Research Article

Perturbation Analysis of Population Growth Rates in Taiwan

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The total fertility rate in Taiwan has fallen below 1.3 since 2003. The objective of the study is to use perturbation analysis with census data from 1992 to 2017 to identify which demographic parameters are most important to target for population management. The research shows that the fertility of older ages plays an important role for declining population in Taiwan. From a practical viewpoint, population management policy having a substantial impact on the survival of prereproductive females and the fertility of older females is likely to produce the most dramatic change in population trends. Therefore, the perturbation analysis is useful in understanding the relative importance of vital rates to increase management effectiveness.

1. Introduction

A quantitative understanding of complex changes in population structure, defining some of the important time scales for population dynamics, is crucial to sustainable population management in a country. Taiwan's population was characterized by both quite high fertility and mortality during the Japanese colonial period [1]. Since the implementation of family planning program in 1964, Taiwan's fertility rate has rapidly declined [2, 3]. Taiwan currently has one of the world's lowest fertility rates and the total fertility rate (TFR) has reached the below-replacement level in 1984. Since 2003, the TFR in Taiwan has fallen below 1.3, which is often described as lowest-low fertility [4], leading to projections of rapid population aging and negative socioeconomic effects.

Many studies pointed out that lowest-low fertility in East Asia is mainly attributable to delayed marriage and child-bearing, and increasing proportion remained single [5–8]. However, to judge the relative importance of demographic parameters for population management and to assess how variation in vital rates contributes to variation in population growth rate (PGR) have been little explored until now. Matrix population models (MPMs) are the primary models used to understand demographics and guide management of populations [9–11]. To model the dynamics of a population requires a link between vital rates and population processes.

That link is provided by age-structured population models since age differences are most important. Furthermore, Fujiwara and Diaz-Lopez [12] recommend using an age-structured MPM for investigating the status of a population when life table data are available. Life tables are now available for nearly all countries and are routinely produced by government statistical offices, insurance companies, and academic demographers. Therefore, the data in life table consisting of the number of births, number of deaths, and the exposed population all by age are employed in this study (see, e.g., [13, 14]).

The life table can be converted to the population projection matrix (PPM) which extends the dichotomy of the life table to consider individuals with different characteristics. The asymptotic population growth rate (PGR) λ , the rate at which population size would change if vital rates remained constant over time, is given by the dominant eigenvalue of a PPM. That is, the multiple vital rates are integrated into a single metric, the PGR. Projection is frequently followed by analytical and numerical perturbation analysis to understand the relative influence of vital rates. The results of perturbation analysis are often expressed as sensitivities and elasticities. Sensitivities quantify the absolute change in λ resulting from an infinitesimal absolute change in transition elements, whereas elasticities quantify the proportional change in λ resulting from an infinitesimal

proportional change in transition elements. Therefore, elasticity can be used to compare the effects of changes in vital rates that are measured on different scales (e.g., survival, which is close to one, and fertility, which is much less than one in this study). Because the elasticities sum to unity, they can be interpreted as the relative contributions of the vital rates to PGR [15, 16]. The effect that each vital rate has on population growth can be used as an index for evaluating the importance of certain vital rates for management and research. In general, management strategies are recommended that improve those vital rates that are most sensitive or elastic with respect to the PGR.

Under constant fertility and mortality rates, the asymptotic population dynamics are independent of initial conditions. This property is known as ergodicity [17, 18], implying that population patterns might reveal something about processes rather than initial conditions. Therefore, in a limited sense, the population management may be explored. The approach employed would be characterized as prospective analysis that focuses on the functional mathematical dependence of λ on parameters.

The objective of the study is to use MPMs with census data from 1992 to 2017 to identify which demographic parameters are most important to target for population management. The status is investigated by comparing differences in important pieces of information with year, such as PGRs and the generation time, calculated using age-structured population matrices and validated using the Euler-Lotka equation [19]. The next section shows a series of matrix approaches to demographic analysis, including the life tables, projections, and perturbation analysis. The schematic framework is applied to the population of Taiwan, followed by a detailed discussion. Finally, the paper concludes with a brief summary of this study.

2. Methods

2.1. Life Tables. In constructing a life table, three assumptions need to be made [20]: (1) no migration or the migrants as a whole have the same mortality regime specified by the life table, (2) annual age-specific death rates do not change over time, and (3) annual number of births remains constant over time. While the life table oversimplifies demographic mechanisms, a rich variety of useful results is based on it [21].

