution is that the political environment could enhance competition and help rent-seekers make decisions.

In this paper, players adjust strategy behaviors in a dynamic game process. We combine game theory with nonlinear dynamics to discuss the stability of Nash equilibrium. The results of this paper which is different from Godwin et al. [7] could be concluded: firstly, Theorem 1 has proved that the Nash equilibrium is unique and symmetrical in the rent-seeking game. In Theorem 2, locally asymptotic stability of Nash equilibrium is summarized. In addition, numerical simulation has investigated that the limit state of the system may be a complex state of period or chaos when the expenditure parameter is not in the stability region. The stability region of Nash equilibrium will enlarge as the competition intensity or policymaker cost increases. As the competition parameter increases, chaos would be weakened in a rent-seeking market. Ultimately, we present strange attractors and the sensitivity on initial values in a chaotic rent-seeking market. Chaos is controlled by the delay feedback control method.

Data Availability

No data were used to support this study. Numerical simulation was used to illustrate the theory results.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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