Research Article

Event-Triggered Average Consensus of Multiagent Systems with Switching Topologies

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Event-triggered average consensus of multiagent systems with switching topologies is studied in this paper. A distributed protocol based on event-triggered time sequences and switching time sequences is designed. Based on the inequality technique and stability theory of differential equations, a sufficient condition for achieving average consensus is obtained under the assumption that switching signal is ergodic and the total period over which connected topologies is sufficiently large. A numerical simulation is presented to show the effectiveness of the theoretical results.

1. Introduction

Recently, the consensus problem of multiagent systems has attracted a great deal of attention in many fields such as multirobot systems [1], sensor networks [2], and unmanned air vehicles (UAVs). This hot topic has been widely discussed in the literature by different methods, such as Lyapunov function methods [3, 4], linear matrix inequality methods [5–7], matrix decomposition approaches [8], and impulsive control methods [9, 10], just to name a few.

In practical engineering applications, connections between agents often change due to obstacles in the environment or limitations of sensor communication distance. Hence, consensus of multiagent systems with switching topologies has attracted considerable attention [11–23]. In particular, the average consensus problem with switching topologies was studied in [17]. The leader-following consensus of second-order agents with switching topologies was studied in [18], which proved that the consensus of multiagent systems was asymptotically reachable and gave an estimate of the convergence rate. In [19], for controllability of multiagent systems with periodically switching topologies was studied and a criterion for m-periodic controllability was proposed. In [20], the guaranteed-performance consensualization for high-order linear and nonlinear

multiagent systems with switching topologies was studied. In [21], the consensus of multiagent systems with switching jointly reachable interconnection was studied. In [22], the consensus problem of multiagent systems with jointly connected switching topologies was studied by adding adaptive control. However, the control protocol used in the above literature requires continuous communication among agents, which is hard to realize due to the limited communication bandwidth. It may also result in the waste of computing resources as well as the consumption of a large amount of energy.

To improve the usage of limited bandwidth resource, event-triggered consensus of multiagent systems with switching topologies has been extensively studied [24–29]. In [30], the event-triggered leaderless and leader-following consensus problems of multiagent systems with jointly connected topology were investigated. In [31], an event-triggered protocol for networks with switching topologies was proposed where the triggering functions were designed based on continuous information. In [32], event-triggered control for pinning cluster synchronization in an array of coupled neural networks was studied. In [33], event-triggered control leader-following consensus problems of multiagent systems with semi-Markov switching topologies were discussed. The event-triggered consensus problems of

multiagent systems with jointly connected switching topologies were also presented in [34–37].

By the above discussion, how to define the control protocol, which updates at both the triggering time instants and switching time instants, is an interesting topic for multiagent systems with switching topologies. In this paper, event-triggered consensus of first-order multiagent systems with switching topologies is studied. The contribution of this paper is as follows: (1) The event-triggered consensus of multiagent systems with switching communication topologies is considered. The update time instants of the controller are determined by both triggering time instants and switching time instants. (2) The Zeno-behavior of the sampling time sequences of the controller can be excluded directly and the infimum of the sampling interval can be bigger than a given positive constant. (3) A sufficient condition on consensus is presented under the assumption that the switching communication topologies are ergodic.

The rest of this paper is organized as follows. Section 2 introduces some preliminaries and the first-order multiagent model. The main results are presented in Section 3. A numerical example is presented in Section 4 to illustrate the effectiveness of theoretical results. Section 5 concludes the paper and offers suggestions for future work.

Notations. let $\|x\|$ be the Euclidean norm of vector x and $\|A\|$ be the corresponding induced matrix norm for a matrix A. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are the minimum and maximum eigenvalues of a symmetric real matrix A. $\mathbf{1}_n = [1, 1, \ldots, 1]^T$.