The life table is used as a unifying technique in demography [21]. In this study, the analysis commences by constructing the female life tables and age-specific fertility schedules using age-specific tabulations of midyear population size, deaths, and births. The following three items are used to construct the life tables and age-specific fertility schedules: midyear population size between age x and $x + n$ (${}_nK_x$), number of deaths between age x and $x + n$ (${}_nD_x$), and number of births at age x (b_x) for age $x \in 0, 1, 4, 9, \dots, 94, 99$. Age one is shown because mortality at age zero is very different from mortality at age one to five. We can directly obtain TFR and period central death rate ${}_nM_x$ as $\text{TFR} = n \sum_i (b_x / {}_nK_x)_i$ and ${}_nM_x = {}_nD_x / {}_nK_x$, respectively.

The life table is constructed using the Keyfitz-Flieger graduation [22] and contains nine columns: age class (x), person-years lived by those dying between age x and $x + n$ (${}_na_x$), period central death rate (${}_nM_x$), interval mortality probability (${}_nq_x = {}_nd_x / l_x$), probability of surviving from birth to age x (l_x), difference in number of survivors for ages n years apart (${}_nd_x = l_x - l_{x+n}$), person-years lived between age x and $x + n$ (${}_nL_x = \int_x^{x+n} l(t)dt$), person-years of remaining life at age x ($T_x = {}_\infty L_x$), and life expectancy at age x ($e_x = T_x / l_x$). Some researchers [18, 20, 23, 24] have provided sufficient discussion on making a life table.

2.2. Projections. Converting age-based life-table data to an age-classified PPM can show how age structure changes during population growth. The PPM **A** based on five-year age structure has survival probability P_i on the subdiagonal and fertility F_j in the first row and zeros elsewhere and can be expressed as

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 & F_8 \\ P_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & P_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & P_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & P_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & P_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & P_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & P_7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{10} & 0 \end{bmatrix}. \quad (1)$$

The asymptotic PGR λ is given by the dominant eigenvalue of **A** ($\lambda > 1$: increasing population; $\lambda < 1$: declining population). The long-term population dynamics are described by λ and the stable age distribution **w**, which is the right eigenvector corresponding to λ . The following Euler-Lotka equation is used to obtain an exact solution for λ [19]:

$$1 = \sum_{i=\alpha}^{\beta} \lambda_1^{-i} \left(\prod_{j=1}^{i-1} P_j \right) F_i, \quad (2)$$

where α is the age at first reproduction and β is the age at last reproduction.

Vector of reproductive values is the ratio of the number of offspring produced by individuals of some specific age or older and the number of individuals of that age in a stable age distribution [25]. The dominant left eigenvector **v** of the PPM is the reproductive value vector [9], which is scaled so that $\mathbf{v}^* \mathbf{w} = 1$ where $\mathbf{v}^*$ is the complex conjugate transpose of **v**.

The net reproduction number R_0 is the number of girl children expected to be born to a girl child just born. Generation time T is the amount of time that it takes a typical female to produce R_0 offspring, or equivalently, the

time it takes a population growing at instantaneous rate r to increase by a factor of R_0 . Thus, the net reproduction number can be defined as $R_0 = \lambda_1^T = e^{nrT}$ over an n -year period [9]. The intrinsic rate of increase measures the rate of increase as a function of absolute time, whereas R_0 measures increase as a function of generation time. Therefore, if we want to calculate intrinsic rate of increase r , we must scale R_0 to account for generation time.

2.3. Perturbation Analysis. The rate of increase of a population is a function of the age-specific vital rates. Demographic perturbation analysis examines the PGR λ of the model to changes in the vital rates a_{ij} (PPM elements). The results of perturbation analysis are often expressed as sensitivities and elasticities. The sensitivity s_{ij} of λ to transition elements a_{ij} is $s_{ij} = (\partial\lambda/\partial a_{ij}) = v_i w_j$. The elasticity e_{ij} of λ to transition elements a_{ij} is $e_{ij} = (a_{ij}/\lambda)(\partial\lambda/\partial a_{ij})$. Caswell [26] derived a formula for the second derivative of λ to investigate the change in sensitivities as follows:

$$\frac{\partial^2 \lambda}{\partial a_{ij} \partial a_{kl}} = s_{il}^{(1)} \sum_{m \neq 1} \frac{s_{kj}^{(m)}}{\lambda - \lambda^{(m)}} + s_{kj}^{(1)} \sum_{m \neq 1} \frac{s_{il}^{(m)}}{\lambda - \lambda^{(m)}}. \quad (3)$$

While sensitivities may be used to compare the absolute effects on λ of the same absolute changes in vital rates, elasticities may be used to predict changes in λ as a result of changes in vital rates. Therefore, elasticity analysis is often used to give directions for population management [9, 10].

Among the modern software package or computer language, for example, MATLAB [27], R [28], SAS [29], SPSS [30], and Stata [31], the high-level programming language R stands out because it allows both statistical and mathematical modeling. In addition, it is free and has excellent graphics capabilities [32]. For all the analysis in this paper, the R package demogR developed by Jones [33] is used to construct and analyze the age-structured population of Taiwan.

3. Results and Discussion

3.1. Vital Rates and Projections. In 1992, the goal of the Guideline for Population Policy was modified from reducing population growth to maintaining a reasonable growth of population. Therefore, six sets of census data from 1992, 1997, 2002, 2007, 2012, and 2017 are selected for this study. The statistical data of midyear population size, number of deaths, and number of births are available from the website of Ministry of the Interior [34] of Taiwan. Table 1 shows all the census counts and death tabulations for female population of Taiwan from the discussed six years.

Age-specific mortality rates for the six populations, as shown in Figure 1, display the pattern of high and declining early mortality. The most rapidly changing portion is the ages under four and especially from birth to the first birthday. The age-specific mortality rates indicate a steady decline trend with year for later ages.

Figure 1 shows that the fertility rate falls more rapidly at age class 25–29 than at others. While the fertility rates of younger ages decrease with year, the fertility rates of older

ages increase with year, indicating a phenomenon of aging of fertility due to the postponement of marriage. The peak in fertility occurred at an older age class 30–34 in 2017 with a lower TFR, compared to the peaks in 1992 at age class 26–30 with higher TFR. The highest fertility values shifted toward older ages with year.

Plotting survivorship against age shows high survivorship during young and intermediate ages, as shown in Figure 2. The age extends only to age 99 because midyear population size and number of deaths beyond that age before 1997 are not available. The decline in survivorship is more moderate before the age of 69. The survivorship is consistent with the trend of age-specific mortality rates. The age-specific life expectancies indicate a steady lengthening trend, as shown in Figure 2. The promotion of National Health Insurance in 1995 has effectively reduced the chances of death among all ages and is reflected in a significant increase in life expectancies. Age-based life-table data are converted to age-classified PPMs, as shown in Table 2.

The Euler-Lotka equation and age-structured model show the similar values for PGRs. This suggests there is no bias introduced by discretizing age. PGR ranges from 0.912 to 0.975. The fluctuation in the intrinsic rate of increase is more substantial than that in TFR, as shown in Figure 3. The intrinsic rate of increase tells us how quickly the declining population over time. Despite the fluctuations, the rebounded fertility in 2012 echoed the effect of the Dragon years on fertility behavior in Taiwan.

From Figure 3, we can see that the generation time is extended over time. The average woman in Taiwan after 1992 replaced herself with less one daughter and took more than 30 years to do so. Therefore, it is also the expected number of offspring produced by an individual during her lifetime.

Since the population is declining, the stable age distribution is skewed toward older ages, as shown in Figure 4. Declining populations have larger proportions of old people. For increasing population, relative frequencies in the stable age distribution always decrease with later ages. That is, newborns are the most common age, and the oldest age is most rare.

The reproductive value vector is usually scaled so that the reproductive value of newborns always equals one, to measure reproductive value relative to a newborn individual. Curves of reproductive value for females in Taiwan show no peak near the age of first reproduction, as shown in Figure 5, reflecting the increased reproduction ages and survival of order ages. Before age class 20–24, the line with highest reproductive value is that with the highest λ ; after age class 20–24, the line with highest reproductive value is that with the lowest λ . That is, the influence on projected λ compared with newborn is larger and larger for older age class.

3.2. Perturbation Analysis. Sensitivities to variation in elements of the PPM are given by Table 3. While the fertility sensitivity increases with age and grows faster with year, the survival sensitivity declines or remains the same with age and shrinks slightly with year, as shown in Figure 6. The PGR

TABLE 1: Midyear population size, number of deaths, and number of births of Taiwan (1992–2017).