2. Preliminaries and Problem Formulation

2.1. Preliminaries. Consider $\Lambda = \{1, 2, \ldots, M\}$ as index set, where M is the total number of all possible interconnection graphs. To consider the consensus problem of multiagent systems with switching topologies, we define $\mathcal{G}_p = (\mathcal{V}, \varepsilon_p, A_p)$, $p \in \Lambda$ as graphs of order n with the set of nodes $\mathcal{V} = \{1, 2, \ldots, n\}$ (denoting the n agents) for any $p \in \Lambda$, the set of edges $\varepsilon_p \subset \mathcal{V} \times \mathcal{V}$, and adjacency matrices $A_p = (a_{ij}^p)_{n \times n}$, where a_{ij}^p is the weight of the edge $(i, j) \in \varepsilon_p$ in the graph $\mathcal{G}_p$. $a_{ij}^p > 0$ if and only if there is an edge (i, j) in $\mathcal{G}_p$. Moreover, we assume $a_{ii}^p = 0$ for all $i = 1, \ldots, n$, $p \in \Lambda$. The degree matrices $D_p = \text{diag}\{d_1^p, \ldots, d_n^p\}$ are diagonal matrices, whose diagonal elements are given by $d_i^p = \sum_{j=1}^n a_{ij}^p$. The Laplacian of the weighted graph is defined as $L_p = D_p - A_p$ for every p. The set of neighbors of node i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V}: (i, j) \in \varepsilon_p\}$.

Let $\mathcal{G}_s$ be the set of connected graphs in $\mathcal{G}_p$ and $\mathcal{G}_u$ be the set of unconnected graphs in $\mathcal{G}_p$. The cardinality of Λ is denoted by $|\Lambda|$. Without loss of generality, we assume that $\mathcal{G}_s = \{\mathcal{G}_p, p = 1, 2, \ldots, \nu\}$ and $\mathcal{G}_u = \{\mathcal{G}_p, p = \nu + 1, \nu + 2, \ldots, |\Lambda|\}$ for some $\nu \in \Lambda$. For any $t > t_0$, let $T_p(t_0, t)$ denote the total activation time of $\mathcal{G}_p$, $p \in \Lambda$, and $T_s(t) = \sum_{p=1}^{\nu} T_p(t_0, t)$, and $T_u(t) = \sum_{p=\nu+1}^{|\Lambda|} T_p(t_0, t)$.

In the following, we assume that switching is "ergodic switching" [13], i.e., each graph will be activated infinite times.

2.2. Problem Formulation. Consider the first-order multiagent systems described by

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \ldots, n, \tag{1}$$

where $x_i(t)$ and $u_i(t) \in R$ denote the position and control input of agent i, respectively.

In order to discuss the consensus problem of multiagent systems (1) and avoid the Zeno-behavior directly, i.e., there is no infinite sampling in a finite interval, the following distributed event-triggered control protocol will be used:

$$u_i(t) = \begin{cases} 0, & t \in [t_k^i, t_k^i + \varsigma), \\ -\sum_{j \in \mathcal{N}_i} a_{ij}^p \left(x_i(t_k^i) - x_j(t_{k'}^j) \right), & t \in [t_k^i + \varsigma, t_{k+1}^i], \end{cases} \tag{2}$$

where $\varsigma > 0$, $t_{k'}^j = \text{argmin}_{l \in \mathcal{N}_i: t \geq t_l^j} \{t - t_l^j\}$. t_k^i is the kth triggering time instant for agent i and is defined iteratively as

$$t_{k+1}^i = \inf_{t > t_k^i + \varsigma} \{t: f_i(t) \geq 0\} \cup t_p, \tag{3}$$

in which t_p is the switching time and $f_i(t)$ is the triggering function defined as follows:

$$f_i(t) = \|e_i(t)\| - \beta \left\| \sum_{j \in \mathcal{N}_i} a_{ij}^p \left(x_i(t_k^i) - x_j(t_{k'}^j) \right) \right\|, \tag{4}$$

for some $\beta > 0$ and $e_i(t) = x_i(t_k^i) - x_i(t)$.

Remark 1. From formulas (2) and (3), one can see that if $t_{k+1}^i - t_k^i < \varsigma$, then $u_i(t) = 0$, which means that the agent will not sample the information of its neighboring agents. The controller update only occurs when $t_{k+1}^i - t_k^i \geq \varsigma$. Thus, the Zeno-behavior of the sampling time sequences of the controller can be excluded directly.