Age Class	1992			1997			2002			2007			2012			2017		
	nK_x	nD_x	b_x	nK_x	nD_x	b_x	nK_x	nD_x	b_x	nK_x	nD_x	b_x	nK_x	nD_x	b_x	nK_x	nD_x	b_x
0	147356	717	0	146330	988	0	115418	650	0	91808	443	0	97887	385	0	91590	363	0
1	157049	155	0	154191	145	0	134812	96	0	99093	49	0	88758	34	0	102500	29	0
2–4	475875	250	0	464325	200	0	414173	145	0	320733	65	0	278539	66	0	308456	35	0
5–9	847953	238	0	774933	168	0	777268	124	0	669329	101	0	519036	77	0	473218	47	0
10–14	978288	262	0	837993	205	0	775539	118	0	776093	88	0	669180	79	0	518601	64	0
15–19	903112	447	15283	974787	453	14987	836336	277	10545	773342	192	4317	775452	174	3115	668830	148	2727
20–24	909546	509	82712	896260	415	71962	976110	392	55974	836237	317	30563	773147	253	19882	774651	211	16196
25–29	951476	616	140963	903078	536	132310	905514	443	91994	989651	476	75434	854840	376	67712	773807	256	45525
30–34	932646	771	67037	949945	703	83029	911575	570	66394	920682	557	68137	1019195	589	99237	866326	436	73660
35–39	849683	933	13929	931417	992	20368	950255	887	19284	918097	747	22287	937997	748	39095	1026472	731	47949
40–44	664160	1084	1414	846357	1275	2228	927101	1247	2461	948103	1182	2872	923424	1054	5399	938471	1010	8182
45–49	432660	1136	67	658309	1508	96	839730	1659	106	921845	1745	101	947141	1711	159	919590	1453	377
50–54	436912	1695	0	416775	1491	0	651546	2007	0	833227	2347	0	916677	2296	0	939609	2260	0
55–59	375098	2304	0	422084	2354	0	408946	2033	0	643321	2746	0	825259	3020	0	904809	3206	0
60–64	319961	3321	0	357237	3279	0	408863	3286	0	398573	2788	0	631597	3753	0	809161	4190	0
65–69	247233	4358	0	296121	4686	0	337926	4639	0	390323	4525	0	384846	3821	0	612043	5071	0
70–74	175615	5559	0	217330	5611	0	269127	6334	0	311168	6140	0	364508	6128	0	361830	5522	0
75–79	114103	6484	0	142027	6658	0	184658	7202	0	233440	7915	0	275107	8444	0	326648	8927	0
80–84	64410	6012	0	80268	6550	0	106527	7371	0	144177	8604	0	187533	10259	0	224814	11514	0
85–89	25460	3717	0	36643	4888	0	48351	5797	0	68872	7239	0	97308	9468	0	130012	12392	0
90–94	8188	1574	0	10482	2099	0	16075	2953	0	22995	4055	0	34755	5920	0	50927	8505	0
95–99	1768	422	0	2451	592	0	3123	777	0	5150	1280	0	7485	1982	0	11997	3114	0

FIGURE 1: Age-specific mortality and fertility rates for the six populations.

FIGURE 2: Age-specific survivorship and life expectancy for the six populations.

is more sensitive to changes in fertility than to changes in survival for all ages. Sensitivities vary by as much as three orders of magnitude in 2007, with the lowest PGR. The highest sensitivities are to change in fertility at age class 45–49. That is, λ is most sensitive to fertility element F_8 , which has larger effect on λ than the most important survival element P_1 . The ratio F_8/P_1 ranges from 1.3 to 2.7. Note that

the value of the sensitivity to first period survival declines with year. The intuition behind this is that, in low-fertility populations, the age structure is more heavily weighted toward the older ages. Demetrius [35] showed that the sensitivity to fertility is a declining function of the age if the PGR is greater than the greatest survival rate. Caswell [9] also showed that the sensitivity to fertility in an increasing

TABLE 2: PPMs with the first row and subdiagonal elements (1992–2017).

Year	Age										
	0	2–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
1992	0.9967	0.9980	0.9986	0.0205	0.1304	0.2891	0.2662	0.1068	0.0224	0.0028	0.0001
1997	0.9970	0.9984	0.9988	0.0186	0.1155	0.2738	0.2825	0.1320	0.0296	0.0034	0.0001
2002	0.9976	0.9987	0.9992	0.0153	0.0846	0.1922	0.2110	0.1127	0.0278	0.0034	0.0001
2007	0.9984	0.9991	0.9994	0.0068	0.0510	0.1365	0.1819	0.1191	0.0331	0.0038	0.0001
2012	0.9986	0.9991	0.9993	0.0049	0.0360	0.1272	0.2140	0.1686	0.0577	0.0073	0.0001
2017	0.9991	0.9995	0.9994	0.0049	0.0303	0.0967	0.1744	0.1598	0.0673	0.0111	0.0002

FIGURE 3: (a) Intrinsic rate of increase and total fertility rate for the six populations. (b) Generation times for the six populations with different net reproduction number.

population modeled by a PPM cannot increase with age. However, in this study, the PGR is smaller than the greatest survival rate and the sensitivity to fertility increases from one age to the next, as shown in Figure 6.

Table 4 shows elasticities to variation in elements of the PPM. The most important coefficients to λ are survivals, which together account for 84.60% to 86.48% of λ and the fertilities for only 13.52% to 15.40%. While the importance of survivals increases with year, the importance of fertilities decreases with year. While the changes in fertility at age class 20–29 have the larger impact on λ before 2007, the changes in fertility at age class 25–34 have the larger impact on λ after 2007. Figure 7 identifies that the elasticity to survival probability consistently declines with age and that the highest elasticities are those of prereproductive survival. That is, the smallest changes in survival at prereproductive will produce the bigger change in PGR. The elasticity to

fertility firstly increases and then declines. The elasticities to survival from younger ages are much greater than the elasticities to fertility from younger ages. However, the elasticity to fertility from older ages is slightly greater than the elasticity to survival from older ages. Caswell [9] showed that the elasticity to survival cannot increase with age and that the elasticity to survival from any age is greater than or equal to the elasticity to fertility from any older age. For a decreasing population, Figure 7 shows that the elasticity to survival is greater than the elasticity to fertility only before middle in the reproductive period. However, the elasticity to survival into the oldest age is equal to the elasticity to fertility from the oldest age. This is to be expected, since these two demographic processes belong to the same life cycles and the elasticities to all processes within a cycle are the same [9, 36]. The peak of the elasticity to fertility shifts to order ages with year, indicating that the

FIGURE 4: Stable age distributions for the six populations.

FIGURE 5: Age-specific reproductive values for the six populations.

TABLE 3: Sensitivity matrices with the first row and subdiagonal elements (1992–2017).

Year	Age										
	0	2–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49
1992	0.0000	0.0000	0.0000	0.1670	0.1717	0.1763	0.1810	0.1857	0.1903	0.1946	0.1982
	0.1501	0.1499	0.1498	0.1464	0.1241	0.0730	0.0247	0.0048	0.0006	0.0000	
1997	0.0000	0.0000	0.0000	0.1624	0.1663	0.1702	0.1741	0.1780	0.1818	0.1853	0.1883
	0.1480	0.1478	0.1477	0.1448	0.1256	0.0789	0.0296	0.0060	0.0006	0.0000	
2002	0.0000	0.0000	0.0000	0.1806	0.1928	0.2057	0.2193	0.2337	0.2488	0.2645	0.2803
	0.1394	0.1392	0.1391	0.1364	0.1202	0.0806	0.0343	0.0078	0.0009	0.0000	
2007	0.0000	0.0000	0.0000	0.1877	0.2056	0.2251	0.2463	0.2693	0.2942	0.3209	0.3491
	0.1305	0.1305	0.1304	0.1292	0.1188	0.0880	0.0432	0.0110	0.0012	0.0000	
2012	0.0000	0.0000	0.0000	0.1693	0.1812	0.1938	0.2071	0.2213	0.2361	0.2517	0.2675
	0.1293	0.1292	0.1292	0.1284	0.1219	0.0973	0.0529	0.0155	0.0019	0.0000	
2017	0.0000	0.0000	0.0000	0.1734	0.1883	0.2045	0.2220	0.2409	0.2611	0.2827	0.3053
	0.1244	0.1244	0.1244	0.1236	0.1179	0.0981	0.0594	0.0208	0.0032	0.0001

FIGURE 6: Sensitivities to age-specific survival in black and fertility in grey.

TABLE 4: Elasticity matrices with the first row and subdiagonal elements (1992–2017).