Noticing that $x_i(t_k^i) = x_i(t) + e_i(t)$, event-triggered control protocol (2) can be rewritten as

$$u_i(t) = -\sum_{j \in \mathcal{N}_i} a_{ij}^p \left(x_i(t) - x_j(t) + e_i(t) - e_j(t) \right). \tag{5}$$

Denoting $x(t) = [x_1(t), \ldots, x_n(t)]^T$ and $e(t) = [e_1(t), \ldots, e_n(t)]^T$, by (1) and (5), we have the following equation:

$$\dot{x}(t) = -L_p x(t) - L_p e(t). \tag{6}$$

Defining $\overline{x}(t) = 1/n \sum_{i=1}^n x_i(t)$, by simple computation, one can easily obtain that $\dot{\overline{x}}(t) = 0$, which shows that $\overline{x}(t)$ is constant.

Denoting $\delta(t) = x(t) - \overline{x}(t)\mathbf{1}_n$, by (6), we have the following equation:

$$\dot{\delta}(t) = -L_p \delta(t) - L_p e(t). \tag{7}$$

Definition 1. The MASs (1) is said to achieve average consensus under designed control protocol, if $\lim_{t \to \infty} \|x_i(t) - \overline{x}(t)\| = 0$ holds for any initial conditions.

3. Main Results

Lemma 1 (see [23]). *Suppose L is the Laplacian of an undirected and connected graph $\mathcal{G}$, then*

$$\left\| e^{-L(t-t_0)} v \right\| \leq e^{-\lambda_2(L)(t-t_0)} \|v\|, \quad t \geq t_0, \tag{8}$$

holds for $v \in R^n$ and $\mathbf{1}_n^T v = 0$.

Theorem 1. *Consider multiagent systems (1) with control strategy (2) and triggering time sequence (3). The average consensus can be achieved asymptotically, if the switching guarantees that*

$$\inf_{t>t_0} \frac{T_s(t)}{T_u(t)} > \frac{\gamma}{\lambda}, \tag{9}$$

where $0 < \lambda < \min_{1 \leq p \leq \nu} \{\lambda_2^p\}$ and λ_2^p is the minimum nonzero eigenvalue of undirected connected graph $\mathcal{G}_p$:

$$\max_{\nu+1 \leq p \leq |\Lambda|} \left\{ \left\| L_p \right\| \right\} < \gamma < \infty,$$

$$0 < \beta < \min_{1 \leq p \leq |\Lambda|} \left\{ \frac{\lambda_2^p}{\sqrt{n} \left\| L_p \right\|^2 + \sqrt{n} \left\| L_p \right\| \lambda_2^p} \right\}. \tag{10}$$

Proof. Since the switching is "ergodic switching," one can see that the estimating process is independent of the order of the activated topology. Without loss of generality, assume that activate topology on $[t_{q-1}, t_q], q = 1, 2, \ldots$, is $\mathcal{G}_p$, where $p \in \Lambda$ and $p = q \mod |\Lambda|$, and set $p = |\Lambda|$ if $p = 0$.

From equation (7), by the variation of parameter formula, for $t \in [t_{p-1}, t_p]$, $p = 1, 2, \ldots, \nu$, we have

$$\delta(t) = e^{-L_p(t-t_{p-1})} \delta(t_{p-1}) + \int_{t_{p-1}}^t e^{-L_p(t-s)} \left(-L_p e(s) \right) ds. \tag{11}$$

Then, since $\mathbf{1}_n^T \delta(t_{p-1}) = 0$, by Lemma 1, we can derive that

$$\left\| \delta(t) \right\| \leq e^{-\lambda_2^p(t-t_{p-1})} \left\| \delta(t_{p-1}) \right\| + \int_{t_{p-1}}^t e^{-\lambda_2^p(t-s)} \cdot \left\| L_p \right\| \cdot \|e(s)\| ds. \tag{12}$$

On the other hand, for $t \in [t_{p-1}, t_p]$, $p = \nu + 1, \nu + 2, \ldots, |\Lambda|$, we have

$$\left\| \delta(t) \right\| \leq e^{\left\| L_p \right\|(t-t_{p-1})} \left\| \delta(t_{p-1}) \right\|$$
$$+ \int_{t_{p-1}}^t e^{\left\| L_p \right\|(t-s)} \cdot \left\| L_p \right\| \cdot \|e(s)\| ds. \tag{13}$$