Year	Age											Total
	0	2–4	5–9	10–14	15–19	20–24	25–29	30–34	35–39	40–44	45–49	
1992	0.0000	0.0000	0.0000	0.0035	0.0230	0.0525	0.0496	0.0204	0.0044	0.0006		0.1540
	0.1540	0.1540	0.1540	0.1505	0.1275	0.0750	0.0254	0.0050	0.0006	0.0000	0.0000	0.8460
1997	0.0000	0.0000	0.0000	0.0031	0.0197	0.0478	0.0505	0.0241	0.0055	0.0006		0.1513
	0.1513	0.1513	0.1513	0.1482	0.1285	0.0807	0.0303	0.0062	0.0007	0.0000	0.0000	0.8487
2002	0.0000	0.0000	0.0000	0.0029	0.0174	0.0422	0.0495	0.0282	0.0074	0.0010		0.1486
	0.1486	0.1486	0.1486	0.1456	0.1282	0.0860	0.0365	0.0084	0.0010	0.0000	0.0000	0.8514
2007	0.0000	0.0000	0.0000	0.0014	0.0115	0.0337	0.0491	0.0352	0.0107	0.0013		0.1429
	0.1429	0.1429	0.1429	0.1415	0.1300	0.0963	0.0472	0.0120	0.0014	0.0000	0.0000	0.8571
2012	0.0000	0.0000	0.0000	0.0009	0.0070	0.0264	0.0475	0.0400	0.0146	0.0020		0.1383
	0.1383	0.1383	0.1383	0.1374	0.1304	0.1040	0.0565	0.0166	0.0020	0.0000	0.0000	0.8617
2017	0.0000	0.0000	0.0000	0.0009	0.0062	0.0215	0.0421	0.0418	0.0191	0.0034		0.1352
	0.1352	0.1352	0.1352	0.1342	0.1280	0.1065	0.0644	0.0226	0.0035	0.0001	0.0001	0.8648

FIGURE 7: Elasticities to age-specific survival in black and fertility in grey.

importance of fertility at order ages to PGR is getting more evident. Note that as fertility shifts to order ages, the relative elasticity of early survival is reduced.

The second derivatives $\partial^2 \lambda / \partial F_i^2$ denoted with small circles in Figure 8 decrease with age, implying nonlinear rates of change in elasticity values, and become negative

FIGURE 8: PGR second derivatives with respect to pairs of fertilities.

soon after the beginning of the life cycle. Note that $\partial^2 \lambda / \partial F_1^2 > 0$ as expected [37]. The rates of increase λ are a convex function of F_j for the age class 10–14 and a concave function of F_j for later ages. The second derivative of λ with respect to changes in the fertility $\partial^2 \lambda / \partial F_i \partial F_j$ decreases with both i and j ; this means that declining fertility at any age shifts the age distribution to older ages, because it decreases λ . Despite the fluctuation in declining rates of the second derivative with respect to the fertility, the rebounded rate in

2012 echoed the effect of the Dragon years on fertility behavior in Taiwan.

The value of $\partial^2 \lambda / \partial P_i^2$ denoted with small circles in Figure 9 is negative for all age, indicating that PGR is a concave function of P_i , especially at young ages. However, these values are getting closer to zero with year. That is, changes in survival at these ages have some effect on the sensitivity of λ to further changes in survival. However, the effect decreases with year. The value of $\partial^2 \lambda / \partial P_i^2$ at old ages is

FIGURE 9: PGR second derivatives with respect to pairs of survival probabilities.

very small; that is, changes in survival at later ages have little or no effect on the sensitivity of λ to further changes in survival. The second derivatives of λ with respect to changes in the survival $\partial^2 \lambda / \partial P_i \partial P_j$ may be positive and decrease with age for early ages. However, the declining rate is getting more and more moderate. At early ages, the survival second derivatives show alternation in sign. That is, as survival at one age increases, further increases in survival sensitivity at that age become less important, and increases in survival sensitivity at other ages become more important.

Overall, the way that survival and fertility combine to influence λ at later ages is highest, as shown in Figure 10, but declines as PGR declines. There is a great deal of diversity in these six populations in the magnitude of the eigenvalue second derivatives to survival and fertility. For example, the curves at age class 20–24 show that an increase in fertility at age class 20–24 reduces the sensitivity to survival at age class 20–24 and all subsequent ages and increases the sensitivity to survival before age class 20–24.