It follows from (4) that $f_i(t) \leq 0$, that is,

$$\left\| e_i(t) \right\| \leq \beta \left\| \sum_{j \in \mathcal{N}_i} a_{ij}^p \left(x_i(t_k^i) - x_j(t_{k'}^j) \right) \right\|$$

$$\leq \beta \left\| \sum_{j \in \mathcal{N}_i} a_{ij}^p \left(\delta_i(t) - \delta_j(t) \right) + \sum_{j \in \mathcal{N}_i} a_{ij}^p \left(e_i(t) - e_j(t) \right) \right\|$$

$$\leq \beta \left\| L_p \right\| \|e(t)\| + \beta \left\| L_p \right\| \|\delta(t)\|. \tag{14}$$

Then,

$$\|e(t)\| \leq \frac{\sqrt{n} \beta \left\| L_p \right\|}{1 - \sqrt{n} \beta \left\| L_p \right\|} \|\delta(t)\|. \tag{15}$$

Due to the fact that $(1 - \sqrt{n} \beta \|L_p\|) \lambda_2^p - \sqrt{n} \beta \|L_p\|^2 > 0$, there is a positive constant λ such that

$$\left(1 - \sqrt{n} \beta \left\| L_p \right\| \right) \left(\lambda_2^p - \lambda \right) - \sqrt{n} \beta \left\| L_p \right\|^2 > 0. \tag{16}$$

In the following, for $p = 1, 2, \ldots, \nu$, it will be proved that

$$\left\| \delta(t) \right\| \leq \left\| \delta(t_{p-1}) \right\| e^{-\lambda(t-t_{p-1})}, \quad t \in \left[t_{p-1}, t_p \right]. \tag{17}$$

In order to prove (17), we first claim that

$$\left\| \delta(t) \right\| < \left\| \mu \delta(t_{p-1}) \right\| e^{-\lambda(t-t_{p-1})} \triangleq \psi(t), \quad t \in \left[t_{p-1}, t_p \right], \tag{18}$$

holds for any $\mu > 1$.

Otherwise, by the continuity of $\|\delta(t)\|$, there must exist a $t^* > t_{p-1}$ such that

$$\|\delta(t)\| < \psi(t), \quad \text{for } t \in \left[t_{p-1}, t^* \right), \|\delta(t^*)\| = \psi(t^*). \tag{19}$$

Then, we have

$$\psi(t^*) = \|\delta(t^*)\| < \mu e^{-\lambda_2^p(t^*-t_{p-1})}\|\delta(t_{p-1})\| + \mu \int_{t_{p-1}}^{t^*} e^{-\lambda_2^p(t^*-s)} \cdot \|L_p\| \cdot \|e(s)\|ds < \mu e^{-\lambda_2^p(t^*-t_{p-1})}\|\delta(t_{p-1})\|$$

$$+ \frac{\mu\sqrt{n}\beta\|L_p\|^2}{1-\sqrt{n}\beta\|L_p\|} \int_{t_{p-1}}^{t^*} e^{-\lambda_2^p(t^*-s)}\|\delta(s)\|ds$$

$$= \mu e^{-\lambda_2^p(t^*-t_{p-1})}\|\delta(t_{p-1})\| + \frac{\mu\sqrt{n}\beta\|\delta(t_{p-1})\|\|L_p\|^2}{(\lambda_2^p-\lambda)(1-\sqrt{n}\beta\|L_p\|)}\left(e^{-\lambda(t^*-t_{p-1})} - e^{-\lambda_2^p(t^*-t_{p-1})}\right)$$

$$= \mu\|\delta(t_{p-1})\|\left(e^{-\lambda_2^p(t^*-t_{p-1})} + \frac{\sqrt{n}\beta\|L_p\|^2}{(\lambda_2^p-\lambda)(1-\sqrt{n}\beta\|L_p\|)}\left(e^{-\lambda(t^*-t_{p-1})} - e^{-\lambda_2^p(t^*-t_{p-1})}\right)\right) \quad (20)$$

$$< \mu\|\delta(t_{p-1})\|\left(e^{-\lambda_2^p(t^*-t_{p-1})} + \left(e^{-\lambda(t^*-t_{p-1})} - e^{-\lambda_2^p(t^*-t_{p-1})}\right)\right)$$

$$= \mu\|\delta(t_{p-1})\|e^{-\lambda(t^*-t_{p-1})}$$

$$= \psi(t^*),$$

which is a contradiction. Hence, (18) holds for any number $\mu > 1$. Let $\mu \longrightarrow 1$, we have

$$\|\delta(t)\| \le \|\delta(t_{p-1})\|e^{-\lambda(t-t_{p-1})}, \quad t \in [t_{p-1}, t_p], \ p = 1, 2, \ldots, v. \quad (21)$$

Therefore, for $t \in [t_0, t_1]$, one can derive that

$$\|\delta(t)\| \le \|\delta(t_0)\|e^{-\lambda(t-t_0)}. \quad (22)$$

Then, for $t \in [t_1, t_2]$, we have

$$\|\delta(t)\| \le \|\delta(t_1)\|e^{-\lambda(t-t_1)} \le \|\delta(t_0)\|e^{-\lambda(t_1-t_0)}e^{-\lambda(t-t_1)}. \quad (23)$$

Repeating the above procedure, for $t \in [t_{v-1}, t_v]$, one has

$$\|\delta(t)\| \le \|\delta(t_0)\|e^{-\sum_{p=1}^{v-1}\lambda(t_p-t_{p-1})-\lambda(t-t_{v-1})}. \quad (24)$$

For $p = v+1, v+2, \ldots, |\Lambda|$, in terms of (13), we have

$$\|\delta(t)\| \le e^{\|L_p\|(t-t_{p-1})}\|\delta(t_{p-1})\|$$
$$+ \int_{t_{p-1}}^{t} e^{\|L_p\|(t-s)} \cdot \|L_p\| \cdot \|e(s)\|ds. \quad (25)$$

Since $\max_{v+1 \le p \le |\Lambda|}\{\|L_p\|\} < \gamma < \infty$ and $\sqrt{n}\beta\|L_p\|^2 < \lambda_2^p(1-\sqrt{n}\beta\|L_p\|)$, then there is γ such that $\sqrt{n}\beta\|L_p\|^2 < (\gamma - \|L_p\|)(1 - \sqrt{n}\beta\|L_p\|)$, $p = v+1, v+2, \ldots, |\Lambda|$. By a similar argument to that in the proof of (17), we have

$$\|\delta(t)\| \le \varphi e^{\gamma(t-t_v)}, \quad t \in [t_v, t_{v+1}], \quad (26)$$

where $\varphi = \|\delta(t_0)\|e^{\sum_{p=1}^{v}-\lambda(t_p-t_{p-1})}$.

Repeating the above procedure, we can derive that

$$\|\delta(t)\| \le \|\delta(t)\| \le \varphi e^{\sum_{p=v+1}^{|\Lambda|-1}\gamma(t_p-t_{p-1})+\gamma(t-t_{|\Lambda|-1})}, \quad t \in [t_{|\Lambda|-1}, t_{|\Lambda|}]. \quad (27)$$

In general, since $\mathscr{G}_p$ is finite and switching is "ergodic switching," by (24), (27), and mathematical induction, we have

$$\|\delta(t)\| \le \|\delta(t_0)\|e^{-\sum_{p=1}^{v}\lambda T_p(t_0,t)+\sum_{p=v+1}^{\Lambda}\gamma T_p(t_0,t)}$$
$$\le \|\delta(t_0)\|e^{-\lambda T_s(t)+\gamma T_u(t)}, \quad t \ge t_0. \quad (28)$$

Noticing that $\inf(T_s(t)/T_u(t)) > (\gamma/\lambda)$, there is a constant $\rho > 0$ such that $(\gamma + \rho)T_u(t) \le (\lambda - \rho)T_s(t)$. Thus, $\gamma T_u(t) - \lambda T_s(t) \le -\rho(T_u(t) + T_s(t)) = -\rho(t - t_0)$. Then, we have

$$\|\delta(t)\| \le \|\delta(t_0)\|e^{-\rho(t-t_0)}, \quad t \ge t_0, \quad (29)$$

which implies that the consensus is reached asymptotically and the exponential convergence rate is ρ. □

Remark 2

(1) Compared with the work in [18], a novel distributed event-triggered consensus protocol with switching topologies is proposed in this paper. Our control protocol does not require continuous communication among agents and a sufficient condition on consensus is presented under the assumption that the switching topologies are ergodic.