FIGURE 10: PGR second derivatives with respect to survival and fertility.

4. Conclusions

This study provides guidance about the relative importance of demographic parameters that are most likely to influence population dynamics in Taiwan. The research results are different from some previously proposed constraints on sensitivity and elasticity for increasing population. For increasing population, elasticity analysis inevitably concludes that management strategies improving survival are best

applied to younger ages and indicates that changes to the fertility of older ages are mathematically constrained to have less impact on population growth than equivalent changes to the fertility of younger ages. However, the fertility of older ages plays an important role for declining population in Taiwan. PGR is most sensitive to fertility at older ages, which is consistent with the result of the elasticity analysis; the importance of fertility at older ages to PGR is getting more evident. The skewness toward older ages is also reflected in

the stable age distribution. The reproductive value vector reveals that reproductive females theoretically contribute proportionally less to future population growth because these females reproduce less than one in their lives. Survival sensitivity declines or remains the same with age and shrinks slightly with year. However, the largest elasticities are those for the probability of prereproductive survival. From a practical viewpoint, population management policy having a substantial impact on the survival of prereproductive females and the fertility of order females is likely to produce the most dramatic change in population trends.

PGR and generation time are two of the most important pieces of information in determining the status of threatened species [38]. The perturbation analysis is useful in understanding the relative importance of vital rates to increase management effectiveness. The survivorships during young and intermediate ages in Taiwan are high and show a steady increasing trend with year for later ages. The long-term PGR in Taiwan is most sensitive to fertility at older ages, suggesting that management should aim to increase such fertility.