(2) Compared with the works in [26, 27, 34], the update time instants of the controller are determined by both triggering time instants and switching time instants. Furthermore, the Zeno-behavior of the sampling time sequences of the controller can be

excluded directly and the infimum of the sampling interval can be bigger than a given positive constant.

4. Simulations

In this section, a numerical example is given to illustrate the feasibility and effectiveness of the theoretical results.

Consider the multiagent systems with five agents, where the communication topologies are described by Figure 1. Obviously, $\mathcal{G}_1, \mathcal{G}_2, \mathcal{G}_3$ are connected and $\mathcal{G}_4, \mathcal{G}_5$ are disconnected. For each communication topology, the corresponding Laplacian is given as follows, respectively:

$$
L_1 = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix},
$$

$$
L_2 = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix},
$$

$$
L_3 = \begin{bmatrix} 1 & -1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & 0 & 0 & -1 & 2 \end{bmatrix}, \qquad (30)
$$

$$
L_4 = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},
$$

$$
L_5 = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}.
$$

Then, we can get $\min_{1 \le p \le v}\{\lambda_2^p\} = 0.382$, $\max_{v+1 \le p \le |\Lambda|}\{\|L_p\|\} = 3.4142$, and $\min_{1 \le p \le v}\{\lambda_2^p/(\sqrt{n}\,\|L_p\|^2 + \sqrt{n}\,\|L_p\|\,\lambda_2^p)\} \approx 0.0118$. We assume $\beta = 0.01 < 0.0118$, $\lambda = 0.3 < 0.382$, and $\gamma = 3.5 > 3.4142$, which satisfies the parameter requirements of Theorem 1. Let $\varsigma = 0.5$ and the dwell time is a random number which is greater than ς. In order to guarantee that
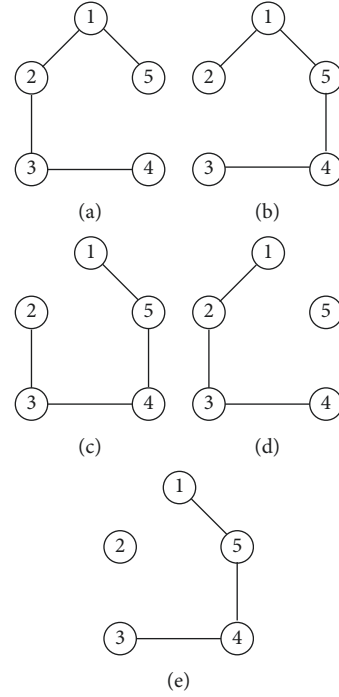

Figure 1: Switching topologies: $\mathcal{G}_p$, $p = 1, 2, 3, 4, 5$. (a) $\mathcal{G}_1$, (b) $\mathcal{G}_2$, (c) $\mathcal{G}_3$, (d) $\mathcal{G}_4$, and (e) $\mathcal{G}_5$.
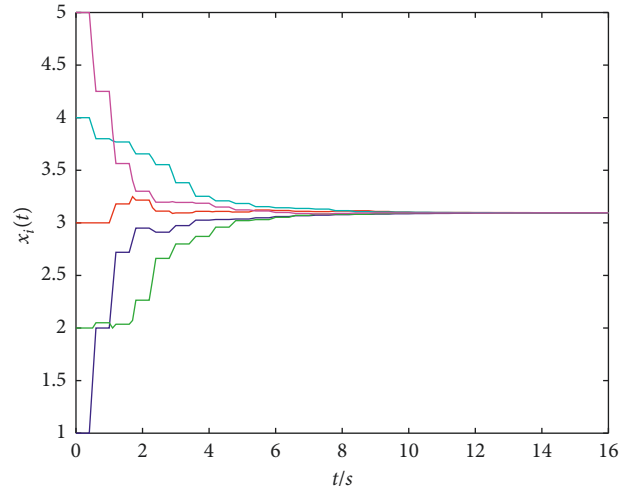

Figure 2: State for each agent.

$$
\inf_{t > t_0} \frac{T_s(t)}{T_u(t)} > \frac{\gamma}{\lambda}, \qquad (31)
$$

we assume that the total activated time for connected graphs is about 93 percent of the total time. Hence, the consensus can be achieved by Theorem 1.