Data Availability

The statistical data of midyear population size, number of deaths, and number of births are available from the website of Ministry of the Interior of Taiwan (<https://www.ris.gov.tw/app/en/3911>).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] R. Freedman, J. Y. Peng, Y. Takeshita, and T. H. Sun, "Fertility trends in Taiwan: tradition and change," *Population Studies*, vol. 16, no. 3, pp. 219–236, 1963.
- [2] W. L. Li, "Temporal and spatial analysis of fertility decline in Taiwan," *Population Studies*, vol. 27, no. 1, pp. 97–104, 1973.
- [3] W.-I. Lin and S.-Y. Yang, "From successful family planning to the lowest of low fertility levels: taiwan's dilemma," *Asian Social Work and Policy Review*, vol. 3, no. 2, pp. 95–112, 2009.
- [4] H.-P. Kohler, F. C. Billari, and J. A. Ortega, "The emergence of lowest-low fertility in Europe during the 1990s," *Population and Development Review*, vol. 28, no. 4, pp. 641–680, 2002.
- [5] G. W. Jones and B. Gubhaju, "Factors influencing changes in mean age at first marriage and proportions never marrying in the low-fertility countries of East and Southeast Asia," *Asian Population Studies*, vol. 5, no. 3, pp. 237–265, 2009.
- [6] M. Lee, "Transition to below replacement fertility and policy response in Taiwan," *The Japanese Journal of Population*, vol. 7, no. 1, pp. 71–86, 2009.
- [7] T. Frejka, G. W. Jones, and J.-P. Sardon, "East Asian child-bearing patterns and policy developments," *Population and Development Review*, vol. 36, no. 3, pp. 579–606, 2010.
- [8] Y. H. Chen, "Trends in low fertility and policy responses in Taiwan," *The Japanese Journal of Population*, vol. 10, no. 1, pp. 78–88, 2012.
- [9] H. Caswell, *Matrix Population Models: Construction, Analysis, and Interpretation*, Sinauer, Sunderland, MA, USA, 2nd edition, 2001.
- [10] W. F. Morris and D. F. Doak, *Quantitative Conservation Biology: Theory and Practice in Conservation Biology*, Sinauer, Sunderland, MA, USA, 2002.
- [11] E. E. Crone, E. S. Menges, M. M. Ellis et al., "How do plant ecologists use matrix population models?" *Ecology Letters*, vol. 14, no. 1, pp. 1–8, 2011.
- [12] M. Fujiwara and J. Diaz-Lopez, "Constructing stage-structured matrix population models from life tables: comparison of methods," *PeerJ*, vol. 5, Article ID e3971, 27 pages, 2017.
- [13] V. Rapisarda, C. Loreto, E. Vitale et al., "Incidence of sharp and needle-stick injuries and mucocutaneous blood exposure among healthcare workers," *Future Microbiology*, vol. 14, no. 9s, pp. 27–31, 2019.
- [14] R. Ragusa, G. Giorgianni, L. Lupo et al., "Healthcare-associated *Clostridium difficile* infection: role of correct hand hygiene in cross-infection control," *Journal of Preventive Medicine and Hygiene*, vol. 59, no. 2, pp. E145–E152, 2018.
- [15] H. Caswell, R. J. Naiman, and R. Morin, "Evaluating the consequences of reproduction in complex salmonid life cycles," *Aquaculture*, vol. 43, no. 1–3, pp. 123–134, 1984.
- [16] H. de Kroon, A. Plaisier, J. van Groenendaal, and H. Caswell, "Elasticity: the relative contribution of demographic parameters to population growth rate," *Ecology*, vol. 67, no. 5, pp. 1427–1431, 1986.
- [17] A. J. Coale, *The Growth and Structure of Human Populations: A Mathematical Investigation*, Princeton University Press, Princeton, NJ, USA, 1972.
- [18] S. H. Preston, P. Heuveline, and F. Guillot, *Demography: Measuring and Modeling Population Processes*, Blackwell, Oxford, UK, 2001.
- [19] M. Kot, *Elements of Mathematical Ecology*, Cambridge University Press, Cambridge, UK, 2001.
- [20] F. Yusuf, J. M. Martins, and D. A. Swanson, *Methods of Demographic Analysis*, Springer, New York, NY, USA, 2014.
- [21] N. Keyfitz and H. Caswell, *Applied Mathematical Demography*, Springer, New York, NY, USA, 3rd edition edition, 2005.
- [22] N. Keyfitz and W. Flieger, *World Population Growth and Aging: Demographic Trends in the Late Twentieth Century*, University of Chicago Press, Chicago, IL, USA, 1990.
- [23] T. Greville, "Short methods for constructing life tables," *American Institute of Actuaries*, vol. 32, pp. 29–42, 1943.
- [24] R. Schoen, *Modeling Multigroup Populations*, Plenum, New York, NY, USA, 1988.
- [25] E. O. Wilson and W. H. Bossert, *A Primer of Population Biology*, Sinauer, Sunderland, MA, USA, 1971.
- [26] H. Caswell, "Second derivatives of population growth rate: calculation and applications," *Ecology*, vol. 77, no. 3, pp. 870–879, 1996.
- [27] MatlabThe MathWorks Inc., Natick, MA, USA, 2019.
- [28] Development Core Team, *A Language and Environment for Statistical Computing*, Foundation for Statistical Computing, Vienna, Austria, 2014.
- [29] SAS Institute, *SAS® 9.4 Statements: Reference*, SAS Institute, Cary, NC, USA, 2013.
- [30] IBM Corp, *IBM SPSS Statistics for Windows*, IBM Corp., Armonk, NY, USA, 2017.
- [31] StataCorp, *Statistical Software: Release*, Vol. 14, StataCorp LP., College Station, TX, USA, 2015.
- [32] F. J. C. Willekens, "A comment on Thomas K. Burch's paper 'does demography need differential equations?'" *Canadian Studies in Population*, vol. 38, no. 1–2, pp. 169–178, 2011.
- [33] J. H. Jones, "A package for evolutionary demographic analysis," *Journal of Statistical Software*, vol. 22, no. 10, pp. 1–28, 2007.

- [34] Ministry of the Interior (MOI), “Table 11 Number of births by age of mother, total fertility rate, average age of mother and average age of first birth, table 16 midyear population by sex and 5-year age group (1975-2017,” Department of Household Registration Department, 2018, <https://www.ris.gov.tw/app/en/3911>.
- [35] L. Demetrius, “The sensitivity of population growth rate to perturbations in the life cycle components,” *Mathematical Biosciences*, vol. 4, no. 1-2, pp. 129–136, 1969.
- [36] J. van Groenendael, H. de Kroon, S. Kalisz, and S. Tuljapurkar, “Loop analysis: evaluating life history pathways in population projection matrices,” *Ecology*, vol. 75, no. 8, pp. 2410–2415, 1994.
- [37] J. E. Cohen, “Derivatives of the spectral radius as a function of non-negative matrix elements,” *Mathematical Proceedings of the Cambridge Philosophical Society*, vol. 83, no. 2, pp. 183–190, 1978.
- [38] IUCN, *IUCN Red List Categories and Criteria*, 2nd edition, 2012.