The initial states are denoted by $x(0) = [1, 2, 3, 4, 5]^T$. The state responses $x_i(t)$ and the controllers $u_i(t)$ are depicted in Figures 2 and 3, respectively. The switching sequence is depicted in Figure 4. The triggering time sequences for each agent are shown in Figure 5.
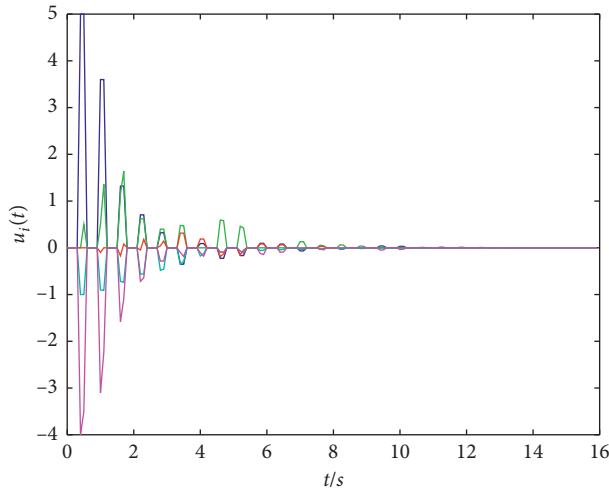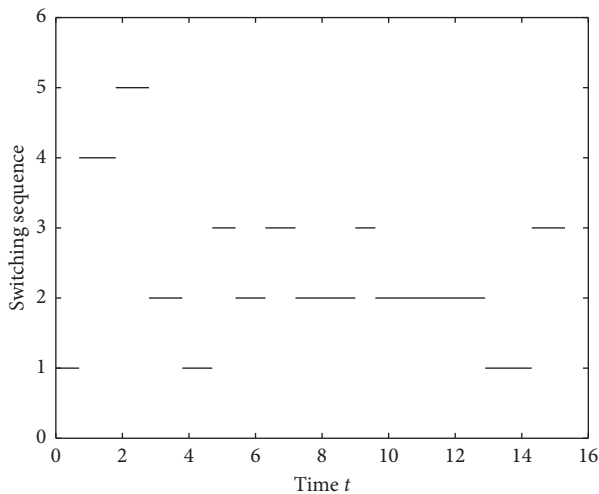
Figure 3: Controller for each agent.
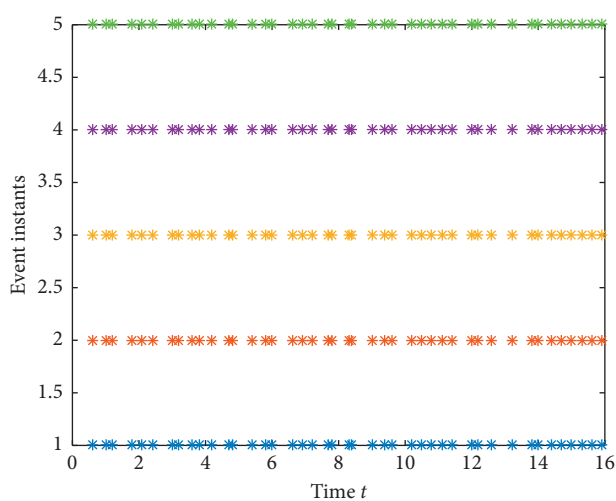

Figure 4: Switching sequence.

Figure 5: Triggering time sequences.

5. Conclusions

Based on the inequality technique and stability theory of differential equations, the event-triggered average consensus of multiagent systems with switching topologies is studied. A sufficient condition for achieving average consensus is obtained under the assumption that switching signal is ergodic and the total period over which connected topologies is sufficiently large. Moreover, the designed control protocol can exclude Zeno-behavior directly. It should be noted that the main results are derived only for a first-order multiagent system and an undirected graph. More general linear system models and directed graph scenarios will be addressed in future study.

Data Availability

The data used to support the findings of this study are included within the article (see Section 4).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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